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Not Relying on the Newton Gravitational Constant Gives More Accurate Gravitational Predictions

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Abstract

The Newton gravitational constant is considered a cornerstone of modern gravity theory. Newton did not invent or use the gravity constant; it was invented in 1873, about the same time as it became standard to use the kilogram mass definition. We will claim that G is just a term needed to correct the incomplete kilogram definition so to be able to make gravity predictions. But there is another way; namely, to directly use a more complete mass definition, something that in recent years has been introduced as collision-time and a corresponding energy called collision-length. The collision-length is quantum gravitational energy. We will clearly demonstrate that by working with mass and energy based on these new concepts, rather than kilogram and the gravitational constant, one can significantly reduce the uncertainty in most gravity predictions.

Keywords

Gravity Predictions, Reduction of Errors, Newton's Gravitational Constant, Collision Space-Time, Cavendish Apparatus, Planck Length, Planck Time

1. Introduction

Gravity depends on mass, as mass curves space-time (according to general relativity theory). To find the mass of the Earth in kilograms, one first needs to determine the gravitational constant G. Once G is known, the mass of the Earth can be found and then G in combination with M can be used to make gravitational predictions, such as predicting gravitational time dilation. Gravitational time dilation and gravitational redshift, for example, play a central role in the GPS system and are therefore of great practical importance.

With the invention of GPS, researchers working with gravity from a more practical perspective have realized that GM could be measured much more precisely than G and M separately. For example, the National Imagery and Mapping Agency (NIMA), states in their technical report on GPS systems [1] that

"*The central term in the Earth's gravitational field (GM) is known with much greater accuracy than either 'G', the universal gravitational constant, or 'M', the mass of the Earth.*" (pages 3-3 WGS 84 third version)

That one should rely on a measure of GM directly[1], rather than on G and M separately and then multiplying them together, has been of great importance in achieving the high precision of the GPS system we have today. Naturally, more precise measurements of GM have also been made possible by better measurement devices. This is also clear from the recent report that says "*Significant improvement in the knowledge of GM has occurred since the original WGS 84 development effort and the original WGS 84 value were updated in 1994*".

The fact that GM is known with much greater accuracy than either G (the universal gravitational constant) or M was not specifically mentioned in WGS 66 or WGS 72, which are two previous versions of the WGS document that is a cornerstone document in the theory of calibrating the GPS system. However, it seems that even then, GPS researchers possibly realized the importance of measuring GM directly, rather than measuring G and M separately and then multiplying them together. In this paper, we will also clearly demonstrate that finding G and M separately gives less accuracy.

Also papers [2] NIMA refer to from 1980 shows that it was not clear yet at that time that finding GM directly is more accurate than finding G and M seperately for so to multiply them together.

Although it is today well known among many if not most gravitational "practitioners" working with GPS that GM is known with much greater accuracy than either G or M, it has never been fully understood or discussed why this is the case from a deeper theoretical perspective, despite considerably published literature on measuring GM, see [3] [4]. However, even when finding GM directly without first determining G and M, one still gets the impression that G is indirectly needed, which we will demonstrate is not the case.

In this paper, we aim to explore this topic in greater depth than done before. Due to recent progress in understanding gravity all the way down to the Planck scale, we now believe that we have the full explanation. We however need to go back all the way to Newton before we go to the very recent progress in gravity research.

2. Background History

Isaac Newton in his *Principia* [5] introduced the following gravitational force formula:

[1] GM is also known as the geocentric gravitational constant, or simply the gravitational parameter ($\mu = GM$).

$$F = \frac{M_n m_n}{R^2} \tag{1}$$

Newton only stated this formula in words, but never clearly introduced a gravity constant. This has also been discussed recently in [6] [7]. Also, Newton's view of mass was very different than today's view and is why we use the symbols M_n and m_n instead of M and m. He only mentioned mass once and said it is the quantity of matter. Further matter, he claimed, consists of indivisible particles; see also his book *Opticks* [8]. Taken literally, this means Newton meant that mass was quantized, as it had to come in quanta of these indivisible particles, and thereby Newton actually gave a hint about quantum gravity already in 1684. Newton, based on his theory of gravity, was able to calculate the relative mass of the Earth, the sun, and a series of planets; see also Cohen [9]. He was also able to find the density of the Earth relative to the sun with very high accuracy. However, Newton described how he was not able to do what he wanted to, which was to find also the density of the Earth relative to a uniform known substance on the Earth, such as water or lead.

Clotfelter [10] in 1987, Hodges [11] in 1998, and Haug [6] in 2021 pointed out that Cavendish [12], using what today is known as a Cavendish apparatus and which he published about in 1798, never measured the gravitational constant nor tried to do so. What Cavendish measured was the density of the Earth relative to a known substance. Sean [13] in 1999 pointed out that a series of authors have incorrectly claimed Cavendish measured the Newton gravitational constant. A series of researchers, including such as Noble prize winner Feynman [14] [15], have incorrectly claimed Cavendish measured the Newton gravitational constant; see, for example, [16] [17]. Despite the many researchers that have looked closely at what Cavendish actually did and did not do, there are still researchers presenting the incorrect information that Cavendish measured G. For example, Rothleitner and Schlamminger [18] in 2017 claimed "*G was first measured in the laboratory by Henry Cavendish in 1798*. Still, later in the same paper they correctly stated: "*Thus, originally, Cavendish did not measure G in his famous experiment but the mean density of the Earth, as his article was titled.*". Why Rothleitner and Schlamminger then stated, earlier in the paper, that G was first measured by Cavendish is confusing at best.

Cornu and Baille [19] in 1873 were likely the first to introduce what today is known as the Newton's gravitational constant, using the notation $F = fMm/r^2$; in other words, the symbol f for the gravity constant. In 1885 König and Richarz [20] suggested a gravity constant G, but it is not fully clear from the paper that they suggested $F = GMm/r^2$ as we know it today. This was first clearly suggested by Boyes [21] in 1894. Max Planck [22] used the notation f for the gravitational constant at least as late as 1928, and Einstein [23] used the notation k in 1916. Whether one uses the notation f, k, or G for the gravitational constant is merely cosmetic, but what is important here is that for several hundred years the Newton gravity theory was used without a gravity constant.

We think it is no coincidence that the gravity constant was introduced in the 1870s, as this was also when the kilogram standard for mass was accepted across Europe. The kilogram had been invented considerably earlier in France, but the use of the kilogram definition of mass in European scientific circles happened first after the meter convention meeting in 1870. The gravitational constant was, in our view, needed as the kilogram mass definition, as we will demonstrate, was now somehow incomplete in relation to gravity, unless one also introduced a gravity constant. Before 1873 one used in France the kilogram and gram already as mass standard for macroscopic and small masses but not for astronomical masses yet. In England one used the pound as described by Maxwell [24] in his 1873 book. Maxwell points out also that that astronomical masses used at that time had the dimensions of $[L^3 \cdot T^{-2}]$ which is naturally very different than the dimensions of the modern mass definition of the kilogram, which simply is said to have dimension equal to mass, but then what is gravitational mass? For scientific simplicity one naturally wanted the same mass standard internationally, but also the same mass standard for standard macroscopic human handable masses as well as astronomical bodies. One decided to go for the kilogram mass, even as we will understand through this paper that it likely would have been better to change also small masses to the astronomical mass standard of mass being $[L^3 \cdot T^{-2}]$.

In 1961 Thüring [25] concluded that the gravitational constant had been introduced somewhat ad hoc. Further he concluded that it could not be associated with a unique property of nature. The gravitational constant is, however, clearly needed when working with kilogram mass. It can be found, for example, by using a Cavendish apparatus. That is, the Cavendish apparatus was not invented to find the gravitational constant, but it can also be used to find the gravitational constant. The gravitational constant is what is missing in the gravitational formula when using the kilogram definition of mass. What is missing is then found by calibration, not by derivation or from some deep fundamental understanding of gravity all the way down to the quantum scale.

Kilogram has traditionally been related to weight, and weight is not mass. However, already in *Principia*, Newton concluded that weight and mass are proportional when the masses are in the same gravitational field; that is, at the same distance from the gravitational object, which explains why the only information one needs to know about the masses in the gravitational force formula from 1873 is the weight of the mass in kilogram. Still, this is not enough information, and the rest of the information about the mass needed for gravity one can actually get through the gravitational constant that is calibrated from a gravitational observation. This view is somewhat controversial, but our view should become clear as you read through the various sections of the paper. When the gravitational constant is first calibrated to one observed gravity phenomena, it can be used in combination with the Newton formula to predict a series of observable gravitational phenomena, and this works well without recalibration.

This strongly supports that the gravity constant is indeed a constant. General relativity theory adopted the Newtonian gravitational constant that had been invented ad hoc in 1873. The gravitational constant is also needed in general relativity theory to predict such things as gravitational redshift, gravitational time dilation and deflection, and other types of gravitational effects. There is, however, also no deep theory in general relativity theory or Newton theory that tells us exactly what the gravitational constant represents physically. It is naturally combined with the mass it is multiplied with somehow linked to gravitational bending of space-time in general relativity theory, but exactly why we need a constant is not clear, except that the 1873 Newton modified theory and general relativity theory do not work without such a constant.

3. All Observable Gravitational Predictions Can Be Done without Knowledge off *G*

We will conjecture that all gravitational phenomena that actually can be observed can be predicted without first knowing G. This has also been recently demonstrated by [7] [26] [27]. This, we will later see, is rooted in a deeper understanding of G and even to its relation to quantum gravity. We will shortly demonstrate in this paper that we can make a series of gravity predictions without knowing G, as this point is important for reducing the prediction uncertainty in gravitational predictions.

Assume we want to make gravitational predictions related to the gravity field from the Earth. We can start out by first making a single gravitational observation. For example, we can find the gravitational acceleration from a drop ball with a single built-in stopwatch. The gravitational acceleration is given by:

$$g = \frac{2H}{T_d^2} \tag{2}$$

where H is the height we dropped the ball from above the ground, and T_d is the time it took for the ball from the drop to hitting the ground.[2] That is, we only need to know the height and the time. Today, there are even specially designed balls for such a purpose, with a built-in stopwatch that stops automatically when the ball hits the ground. There is no need to know the mass of the Earth or the gravity constant to measure the gravitational acceleration this way. On the other hand, to predict the gravitational acceleration in standard theory, we naturally have $g = \frac{GM}{R^2}$. Here, both the gravitational constant and kilogram mass of the Earth M are needed, as well as the radius of the Earth. However, here we are not predicting the gravitational acceleration, but simply measuring it, and that requires no knowledge of G or M. To find G and M, we need to make a series of measurements that are totally unnecessary to predict observable gravity pheno-

[2]We could naturally have done this more accurately with an advanced gravimeter, but this is not the point here. The important point is that using G, as we soon will see, leads to more uncertainty in predictions than needed.

mena, as G and M are never needed to predict any gravity phenomena. We will later return to why this is the case. This does not mean that we do not need to know the mass of the gravitational mass, but that the gravitational mass, as we will understand it, is not defined by the kilogram mass.

Next, we can use the observed g to predict a series of other gravity phenomena. For example, the orbital velocity of the moon is given by:

$$v_o = \sqrt{\frac{g}{R_2} R_1^2} \tag{3}$$

where R_1 is the radius of the Earth and R_2 is the distance between the center of the moon and center of the Earth. Note that G and M are not needed to predict this. Table 1 similarly shows how we can predict a series of gravity phenomena and how many of these predictions we can easily check with observations. The predictions in Table one are related both to gravity phenomena that can be predicted with Newton's gravitational theory and Einstein's general relativity theory. The important point here is that we make all these predictions without needing to know G or M.

We do not need to start out with the gravitational acceleration to do this, as shown in Table 1. We could just as well have started out with, for example, the orbital time of the moon, simply by counting the number of days it takes for the moon to orbit the Earth (then convert this to seconds if that is the time unit used). Next, we can use this to predict a series of gravitational effects on the Earth; for example, we can get orbital velocity, which is equal to:

$$v_0 = \frac{2\pi R_2}{T} \tag{4}$$

where R_2 is the distance from the center of the Earth to the center of the moon. This we can find by parallax and knowing the circumference of the Earth; these measurements that are all totally independent of any knowledge of gravity, and naturally there is no use of G or M. To predict this from Newton's theory or general relativity theory is done by $v_o = \sqrt{\frac{GM}{R_2}}$ so here we need to know G and the kilogram mass of the Earth. This requires measuring many more input parameters, as will be demonstrated soon. Further, the gravitational acceleration on the Earth is then given by:

$$g = \frac{4\pi^2 R_2^3}{T^2 R_1^2} \tag{5}$$

where R_1 is the radius of the Earth and R_2 the center-to-center distance between the Earth and the moon. Again, we see there is no need to know G or M to predict this. Table 2 shows a series of gravity predictions we can make on Earth from first finding just the orbital time of the moon. The approach is valid for any gravitational object. We could have done the same with the sun as the gravitational object and then instead used the orbital time of the Earth as the orbital time. This could then again be used to make a long series of predictions about

Table 1. The table shows various gravitational predictions we can make without any knowledge of G and M.

Observable:	Formula	Input parameters to be measured:	Constant needed:
gravitational acceleration	$g=\frac{2H}{T_b^2}$	H, T_b	
Orbital velocity	$v_o=\sqrt{\frac{g}{R_2}R_1^2}$	H, T_b, R_2, R_1	
Orbital time	$T=\frac{2\pi R_2}{\sqrt{\frac{g}{R_2}R_1^2}}$	H, T_b, R_2, R_1	
Light bending	$\delta=\frac{4gR_1^2}{c^2R_2}$	H, T_b, R_2, R_1	c
Gravitational redshift	$\delta=\frac{gR_1^2}{c^2R_2}$	H, T_b, R_2, R_1	c
Time dilation	$T_2=T_1\sqrt{1-\frac{2gR_1^2}{R_2c^2}}$	H, T_b, R_2, R_1	c

Table 2. The table shows various gravitational predictions we can make without any knowledge of G and M.

Observable	Formula	Parameters needed to be measured	Constant needed.
Orbital time	T	T	
gravitational acceleration	$g=\frac{4\pi^2R_2^3}{T^2R_1^2}$	R_2, R_1, T	
Orbital velocity	$v_o=\frac{2\pi R_2}{T}$	R_2, R_1, T	
Light bending	$\delta=\frac{16\pi^2R_2^3}{T^2R_1c^2}$	R_2, R_1, T	c
Gravitational redshift	$\delta=\frac{4\pi^2R_2^3}{T^2R_1c^2}$	R_2, R_1, T	c
Time dilation	$T_2=T_1\sqrt{1-\frac{8\pi^2R_2^2}{T^2c^2}}$	R_2, R_1	c

the effects of the sun's gravitational field. For example, we could use this to correctly predict the orbital time of the other planets, or the light bending of the sun, or the gravitational redshift from the sun.

However, if we want to know the kilogram mass of the Earth and use g to find it, then we also need to know G, as we have:

$$M = \frac{gR^2}{G} \tag{6}$$

And if we want to find the kilogram mass of the Earth from the orbital time of the moon, we have:

$$M = \frac{R_2^3 2\pi}{T^2 G} \tag{7}$$

That is, we again need to know G.

We can also find G from g as we also have:

$$G = \frac{gR^2}{M} \tag{8}$$

And we can find G from the orbital velocity; it is given by:

$$G = \frac{R_2^3 2\pi}{T^2 M} \tag{9}$$

The problem here is that we need to know M and we were going to use G to find M. This means we need another way to find G. We can find G in a Cavendish apparatus, and this is done by[3]:

$$G = \frac{L 2\pi^2 R_c^2}{M_c T_c^2}\theta \tag{10}$$

That is, in the Cavendish apparatus we need to know the distance between the two small balls in the Cavendish apparatus L, the distance between the small and the large balls from center to center, which is R_c, and also the angle of the arm θ. Further, we need to know the oscillation time of the arm in the apparatus: T_c. In addition, we need to know the kilogram mass of M_c of one of the large balls in the Cavendish apparatus. For all these input parameters that can all be measured and that we can calculate G from, naturally measurement errors will occur. The errors will cause uncertainty in the measured (extracted) value of G. So, it is well known that there will be considerable measurement error in G. There are a series of ways to measure the gravitational constant, even using a Cavendish apparatus as well as other methods, but the constant has a known large uncertainty in its value and is part of the reason why there is still considerable ongoing work going to make improvements in measurements of G, see [18] [31] [32] [33] [34] [35]. For example, the NIST CODATA 2019 value for the gravitational constant is $6.67430 \times 10^{-11}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$ and with a one standard uncertainty off $0.00015 \times 10^{-11}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$. Now, after we have measured G, we can find the mass of the Earth (in kilogram), given by Equation (6). That is, in addition, we need to measure g and the radius of the Earth. Now we know the mass of the Earth.

We now know G and M. To predict the orbital velocity of the Moon, we can now use the well-known formula:

[3]There are several different ways to find G using a Cavendish apparatus and similar torsion pendulum inspired apparatuses (se for example [28] [29] [30] [31]), but this is one of the well known ways to do it.

$$v_o = \sqrt{\frac{GM}{R_2}} \tag{11}$$

Here, we will get measurement error in the distance from the Earth to the moon, R_2; this in addition to measurement errors in R_c, T_c, θ, M_c, T_b, H, and R_1 that are needed to find the mass of the Earth if using the gravity constant as measured by a Cavendish apparatus. Be aware if we predict the orbital velocity directly from just the observed gravitational acceleration, or from just the orbital time, then we are only dealing with measurement errors in R_1, R_2, and T. Avoiding being dependent on G clearly leads to considerably lower prediction errors, as there are far fewer input variables that need to be measured when we do not rely on G. We will go as far as to conjecture that it can therefore be seen as a mistake to use the gravitational constant G if one wants gravity predictions as accurate as possible. **Table 3** shows gravitational predictions as predicted in the normal framework of using G and M. This requires knowledge of many more input parameters than the approach that does not use G and M. Here, we assume G is found by a Cavendish apparatus. Compare **Tables 1-3**. In **Table 3**, when using G to make predictions, we clearly need many more input parameters. There are other ways to find G than the Cavendish apparatus, but we will claim that all these methods lead to bigger uncertainty than does not relying on G, as

Table 3. The table shows various things we can find from a Cavendish apparatus and what leads to uncertainty.

Observable	Formula	Parameters needed to be measured	Needed constants
Gravity constant	$G = \frac{L2\pi^2 R_c^2}{M_c T_c^2}\theta$	M_c, L, R_c, T_c, θ	
Mass M (Earth)	$M = \frac{gR_1^2}{G} = \frac{2HR_1^2}{T_b^2 G}$	H T_b, R_1, M_c, L, R_c, T_c, θ	
Orbital velocity	$v_o = \sqrt{\frac{GM}{R_2}}$	H, T_b, R_2, R_1, M_c, L, R_c, T_c, θ	
Orbital time	$T = \frac{2\pi R_2}{\sqrt{\frac{GM}{R_2}}}$	H, T_b, R_2, R_1, M_c, L, R_c, T_c, θ	Orbital time
Light bending	$\delta = \frac{4GM}{c^2 R_2}$	H, T_b, R_2, R_1, M_c, L, R_c, T_c, θ	c
Gravitational red-shift	$\delta = \frac{GM}{c^2 R_2}$	H, T_b, R_2, R_1, M_c, L, R_c, $T_c\theta$	c
Time dilation	$T_2 = T_1\sqrt{1 - \frac{2GM}{R_2 c^2}}$	H, T_b, R_2, R_1, M_c, L, R_c, T_c, θ	c
Escape velocity	$v_o = \sqrt{\frac{2GM}{R_2}}$	H, T_b, R_2, R_1, M_c, L, R_c, T_c, θ	

they always require measurements of more input variables. The very simple way we have described avoiding use of G will therefore reduce prediction uncertainty in gravity predictions.

4. Finding *GM* Requires Much Less Input Information than Does Finding *G* and *M*

All observable gravitational phenomena can be predicted from GM. None of them only need G or only M, nor do they need GMm. The small mass m ($m \ll M$) in the Newton force formula is always canceling out in gravitational derivations that lead to formulas that can be checked with observable gravitational observations. The gravity force itself is never observed. In real, two-body problems where the gravity force of m also plays a significant role, the gravitational parameter is $\mu = G\left(M_1 + m_2\right) = GM_1 + Gm_2$, so again we always have G multiplied by a mass when doing gravity predictions.

The output units of M are kilogram, and the output unit of G is $m^3 \cdot kg^{-1} \cdot s^{-2}$. This means that when G is multiplied by M, the kilogram output unit of M cancels out with the kg^{-1} in G. Thus for no observable gravitational observation is the output unit (dimension) kilogram needed. On the other hand the dimensions $L^3 \cdot T^{-2}$ as was in the original Newton astronomical mass is preserved as it is needed for gravitation, when using meters and seconds unit system (S.I.) then what is preserved of units after multiplying G with M is $m^3 \cdot s^{-2}$. This is obvious when pointed out, but has hardly got attention in the gravitational literature. The kilogram is originally a humanly-constructed amount of matter. Since 2019, the kilogram has been re-defined in terms of the Kibble balance and the Planck constant (see, for example, [36] [37]). Still, the kilogram is a human "arbitrary" choice of quantity of matter, and it is well known that even after the 2019 re-definition and use of the Watt balance (Kibble balance) to measure the kilogram, there is still uncertainty around its measurement. NIST 2019 CODATA reports a standard uncertainty of 3×10^{-10} in the kilogram-atomic mass unit relationship. So, to find G and M separately means that G and M separately will contain an additional uncertainty related to the uncertainty in kilogram, and we will see this is not needed. If G is found from the same mass that it is later used to calculate gravity predictions for, then this does not cause any problem, as the additional uncertainty in M will then perfectly cancel out with the additional (not all, just the additional) uncertainty in G. That is, if you find G from the mass of the large ball in a Cavendish apparatus M_c, and then use GM_c to predict something about the gravitational field of M_c, this does not cause additional errors from using the kilogram definition of mass. It is when you take G found from one mass, for example M_c (the large ball in the Cavendish apparatus) and then use it to find the mass of another object (for example, the mass of the Earth), and then combine G with M, that you add uncertainty that could be avoided. The gravitational constant must, in general, be found from a small mass that we know the kilogram mass of, and is then later used to make predictions of astronomical gravitational objects. This is not needed, as G is never needed to make predictions of gravity phenomena that are observable and can therefore be checked. In addition, using the gravitational constant G adds additional uncertainty to our predictions, when G is used on another mass than it has been calibrated from. This has nothing to do with G changing with mass (something it does not do), but relates to the fact that to measure G requires a series of measured inputs that are otherwise not needed.

Table 4 shows how we can find GM directly without going through the much longer process to find G and M separately first and then multiplying them with each other, as is standard methodology in modern physics; see Table 3. To find G and M separately drags in the uncertainty in the kilogram measure, in addition to other uncertainties such as the uncertainties in the different parameters one needs to measure when finding G from a Cavendish apparatus. There are also other ways to find G, but they all require some additional measurements that are not needed when we directly find GM as a single entity.

If one wants to reduce uncertainty in the predictions, one should never, in general, try to calculate G and M separately; at least, not for any astronomical bodies, such as planets, suns, galaxies, or even the whole observable universe. If one wants to make gravitational predictions from the sun then one needs one precise gravitational observation from the sun. From this, one can get out GM directly without knowing G or M separately first. Then GM can be used for all other gravitational predictions from the sun. If one wants to make gravitational predictions from the gravitational field of the Earth then GM should be calculated directly from one observable gravitational effect from Earth, such as the orbital time of the moon or the gravitational acceleration, which can both be easily measured with no knowledge of G and M. This is a general principle that holds for any gravitational prediction. For any gravitational mass, one can make one gravitational observation and this should then be used to estimate GM directly without finding G or M separately, and then one would use the found GM to predict other gravity phenomena for that gravity object. This approach will, in

Table 4. The table shows various gravitational observations we can use to find GM. To find G and M separately requires much more input information and therefore has more uncertainty than GM.

Observable	Formula	Parameters needed to be measured	Constant needed
Orbital time	$GM=\frac{4\pi^2R_2^3}{T^2}$	R_2, T	
gravitational acceleration	$GM=\frac{2HR_1^2}{T_b^2}$	H, R_1, T_b	
Light bending	$GM=\delta\frac{Rc^2}{4}$	δ, R	c
Gravitational redshift	$GM=zRc^2$	z, R	c

general, reduce the prediction uncertainty compared to any calculation relying on the value of first finding the value of G and M. Why this is the case we will better understand when we dive deeply into G and M in the next sections.

5. Kilogram Mass versus Collision-Time Mass

As we showed above, to find the kilogram mass of the Earth, we need to know G. Further, to find G, we need to know the kilogram mass of the large balls in the Cavendish apparatus. But to predict observable gravitational phenomena, we do not need to know either G or M. So how is it possible that we do not need to know the kilogram mass to make gravity predictions, when gravity is known to be linked to mass?

In our view, the kilogram mass is an incomplete mass definition that never should have replaced Newtons mass definition. However, the incomplete kilogram mass gets fixed by multiplying it with the gravity constant G. The kilogram mass is then, as we have shown in previous papers and will show again here, re-constructed (by multiplying G with M) into a more complete mass definition that we will call collision-time. To understand this, we must go to the Planck scale. Max Planck [38] [39] introduced in 1899 a unique length: $l_p=\sqrt{\frac{G\hbar}{c^3}}$, time: $t_p=\sqrt{\frac{G\hbar}{c^5}}$, mass: $m_p=\sqrt{\frac{\hbar c}{G}}$, and temperature $T_p=\sqrt{\frac{\hbar c^5}{Gk_b^2}}$ (The Planck energy is simply T_pk_b, but Max Planck did not describe it directly, only indirectly through Planck temperature that is basically another way to describe energy).

One can also solve the Planck length formula of Max Planck for G and this gives:

$$G=\frac{l_p^2c^3}{\hbar} \tag{12}$$

In 1987, Cohen [40] pointed out that solving Planck units with respect to G only would lead to a circular problem as one first had to know G to find the Planck unit. Trying to express G in this way would just lead, in circular fashion, back to needing to know G. This view has been repeated as recently as 2016; see McCulloch [41]. However, in recent years it has been demonstrated that we can find the Planck length as well as other Planck units totally independent of any knowledge of G; see [26] [42] [43] [44] [45]. This means the Newton's gravitational constant is, in reality, a composite constant; see [46] for an in-depth discussion and literature review study about this view.

This means that to measure G can be seen to entail measuring the Planck length (squared) as well as the Planck constant and the speed of light. The uncertainty in G comes from the uncertainty in the Planck length, as the Planck constant and the speed of light are exact (by definition at least). Before we can understand the consequences and reasons for this fully, we must also investigate the kilogram mass from a "new" perspective. Still, it is worth mentioning that it is no surprise that there is a big uncertainty in measures of G as it is indirectly related to measuring the square of the smallest distance there is: the Planck length.

The Compton [47] wavelength formula is given by $\lambda = \frac{h}{mc}$. This formula we can solve for the mass, and this gives:

$$m = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{13}$$

where λ is the Compton wavelength and $\bar{\lambda}$ is the reduced Compton wavelength, further h is the Planck constant, and $\hbar$ is the reduced Planck constant (also known as the Dirac constant). This way to express mass was likely first suggested by Haug [48] [49]. One could mistakenly think this way to express the kilogram mass only can be used for electrons, as the Compton wavelength formula was developed for the electron, even if it has also been discussed for the proton; see [50] [51]. We have demonstrated in a series of papers [6] [26] that this form of expressing the kilogram mass, based on its Compton wavelength (Equation (13)), can be done for any mass, even for astronomical objects like stars and galaxies. We naturally do not claim macroscopic masses have a single Compton wavelength. However, the aggregates of all the Compton wavelengths of all the elementary particles in the mass of interest can be expressed as an aggregate, which is given by:

$$\lambda = \frac{1}{\sum_i^n \frac{1}{\lambda_i}} \tag{14}$$

We can even find the equivalent Compton wavelength of all the mass (equivalent) in the universe, as has recently been demonstrated by Haug [52]. Recently, we [53] also demonstrated that the Compton wavelength is identical to what can be called the rest-mass photon wavelength. Furthermore, we found that the de Broglie [54] [55] wavelength is likely only a mathematical derivative of the Compton wavelength. The Compton frequency appears to be directly related to the quantization of mass. Moreover, the Compton frequency plays a central role in the Schrödinger [56] equation, the Dirac [57] equation, and the Klein-Gordon equation, and even in the understanding of the Planck constant from a deeper perspective. This is discussed in detail in the paper just mentioned.

If any kilogram mass can be expressed with Equation (13) then clearly the only thing that distinguishes one kilogram mass from another is the Compton wavelength (when we are only interested in how many kilogram). This is because the Planck constant and the speed of light are exact constants that cannot vary from object to object. As we have pointed out in a series of papers, the kilogram mass is lacking information needed to describe gravity. However, this is fixed in standard gravitational theory (without one having had that in mind) by multiplying the kilogram mass with G. Be aware that in predictions of all observable gravitational phenomena, we always have GM in the prediction formula and never GMm. The small mass in the Newton gravity formula always cancels out in derivations to arrive at a formula predicting something observable. Again, in real, two-body problems where the small mass also plays a significant role, the gravity parameter is $\mu = G\left(M_1 + m_2\right) = GM_1 + Gm_2$. That is, the kilogram mass is always multiplied by G and, in our view, this makes the incomplete kilogram mass definition complete, as we will discuss.

Based on the analysis above, we can look at this from a deeper perspective, namely by replacing G with its composite $G = \frac{l_p^2 c^3}{\hbar}$, and M with $M = \frac{\hbar}{\bar{\lambda}} \frac{1}{c}$. This gives:

$$GM = \frac{l_p^2 c^3}{\hbar} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = c^3 \frac{l_p}{c} \frac{l_p}{\bar{\lambda}} \tag{15}$$

The embedded Planck constant cancels out from the mass between G and M, and the Planck length is introduced into the mass. This actually corresponds to what we have coined collision-time mass [58]. The collision-time mass is defined as:

$$\frac{GM}{c^3} = M_g = \frac{l_p}{c} \frac{l_p}{\bar{\lambda}} = t_p \frac{l_p}{\bar{\lambda}} \tag{16}$$

where t_p is the Planck time, and the integer part of $\frac{l_p}{\bar{\lambda}}$ represents the number of certain Planck events per Planck time, and the fractional part is the frequency probability for one more such event; see [58]. For any macroscopic mass, the integer part is so large that the fractional part can be ignored. For example, for the Earth we have $\frac{l_p}{\bar{\lambda}} \approx 275^{30}$, which is an enormous amount of Planck events per Planck time. This means GM is identical to c^3 multiplied by the collision-time mass. To find G multiplied by M, that is GM, requires much less information than finding G and M separately. One can, for example, estimate GM from only orbital time of the moon, as given by:

$$GM = \frac{4\pi^2 R_2^3}{T^2} \tag{17}$$

where R_2 is the distance from the center of the moon to the center of the Earth, and T the orbital time of the moon around the Earth; see also the previous section. This only requires one constant: the speed of light. To find G and M separately requires much more information and measurements than finding just GM, and all observable gravitational phenomena can be predicted by GM.

To find the kilogram of a mass is indirectly related to finding the Compton wavelength of that mass, and to find G is indirectly linked to finding the Planck length that, in our view, is also embedded in the collision-time mass which is the real mass. The only time we need to extract G is when we want to calculate Planck units separately, when relying on using the standard Max Planck method. The Newton gravitational constant is never needed to make gravitational predictions for any observable gravity phenomena. This also means we can always get a more accurate prediction of the gravity mass (collision-time mass) than the kilogram mass for large objects. To find the kilogram mass from, for example, the orbital time of the moon, we had $M = \frac{R_2^3 4\pi^2}{T^2 G}$. To do the same for the collision-time mass, we have:

$$M_g = \frac{R_2^3 4\pi^2}{T^2 c^3} \tag{18}$$

We need knowledge of the speed of light instead of the gravity constant. There is no uncertainty in the speed of light, but the uncertainty in measurements of the gravitational constant is considerable. Again, M_g is nothing more than $\frac{GM}{c^3}$ so this naturally relates to needing considerably less input information to find GM than to find G and M separately, as shown in the previous sections. Still, in our view, there is more to it than that, as G is not related to anything physical, but a composite constant, not by assumption, but by calibration and based on how mass in gravity has been defined since 1873. The gravity constant is needed to fix the incomplete kilogram mass.

It is possible to express the Newton gravity force as:

$$F = c^3 \frac{M_g m_g}{R^2} \tag{19}$$

or even as

$$F = c_g \frac{E_g E_g}{R^2} \tag{20}$$

where $E_g = l_p \frac{l_p}{\bar{\lambda}}$ is the gravitational energy (collision-length) as described in detail by [59] [60]. Both these formulas for the gravity force give different output units than the 1873 Newton gravitational formula used in modern physics today. However, no one has ever measured the gravitational force, only its consequences.

When it comes to predictions of observable gravitational phenomena, these three gravitational force formulas all give the same output predictions in both numbers and units, but they require different input. So should the scientific community prefer a model that requires a constant G with large uncertainty in it to predict gravity phenomena, as well as the constant that isn't linked to anything physically observable, or should one prefer a model where the constant is exact and therefore doesn't lead to additional uncertainty and is linked to something physical, namely the speed of light? We are in no doubt we would prefer the latter. **Table 5** demonstrates that the 1873 modified Newton gravity formula, as well our new gravity force formula that never relies on G and therefore gives more precise predictions, are the same at the depth of reality, but this is only true when one understands G is a composite constant and that the kilogram mass also can be written as a composite form.

In **Table 5**, pay attention to how, when replacing G with its composite form, and replacing the kilogram mass with the way we can write it based on the

Table 5. The table shows several important things. All gravitational predictions in the table that can actually be observed depend on GM and not on GMm. Further, at a deeper level, $GM = c^2 \frac{l_p^2}{\bar{\lambda}}$, the c^2 in some predictions, cancel out, but all gravitational predictions at a deeper level contain $\frac{l_p^2}{\bar{\lambda}}$ so the Planck length and the reduced Compton wavelength of the mass should tell us something essential. Further, the Newton gravity force written as $F = c_g \frac{E_g E_g}{R^2}$ gives exactly the same predictions in numbers and output units as the well-known 1873 modified Newton force formula. This is in spite of the gravity force formulas having different output units.

	Standard from deeper level:	**Alternative:**
Mass	$M = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ (kg)	$M_g = t_p \frac{l_p}{\bar{\lambda}}$ (collision-time mass)
Energy	$E = Mc^2$ (joule)	$E_g = l_p \frac{l_p}{\bar{\lambda}}$ (gravitational energy)
Gravitational constant	$G = \frac{l_p^2 c^3}{\hbar}$	c_g m/s
	Non observable (contains GMm)	
Gravity force	$F = G\frac{Mm}{R^2}$ ($\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$)	$F = c_g \frac{E_g E_g}{R^2}$ m/s $F = c_g^3 \frac{M_g m_g}{R^2}$ m/s
	Observable predictions, identical for the two methods: (contains only GM)	
Gravity acceleration	$g = \frac{GM}{R^2} = c^2 \frac{l_p^2}{\bar{\lambda} R^2}$	$g = c_g^2 \frac{E_g}{R^2} = c_g^2 \frac{l_p^2}{\bar{\lambda} R^2}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c\sqrt{\frac{l_p^2}{\bar{\lambda} R}}$	$v_o = c_g \sqrt{\frac{E_g}{R}} = c_g \sqrt{\frac{l_p^2}{\bar{\lambda} R}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{c\sqrt{\frac{l_p^2}{\bar{\lambda} R}}}$	$T = \frac{2\pi R}{c_g \sqrt{\frac{E_g}{R}}} = \frac{2\pi R}{c_g \sqrt{\frac{l_p^2}{\bar{\lambda} R}}}$
Velocity ball Newton cradle	$v_{out} = \sqrt{2\frac{GM}{R^2}H} = \frac{c}{R}\sqrt{2\frac{l_p^2}{\bar{\lambda}}H}$	$v_{out} = \frac{c_g}{R}\sqrt{E_g H} = \frac{c_g}{R}\sqrt{2\frac{l_p^2}{\bar{\lambda}}H}$
Periodicity Pendulum (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi R\sqrt{\frac{L}{GM}} = \frac{2\pi}{c}R\sqrt{\frac{L\bar{\lambda}}{l_p^2}}$	$T = \frac{2\pi R}{c_g}\sqrt{\frac{L}{E_g}} = \frac{2\pi}{c_g}R\sqrt{\frac{L\bar{\lambda}}{l_p^2}}$
Frequency Newton spring	$f = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}} = \frac{c}{2\pi R}\sqrt{\frac{l_p^2}{\bar{\lambda} x}}$	$f = \frac{c_g}{2\pi R}\sqrt{\frac{E_g}{x}} = \frac{c_g}{2\pi R}\sqrt{\frac{l_p^2}{\bar{\lambda} x}}$
Gravitational red shift	$z = \frac{\sqrt{1-\frac{2GM}{R_1 c^2}}}{\sqrt{1-\frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1-\frac{2l_p^2}{\bar{\lambda} R_1}}}{\sqrt{1-\frac{2l_p^2}{\bar{\lambda} R_2}}} - 1$	$z = \frac{\sqrt{1-\frac{2E_g}{R_L}}}{\sqrt{1-\frac{2E_g}{R_h}}} - 1 = \frac{\sqrt{1-\frac{2l_p^2}{\bar{\lambda} R_1}}}{\sqrt{1-\frac{2l_p^2}{\bar{\lambda} R_2}}} - 1$

Continued

Time dilation	$T_R \approx T_f\sqrt{1-\sqrt{\frac{2GM}{R}}^2 \Big/ c^2} = T_f\sqrt{1-\frac{2l_p^2}{\bar{\lambda}R}}$	$T_R \approx T_f\sqrt{1-\frac{2E_g}{R}} = T_f\sqrt{1-\frac{2l_p^2}{\bar{\lambda}R}}$
Gravitational deflection (GR)	$\delta = \frac{4GM}{c^2R} = 4\frac{l_p^2}{\bar{\lambda}R}$	$\delta = \frac{4E_g}{R} = 4\frac{l_p^2}{\bar{\lambda}R}$
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi l_p^2}{a(1-e^2)\bar{\lambda}}$	$\sigma = \frac{6\pi E_g}{a(1-e^2)} = \frac{6\pi l_p^2}{a(1-e^2)\bar{\lambda}}$
Micro lensing	$\theta = \sqrt{\frac{4GM}{c^2}\frac{d_s-d_L}{d_s d_L}} = \sqrt{\frac{4l_p^2}{\bar{\lambda}}\frac{d_s-d_L}{d_s d_L}}$	$\theta = 2\sqrt{E_g\frac{d_s-d_L}{d_s d_L}} = \sqrt{\frac{4l_p^2}{\bar{\lambda}}\frac{d_s-d_L}{d_s d_L}}$

Compton wavelength formula, then what we have in every formula is $\frac{l_p^2}{\bar{\lambda}}$. Also, note that the Planck length is a constant, and $\bar{\lambda}$ is a variable depending on the gravitational mass size. In addition, in some gravitational predictions we need one more constant, namely the speed of light (gravity). Further, we naturally need the distance from the gravity mass center to where we want to make the predictions for, so R.

It is important to understand we do not need to calculate or find the Planck length and the Compton wavelength separately to make these gravity predictions despite them being dependent on both the Planck length and the Compton wavelength. What we need for all gravity predictions related to observable phenomena is $\frac{l_p^2}{\bar{\lambda}}$ (or in some cases $\frac{\bar{\lambda}}{l_p^2}$). This parameter can be found easily from basically any observable gravity observation without any knowledge of the Planck length, the Compton wavelength, the gravity constant, or the Planck constant. This is demonstrated in **Table 6**.

To separate out the Planck length, we need to know the Compton wavelength. This we can measure for any mass, even large ones, but it requires much more input than just measuring $\frac{l_p^2}{\bar{\lambda}}$. To separate out l_p and $\bar{\lambda}$ is similar to separating out G and the kilogram mass M. However, even if such separation is not needed for predicting any observable gravitational observation, it is needed to understand and study the quantum level of gravity and matter. The factor:

$$l_p\frac{l_p}{\bar{\lambda}} \tag{21}$$

should, in our view, be interpreted as the gravitational collision-length energy. This is directly related to gravity and can also be called quantum gravitational energy. The last term $\frac{l_p}{\bar{\lambda}}$ should again be interpreted as the number of Planck events in the mass M per Planck time; see [58] [59]. It is also the reduced

Table 6. The table shows various gravitational observations we can use to find $\frac{l_p^2}{\bar{\lambda}}$. To find l_p and $\bar{\lambda}$ separately requires much more input information and therefore there is more uncertainty in them than $\frac{l_p^2}{\bar{\lambda}}$.

Observable	Formula	Parameters needed to be measured	Constant needed
Orbital time	$\frac{l_p^2}{\bar{\lambda}} = \frac{4\pi^2 R_2^3}{c^2 T^2}$	R_2, T	c
gravitational acceleration	$\frac{l_p^2}{\bar{\lambda}} = \frac{2HR_1^2}{c^2 T_b^2}$	H_1, R_1, T_b	c
Light bending	$\frac{l_p^2}{\bar{\lambda}} = \delta \frac{R}{4}$	δ, R	
Gravitational redshift	$\frac{l_p^2}{\bar{\lambda}} = zR$	z, R	

Compton frequency per Planck time. It is, for any macroscopic mass, an enormous number as the reduced Compton wavelength of a macroscopic mass is much smaller than the Planck length. No real wavelength can be smaller than the Planck length, in our view. The reason the equivalent Compton wavelength for a macroscopic mass is smaller than the Planck length is because it is an aggregate, and the aggregate follows the formula 14. It is therefore not a physical Compton wavelength, but we could call it an equivalent aggregated Compton wavelength.

There is much that is new in this presented interpretation, and it can seem controversial, but many of the answers to questions that some readers likely have can be found in a series of papers we have now published about collision space-time. Even if one ignores that theory here, it is still quite clear that one gets more accurate gravitational predictions by not relying on measuring G and M separately and then multiplying them together to make gravitational predictions. It is always more accurate to just find GM directly as well as $c^2 l_p \frac{l_p}{\bar{\lambda}}$ directly than to find G and M separately, or l_p and $\bar{\lambda}$ separately. Gravity is, at the deepest level, linked to both Compton frequency and the Planck length, but to make gravity predictions we can find both of these in the needed combination at the same time. This will, in general, lead to considerably less uncertainty in the gravitational predictions than those one gets by first finding G, as is the case in standard gravitational physics.

6. The Uncertainty in $\frac{l_p^2}{\bar{\lambda}}$ from Gravity Observations Is Smaller than for Only the Planck Length

In this section, we show how we can find the Planck length independent of knowledge off G and $\hbar$. This has been shown previously in some of our other papers, but here our focus is on first finding the Planck length squared, divided by the Compton wavelength; this needs much less information and therefore gives less uncertainty in the predictions. Further, we discuss why there is much more uncertainty in the Planck length than the Planck length squared, divided by the Compton wavelength.

From the previous section, it is clear that we can find the term $l_p \frac{l_p}{\bar{\lambda}}$ easily from a series of gravity observations with no knowledge of G or $\hbar$. For example, we can find this term by simply measuring the orbital time of the moon; it is about 27.3 days, which correspond to $27.3 \times 24 \times 60 \times 60 \approx 2358720$ seconds. Further, we need the distance from the Earth to the moon and this is approximately 38,440,0000 meters. We also need to know the speed of light, which we can measure totally independent of any knowledge about gravity, or we can just look it up. It is $c = 299792458\ \text{m/s}$. Now we can plug this into the first formula in Table 6:

$$\frac{l_p^2}{\bar{\lambda}} = \frac{4\pi^2 R_2^3}{c^2 T^2} = \frac{4\pi^2 \times 384400000^3}{299792458^2 \times 2358720^2} \approx 0.0045\ \text{m}$$

However, to extract the Planck length we need to know the reduced Compton wavelength of the Earth. This we can find by the Compton wavelength formula $\bar{\lambda} = \frac{\hbar}{Mc}$, but this we can only do if we know the kilogram mass of the Earth M. To find this mass, we can go through the procedure of first finding the gravitational constant from a Cavendish apparatus, then calculating the mass of the Earth by solving the orbital time formula $T = \frac{2\pi R_2}{\sqrt{\frac{GM}{R_2}}}$ with respect to M. But again, we then need to rely on G. We can naturally do this if we do not care about being dependent on G. Still, this means there will be more uncertainty in the Planck length estimate than in $\frac{l_p^2}{\bar{\lambda}}$, which we could easily extract without knowledge of G or M. There is, however, another way to find the Compton wavelength independent of both knowledge of G and $\hbar$. First, however, we show a way to find the Planck length independent of G. In a Cavendish apparatus we have:

$$\frac{l_p^2}{\bar{\lambda}} = \frac{L 2\pi^2 R_c^2 \theta}{T_c^2 c^2} \tag{22}$$

That is also in a Cavendish apparatus we need no knowledge off G to find $\frac{l_p^2}{\bar{\lambda}}$.

Next, we can re-formulate it to only have the Planck length on the left side, and this gives:

$$l_p^2 = \frac{L2\pi^2 R_c^2 \theta \bar{\lambda}}{T_c^2 c^2}$$

$$l_p^2 = \frac{\hbar L2\pi^2 R_c^2 \theta}{T_c^2 c^3 \frac{\hbar}{\bar{\lambda}} \frac{1}{c}} \tag{23}$$

Since $\frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ is the formula we can use to express the kilogram mass of the large ball in the Cavendish apparatus, since $\bar{\lambda}$ is the reduced Compton wavelength of this mass, then we can replace this with *m*. In other words, we just need a standard tool to measure weight to find the kilogram mass of *m*. We get:

$$l_p^2 = \frac{\hbar L2\pi^2 R_c^2 \theta}{T_c^2 c^3 m}$$

$$l_p = \sqrt{\frac{\hbar L2\pi^2 R_c^2 \theta}{T_c^2 c^3 m}} \tag{24}$$

A more detailed derivation and analysis of this can be found in the appendix of our 2017 paper [42], which was the first paper clearly demonstrating how to find the Planck length independent of *G*. However, here our main focus is that to find the Planck length leads to higher uncertainty than does finding $\frac{l_p^2}{\bar{\lambda}}$, because it is clear we need to measure the kilogram weight of the large ball in the Cavendish apparatus if we want to find the Planck length. On the other hand, to find $\frac{l_p^2}{\bar{\lambda}}$ (Equation (22)), we do not need to know the kilogram mass of the large ball (or any object) in the Cavendish apparatus. This is very similar to how, to find *G*, we also need to know the kilogram mass of the large balls in the Cavendish apparatus, but not when we want to find *GM*. Predictable gravitational phenomena do not need *G* but they need *GM*, just as they do not need the l_p but $\frac{l_p^2}{\bar{\lambda}}$. From a deeper understanding of gravity, this is naturally the same thing as $GM = c^2 \frac{l_p^2}{\bar{\lambda}}$.

We can also find the Compton wavelength without knowledge of the Planck constant $\hbar$, by observing Compton scattering of the electron, and this gives:

$$\lambda_e = \frac{\lambda_{2,\gamma} - \lambda_{1,\gamma}}{1 - \cos\theta} \tag{25}$$

where λ_e is the Compton wavelength of the electron. All we need to observe, to find this, is the wavelength of the photon when shot out towards the electron $\lambda_{1,\gamma}$, and the wavelength of the photon scattered by the electron $\lambda_{1,\gamma}$, as well as the angle between the incoming and outgoing photon. In other words, we need no knowledge of the Planck constant or *G* to measure this.

Next, we can find the Compton wavelength of the proton using a cyclotron. The ratio of the Compton wavelength of the electron and proton is equal to their cyclotron frequency ration. This is because the proton and electron have the same absolute value of charge. The cyclotron frequency is given by:

$$f = \frac{qB}{m} \tag{26}$$

where q is the charge, B is the magnetic field, and m is the mass of the particle one runs in the cyclotron. The cyclotron frequency ratio is therefore given by:

$$\frac{f_e}{f_P} = \frac{\frac{q_e B}{m_e}}{\frac{q_P B}{m_P}} = \frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \tag{27}$$

Experimental cyclotron frequency research shows that this ratio is about 1836.15; see, for example, [61] [62]. This is the well-known proton electron mass ratio that is identical to the electron proton Compton wavelength ratio. So, the Compton wavelength of the proton is simply the Compton wavelength of the electron divided by the proton to electron cyclotron frequency ratio. We do not claim the proton has one single physical Compton wavelength, but that the Compton wavelength we derive can still be used to find the mass of the proton.

Next, we will find the Compton wavelength of a macroscopic object from which we can measure effects from a gravitational field. That is, we "simply" need to count the number of protons and neutrons in one of the two large (identical) balls in the Cavendish apparatus; that is, to count the number of atoms. The Compton wavelength of the mass of the whole ball is then simply the Compton wavelength of the proton divided by the sum of the number of protons and neutrons in that mass. For simplicity, we can treat protons and neutrons as having the same mass, which is close to reality. To accurately count the number of atoms in a sphere of macroscopic size is not easy, but fully possible. One of the competing methods for a new kilogram standard consisted of actually counting atoms in silicon spheres' (^{28}Si) crystals; see [63] [64] [65]. Also, other methods to count atoms exist [66] [67]. So even though it is hard and quite expensive, it is fully possible to count the number of atoms in such a sphere.

In addition, it is well known that the mass of a bound element (mass) is typically slightly different than the aggregate of the mass of the individual atoms. The main component of this is nuclear binding energy; see, for example, [16] [68]. The nuclear binding energy is always less than 1% for any known atom and therefore the maximum error in the Compton wavelength, created by ignoring binding energy in its estimation, will be one percent. We can naturally quite easily incorporate the binding energy. However, for gravity predictions, we need $\frac{l_p^2}{\bar{\lambda}}$ and not the Planck length or Compton wavelength as separate entries. When we estimate $\frac{l_p^2}{\bar{\lambda}}$ directly from a gravitational observation, then the binding energy is automatically accounted for as then there is no need to count atoms in that case.

7. Even Newton's Original Formula Is More Accurate than Using *G* in the 1873 Formula

Modern physics uses the 1873 modified version of Newton's gravitational formula and thinks G is essential for gravity predictions. As we have shown in the sections above, when first extracting G from one mass by, for example, using a Cavendish apparatus for using G in combination with another mass than one calibrated it from, leads to unnecessary additional uncertainty in the gravity predictions. If one fully understands the 1873 formula, and only relies on calibrating GM from one gravity observation from the gravitational mass one plans to do other gravity predictions for, then this is not an issue. However, this is not normally what is done as the gravity community has not thought carefully about this.

Very few papers have even questioned why G came into place and what it truly represents. University textbooks have, for decades, also not made physics students aware that the so-called Newton gravity force formula they learn about is a modified version of Newton's original formula. Assume we just use Newton's original formula:

$$F = \frac{M_n m_n}{R^2} \tag{28}$$

then gravitational acceleration must be:

$$m_n a = \frac{M_n m_n}{R^2}$$

$$a = \frac{M_n}{R^2} \tag{29}$$

Assume we measure gravitational acceleration on the surface of the Earth with a drop ball to be $g = \frac{2H}{T^2} \approx 9.81\ \mathrm{m/s^2}$. This means the Newton mass of the Earth must be:

$$9.81\ \mathrm{m/s^2} = \frac{M_n}{R^2}$$

$$M_n = R^2 \times 9.81\ \mathrm{m/s^2}$$

$$M_n = 6371000^2 \times 9.81\ \mathrm{m/s^2} \approx 3.98 \times 10^{14}\ \mathrm{m^3 \cdot s^{-2}} \tag{30}$$

The Newton mass is equal to meters cubed, divided by seconds squared. At the time of Newton, the meter and second were not yet invented as units, but at least it would be "length-unit" cubed, divided by "time-units" squared. Now from this we can, for example, calculate the orbital speed of the moon. It must be:

$$T = \frac{2\pi R}{\sqrt{\frac{M_n}{R_2}}} \approx \frac{2\pi \times 384400000}{\sqrt{\frac{3.98 \times 10^{14}}{384400000}}} \approx 2373632\ \mathrm{s} \tag{31}$$

which corresponds to 27.47 days after we divided the number above by the number of seconds per day 24 × 60 × 60. Every observable gravity phenomena that can be predicted by the 1873 modified Newton gravity force formula that is used today can be predicted by the original Newton gravity force formula; see [7].

Actually, the original Newton force formula can be interpreted in several different ways. It can be interpreted as that $M_n = GM$. This view is correct in relation to the numerical example just given above. And again, gravity predictions do not depend on *G* and *M* separately, or *GMm*, but on *GM* that can be extracted directly from one gravitational observation. This would mean the mass was defined as $M_n = GM = c^2 \frac{l_p^2}{\bar{\lambda}}$. The output unit of the mass is then $m^3 \cdot s^{-2}$ which, in our view, is no stranger than kilogram, but still it seems to lack logical sense for something physical.

Another potential interpretation that we like much better is that the Newton force formula is identical to:

$$F = \frac{M_n m_n}{R^2} = c^3 \frac{M_g m_g}{R^2} \tag{32}$$

This would be true only if one has linked the distance to time through the speed of light, which means $c = c_g = 1$. In that case, the mass is the collision length, namely $M_n = M_g = t_p \frac{l_p}{\bar{\lambda}}$, but we cannot, with this interpretation of the original Newton formula, work with meters and seconds. It is unlikely Newton had thought about this possible interpretation, but theoretically he could have as he knew well the speed of light, mentioning in *Principia* it takes about eight minutes for the light to go from the sun to the Earth.

The original Newton formula, the 1873 modified formula, as well as the recent Newtonian type of gravity force formulas that we have recently introduced, all predict the same if used consistently. However, the 1873 modified formula, if not understood from a deeper perspective, can lead to less accurate predictions than necessary because one thinks one needs to rely on *G*. Again, *G* is never needed to predict a single observable gravity phenomena, for it is *GM* that is needed and that, at a deeper level, is: $c^2 \frac{l_p^2}{\bar{\lambda}}$.

8. Field Equation That Give All the Same Results as General Relativity But without the Need of *G*

Einstein's [23] field equation is given by

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{33}$$

It is clear that this equation requires the gravitational constant. The field equation itself does not give gravitational predictions that can be tested against observations without first solving the field equation based on chosen boundary conditions. Schwarzschild [69] assumed that the cosmological constant was zero, as Einstein had also assumed at that time (1916), and that one was dealing with a spherical, non-rotating gravitational mass. Based on this, he derived the metric using spherical polar coordinates, and got the following equation

$$\mathrm{d}s^2 = \alpha\left(1-\frac{k}{R}\right)\mathrm{d}t^2 - \left(1-\frac{k}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right). \tag{34}$$

Next one must find α and k, where one end up with two equations that must be solved (see, for example, [70]),

$$A(R) = \alpha\left(1+\frac{k}{R}\right) \text{ and } B(R) = \left(1+\frac{k}{R}\right)^{-1}. \tag{35}$$

The standard is then to find k and α by considering the weak-field limit where $\Phi = -\frac{GM}{R}$ is the Newton gravitational potential, which means we get

$$\frac{A(R)}{c^2} \to 1 + \frac{2\Phi}{c^2}, \tag{36}$$

This gives $\alpha = c^2$ and $k = -\frac{2GM}{c^2}$. Inserting this into Equation (34), we get the well-known "textbook" [71] Schwarzschild metric:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{Rc^2}\right)c^2\mathrm{d}t^2 - \left(1-\frac{2GM}{Rc^2}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \tag{37}$$

This appears to require knowledge of G. However, as we have pointed out in Section 5, the gravitational constant can be written as $G = \frac{l_p^2 c^3}{\hbar}$ (see [46]) and the kilogram mass M as $M = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. By replacing G and M with these in the Schwarzschild metric, we obtain:

$$\mathrm{d}s^2 = \left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)c^2\mathrm{d}t^2 - \left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

$$\mathrm{d}s^2 = \left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)c^2\mathrm{d}t^2 - \left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \tag{38}$$

As we have demonstrated in Section 5, the term $l_p\frac{l_p}{\bar{\lambda}}$ can be found without knowledge of G. This means that we only need to know the speed of light constant c to make all predictions from the Schwarzschild metric, provided that the metric is understood from a deeper level. The term $\frac{l_p}{\bar{\lambda}}$ is the reduced Compton frequency per Planck time, so this can actually be seen as a quantization of general relativity theory, where one has even incorporated the Planck scale, as recently suggested [72].

Still, why do the field equations contain G? We have recently shown in [60] that there is another field equation that gives all the same predictions as Einstein's field equation, but it is given by:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi E_{\mu\nu}, \tag{39}$$

The term: $E_{\mu\nu}$, is now identical to Einstein's stress-energy tensor except that any energy in it is linked to what we have defined as collision-length energy $E_g = l_p \frac{l_p}{\bar{\lambda}}$ and collision-time mass $M_g = t_p \frac{l_p}{\bar{\lambda}}$. This should not be mistaken as simply setting just $G = c = 1$. This field equation does not depend on G and the speed of light indirectly comes in when we develop the metric and calibrating to Newton. If we follow the same procedure as Schwarzschild, we end up with the metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2M_g c}{R}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2M_g c}{R}\right)^{-1} \mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right), \tag{40}$$

or alternatively:

$$\mathrm{d}s^2 = \left(1 - \frac{2E_g}{R}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2E_g}{R}\right)^{-1} \mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right), \tag{41}$$

As $E_g = l_p \frac{l_p}{\bar{\lambda}_M}$ and $M_g = t_p \frac{l_p}{\bar{\lambda}_M}$, we end up with:

$$\mathrm{d}s^2 = \left(1 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1} \mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right), \tag{42}$$

which is the same as the Schwarzschild metric coming from Einstein's field equation when understood from a deeper perspective. That is Equation (38) and Equation (42) are identical. However, our new approach has the advantage that nowhere in our theory is G needed; still, we can give the same predictions as general relativity theory. This in our view ultimately demonstrated G is not needed in gravitational physics when understood from a deeper perspective.

We also think there is no coincident that the $\frac{G}{c^4}$ part of the Einstein constant when multiplied by standard joule defined energy turn the joule energy into what we have defined as collision-length energy, that is

$$\frac{G}{c^4} E = \frac{l_p^2}{\hbar c} \hbar \frac{c}{\bar{\lambda}} = E_g = l_p \frac{l_p}{\bar{\lambda}} \tag{43}$$

See [60] for a more in depth discussion on this.

Modern Newtonian theory, which is the post-1873 version after the gravitational constant was inserted into Newton's formula, as well as the general theory of relativity and various alternative gravitational theories, all rely on the gravitational constant (denoted as G). Our collision space-time theory, as well as the original Newtonian theory, does not rely on the gravitational constant and can explain why one can achieve higher precision in predicting gravitational observations without using G. Aside from the improved precision in predictions, the original Newtonian theory provides the same predictions as the modern post-1873 version.

Based on recent progress in understanding the Planck scale and demonstrations of ways to find the Planck length independent of G (see [46]), a new avenue has opened up to quantize and link general relativity to the Planck scale, as seen in [73]. By carefully studying the newly quantized version of the general relativity theory, it becomes clear that all predictable gravity phenomena depend on knowing the factor $l_p \frac{l_p}{\bar{\lambda}}$, where the last part, $\frac{l_p}{\bar{\lambda}}$, represents the reduced Compton frequency in the mass M_g per Planck time. The reduced Compton frequency in matter can only be observed in quanta of integers, where the smallest observable frequency is 1, within the observational time window of the Planck time. A frequency lower than that can be seen as a probability. This implies that gravity is dominated by probabilistic effects for masses smaller than the Planck mass and by determinism for masses $m \gg m_p$. As demonstrated in Section 6, it is also more precise to find $l_p \frac{l_p}{\bar{\lambda}}$ directly rather than l_p and $\bar{\lambda}$ separately.

GM Is the Newton Mass

As stated at the beginning of the paper, the original formula for the Newtonian gravity force is:

$$F = \frac{M_n m_n}{R^2} \tag{44}$$

This formula was used all the way up to 1873, as can be seen, for example, by reading Maxwell's [24] book "*A Treatise on Electricity and Magnetism*". In the book, Maxwell points out that astronomical masses have the dimensions of $[L^3 \cdot T^{-2}]$. The dimensions of GM are also $[L^3 \cdot T^{-2}]$, which means that the kilogram mass cancels out when we multiply G with M as demonstrated in section 5. Therefore, there is no need for kilograms in gravitational predictions. Maxwell also explains how M_n can easily be found, for example, from gravitational acceleration. In the original Newtonian theory, gravitational acceleration is given by:

$$g = \frac{M_n}{R^2} \tag{45}$$

As g can be found, for example, by simply dropping a ball from a height H and measuring the time it takes to drop, and inserting this into $g = \frac{2H}{T_d^2}$ to measure g, one can easily find M_n without G. This is because we must have $M_n = \frac{2HR^2}{T_d^2}$. The gravitational constant was not even invented at the time Maxwell published his book; it was invented several months after Maxwell's book was published.

If one should use G and M, then G has to be found, for example, first by using a Cavendish apparatus. However, if G is applied to any other mass than the large ball in the Cavendish apparatus, then the error in finding GM by first finding G and then M, and then multiplying these together, will be larger than finding GM directly. To find GM directly is, in reality, to find the Newtonian mass, which requires less input than finding the kilogram mass M. The reason is that the kilogram mass contains information that is not needed for gravity, and in addition, it lacks information that is needed for gravity. Therefore, it needs to be multiplied by G to extract information that is not needed and to introduce information into the mass that is needed. In the Newtonian gravitational mass, there is no information that is not needed, and all the information needed about mass to make gravity predictions is present. See also [72] that gives an overview of many possible mass definitions, and how the Newton mass M_n as well as the collision-time mass are complete masses for gravitational purposes while the kilogram mass and many other possible mass definitions are not.

The mass in the modified Newton formula of 1873, which is now known as the Newtonian gravitational force formula, is measured in kilograms. So, if the kilogram falls out of the formula, does this mean that gravity is not related to mass? We have indeed demonstrated that all observable gravitational phenomena can be predicted without knowing the mass (M) or the gravitational constant (G) in the force formula, see [27]. The reason for this is not that gravity does not depend on mass, as it naturally does, but that it depends on the collision-time mass. If we multiply the collision-time mass with c^3, we get the Newtonian mass (M_n). In fact, we have:

$$GM = c^3 M_g = M_n \tag{46}$$

Therefore, the method of finding GM directly as is used in the GPS system, rather than first finding G and then M, is essentially to find M_m, which is the Newtonian mass directly. This can be done from most gravitational observations as was pointed out even by Maxwell, but naturally with much higher accuracy today due to much better measurement devices. We have recently presented yet another field equation consistent with the original Newtonian gravity force formula as the weak field limit. This field equation is given by [60]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4} E_{\mu\nu}, \tag{47}$$

where $E_{\mu\nu}$ now is the stress-energy tensor, but now linked to the Newton mass M_n and its corresponding Newton energy $E_n = M_n c^2$ rather than the kilogram mass M and the joule energy $E = Mc^2$.

Once again, there is no need for G in this field equation or its solutions, as the only constant required is c. For example the Schwarzschild type solution in this field equation is simply

$$\mathrm{d}s^2 = \left(1 - \frac{2M_n}{c^2 R}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2M_n}{c^2 R}\right)^{-1} \mathrm{d}R^2 - R^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{48}$$

This means that the way practitioners that have developed the GPS system that find GM directly without first finding G and M, is in reality finding the old Newton mass: M_n. To find M_n requires no information about the kilogram. If both general relativity theory and the 1873 modified Newton theory were giving a full understanding of gravity, then there should be no difference in precision by first finding G and then M, and multiplying them together to get GM compared to finding GM without first finding G and M.

While the theory of general relativity may be more complicated compared to our two new field equations, these equations are strongly linked to Einstein's theory since they yield identical predictions. Two of the field equations can explain why finding GM directly, rather than G and M, gives greater precision: G is not needed when using the correct mass, which is also embedded in GM. The method practitioners use to find GM directly is consistent with these equations and cannot be fully explained within general relativity theory. That is, general relativity theory cannot predict why this is the case without modification. However, at the deepest quantum level, our quantum gravity version of general relativity can provide an explanation. In relation to the Schwarzschild metric, both general relativity theory and the two new field equations are the same at the deepest level, as demonstrated in [60] and also in section 8. While general relativity theory is fully correct, it may be slightly overly complicated. That is we can get rid of the gravitational constant G, but we then need to switch to a different energy and mass definition, see [72].

9. Practical Implications

All technology based on predictions from gravity phenomena where high precisions is needed, such as GPS, should rely only on either Newton mass M_n or collision-time mass M_g for maximum precision, and not on the kilogram mass M and the gravitational constant G. Currently, this is done ("unknowingly") indirectly by gravitational practitioners, such as for GPS, by relying on finding GM directly instead of finding G and M independently and then multiplying them together. The latter method will result in unnecessary measurement errors as is well known by GPS agencies, such as the National Imagery and Mapping Agency as is quite clear from their document WGS 84 version 3.

Therefore, $GM = M_n = c^3 M_g$ should be used for GPS on Earth, as it is currently done, but also in spaceship navigation, where gravity measurements are relied upon for navigation, such as in the solar system. In this case, the Heliocentric Gravitational Constant $GM_{g,\odot}$ should be used see also [74]. The Heliocentric Gravitational Constant is when fully understood essentially simply the original Newtonian mass of the Sun $M_{n,\odot}$, which again is the collision-time mass of the Sun $M_{g,\odot}$ multiplied by c^3.

The same is true for any gravitational object where predictions of gravitational effects are important, such as navigation, or actually for any gravitational prediction. It will always be more accurate to find GM directly, which is simply the

Newtonian mass M_n, which is identical to $c^3 M_g$, rather than to first find G and then the kilogram mass M of the gravitational object of interest for making predictions. For example, when navigating around Mars, one should use the M_n of Mars in all predictions, which is identical to GM and $c^3 M_g$.

The reason why finding GM directly is much more accurate than finding first G and then M cannot be fully explained inside General Relativity Theory. General Relativity Theory is also linked to the 1873 ad hoc modified Newton formula when the gravity constant was introduced. However, the explanation for why finding GM directly without first finding G and then M is possible within our new gravitational model, which is based on two new Einstein-inspired field equations.

Even if our new and deeper understanding of gravity does not necessarily lead to any practical improvements in geodetic technology, it provides a solid and profound theoretical understanding of why methods that rely on GM directly are superior in accuracy to those that involve finding G and M separately and then multiplying them together. The only exception is when both G and M are determined from the mass in the Cavendish apparatus, that is when G is used on the same mass it was found from, the additional measurement errors in G and M will in that special case cancel out. However, in practice, for geodetic technology, one would first need to find G from one mass (for example in a Cavendish apparatus) and next to apply G on a different mass, the Earth. Ultimately, the way practitioners use gravity theory in practice for GPS (by finding GM directly) is fully consistent with our two new field equations, but cannot be fully explained in Einstein's general relativity theory. General relativity theory cannot predict that finding GM directly gives higher precession, but our new field equations can. We know all this may sound controversial, but we ask the gravity research community to think carefully about this before jumping to prejudiced or rapid conclusions. That said, we are naturally open to criticism, discussions and other viewpoints.

10. Conclusions

We have demonstrated that by not relying on finding the gravity constant G as well as the kilogram mass of the gravitational object, we will get more accurate gravitational predictions. The gravitational constant G was invented in 1873, about at the same time as the kilogram definition of mass in gravity came into use. The kilogram unit is also embedded in G, as can be seen from its output units. However, the kilogram output unit cancels with the kilogram output units in the kilogram mass. No observable gravitational phenomena cares about the kilogram definition of mass as it is not the essence of mass related to gravity. At a deeper level, the gravitational constant contains the Planck length and the Planck constant as well as the speed of light. From a deeper perspective, the gravitational constant must be multiplied by the kilogram defined mass of the gravitational object to get the Planck constant (related to kilogram) out of the kilogram mass, and the Planck length into the kilogram mass. At a deeper level, all observable gravitational observations are dependent on $l_p \frac{l_p}{\bar{\lambda}}$ and some are, in addition, dependent on the speed of gravity c_g, which is identical to the speed of light c. This means all gravitational predictions only need two constants: c and the Planck length. However, the Planck length is not needed as a separate constant to make gravity predictions that also can be observed, but it is needed in the form $l_p \frac{l_p}{\bar{\lambda}}$, which can be extracted easily from any observable gravity phenomena with no knowledge of any constant.

We will claim that our insight is an important step for understanding quantum gravity from a theoretical point of view, but also because it has practical implications for gravity predictions. Quantum gravity is, in this view, remarkably already embedded in even standard Newton gravity and also general relativity, but this cannot be seen from its surface. Well, it is likely more correct to say our quantum gravity theory has Newton gravity as a good first approximation. However, our new insight also has important practical implications; namely, that one should never estimate G first, and then M, and then depend on G to estimate gravity effects, with the exceptions of if G is only used in connection to the mass it was calibrated to. However, if one calibrates G from a Cavendish apparatus, *i.e.*, it is calibrated to the mass of the large balls in the apparatus, then if next used to predict gravitational effects for the mass of the Earth or any other mass, it will, as we have demonstrated, lead to unnecessary additional uncertainty. The way to minimize uncertainty in gravity predictions is to calibrate GM, M_n, $c^3 M_g$ (they are all the same from a deeper understanding $c^2 l_p \frac{l_p}{\bar{\lambda}}$, but of these, only GM can give the incorrect impression one which is dependent on G) directly from one gravitational observation of the mass of interest, and to switch to the new Einstein-inspired gravitational field equation presented in this paper. This yields the same predictions as general relativity theory, but with lower uncertainty.

While it is true that the accuracy of our gravity predictions, which no longer rely on the gravitational constant G, has to some degree already been tested by the National Imagery and Mapping Agency (NIMA) independently of our work, as documented in the Department of Defense World Geodetic System 1984 document, we also encourage other gravity laboratories to rigorously assess our claims and methods. Notably, there is little published information, to our knowledge, that quantifies the extent of improvement in gravity predictions achieved by eliminating reliance on G.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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