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An Investigation of Purely Azimuthal Passive Localization and Position Adjustment in Attempted UAV Formation Flights

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Abstract

When a cluster of unmanned aerial vehicles (UAVs) is flying in formation, it is crucial to maintain the formation and not to be interfered by external electromagnetic wave signals. In order to maintain the formation, this paper proposes to use pure azimuth passive positioning to adjust the position of UAVs, *i.e.*, certain UAVs in the formation transmit signals, the rest of the UAVs receive the signals passively, and extract the orientation information from them to adjust the position of the UAVs [1] [2] [3]. In this paper, the position adjustment problem of UAVs in "circular" formation flight under three models is investigated. To address the problem of "how to obtain the position of the receiving UAV when there are two UAVs with known numbers and evenly distributed on the circumference in addition to the UAV transmitting at the known center of the circle, and the rest of the UAVs with slight deviations in their positions are receiving the signals", two purely mathematical geometric methods, namely, triangular localization method and polar co-ordinate method, are proposed respectively. We have determined the position of the receiving UAV; we have used the exhaustive method and the construction and disproof method to solve the problem of "how many UAVs are needed to transmit signals in order to realize the effective positioning of the UAVs when it is known that a certain UAV with a slight deviation in its position receives the signals emitted by two UAVs at the same time", and the results show that: in addition to the known signals emitted by two UAVs, it is also necessary to transmit the signals emitted by two UAVs. The results show that in addition to the known two UAVs transmitting

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signals, two additional UAVs are required to transmit signals for precise positioning. When the position of UAVs has deviation at the initial moment, the ideal approximation method and the target delimitation method are proposed, and the target of nine UAVs uniformly distributed on a circle of a specific radius is achieved through several adjustments, after which the advantages and disadvantages of each model are analyzed, and suggestions for improvement are put forward. The purely azimuthal passive localization method and the constructed model approach proposed in this paper can be extended to other fields, such as spacecraft formations in space and battleship formations at sea, as well as other formation flight position adjustment problems.

Keywords

Pure Azimuth Passive Positioning, Unmanned Aerial Vehicle (UAV) Position Adjustment, Electromagnetic Silence

1. Introduction

In recent years, UAV clusters have gained increasing attention for their potential applications in a variety of fields, including surveillance, search and rescue, and monitoring, and UAV formation flights have demonstrated significant application potential and value in a number of fields. In UAV formation flying, the positional adjustment of the UAV formation is crucial to ensure the overall stability and mission execution capability of the formation. Each UAV in the formation should maintain an even distribution over a defined circumference to ensure the compactness of the formation and flight safety. If the positions of the UAVs deviate, the shape of the formation will become irregular, which may lead to collision risk, communication interference, and mission execution failure. Therefore, adjusting the position of UAV formations is crucial to ensure formation flight stability and mission success. However, in existing UAV formation flights, emitting electromagnetic wave signals actively and performing ranging or angular measurements is usually necessary to achieve UAV localization and navigation. This method increases the electromagnetic wave signal emission of the formation, which causes potential interference and safety issues, and is not feasible under the requirement of electromagnetic silence.

When flying in formation, UAV clusters must minimize electromagnetic emissions to avoid external interference. Therefore, maintaining electromagnetic silence is key for efficient and safe UAV operations. Therefore, the main challenge of this study is to extract accurate orientation information by relying only on passively received signals without transmitting electromagnetic wave signals, and to adjust UAV positions based on this information to achieve formation flight stability and formation maintenance. We consider a purely azimuthal passive localization method to adjust UAV positions, *i.e.*, localization and position adjustment by having some UAVs transmit signals and the remaining UAVs passively receive signals and extract orientation information from them [1] [2]

[3]. However, researchers have studied purely azimuthal passive localization methods relatively little to date, and they have not yet widely used them in UAV formation flights. Therefore, the uniqueness of this study is to explore the practical feasibility of pure azimuth passive localization methods in UAV formation flying. Meanwhile, by adjusting the position of UAVs based only on the received orientation information, this study can provide a safer and more efficient method of formation flying, which has a wide application potential.

In this study, we examine a "circular" formation of 10 UAVs (FY00 - FY09) with fixed numbers, of which 9 UAVs (FY01 - FY09) are uniformly distributed on a circle circumference, and one UAV is located at the center of the circle (as in **Figure 1**) [1] [2] [3]. The relative position of each UAV in relation to other UAVs in the formation remains constant. We assume that the UAV receiving the signal can extract orientation information from the angle α of the line between the two UAVs connected to it that transmit the signal.

Against this background, we ask the following three questions. First, for problem 1, it is known that the UAV at the center of the circle (FY00) and 2 other UAVs (with known numbers) transmit signals, and there is no deviation in their positions. The remaining 7 UAVs receive signals passively and there is a slight deviation in the positions of these 7 UAVs. It is now required to determine the location of the receiving UAV based on passive signal reception. Secondly, for Problem 2, the UAVs with known numbers FY00 and FY01 and several other UAVs in the formation with unknown numbers transmit signals and there is no deviation in their positions. However, there is a deviation in the positions of the UAVs that receive these signals. It is now required to determine the minimum number of additional signal-transmitting UAVs, in addition to FY00 and FY00, required for effective positioning. Finally, for Problem 3, consider that one UAV needs to be positioned at the center, while the remaining 9 UAVs are distributed around a circle with a radius of 100 m. It is known how many UAVs will be positioned at the initial moment. It is now known that the positions of the UAVs at the initial moment have slight deviation (data in **Table 1**), under the condition

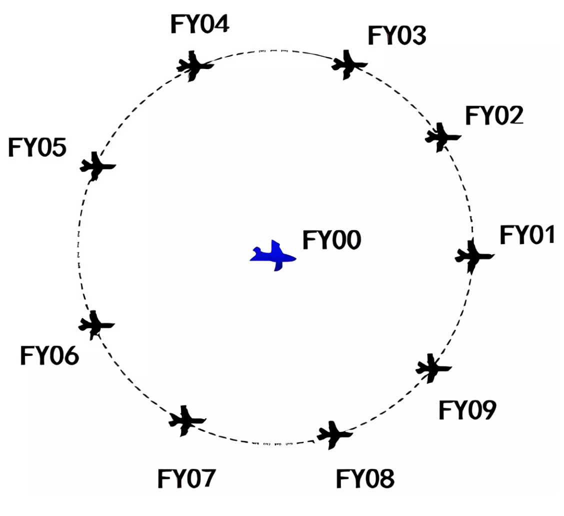


Figure 1. Schematic of a circular formation of drones.

Table 1. Initial position of the drone.

Drone number	Polar coordinates (m, °)
0	(0, 0)
1	(100, 0)
2	(98, 40.10)
3	(112, 80.21)
4	(105, 119.75)
5	(98, 159.86)
6	(112, 199.96)
7	(105, 240.07)
8	(98, 280.17)
9	(112, 320.28)

that the adjustment time is negligible, each time we choose the UAV with FY00 number and at most 3 UAVs on the circumference to transmit signals, and the other UAVs to receive the signals [3] [4]. It is now required to propose a reasonable UAV position adjustment scheme, through multiple adjustments, each time selecting the center UAV FY00 and at most three UAVs on the circumference to transmit signals, and the rest of the UAVs will make position adjustments according to the received direction information [1] [2] [3] [4], so that these nine UAVs can finally realize a uniform distribution on a certain circumference.

The three problems are interrelated and are predicated on a circular formation flight of 10 UAVs. Each problem addresses specific aspects of positional adjustment and localization within the formation. Problem 1 lays the foundation for understanding the localization process within the formation; Problem 2, by exploring the minimum number of additional UAVs required, further investigates the dependencies between the various UAVs for precise localization; and Problem 3 encompasses the practical issue of adjusting the UAVs' positions based on the information received, which contributes to the formation achieving the desired configuration. By addressing these three problems, this paper provides a comprehensive analysis of position adjustment and localization in formation flight. Each issue contributes to the overall goal of maintaining formation integrity and demonstrates different aspects of the localization process, including vehicle position adjustment and localization.

By adopting a purely azimuthal passive localization method, we are able to adjust the position of the UAV and realize the formation requirements of the formation while maintaining electromagnetic silence. This is of great significance for UAV formation flying in scenarios with high electromagnetic environment interference. Our results are important for the safety and reliability of UAV formation flying and have potential generalization value in other fields.

The remainder of the paper is organized as follows. Sections II through IV present our research methodology and modeling process. Then, we present the experimental results in Part V and analyze and discuss the results. Finally, in Section VI, we summarize the application of purely azimuthal passive localization methods in UAV formation flying, and in Section VII, we evaluate the advantages and disadvantages of the models involved in this paper and suggest improvements and look forward to the future directions of further research and potential extended applications. As shown in Figure 2.

2. Problem 1: No Deviation in the Position of the Transmitting Signal Drone

2.1. Formulation of the problem

In the context of a "circular" formation, it is known that one UAV at the center of the circle (FY00) and two other UAVs transmit signals, while the rest of the UAVs with slightly deviated positions passively receive signals [3]. We need to build a localization model that uses the received direction information to determine the position of the receiving UAVs.

2.2. Analysis of the Problem

The known conditions are: 1) A drone with the number transmits a signal. 2) There are two other drones with known numbers that transmit signals. 3) The drones transmitting signals are all positioned without deviation [3], except at the center of the circle, and the other two are on the circumference of the circle. 4)

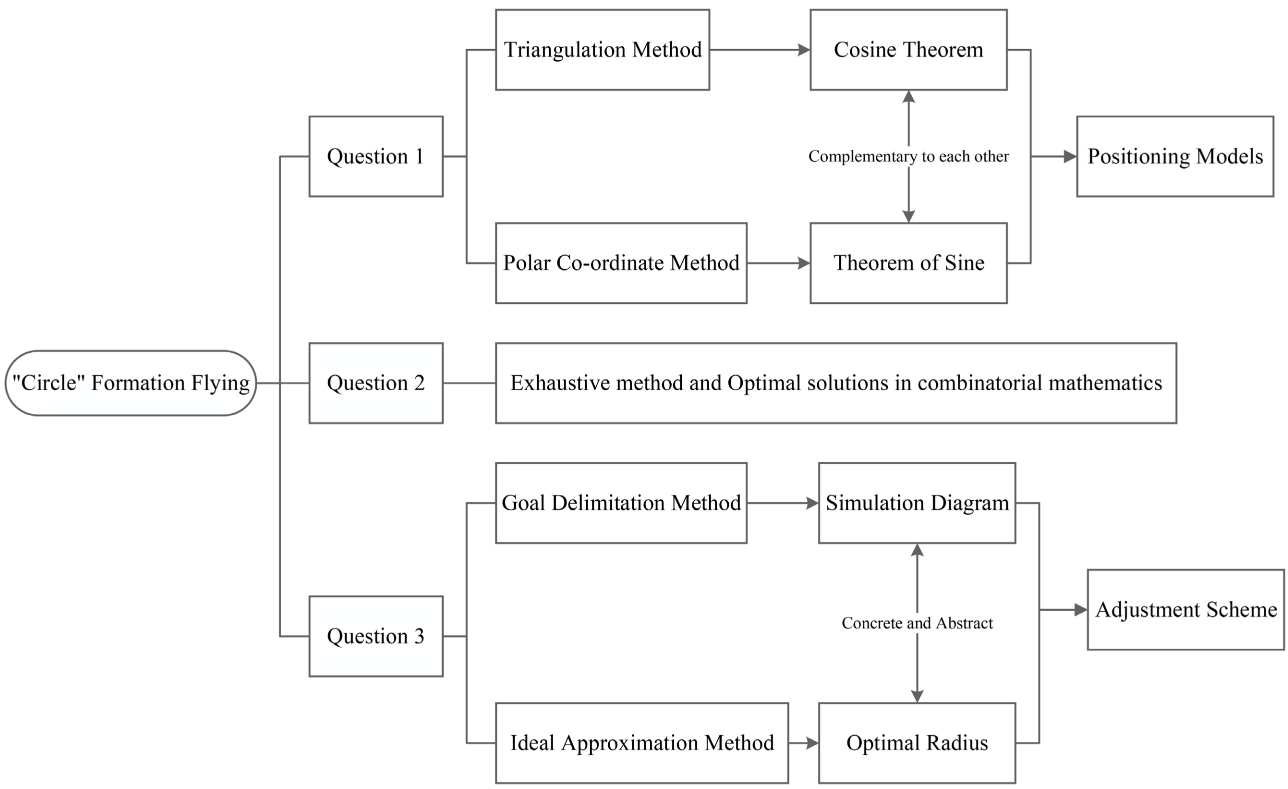


Figure 2. Framework of ideas for the full text.

The position of the drone receiving the signal is deviated and is not on the circumference of the circle.

Problem 1 requires us to model the position of a UAV (FY00) at the center of a known circle and two other UAVs in the formation with known numbers and no deviation in their positions, to model the localization of a UAV that receives signals passively [5]. The problem is a purely mathematical one. This problem is a purely mathematical problem where a mathematical model is developed to calculate the relationship between the received direction information and the position of the UAV. By observing and analyzing the received direction information, we can find that the angle with each UAV transmitting signal is determined, so the position of the UAV can be deduced from these angles. To solve this problem, we adopt two methods.

The first method adopts the trigonometric method, from the principle of "the common intersection of three circles is at most one", we can know that the distance from the unknown point to the known three points is fixed, so that we can uniquely determine the three distances by the cosine theorem, and then finally determine the unique position of the unknown point according to the ternary Monge's theorem. Specifically, in the aircraft formation, we start with the UAV (set as a point P) that receives signals passively in relation to the position of the circular formation is discussed to obtain the abstract orientation information. Then we classify the discussion with different values to get concrete orientation information and determine the UAV positioning model (FY0N is set to be the number of one of the UAVs whose number is known to transmit signals).

The second method uses polar coordinates and sinusoidal theorem method, by discussing the positional relationship between the UAVs receiving signals passively and the circular formation, first, the abstract orientation information is obtained, and then the concrete orientation information is obtained by discussing with different values of k of FY0k.

2.3. Modeling and Solving

2.3.1. Model Solving Based on Triangulation Method

As shown in **Figure 3**, let the drone with a known number be FY0k_1, FY0k_2, set to the point A, B. Where there is no harm in setting $k_1 < k_2$, $k_1, k_2 \in \{1,2,3,4,5,6,7,8,9\}$. Let the drone receiving the signal be P, it is only necessary to find the coordinates of the drone P. The specific steps are shown in **Figure 4**.

We just have to take FY00 as the origin and FY01 as the $(r, 0)$ point to set up a planar right-angle coordinate system. And we have to set: $\mathrm{FY0}k_1 = (a_1, b_1)$, $\mathrm{FY0}k_2 = (a_2, b_2)$, ($k_1$ and k_2 known shows that a_i, b_i known and $i \in \{1,2\}$. And let the radius of the standard circle be r).

Step1: Calculate $l = \left|d(\mathrm{FY01}, \mathrm{FY02})\right| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$. It is now known that $\angle \mathrm{APO} = \alpha_1$, $\angle \mathrm{BPO} = \alpha_2$, $\angle \mathrm{APB} = \alpha_3$. Let $|\mathrm{BP}| = m$, $|\mathrm{OP}| = n$, $|\mathrm{AP}| = p$, then the formula for the cosine theorem of a triangle can be obtained as in the following equation:

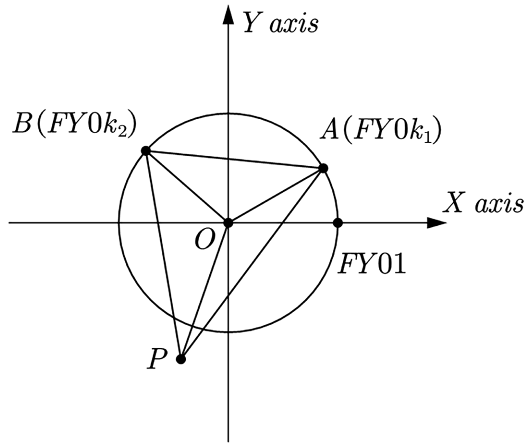


Figure 3. Schematic diagram of two-point localization.

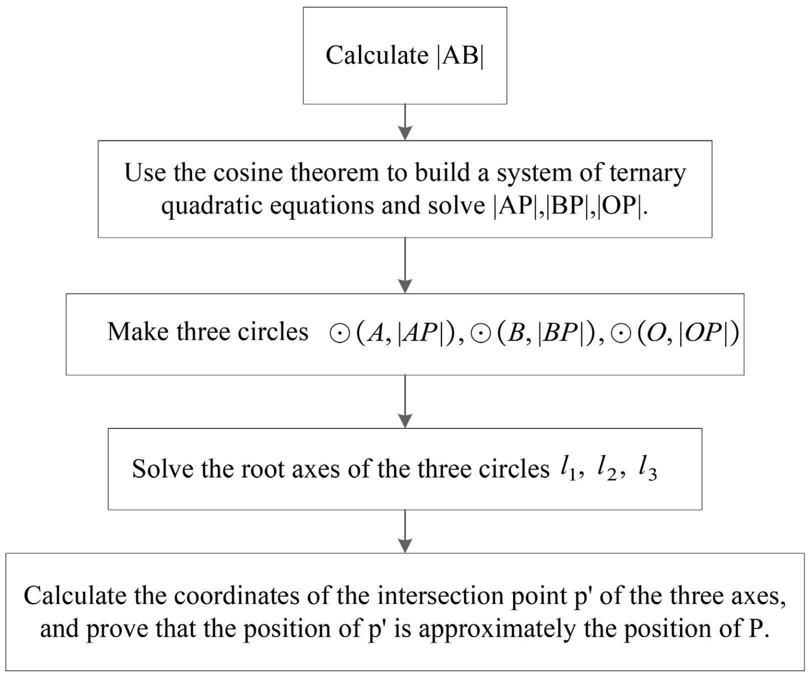


Figure 4. Triangle positioning method idea diagram.

$$\begin{cases} m^2+n^2-2mn\cos\alpha_1=r^2 \\ m^2+p^2-2mp\cos\alpha_3=l^2=\left(a_1-a_2\right)^2+\left(b_1-b_2\right)^2 \\ n^2+p^2-2np\cos\alpha_2=r^2 \end{cases} \tag{1}$$

This system of ternary quadratic equations is computed by MATLAB to obtain m, n, p. The exact value can be obtained when there is explicit arithmetic data.

Step2: We respectively take A, B, O as the center of the circle and m, n, p as the radius to make a circle (**Figure 5**).

Introduce Monge's theorem: any three circles in the plane, if the three circles are not co-centered, then the three root axes intersect at a point, which is called

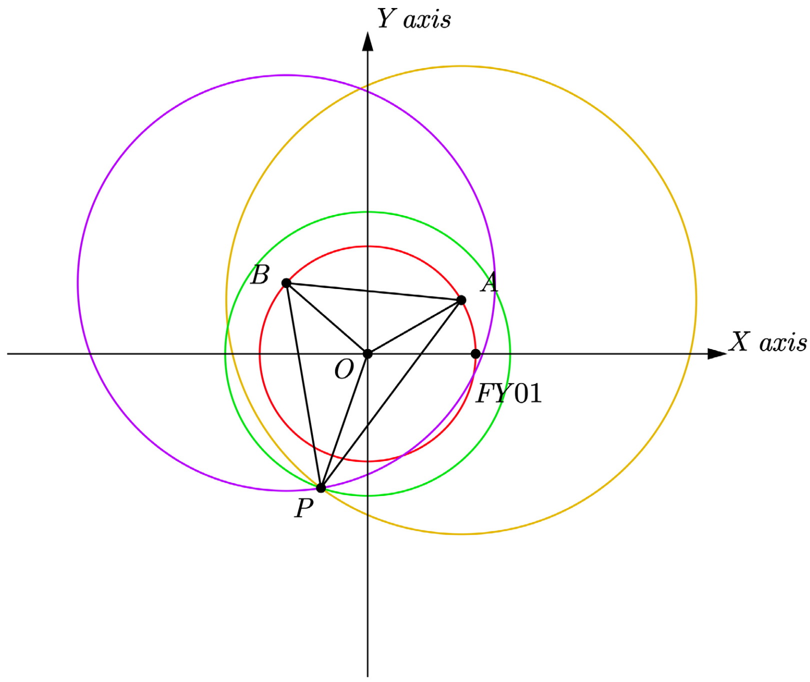


Figure 5. Three circles co-point schematic.

the center of their roots; if the three circles are co-centered, then the three root axes are parallel to each other [6]. Then, by the Monge theorem, two two root axes of the three circles (a total of three) common point (not parallel) [7], then the only point of co-occurrence is the position of p'. Within the tolerance of error, the position of point p' can be considered as the position of P. The exact value can be obtained when clear arithmetic data are available.

2.3.2. Model Solution Based on Polar Co-Ordinate Method

The specific steps are shown in **Figure 6**.

Step1: Get the abstract orientation information.

Set A, B, D be the points at which the drone transmits signals, respectively C, P be the points at which the drone receives signals, respectively, and $\alpha_1,\alpha_2,\alpha_3$ is the angle set in the question. According to the conditions of the question, if there is no deviation between the drones transmitting and receiving signals, the standard schematic is shown in **Figure 7**.

The setup of this problem is that the drone at point P is slightly deviated, and the direction of the new information formed when it is deviated from the original position is correlated with the direction of the original information. So on the basis of **Figure 1** we can have the following discussion situation:

1) When $\alpha_1 < \alpha_{1fixed}$, the UAV receiving the signal (point P) is outside the circle, as in **Figure 8(1)**.

Proof: Because A, B, C, D are on the circle O and $\angle$ACB has a common arc AB with $\angle$ADB, so $\angle ACB = \angle ADB = \alpha_{1fixed}$. And because in ΔPDB, $\angle ACB = \angle ADB + \angle PBD = \alpha_1 + \angle PBD$. Therefore, $\alpha_{1fixed} = \alpha_1 + \angle PBD$. The same can be proved for the other cases, which will not be proved in detail below.

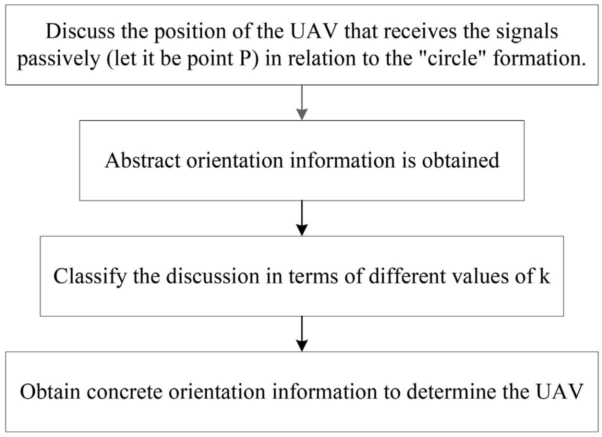


Figure 6. Polar coordinate method.

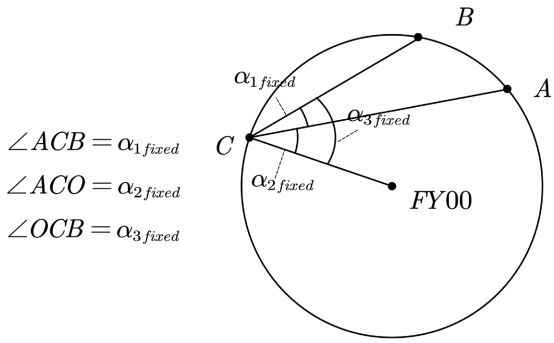


Figure 7. Angle diagram.

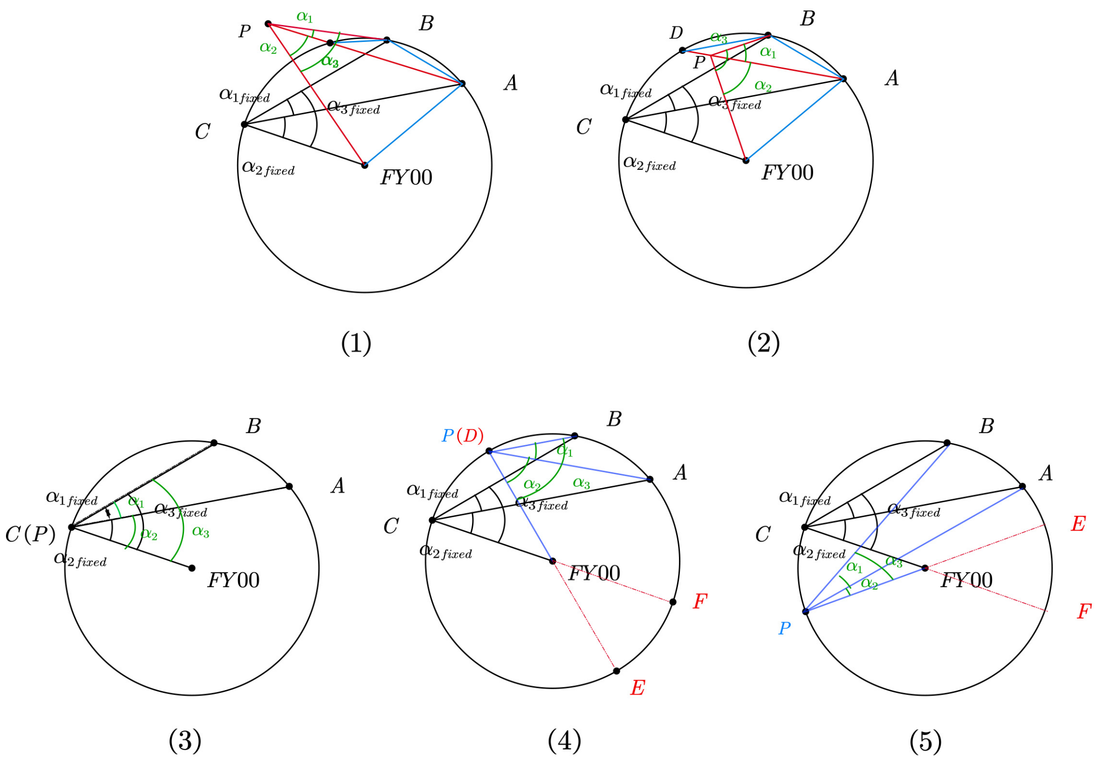


Figure 8. Deviation diagram.

2) When $\alpha_1 > \alpha_{1fixed}$, the UAV receiving the signal (point P) is inside the circle, as in **Figure 8(2)**.

3) When $\alpha_1 = \alpha_{1fixed}, \alpha_2 = \alpha_{2fixed}$, the point P coincides with the point C and there is no deviation, as in **Figure 8(3)**. The case is not discussed in this question because the question sets P to have a deviation.

4) When $\alpha_1 = \alpha_{1fixed}, \alpha_2 > \alpha_{2fixed}$, the point P is coincident with point D with point P on the circle and the deviation is to the left, as in **Figure 8(4)**.

5) When $\alpha_1 = \alpha_{1fixed}, \alpha_2 < \alpha_{2fixed}$ the point P is coincident with point D with point P on the circle and the deviation is to the right, as in **Figure 8(5)**.

Step2: Getting Specific Direction Information

Since the remaining 9 points except point FY00 are uniformly distributed on a certain circle, it is useful to assume that the radius of the circle is R and the angle between the lines connecting each neighboring drone and the drone at the center of the circle (FY00) is $\theta = 2\pi/9$.

From the equivalence of uniformly distributed points on a circle, it is useful to set one of the drones transmitting a signal as FY01, with the point A instead, and the drone FY00 at the center of the circle is replaced by point O. Since there is a fixed number of drones with known numbers, let the other drone transmitting signals be FY0*k*, and replace it with the point B where $2 \le k \le 9$, $k \in N^+$. Then let the drone receiving the signal be the point P. Now, on the circumference of the circle with the point O as the pole, and OA direction is the polar axis direction to establish a polar coordinate system [8] [9] [10]. Then have the point O(0, 0) point A(R, 0) and point $\mathrm{B}\left(R,(k-1)\theta\right)$, and then set the point $\mathrm{P}(x,\alpha)$.

Let the angle between the line between point P and point O and point A be α_1, the angle between the line between point P and point A and point B be α_2, and the angle between the line between point P and point A and point B be α_3. The different model cases are discussed in terms of different values of k. Since the circle is axisymmetric about the OA line (FY00 and FY01), k can be divided into three categories:

i) $k = 2$, *i.e.*, FY0N is adjacent to FY01 adjacent.

ii) k = 3, 4, 5, *i.e.*, the points in the upper semicircle that is not adjacent to FY01.

iii) $5 < k \le 9$, *i.e.*, the points of the lower semicircle that is symmetrical to the upper semicircle.

1) When k =2, the drones transmitting the signal are point O(FY00), point A and point B.

a) When $\alpha_1 > \alpha_2$, the specific case is in **Figure 9(1)**. The sine theorem for ΔOPA, ΔOPB respectively has:

$$\begin{cases} \dfrac{\sin\alpha_1}{R} = \dfrac{\sin\left((\pi-\alpha_1)-(2\pi-\alpha)\right)}{x} = \dfrac{\sin(\pi-\alpha_1-\alpha)}{x} \\ \dfrac{\sin\alpha_2}{R} = \dfrac{\sin\left((\pi-\alpha_2)-(2\pi-\alpha)-(k-1)\theta\right)}{x} = \dfrac{\sin\left(\pi-\alpha_2-\alpha+(k-1)\theta\right)}{x} \end{cases} \tag{2}$$

obtain a solution:

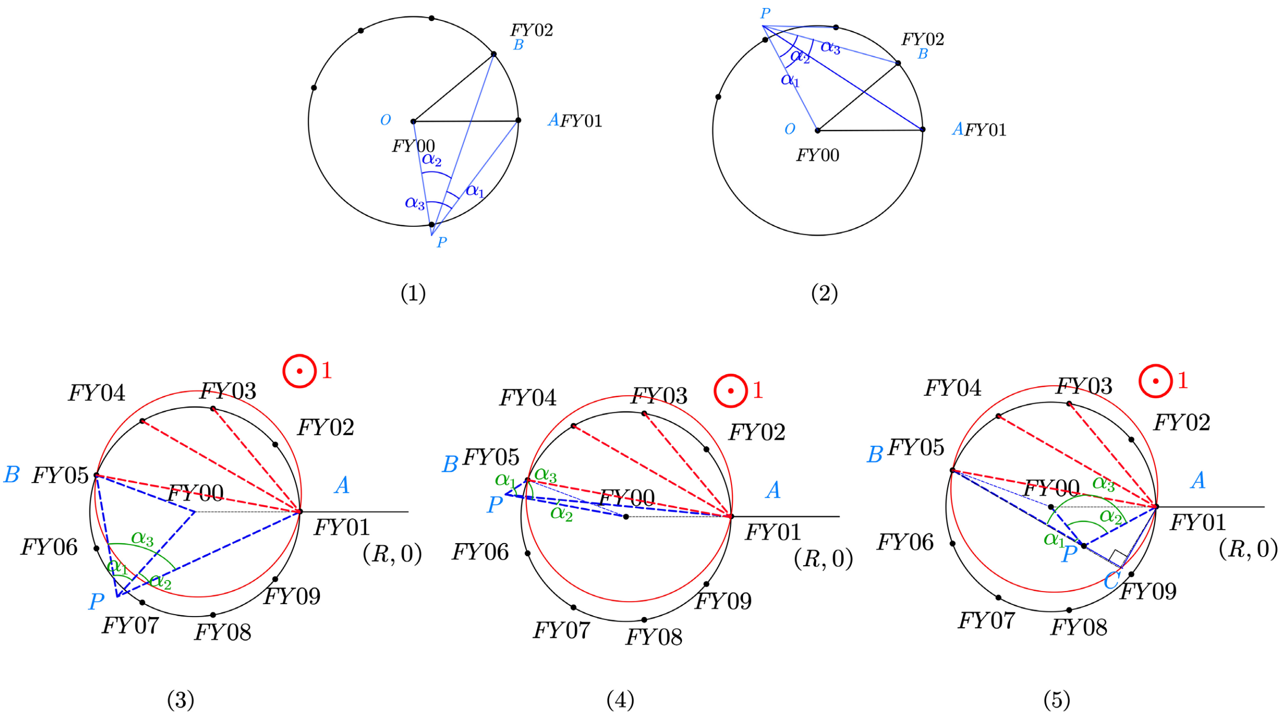


Figure 9. P point position deviation diagram.

$$
\begin{cases}
\alpha = \arctan\left(\dfrac{\sin(\alpha_1)\sin(\alpha_2) - \sin(\alpha_1)\sin(\alpha_2 - (k-1)\theta)}{\cos(\alpha_2 - (k-1)\theta) - \sin(\alpha_1)\sin(\alpha_2)\cos(\alpha_1)}\right) \\
x = \dfrac{R\sin(\alpha_1 + \alpha_2)}{\sin(\alpha_1)}
\end{cases} \tag{3}
$$

b) When $\alpha_1 \le \alpha_2$, the specific case is in **Figure 9(2)**. The sine theorem for ΔOPA, ΔOPB respectively has:

$$
\begin{cases}
\dfrac{\sin\alpha_1}{R} = \dfrac{\sin(\pi - \alpha_1 - \alpha)}{x} \\
\dfrac{\sin\alpha_2}{R} = \dfrac{\sin(\pi - \alpha_2 - \alpha + (k-1)\theta)}{x}
\end{cases} \tag{4}
$$

obtain a solution:

$$
\begin{cases}
\alpha = \arctan\left(\dfrac{\sin^2(\alpha_1)\sin(\alpha_2) - \sin(\alpha_2 - (k-1)\theta)}{\cos(\alpha_2 - (k-1)\theta) - \sin(\alpha_1)\sin(\alpha_2)\cos(\alpha_1)}\right) \\
x = \dfrac{R\sin(\alpha_1 + \alpha_2)}{\sin(\alpha_1)}
\end{cases} \tag{5}
$$

2) When $k = 3, 4, 5$, we will use the magnitude relation of α_3 to determine whether the drone with deviation in position (point P) is between two determined drones (points A, B) on the circumference.

Since when $N = 5$, AB is longest and the circle made with |AB| as the diameter has the largest range, set to ⊙1. It may be useful to use the circle to determine the size of α_3, which is the angle between the line connecting the point P with

A and B. From the knowledge of the circle, the angle ∠BCA = π/2 because the angle ∠BCA and angle ∠BPA (α_3) have the same arc. So if point P is outside of ⊙1 then $\alpha_3 < \pi/2$; if point P is inside of ⊙1 or on a circle then $\alpha_3 \geq \pi/2$.

a) When $\alpha_3 < \frac{\pi}{2}$, $\alpha_3 = \alpha_1 + \alpha_2$, the specific case when point P is not in the middle of point A and B is in **Figure 9(3)**. The sine theorem for ΔOPA, ΔOPB respectively has:

$$\begin{cases} \dfrac{\sin \alpha_1}{R} = \dfrac{\sin(\pi - \alpha_1 - \alpha)}{x} \\ \dfrac{\sin \alpha_2}{R} = \dfrac{\sin(\pi - \alpha_2 - \alpha + (k-1)\theta)}{x} \end{cases} \tag{6}$$

The solution is obtained as in (5).

b) When $\alpha_3 < \frac{\pi}{2}$, $\alpha_1 = \alpha_2 + \alpha_3$, the specific case when point P is not in the middle of point A and B is in **Figure 9(4)**. The sine theorem for ΔOPA, ΔOPB respectively has:

$$\begin{cases} \dfrac{\sin \alpha_1}{R} = \dfrac{\sin(\pi - \alpha_1 - \alpha)}{x} \\ \dfrac{\sin \alpha_2}{R} = \dfrac{\sin(\pi - \alpha + (k-1)\theta - \alpha_2)}{x} \end{cases} \tag{7}$$

The solution is obtained as in (5).

c) When $\alpha_3 \geq \frac{\pi}{2}$ the specific case when point P is not in the middle of point A and B and P is in the ⊙1 interior is in **Figure 9(5)**. The sine theorem for ΔOPA, ΔOPB respectively has:

$$\begin{cases} \dfrac{\sin \alpha_1}{R} = \dfrac{\sin(\pi - \alpha_1 - (2\pi - \alpha))}{x} = \dfrac{\sin(\pi + \alpha_1 - \alpha)}{x} \\ \dfrac{\sin \alpha_2}{R} = \dfrac{\sin(\pi - (2\pi - \alpha) - (k-1)\theta - \alpha_2)}{x} \end{cases} \tag{8}$$

Getting the solution:

$$\begin{cases} \alpha = \arctan\left(\dfrac{\sin(\alpha_1)\sin(\alpha_2) - \sin(\alpha_1)\sin(\alpha_2 + (k-1)\theta)}{\cos(\alpha_1)\sin(\alpha_2) - \sin(\alpha_1)\cos(\alpha_2 + (k-1)\theta)} \right) \\ x = \dfrac{\sin(\alpha_1 - \alpha_2)}{\sin \alpha_1} \end{cases} \tag{9}$$

3) When $5 < k' \leq 9$, by the symmetry of the circle, this case is similar to the first two cases. Let $k' = 11 - k$ ($2 \leq k < 6$), which is also known as ($1 < k \leq 5$). Then it is sufficient to replace the equation containing the *k*-variable in the first two cases by variable $k' = 11 - k$, *i.e.*, $(k-1)\theta$ is replaced by $(10 - k')\theta$.

Therefore, all possible scenarios have been discussed.

In summary, we can determine the position of the UAV by establishing a localization model and using the received direction information. In the case that we know the position of the transmitting signal UAV and there is no deviation [11], we can realize the localization by the angle between the passive receiving signal UAV and the transmitting signal UAV.

3. Problem 2: Determining the Number of Additional UAVs for Effective Localization

3.1. Formulation of the Problem

It is known that a UAV with a slightly deviated position receives signals emitted by UAVs numbered FY00 and FY01, as well as signals emitted by a number of other UAVs with unknown numbers [11]. With no deviation in the position of the UAVs transmitting signals, we need to determine how many other UAVs besides FY00 and FY01 need to transmit signals in order to achieve effective localization.

3.2. Analysis of the Problem

This problem is strictly a combinatorial problem that requires us to determine the minimum number of additional drones needed to achieve effective localization. First we use the inverse method to determine the highest criterion for effective localization. Second, from the fact that the three circles intersect to one more intersection point, we can roughly determine that at least two drones are required. Finally, using the exhaustive method, we determine the relative position of the drones to the origin for localization.

3.3. Modeling and Solving

Consider Fact 1: No matter how many additional signaling aircraft are added, it is impossible to determine uniquely the position of the receiving aircraft. This fact is demonstrated below:

(1) There are nine remaining points on the circumference of the circle, then removing the point that has been fixed. The remaining points are symmetrical about a straight line, as in **Figure 10**.

Obviously, the axis of symmetry l is the line between the points FY00 and FY01, and we number the remaining points as shown in **Figure 10(1)**.

(2) Then for any k point ($k \le 7$), there is no harm in setting this k coordinates of the point $\{(a_k, b_k)\}$. Then it is proved by the converse, *i.e.* If there exists a function f such that at k points, after k point operations, it is guaranteed that there exists a unique solution x to f, then it can be proved by hypothesis that for any other k points, they have no solution by the action of f.

But if you take all k points about the axis of symmetry l for symmetry treatment, then consider $\{(a'_k, b'_k)\}$ where $a'_k = a_k$, $b'_k = -b_k$. At this point, the k points have been acted upon by f. After the action, since P also has the symmetry, we can obtain P' where P and P' are symmetric about l. As in **Figure 10(2)**, if the target point is on the symmetry axis l, the position can be uniquely determined at this point, but it is meaningless because the symmetry results are the same.

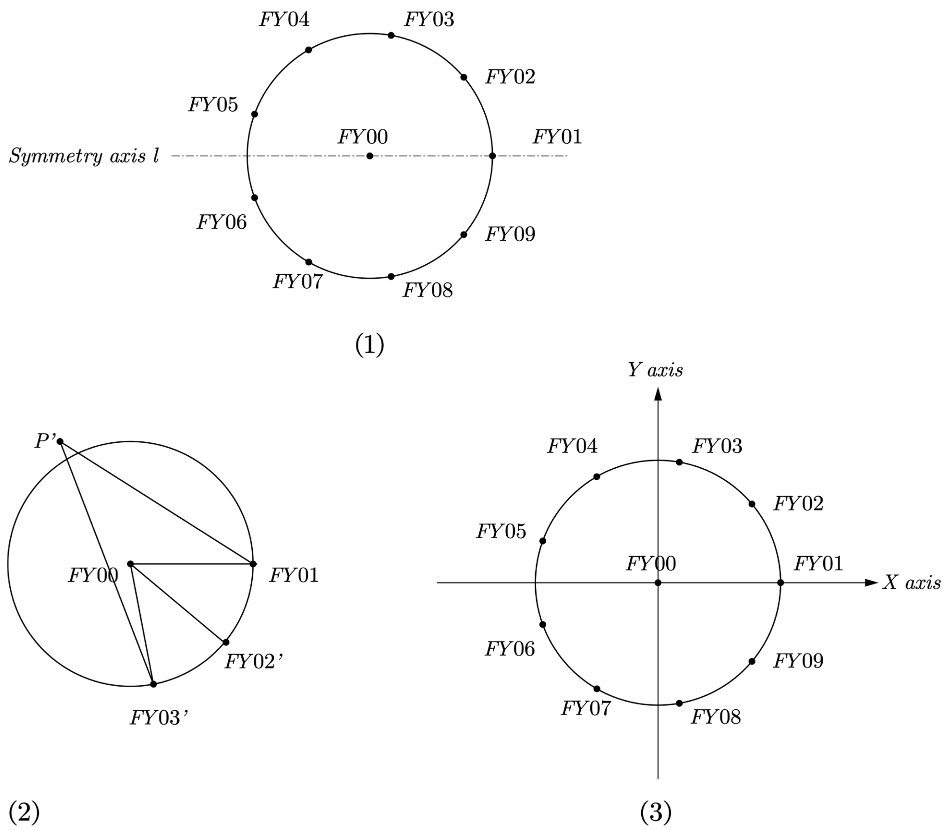


Figure 10. Symmetry processing diagram.

This contradicts *f*'s conclusion about the existence of uniqueness of the *k*-point action.

Thus, from (1) (2) *i.e.*, no matter how many signaling aircraft are selected, it is impossible to uniquely determine the unique coordinates of the receiving aircraft.

Based on Fact 1, we agree that a valid measurement in this question is defined as if the position of the target point relative to the origin can be determined from the known information, as shown by OP with cos∠PO1 where P remains the target point in (1), then it is said to be a valid measurement. The basis and reason for this definition can be based on the fact 1, as shown in **Figure 10(3)**. We can take FY00 as the origin and |FY00FY001| as the radius. Establish a plane rectangular coordinate system and agree that the vertical coordinates of FY02, FY03, FY04, FY05 are greater than zero. Set the coordinates of FY02, FY03, FY04, FY05 as (a_1,b_1), (a_2,b_2), (a_3,b_3) *and* (a_4,b_4), and the corresponding coordinates of FY06, FY07, FY08, FY09 as (a_5,b_5), (a_6,b_6), (a_7,b_7) *and* (a_8,b_8). Then, firstly, by FY00, FY01 both send signals and the position of the receiving aircraft is located above the X-axis so as to know that the set of all possible positions of the receiving aircraft, s_O is a section of circular arc as shown in **Figure 11(1)**.

The radius of this circular arc is $r_1 = \dfrac{r}{\sin \angle OP1}$, then consider the coordinates of the center of circle O in (1) are known and can be solved for unknown.

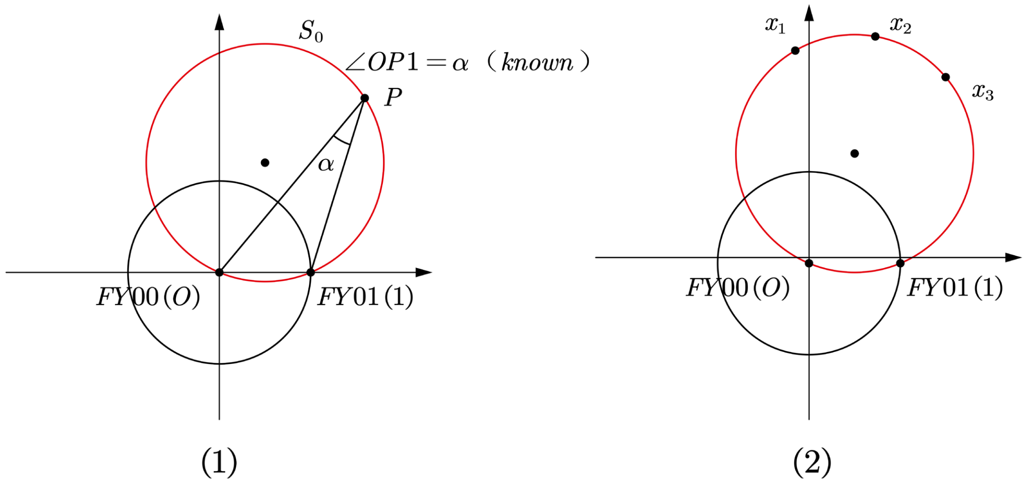


Figure 11. Receiver primary and secondary positioning (possible x-point position) schematic diagrams.

Now continue to abstract an airplane to a point and consider the additive point problem. By calculation and proof, we know that the minimum number of points to be added is 2, *i.e.*, at least 2 more drones are needed to transmit signals. Our ideas and proofs are given below:

(1) Calculation of the feasibility of the number of two planes: If two drones have been added, and we take one of them, there are at most eight cases of its position. Thus, from 2.1, any one of the planes is fixed to a position, and the direction and position of the localized plane can be uniquely determined. From this list, we remove the points with vertical coordinates less than 0 (up to 8 points). Finally, ∠OP3 is computed for all the filtered points. There are 6 possible locations for the last point, which results in a 6-element array for x_i, as shown in **Figure 11(2)**. Consider whether the 6 points have correspondence with the last angle data. If there is a correspondence, the corresponding x_i is the desired one, and it is sufficient to compute x_i with $\sin\angle x_i O1$; if there is not any correspondence, replace the serial number of the first point and continue the computation until the result is computed.

(2) Rationalization: If only one point is added. Let O and 1 be the points with known emitted signals and point *l* be the point with unknown emitted signal. Then by (1) and the relative relationship between *l* and 1, we can solve at least 4 possible solutions. But since no other information exists, we cannot solve for these possible solutions exactly, and thus cannot determine the range of impossible solutions (in general). So it is not possible to add just one airplane; and if two points are added. Let the two points added be 2 and 3, define the three-point system as follows: Based on the above proof, we can take any two points on the circumference (except 1), denoted as k_1, k_2, then we can get three angles $\alpha_1, \alpha_2, \alpha_3$ from the measurements, then we can define $(1, k_1, k_2, \alpha_1, \alpha_2, \alpha_3)$ as a three-point system. Denote a three-point system by the q_λ symbol, where $|\Lambda|$ = |{set of all 3-point systems}|, $\lambda \in \Lambda$. Let $f : \{q_\lambda\}_{\lambda\in\Lambda} \to R^2$, then $f(q_\lambda)$ are the coordinates of the positions of the points identified with the 3-point system denoted by q_λ.

Since any three circles intersect at most one common point, we know that f is a mapping such that a three-point system can determine points and at most one point. Further reasoning leads us to the conclusion that with 2 more points, we can uniquely determine the position of the drone at the unknown location. Therefore, this scheme is reasonable and the proof is complete. In summary, in addition to FY00 and FY01, we need 2 more drones to transmit signals.

In summary, if the positions of the UAVs transmitting signals are known to be without deviation, in addition to FY00 andFY01, two additional UAVs are required to transmit signals to achieve effective localization. By analyzing the received direction information, we can calculate the number of additional signal transmitting UAVs required, *i.e.*, 2, based on the distribution characteristics of the UAV swarms on the fixed circumference using the principle of geometry.

4. Problem 3: Position Adjustment for Uniform Distribution

4.1. Formulation of the Problem

It is known that the position of the UAVs at the initial moment is slightly deviated, and we need to adjust the position of the UAVs by several times so that the 9 UAVs are eventually evenly distributed on a certain circumference. For each adjustment, FY00 and up to 3 UAVs on the circumference are selected to transmit signals, and the positions of other UAVs are adjusted according to the received direction information [4].

4.2. Analysis of the Problem

This problem requires that when there is a deviation in the position of the drones at the initial moment, multiple adjustments are made according to the position information received by the drones [4], so that the final 9 drones are uniformly distributed in a certain circle. According to the conditions of the problem, the objective of Problem 3 is split into two:

i) Objective 1: 9 drones are distributed around a circle.

ii) Objective 2: All 9 UAVs were evenly distributed around the circle.

We can utilize the ideal approximation method to keep approximating the radius of the optimal solution and get a generalized algorithm that can be used for UAV position adjustment. At the same time, we also utilize the second method, *i.e.*, the target delimitation solution method, which is equivalent to the specific arithmetic example of the first method, to classify the UAVs into four categories, and set the final target as a circle with radius of 98 m, 105 m, and 112 m, respectively, and adjusted them many times through iterations to find a definite adjustment scheme so that the UAVs can be uniformly distributed on the target circumference in the end. The specific steps are shown in **Figure 12**.

Based on the radius of each UAV in **Table 1**, they are (except for the FY00) into the following four categories: the first category (FY01); the second category (FY02, FY05, FY07); the third category (FY03, FY06, FY09); category IV (FY04, FY07).

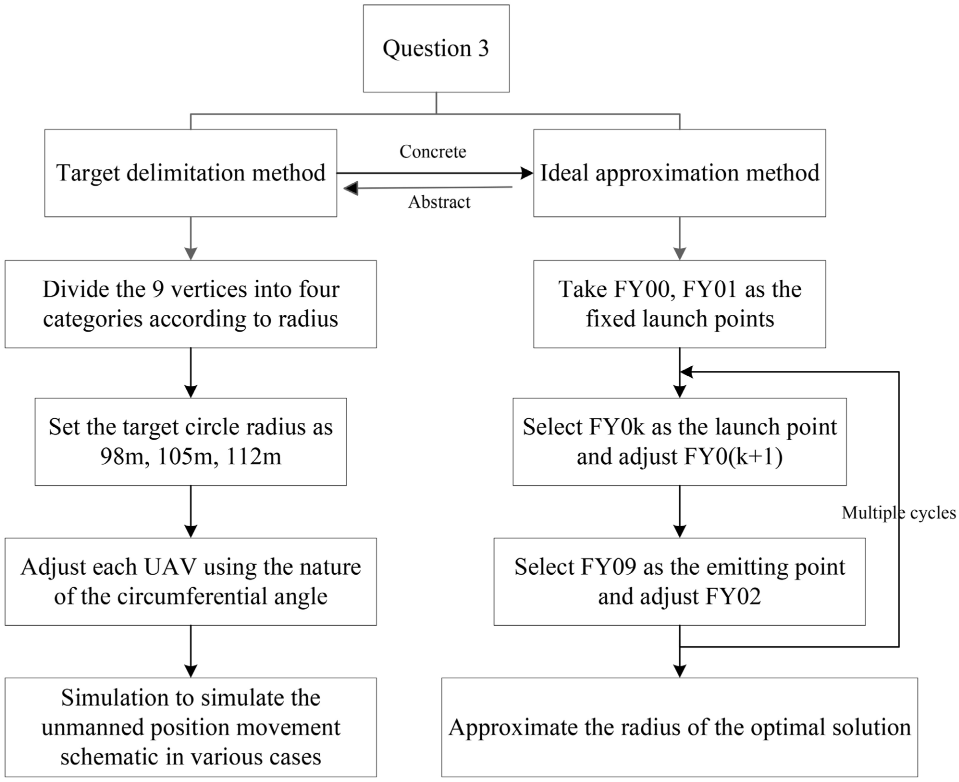


Figure 12. Problem 3 solution idea diagram.

4.3. Modeling and Solving

4.3.1. Targeted Delimitation Method

∠CEA and ∠COA are in the same arc. Since the size of the central angle of the same arc is twice the circumferential angle, we have ∠C0A = 2∠CEA. If the drones are uniformly distributed on the circle, then ∠ADC = 80*n*, ∠CEA = 40*n*. According to the meaning of α_3 as specified earlier, we get $\angle CEA = \alpha_3$, $\alpha_3 = 40n$, $n \in N^+$.

E is the desired location of point D. It may be stipulated that:

i) When $\angle ADC < \alpha_3$, the point D moves outward along the polar axis;

ii) When $\angle ADC = \alpha_3$, the point D does not move;

iii) When $\angle ADC > \alpha_3$, the point D moves inward along the polar axis;

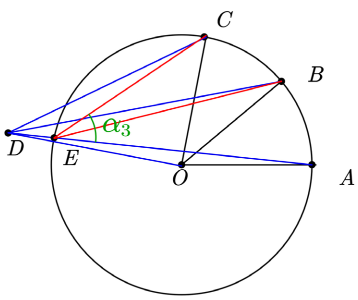
There are three possible scenarios for this question as follows:

1) A circle with radius 98 m and center (0, 0) can be determined from drones numbered 2, 5, and 8, and the positions of the remaining numbered drones can be adjusted to this circle.

(Objective 1): Satisfy that 9 drones are on the circle. Take point 4 as an example, connect 2-5, 2-4, 4-5, and intersect the circle O at point A. Since the coordinates of points 2, 4, and 5 are known, the exact value of ∠2A5 can be found, which can be seen from Figure 13, and ∠542 > ∠2A5. 4 should move inward in the direction of the polar path, gradually approaching the point A, and when 4 coincides with A, it arrives at the circumference of the circle O, as shown in Figure 13. The other 5 points can be moved in the same way.

(Objective 2): Satisfy the uniform distribution of the 9 UAVs on circle O. The UAVs will be distributed uniformly. Since the angles of the same arc are equal in size, points 2 and 5 are fixed, and point 4 can be moved arbitrarily on the circumference of the circle. When point 4 moves to $\angle 405 = 40^{\circ}$, the position of point 4 and point 5 meets the requirement of Objective 2. Fix these two points, and then follow the above steps to handle points 3, 2, 9, 8, 7, 6. Finally, all the points are uniformly distributed on the circle with (0, 0) as the center and radius r = 98 m. The following **Figure 14** is the point displacements obtained by the simulation method.

2) If the UAVs numbered 3, 6, and 9 determine a circle with a radius of 112 m and (0, 0) as the center. The positions of the remaining UAVs are then adjusted to that circumference. Take point 4 as an example. Since the coordinates of 3, 6, and 4 are known, ∠6A3 can be given a specific value. Point A is the ideal position of 4, as shown in **Figure 15(1)**, and ∠643 > ∠6A3. So point 4 should move



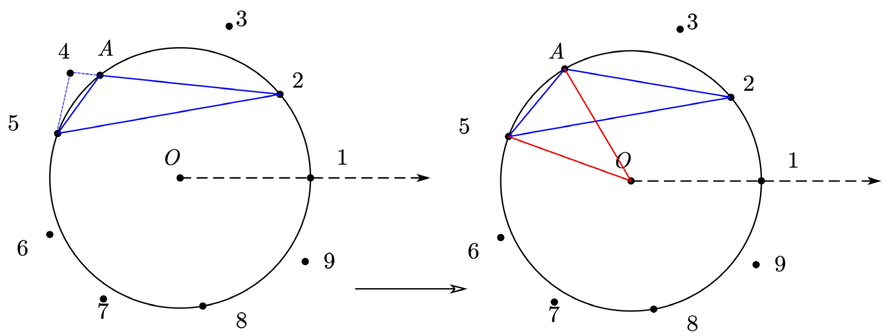


Figure 13. UAV position shift diagram.

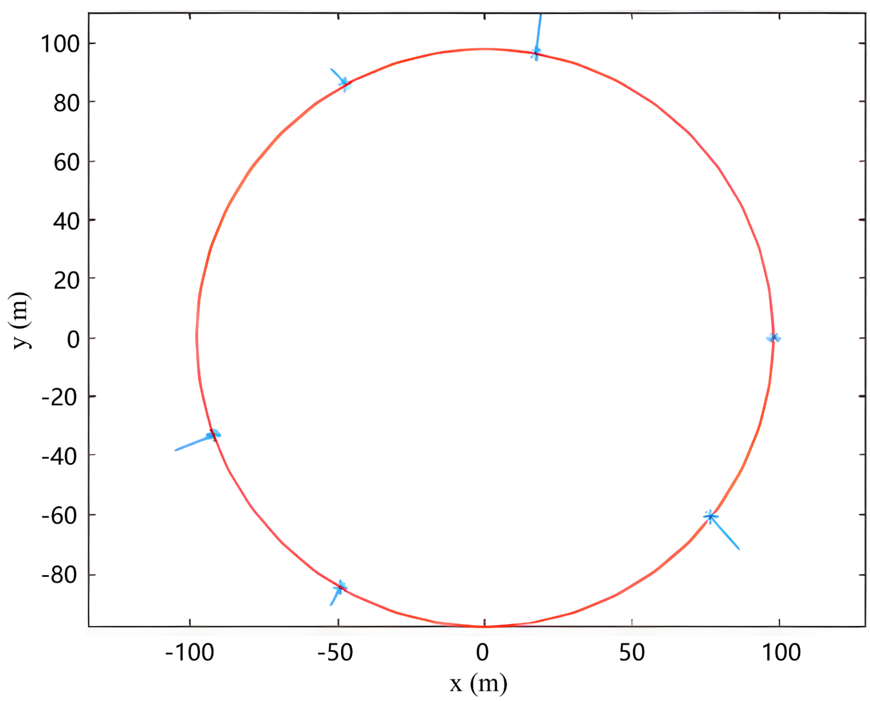


Figure 14. Point displacement simulation mockup (radius of 98 m).

outward along the polar path and approach point A. When 4 coincides with A, 4 is on the circumference of circle O, and the other 5 points can be moved in the same way. The next steps are the same as in Discussion 1, and finally we can realize that 9 UAVs are uniformly distributed on the circle with (0, 0) as the center and 112 m as the radius. The following **Figure 16** is a diagram of the point displacements obtained by the simulation method.

3) If the UAVs numbered 4 and 7 determine a circle with radius 105 m and center at (0, 0), then adjust the positions of the rest of the UAVs to the circle. Take point 5 as an example, since the coordinates of 4, 7, and 5 are all known, as in **Figure 15(2)**, the specific value of ∠7A4 can be obtained. Point A is the ideal location for 5, and ∠754 > ∠7A4, so point 5 should move outward in the polar

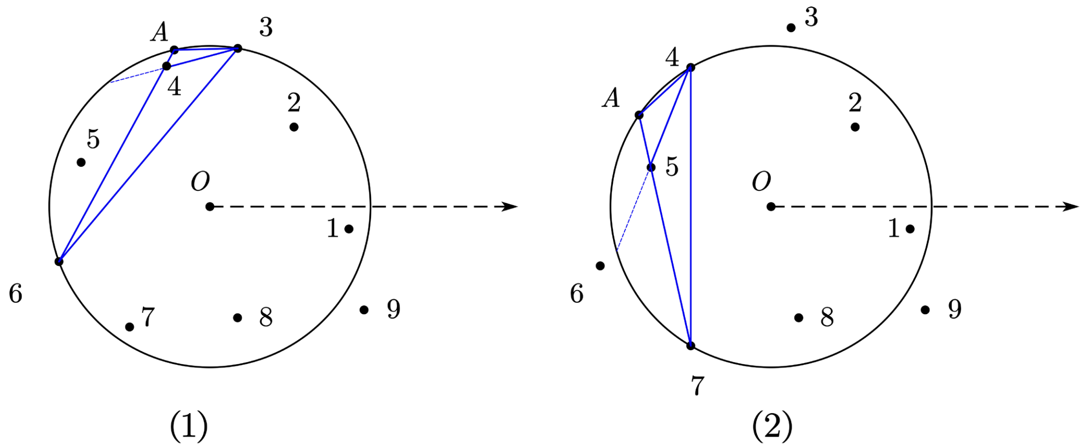


Figure 15. Point map.

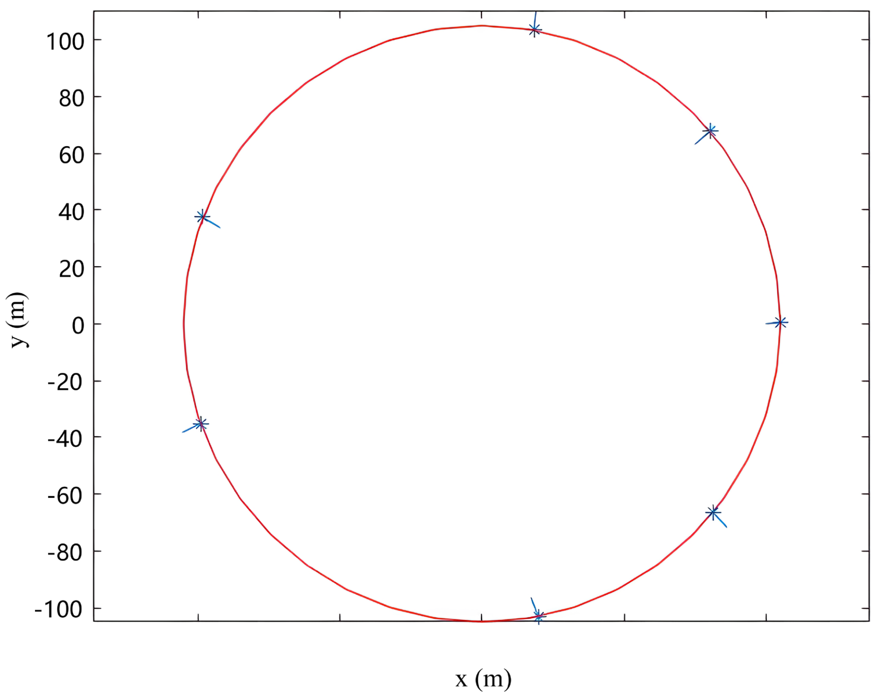


Figure 16. Point displacement simulation mockup (radius of 112 m).

direction, gradually approaching point A. When point 5 coincides with A, point 4 is on the circumference of circle O, and the other 6 points can be moved in the same way. The next steps are the same as in Discussion 1, and finally we can realize that 9 UAVs are uniformly distributed on the circle with (0, 0) as the center and 105 m as the radius. The following **Figure 17** is a diagram of the point displacements obtained by the simulation method.

4.3.2. Ideal Approximation

It is assumed that the UAV re-orientation can be adjusted to any orientation, *i.e.*, adjusting the UAV is possible to move the to-be-adjusted UAV to any of the FY orientations. Notice that no two of the nine points from YF01 to YF09 are on the same circumference and have relative angles that are an integral multiple of 40. Thus, it is not possible to select the appropriate transmitter in such a way that the orientation of the transmitter is ideal enough to allow the receiver to accurately compute its own orientation from the azimuthal angle information alone. For any UAV (no harm in setting it to be YF0k ($k \in \{1,2,3,4,5,6,7,8,9\}$, it has the following worst-case scenario: for any communication localization (transmitter transmits a signal, receiver receives a signal), it is impossible for that receiver to accurately compute its own bearing. Therefore, for any finite positive integer $\alpha \in (1,\infty)$, after α adjustments, it is impossible for us to make that UAV in the desired position.

Next, we consider the following error equation.

$$a_n = \sum_{i=1}^{9} \left| x_{i,real} - x_{i,dream} \right| \tag{10}$$

where $x_{i,real}$ is the actual position of the drone numbered YF0*i*, and $x_{i,dream}$ is

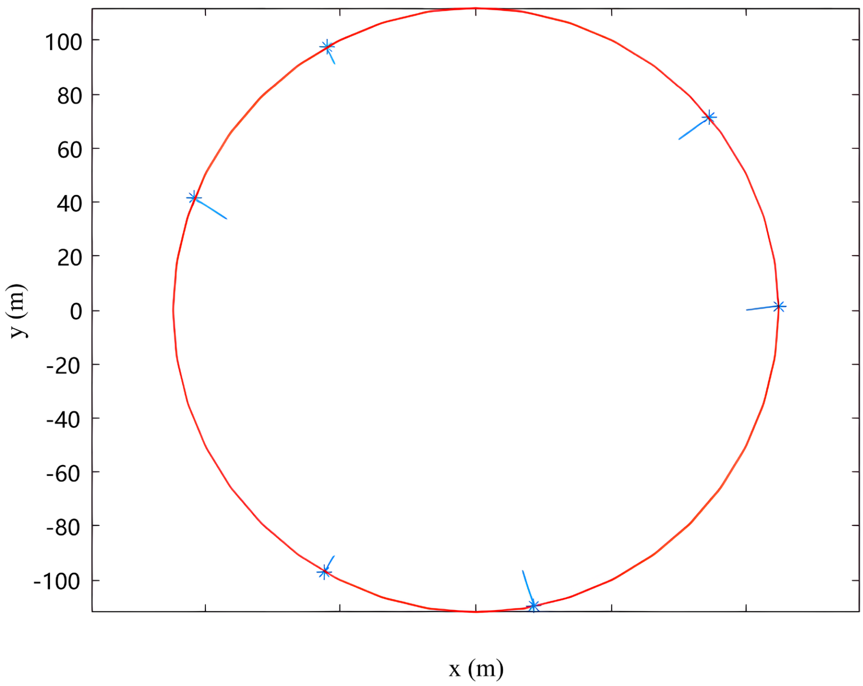


Figure 17. Point displacement simulation mockup (radius of 105 m).

the ideal position of the drone numbered YF0*i*, and *n* is the number of adjustments. By the previous note, we have the possibility of some special selection situation that makes there:

$$\forall n \in Z^{+}, a_n \neq 0$$

From there, there is at most:

$$0 = \lim_{n \to \infty} a_n \tag{11}$$

Therefore, we fix the two UAVs YF00, YF01 to transmit signals all the time, using YF0i ($k \in \{2,3,4,5,6,7,8,9\}$) to take turns transmitting signals. Each time when YF0i transmits a message, the information received by YF0(I + 1) is used to adjust the position of the aircraft, *i.e.*, to move to the standard distance based on the actual computed coordinates of the aircraft (with errors). In particular, if k = 9, the position of UAV YF02 is adjusted. After several adjustments, we can approximate the adjustment of all UAVs to the fixed circle. (FY00 and FY01 are fixed transmitter points. Assuming that, in some cases, FY02 is the third transmitter and FY03 is the target receiver, FY03 may have a localization error, *i.e.*, the UAVs will be localized at point FY03', since FY02 is not at the standard position. At this time, our adjustment strategy is to move the FY03' point to the FY03" point. (At this point in the real situation, the UAV will move in the same direction from point FY03, which may form an adjustment bias.)

When determining position, from Problem 1(1), we know that given two points (known positions), we can uniquely determine the target point coordinates. Based on this conclusion, we can calculate the coordinates of the target point (with deviation) and thus make multiple adjustments to minimize the error.

From the final result, we can see that after two adjustments, the error is reduced and is smaller than the original error (52) (only approximations are used here), and the polar angles are closer to a uniform distribution.

To summarize, I can conclude that when the position of the drone is slightly deviated at the initial moment, we can choose FY00 and up to 3 drones on the circumference to transmit signals and adjust the position of other drones according to the received direction information and adjust the position of the drones through many iterations to achieve the final uniform distribution.

5. Experimental Results and Analysis

In this study, we address the need to maintain electromagnetic silence in formation flight by using purely azimuthal passive localization to adjust the position of UAVs. Based on the requirements of Problem 1, Problem 2 and Problem 3, we conducted a series of experiments and produced the following results and analysis.

For Problem 1, in order to establish the localization model of the passive receiving signal UAV, we chose the FY00 UAV located in the center of the circle and the two transmitting signal UAVs numbered FY01 and FY02. We kept the position of the transmitting signal UAVs without deviation during the experiment and recorded the angles between the passive receiving signal UAVs and the lines connecting these two transmitting signal UAVs. By measuring multiple sets of angle data, we obtained the relationship between the pinch angle and the position of the passive receiving UAVs and established a localization model. Through calculation and verification, we found that the position of the passive receiving UAVs can be calculated according to Equations (2) to (9).

For Problem 2, we examined two signaling UAVs with numbers FY00 and FY01, and the existence of other signaling UAVs with unknown numbers. Through experiments and calculations, we explored the number of requirements of the transmitting signaling UAVs. According to our results, when the UAVs with slightly deviated positions receive the transmitting signals numbered FY00 and FY01, at least two more UAVs transmitting signals are required in order to achieve effective UAV localization.

To solve problem 3, we designed a reasonable UAV position adjustment scheme based on the formation requirements. We require 1 UAV to be located in the center of the circle, while the remaining 9 UAVs are uniformly distributed on the circumference of the circle with radii of 98 m, 105 m, and 112 m, respectively. According to the experimental data **Table 1**, we found that by adjusting the position several times, selecting the UAV numbered FY00 and at most 3 UAVs on the circumference to transmit signals each time, and the other UAVs adjusting their positions according to the received direction information, the uniform distribution of 9 UAVs can be finally realized [12].

In summary, our experimental results and analysis show that our proposed purely azimuthal passive localization method can achieve UAV position adjustment and can maintain formation alignment while maintaining electromagnetic silence.

6. Summary of the Findings

This study proposes a solution to the pure orientation passive localization problem in UAV formation flight. We can use the received orientation information to determine the position of the UAV by triangulation method and polar coordinate method to establish the localization model. In addition, we can utilize the exhaustive method to calculate the number of additional signal-transmitting UAVs needed, *i.e.*, 2, when the positions of the signal-transmitting UAVs are known to be free of deviation. Finally, we utilize the ideal approximation method and the target delimitation method, and through multiple adjustments, each time we select the UAV with the number FY00 and at most 3 UAVs on the circumference for launching, and the other UAVs adjust their positions according to the received direction information, we can obtain a reasonable UAV position adjustment scheme [12], which further makes the UAVs eventually evenly distribute on the target circumference. In summary, our study addresses the need to maintain electromagnetic silence during UAV formation flight and successfully adjusts the UAV positions by means of purely azimuthal passive localization. Our results have important practical application significance and prospects for improving the safety and efficiency of UAV formation flight and provide an effective reference and guidance for the application of pure azimuth passive localization methods in UAV formation flight.

7. Evaluation, Improvement and Extension of the Model

7.1. Advantages of the Model

1) Problems1 and 3 use the triangulation method, which is easy to implement and give specific location information.

2) Problem 1 uses both the sine and cosine theorems to simplify a complex geometric problem into a simpler computational problem, and the sines and cosines make up for the deficiencies of their respective curves, making the results more accurate.

3) Problem2 is solved using a combinatorial mathematical optimal solution problem, thus making the process more rigorous and credible.

4) The purely azimuthal passive localization method used in this paper reduces the need for UAVs to transmit electromagnetic wave signals, reduces the risk of electromagnetic interference, and improves the safety of formation flying.

7.2. Disadvantages of the Model

The model requires a priori knowledge, *i.e.* that the position of the transmitting signaling UAV is unbiased and that the numbering is known. In practical applications, these prerequisites may not be satisfied. In addition, the model only considers orientation information and ignores the utilization of other sensor data, limiting the positioning accuracy.

7.3. Modeling Improvements

1) Problem 3 was simulated only for circles with radii of 98 m, 105 m and 112 m, and more possible radii can be discussed in future research to achieve simulation of UAV position adjustment.

2) The introduction of data from other sensors, such as distance information, could be considered improving the accuracy of UAV localization.

3) It can be investigated how to localize the UAV with a slight deviation in its position and address the dependence on the requirement of no deviation in the position of the launching UAV.

7.4. Extension of the Model

The model can be applied to electromagnetic interference management in UAV formation flight to improve the stealth and safety of the formation. At the same time, we can generalize all the above models to a wider range of fields, such as spacecraft formations in space, battleship formation studies at sea, wireless communications, wireless sensor networks, etc. Other than that, this thesis only discusses the case of UAV clusters forming circular formations. However, in actual flight, UAV clusters can also be in other formations, such as conical formations, and we can discuss and design UAV position adjustment schemes for other formations by applying the models applied in this paper.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Pall, Y.F., Zhu, D.G., Gao, P., *et al.* (2023) Pure Azimuth Passive Localization in UAV Attempted Formation Flight. *Times Automobile*, No. 6, 25-28.

[2] Zhou, X.W. (2022) Pure Orientation-Based Passive Localization in UAV Attempted Formation Flight. *China New Technology and New Products*, No. 24, 34-37.

[3] Yang, Y.H., Yu, A.J., Qiao, C. (2023) Research on Pure Azimuthal Passive Localization Scheme in UAV Attempted Formation Flight. *Mathematical Modeling and Its Applications*, **12**, 60-68.

[4] Li, H.S., Cao, J.M., Zhang, Y.T., *et al.* (2023) A Pure Orientation Passive Adjustment Strategy for Unmanned Aerial Vehicles Based on Sliding Mean Method. *Digital Technology and Application*, **41**, 39-43.

[5] Gong, Y.-B. (2020) Research on Interference Source Localization Method for UAV Network. Beijing University of Posts and Telecommunications, Beijing.

[6] Shu, C. (2020) Exploration of Geometric Problems in Math Competitions. Tianjin Normal University, Tianjin.

[7] Liu, Z.L., Xu, C.J. and Yu, H. (2016) Research on Position Solving in Positioning System Based on Ranging. *Journal of Shanghai Normal University* (*Natural Science Edition*), **45**, 209-214.

[8] Dai, H.-J. (2007) Polar Coordinate Equations of Curves Example Problem Tracking. *New College Entrance Examination* (*Sophomore Edition*), No. Z1, 57-59.

[9] Li, D.X., Song, S.Y. and Liu, H.T. (2023) Trajectory Optimization Method for UAV Cascade Relay Broadcast Communication System. *Journal of Northwestern Polytechnical University*, **41**, 579-586. https://doi.org/10.1051/jnwpu/20234130579

[10] Cheng, Y.J., Liu, Z.T., Lv, S., *et al.* (2014) Research on Modeling and Simulation of Distributed Configuration Jamming Radar for Electronic Warfare UAV. *Ship Electronic Engineering*, **34**, 111-113+197.

[11] Yi, Y.F., Tan, W.T. and Ling, C.Z. (2023) Mathematical Modeling Analysis in UAV Positioning. *Integrated Circuit Applications*, **40**, 268-269.

[12] Nie, C.H., Zeng, H.L., Huang, Y.H., *et al.* (2023) Pure Azimuth Passive Localization in UAV Attempted Formation Flight. *Journal of Hunan Institute of Technology* (*Natural Science Edition*), **36**, 17-20.