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Analysis of the USD/JPY and EUR/JPY Exchange Rates Using Multifractal Analysis and Extreme Value Theory

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Abstract

We performed a multifractal analysis using wavelet transform to detect the changes in the fractality of the USD/JPY and EUR/JPY exchange rates, and predicted their extreme values using extreme value theory. After the 1997 Asian financial crisis, the USD/JPY and EUR/JPY became multifractal, then the USD/JPY became monofractal and stable, and yen depreciation was observed. However, the EUR/JPY became multifractal and unstable, and a strong yen depreciation was observed. The coherence between the USD/JPY and EUR/JPY was strong between 1995 and 2000. After the 2007-2008 financial crisis, the USD/JPY became monofractal and stable, and yen appreciation was observed. However, the EUR/JPY became multifractal and unstable, and strong yen appreciation was observed. Various diagnostic plots for assessing the accuracy of the GP model fitted to USD/JPY and EUR/JPY are shown, and all the diagnostic plots support the fitted GP model. The shape parameters of USD/JPY and EUR/JPY were close to zero, therefore the USD/JPY and EUR/JPY did not have finite upper limits. We predicted the maximum return level for the return periods of 10, 20, 50, 100, 350, and 500 years and their respective 95% confidence intervals (CI). As a result, the 10-year and 100-year return levels for USD/JPY were estimated to be 149.6 and 164.8, with 95% CI [143.2, 156.0] and [149.4, 180.1], respectively.

Keywords

Wavelet, Multifractal, Extreme Value Theory, GP, USD/JPY and EUR/JPY Exchange Rates

1. Introduction

Foreign exchange forecasting is important in all forms of foreign investments and transactions, and is a skill that complements the field of finance and related disciplines. It is widely recognized that financial market series exhibit non-linearity [1] [2]. In foreign exchange markets, this nonlinear variability exhibits fractal properties [3] [4]. Statistically, this corresponds to intermittency or inhomogeneity in a time series.

Self-similarity, alternatively known as the fractal property, exists in various objects in nature. Monofractality shows an approximately similar pattern at different scales and is characterized by a fractal dimension. Multifractality is non-uniform, complex, and decomposed into many subsets characterized by different fractal dimensions. Fractal properties can be observed in a time series that represents the dynamics of complex systems. A change in fractality is accompanied by a phase transition and changes in the state. It is important to clarify whether the exchange rate changes exhibit fractal properties.

Extreme value theory (EVT) has emerged as an important statistical discipline in applied science. Extreme value techniques are widely used in several disciplines. Examples include portfolio adjustments in the insurance industry, risk assessment in financial markets, and traffic prediction in telecommunications [5].

Statistical approaches focused on extreme values have shown promising results in unusual forecasting events in earth sciences, genetics, and finance. For instance, EVT was developed in the 1920s [5] and has been used to predict the occurrence of events, such as droughts and flooding [6] or financial crashes [7]. Additionally, extreme value modeling has been applied in the fields of ocean wave modeling [8], wind engineering [9], biomedical data processing [10], earthquake thermodynamics [11], and public health [12].

In this study, we performed a multifractal analysis using wavelet transform to detect the changes in the fractality of the USD/JPY and EUR/JPY exchange rates and predicted their extreme values using extreme value theory.

2. Data and Method of Analysis

2.1. Data

The monthly USD/JPY and EUR/JPY exchange rates for 1990-2022 [13] were used.

2.2. Wavelet-Based Multifractal Analysis

For the examination, we used the Daubechies wavelet, which is widely used to solve a broad range of problems, such as the self-similarity properties of a signal and signal discontinuities. We used a discrete signal fitted with the Daubechies mother wavelet with the capacity of the correct inverse transformation. Thus, we can precisely calculate the following best $\tau(q)$, which can be regarded as a characteristic function of the fractal behavior. We can define $\tau(q)$ from the power-law behavior of the partition function, as shown in Equation (3). We then computed the scaling of the partition function $Z_q(a)$, which is defined as the sum of the q-th powers of the modulus of the wavelet transform coefficients at scale

a, where q is the q-th moment. In our computation, the wavelet-transform coefficients do not grow to zero. Thus, for correct calculation, the summation was considered for the entire set. Muzy *et al.* [14] defined $Z_q(a)$ as the sum of the q-th powers of the local maxima of the modulus to avoid dividing by zero. We obtain the following partition function, $Z_q(a)$:

$$Z_q(a) = \sum \left| W_\varphi[f](a,b) \right|^q , \tag{1}$$

where $W_\varphi[f](a, b)$, a, and b are the wavelet coefficients of function f, scale parameter, and space parameter, respectively. $W_\varphi[f](a,b)$ is defined as below.

$$W_\varphi[f](a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} f(t)\, \varphi^*\left(\frac{t-b}{a}\right) \mathrm{d}t , \tag{2}$$

where $f(t)$ is the data and φ is the wavelet function. For small scales, we expect

$$Z_q(a) \sim a^{\tau(q)} . \tag{3}$$

First, we examined the changes in $Z_q(a)$ in the time series at different scales for each moment q. We plotted the logarithm of $Z_q(a)$ against that of the time scale a. Where $\tau(q)$ is the slope of the fitted straight line for each q. Next, we plotted $\tau(q)$ as a function of q. The time window was advanced by one year, which was repeated. The time window was fixed at six years, when a moderate change in fractality was observed. The monofractal and multifractal signals are defined as follows. For $\tau(q)$, a monofractal signal corresponds to a straight line, whereas a multifractal signal is nonlinear [15]. We calculated the R^2 value, which is the coefficient of determination of the fitted straight line. If $R^2 \geq 0.98$, the time series is monofractal, and if $0.98 > R^2$, it is multifractal.

We calculated $\tau(q)$ for moments $q = -6, -5, -4, -3, -2, -1, -0.5, 0.5, 1, 2, 3, 4, 5, 6$ for individual records for USD/JPY and EUR/JPY. We plotted the value of $\tau(-6)$ for each index. A large negative value of $\tau(-6)$ indicates strong multifractality. For $\tau(q)$, $q = -6$ is an appropriate number to show the change in τ.

2.3. Extreme Value Theory

2.3.1. Generalized Pareto (GP) Distributions

Modeling only block maxima is a wasteful approach to extreme value analysis if other data on extremes are available. In this technique, the data are collected over a specific threshold value. Modeling the extremes using this method enables a more efficient usage of extreme value information than that given by an analysis of annual maxima data, which excludes many extreme events that did not happen to be the largest annual event. In this study, the data were fitted to the GP distribution:

$$G(z) = 1 - \left[1 + \xi\left(\frac{z-u}{\sigma}\right)\right]^{-1/\xi}, \text{ for } \xi \neq 0,$$

$$G(z) = 1 - \exp\left(-\left(\frac{z-u}{\sigma}\right)\right), \text{ for } \xi = 0, \tag{4}$$

where z is the extreme value from the blocks, u is the known threshold, σ is the scale parameter, and ξ is the shape parameter. For $\xi > 0$ and 0, we do not obtain a finite upper limit, whereas for $\xi < 0$, we obtain a finite upper limit.

2.3.2. Return Levels

The level of return for the GP distribution is formed by the geometric locations of the points (m, x_m) for large values of m, where x_m is the return level estimated from the m-observation:

$$x_m = u + \frac{\sigma}{\xi}\left[\left(m\zeta_u\right)^{\xi} - 1\right], \text{ for } \xi \neq 0,$$

$$x_m = u + \sigma \log\left(m\zeta_u\right), \text{ for } \xi = 0, \tag{5}$$

where u is the selected threshold value, $\zeta_u = \Pr\left(x > u\right) = k/n$, k is the number of exceedances, and n is the number of observations.

Modeling was performed using the evd package in R for GP distribution calculations.

3. Results and Discussion

3.1. Wavelet-Based Multifractal Analysis

The monthly USD/JPY and EUR/JPY exchange rates are shown in **Figure 1**.

The slope of the approximately straight line was -2.727×10^{-2} and was decreasing.

In both cases, a peak was observed around 1998. The USD/JPY 2007 peak preceded the EUR/JPY 2008 peak. Yen depreciation for the EUR/JPY was stronger than that for the USD/JPY. In the 2010s, these changes were similar. The 1997 Asian financial crisis, which was a period of the financial crisis that gripped much of East and Southeast Asia during the late 1990s, occurred. The crisis began in Thailand in July 1997, and the recovery in 1998-1999 was rapid. The 2007-2008 financial crisis, which was a severe worldwide economic crisis that occurred in the early 21st-century occurred. This was the most serious financial crisis since the Great Depression (1929).

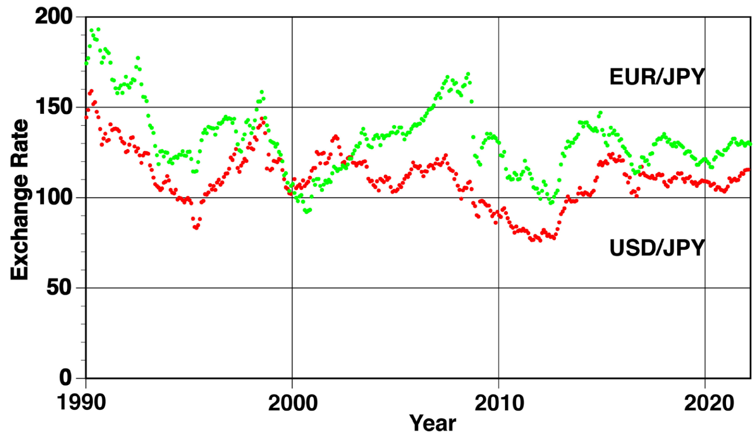

Figure 1. Plot of USD/JPY and EUR/JPY exchange rates.

The monthly USD/JPY vs. EUR/JPY and an approximately straight line are shown in Figure 2. The correlation coefficient was 0.62 with a p-value of < 0.01.

Figure 3 shows the wavelet power spectrum obtained using the Morlet wavelet of USD/JPY. There were an eight-year cycle for 2007-2008 and a four-year cycle for 1996-2002. Figure 4 shows the wavelet power spectrum of the EUR/JPY. There was a strong eight-year cycle from 2007 to 2008.

The $\tau(-6)$ values for USD/JPY and EUR/JPY are shown in Figure 5. The red square shows monofractality, and the green circle shows multifractality for the six years centered on the year plotted. For instance, the green circle for 2010 in USD/JPY shows the multifractality between 2007 and 2012. The data were excluded from Figure 5 for cases in which we could not distinguish between monofractality and multifractality. In 1997, both fractal properties were monofractal. After the 1997 Asian financial crisis, both became multifractal, and then the

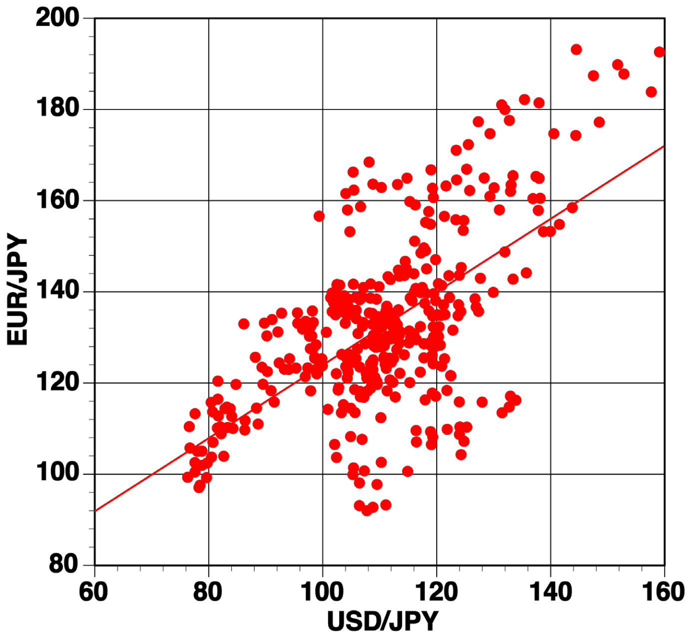

Figure 2. USD/JPY vs. EUR/JPY exchange rates and an approximately straight line.

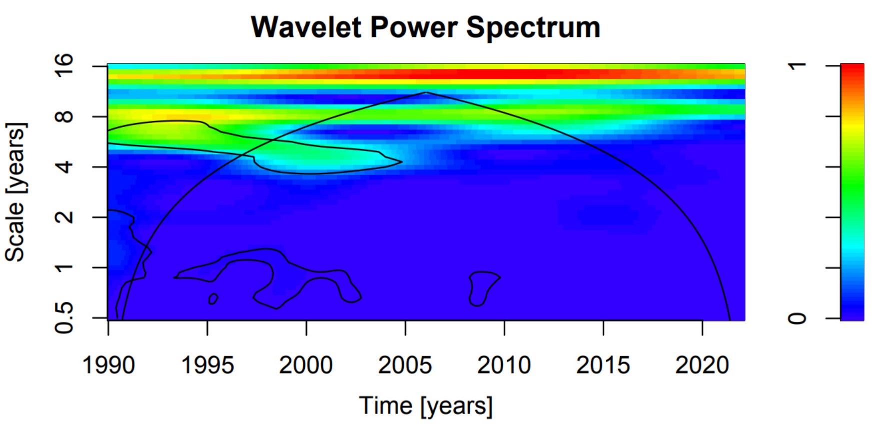

Figure 3. Wavelet power spectrum of USD/JPY exchange rate.

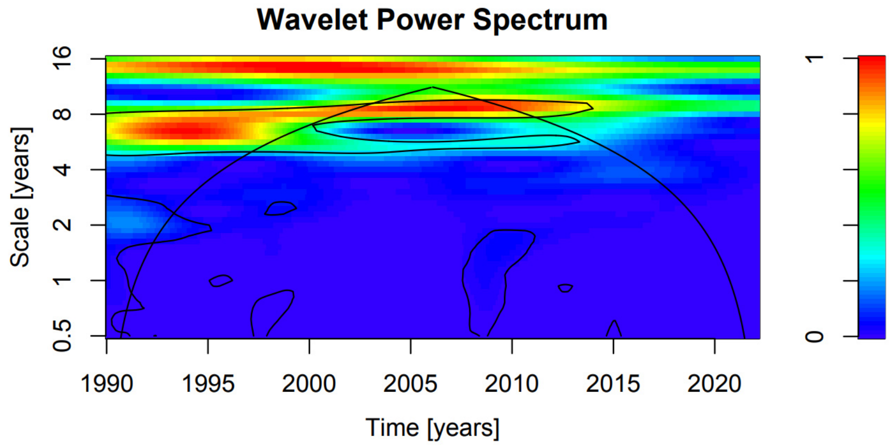

Figure 4. Wavelet power spectrum of EUR/JPY exchange rate.

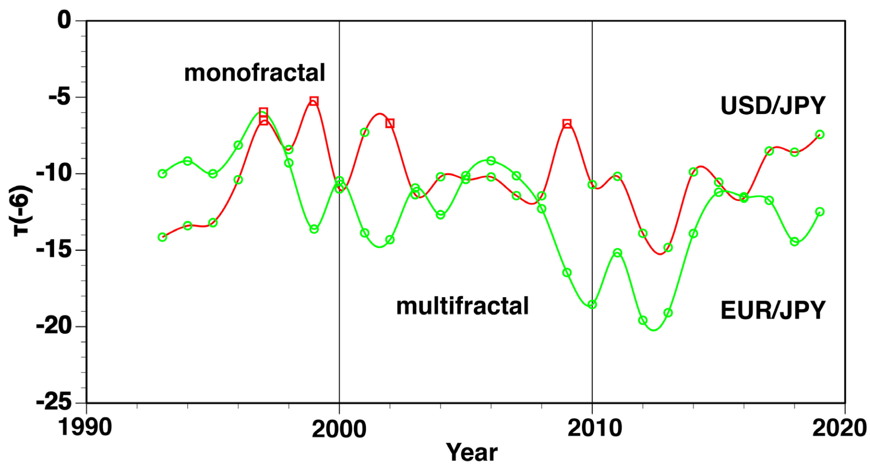

Figure 5. τ(−6) for USD/JPY and EUR/JPY exchange rates. The red square shows monofractality and the green circle shows multifractality for the six years centered on the year plotted.

USD/JPY became monofractal and stable, and yen appreciation was observed. However, EUR/JPY became multifractal and unstable, and strong yen appreciation was observed. For 2007-2008, both fractal properties were multifractal, after the 2007-2008 financial crisis, the USD/JPY became monofractal and stable, and yen appreciation was observed. However, EUR/JPY became multifractal and unstable, and strong yen appreciation was observed. After both crises, for USD/JPY, the yen appreciation was small, and a change from multifractal to monofractal was observed. For EUR/JPY, the yen appreciation was large, and multifractality became strong. The changes in the fractality of USD/JPY and EUR/JPY were similar in the 2010s, and the changes in the USD/JPY and EUR/JPY were related.

We applied the Morlet wavelet to show the wavelet coherence and phase between USD/JPY and EUR/JPY in Figure 6 (top and bottom, respectively). The coherence between USD/JPY and EUR/JPY was strong for 1995-2000, which was the 1997 Asian financial crisis period. As an overall trend, a lead in USD/JPY was observed.

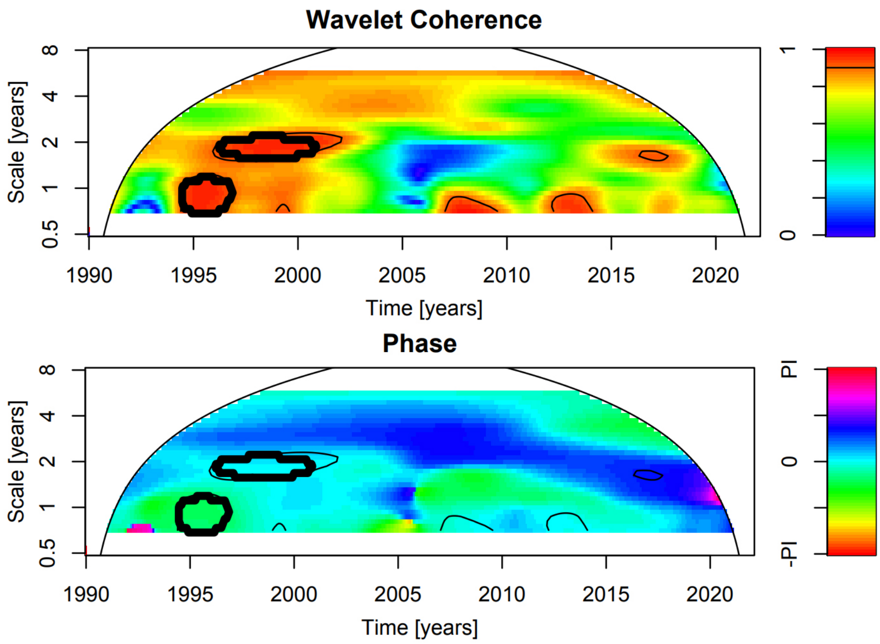

Figure 6. Wavelet coherence (top) and phase (bottom) between USD/JPY and EUR/JPY exchange rates. The thick black contour encloses regions of greater than 95% confidence. The thin black contour encloses regions of greater than 90% confidence. The cone of influence, which indicates the region affected by the edge effects, is shown by a black line. In the wavelet phase, the positive values shown by the blue and pink shading indicate that USD/JPY leads EUR/JPY and the negative values shown by the green, yellow, and red shading indicate that EUR/JPY leads USD/JPY.

3.2. Extreme Values of Exchange Rates

Table 1 shows the results of GP modeling for USD/JPY. The model has the scale parameter σ, and shape parameter ξ. ξ was close to zero (−0.1054) and included zero as the confidence interval. Therefore, the USD/JPY did not have a finite upper limit, and the probability of taking a large value was small.

Table 2 shows the predicted maximum return levels for the return periods of 10, 20, 50, 100, 350, and 500 years and their respective 95% confidence intervals (CI). The 10-year return level was estimated to be 149.6 with 95% CI [143.2, 156.0]. The 100-year return level was estimated to be 164.8, with 95% CI [149.4, 180.1]. Another way to interpret the plot is to say that there is an approximately 1% chance (1/100) each year that USD/JPY will exceed 164.8. There is an approximately 10% chance (1/10) each year that USD/JPY will exceed 149.6.

The various diagnostic plots for the GP distribution fitted to the USD/JPY exchange rates are shown in Figure 7. Straight lines and curves represent the estimated functions. Each point plot represents a realization value. The lines on both sides represent the 95% CI. The output provides little reason to doubt the validity of the GP model. Neither the probability plot nor the quantile plot doubts the validity of the fitted model. In the return level curve, the estimated curve is linear because ξ is close to zero. Finally, the corresponding density estimate is consistent with the data. Consequently, the diagnostic plots supported

Table 1. GP parameter estimates for USD/JPY exchange rate.

	σ	ξ
Parameter estimate	11.095	−0.1054
Standard errors	1.328	0.08534
95% CI	[8.491, 13.70]	[−0.2726, 0.06186]

Table 2. GP return-level estimates for USD/JPY exchange rate.

Return period (year)	10	20	50	100	350	500
Return level	149.6	154.6	160.6	164.8	171.8	173.6
Standard errors	3.265	4.421	6.278	7.839	11.13	12.12
95% CI	[143.2, 156.0]	[145.9, 163.2]	[148.3, 172.9]	[149.4, 180.1]	[149.9, 193.6]	[149.8, 197.4]

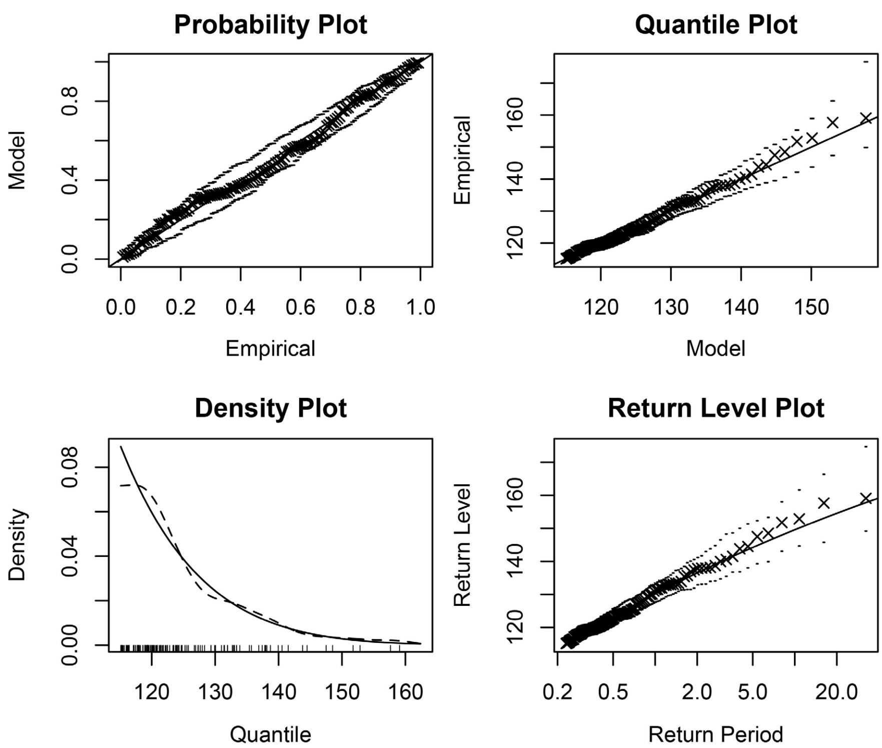

Figure 7. Diagnostic plots for the threshold excess model fitted to USD/JPY exchange rate.

the fitted GP model. As there were 140 exceedances of the threshold $u = 115$ in the complete set of 385 observations, the maximum likelihood estimate of the exceedance probability was 0.3636.

Table 3 shows the results of the GP modeling for EUR/JPY. ξ was close to zero (−0.06784) and included zero as the confidence interval. Therefore, the EUR/JPY did not have a finite upper limit, and the probability of taking a large value was small.

Table 4 shows the predicted maximum return levels for the return periods of 10, 20, 50, 100, 350, and 500 years and their respective 95% CI. The 10-year return

Table 3. GP parameter estimates for EUR/JPY exchange rate.

	σ	ξ
Parameter estimate	16.95	−0.06784
Standard errors	2.025	0.09644
95% CI	[12.98, 20.92]	[−0.2568, 0.1211]

Table 4. GP return-level estimates for EUR/JPY exchange rate.

Return period (year)	10	20	50	100	350	500
Return level	191.1	200.0	211.0	218.9	233.7	240.8
Standard errors	6.820	9.710	14.20	18.05	27.10	32.12
95% CI	[177.7, 204.4]	[180.9, 219.0]	[183.2, 238.9]	[183.5, 254.3]	[180.5, 286.8]	[177.8, 303.7]

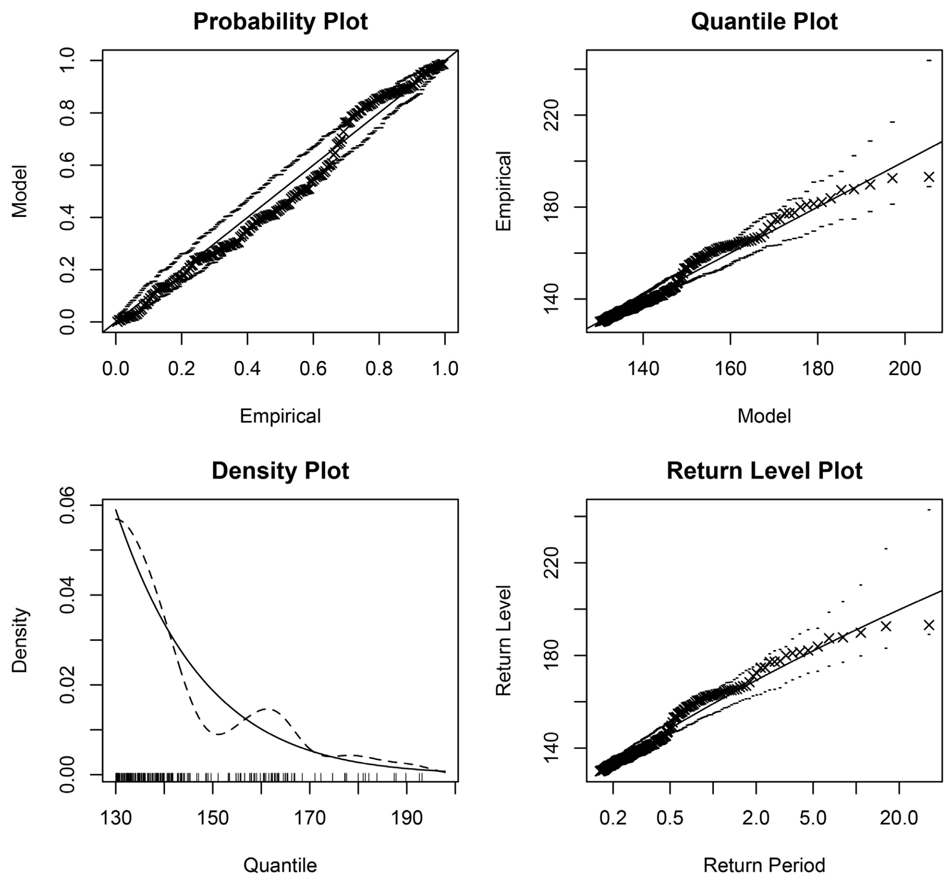

Figure 8. Diagnostic plots for the threshold excess model fitted to EUR/JPY exchange rate.

level was estimated to be 191.1, with 95% CI [177.7, 204.4]. The 100-year return level was estimated to be 218.9, with 95% CI [183.5, 254.3].

The various diagnostic plots for the GP distribution fitted to the EUR/JPY exchange rates are shown in Figure 8. The output gives little reason to doubt the validity of the GP model. Neither the probability plot nor the quantile plot doubts the validity of the fitted model. In the return level curve, the estimated

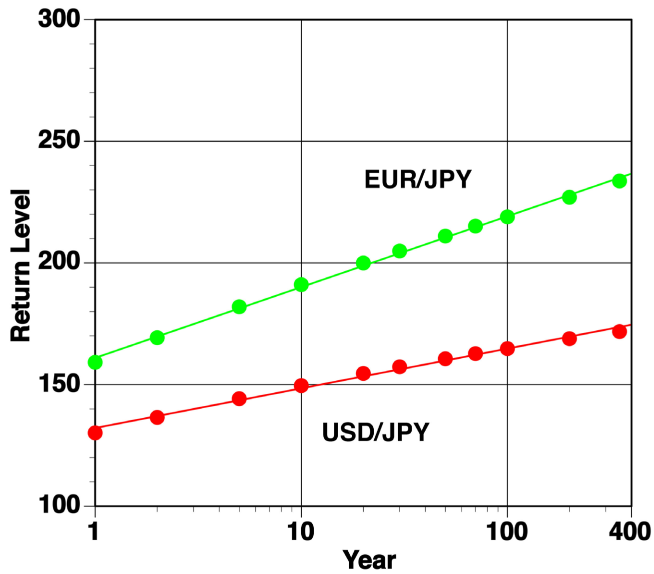

Figure 9. Return-level plot for USD/JPY and EUR/JPY exchange rates.

curve is linear because ξ is close to zero. Finally, the corresponding density estimate is consistent with the data. Consequently, all the diagnostic plots supported the fitted GP model. As there were 200 exceedances of the threshold u = 130 in the complete set of 385 observations, the maximum likelihood estimate of the exceedance probability was 0.5181.

The return levels for USD/JPY and EUR/JPD for each return period are shown in Figure 9. The 10-year return level for USD/JPY was 149.6, which was close to the value of 146.02 on August 31, 2023. In contrast, the EUR/JPY on August 31, 2023 was 159.45, which was smaller than the 10-year return level for EUR/JPY, 191.1.

4. Conclusions

We performed a multifractal analysis using wavelet transform to detect the changes in the fractality of the USD/JPY and EUR/JPY exchange rates, and predicted their extreme values using extreme value theory. The main findings are summarized as follows.

1) After the 1997 Asian financial crisis, the USD/JPY and EUR/JPY became multifractal, then the USD/JPY became monofractal and stable, and yen depreciation was observed. However, the EUR/JPY became multifractal and unstable, and strong yen depreciation was observed. The coherence between the USD/JPY and EUR/JPY was strong from 1995 to 2000.

2) After the 2007-2008 financial crisis, the USD/JPY became monofractal and stable, and yen appreciation was observed. However, the EUR/JPY became multifractal and unstable, and strong yen appreciation was observed.

3) Various diagnostic plots for assessing the accuracy of the GP model fitted to USD/JPY and EUR/JPY are shown, and all the diagnostic plots support the fitted GP model. The shape parameter of USD/JPY and EUR/JPY was close to

zero, therefore the USD/JPY and EUR/JPY did not have a finite upper limit.

4) We predicted the maximum return level for the return periods of 10, 20, 50, 100, 350, and 500 years and their respective 95% confidence intervals (CI). As a result, the 10-year and 100-year return levels for USD/JPY were estimated to be 149.6 and 164.8, with 95% CI [143.2, 156.0] and [149.4, 180.1], respectively.

5) The 10-year return level for USD/JPY was 149.6, which was close to the value of 146.02 on August 31, 2023.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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