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Fuzzy-Bayes Decision Making with Reserved Judgement

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Abstract

This paper discusses a method for identifying states in a multistage Decision Making Problem in which an Indifferent Event is either predetermined or can be automatically derived after the fact. First, when they are pre-set, the amount of possible information about Indifferent Event tends to be large. Therefore, since the decision is risk tolerant, the Max-Product method of Tanaka *et al.* is used to calculate the expected utility possibility. Next, in the case of automatic derivation after the fact, the amount of information on the possibility of Indifferent Event is relatively small, so the expected utility possibility is derived using Zadeh's Fuzzy Event Possibility Measure. Here, it is assumed that the setting of the utility function is independent of the information on the occurrence of the Indifferent Event and is identified by the decision maker by lot drawing using the certainty equivalence method. As a concrete example, we focus on the pass/fail decision of a recommendation test, which is a two choice question in the No-Data Problem, and illustrate the multistage state identification method.

Keywords

Fuzzy Event, Reserved Judgment, Indifferent Event, Expected Utility, Max-Product

1. Introduction

Fuzzy events were defined by Zadeh [1]. Subsequently, Okuda *et al.* formulated a Decision Making Problem in an ambiguous environment [2] [3]. Here, this fuzzy environment is called Fuzzy Event. On the other hand, the Decision Making Problem proposed by Hori *et al.* [4] [5] is another Decision Making in a fuzzy event, and this fuzzy event is called a Vague Event to distinguish it from a Fuzzy Event. Note that Fuzzy Event deals with the possibility of an ambiguous environment in a natural state, while Vague Event is defined by the degree of attribution of an ambiguous event in a natural state. As for the state identification problem, since we deal with state possibilities, this paper will focus on the Decision Making Problem in Fuzzy Event, following Zadeh and Okuda *et al.* [1] [2]. Furthermore, we also refer to the Decision Making Problem with Indifferent Events proposed by Uemura [6].

In this paper, the discussion proceeds under the assumption that prior information on the occurrence of a state is given. For example, if, in the prior information on the occurrence of this state, Fuzzy events that are not directly summed are set in advance, the posterior possibility distribution of Indifferent Event is automatically derived by subtracting the sum of their possibility distributions from 1. Conversely, if Indifferent Events are predetermined, two Fuzzy events can be automatically specified orthogonal to the prior possibility distribution of the Indifferent Event. In this paper, we focus on these differences between the prior and posterior information of the occurrence and propose a state identification method in two different fuzzy environments.

2. Possibility Distribution of Indifferent Event

Let (S,K,Π) denote the usual possibility space. S is a state of nature,、K is a fully additive family consisting of subsets of S and Π is a possibility measure. A Fuzzy Set characterized by a measurable possibility distribution $\Pi_F(S): S \to (0,1)$ on S is called a Fuzzy Event. Note that the probability of fuzzy events defined by Zadeh [1] requires the orthogonality condition of Fuzzy Event, *i.e.*, the orthogonality condition of the possibility distribution of Fuzzy Event.

2.1. Fuzzy Events that Are Not Orthogonal Sums Are Pre-Set

Suppose that the possibility distribution $\Pi_{FK}(K=1,\cdots,n)$ of two or more non-orthogonal Fuzzy Events is pre-set by the decision maker

In this section, we consider the case where $\sum_{k=1}^{n}\Pi_{Fk}(S)\le 1 \quad \forall_S \in S$. Here, we introduce the concept of Indifferent Event F_e in order to avoid the risk of decision-making arising from the lack of information in Fuzzy Events. The possibility distribution of this Indifferent Event can be automatically derived by the following equation.

$$\Pi_{Fe}(S) = 1-\sum_k \Pi_{Fk}(S) \tag{1}$$

To briefly illustrate the Decision Making Problem in an ambiguous environment, we take the example of recommended admission test having two choice question in the No-Data Problem. The state of nature is assumed to be from 0 to 100 points in the internal score. Assume that F_1 = {good internal score} and F_2 = {bad internal score} as Fuzzy Event are set by the decision maker as shown in **Figure 1**. Here, the Indifferent Event F_e in **Figure 1** is F_e = {not knowing if the internal score is bad}, and the possibility distribution of this Indifferent Event is automatically derived.

The Indifferent Event F_e is divided into zones of the state of nature to make sense of it. In Zone X = {0 ≤ s < 20}, it is completely F_2, In Zone a = {20 ≤ s < 45}, it is a conditional Indifferent Event known to be a fuzzy event F_2. In Zone b = {45 ≤ s < 70}, it is an Indifferent Event that is neither fuzzy event F_1 nor fuzzy event F_2. However, the relationship between the magnitude of fuzzy event F_1. and fuzzy event F_2 is known. Zone c = {70 ≤ s < 80} is a conditional Indifferent Event that is known to be fuzzy event F_1. Zone Y = {80 ≤ s ≤ 100} is completely F_1. Here, each zone has different characteristics, so it is necessary to analyze each zone individually. However, decomposing and recomposing the system is very risky. In this decision problem, F_1, F_2 and F_e are orthogonal sum events, so there is no need to decompose and recompose the system.

Also, since this is a two choice question, we take D_1 = {pass} and D_2 = {fail} as decision D. Here, the adopter wants to make a risk-neutral decision. In addition, D_3 ={decision withholding} is added because of the risk of lack of information in Fuzzy Event due to the introduction of Indifferent Event. For example, a concrete example of decision withholding is when an interview is conducted after the recommendation test. Here, the utility function of decision withholding D_3 is automatically derived as a utility function orthogonal to the utility functions of passing D_1 and failing D_2. Assume that the utility function $\cup(S \mid D)$ is set up as in **Figure 2**. The risk-neutral utility function is a linear function and is identified by the certainty equivalent method by drawing lots [7].

2.2. The Case Where an Indifferent Event Is Pre-Set

This case is a Decision Making Problem in which the possibility distribution of

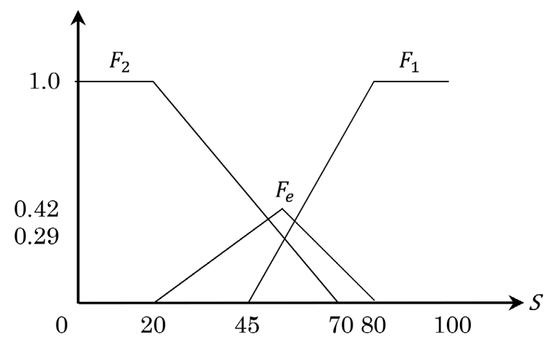

Figure 1. Indifferent Event (No.1).

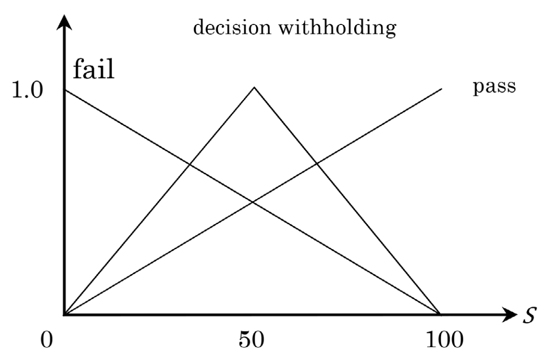

Figure 2. Utility function (risk-neutral).

the Indifferent Event F_e is set by the decision maker as a triangular type possibility distribution using the three-point estimation method. The posterior Fuzzy Events are automatically derived to be in orthogonal sum with the Indifferent Events, and their possibility distribution is obtained as a possibility distribution orthogonal to the possibility distribution of the Indifferent Events. In the example of the recommended admission test in the previous section, the possibility distributions F_1 = {good} and F_2 = {bad} are automatically derived as shown in **Figure 3**. Note that the utility function is the same value as in **Figure 2** shown in section 2-1, since it is unrelated to the information on the occurrence of Fuzzy Events.

2.3. Different Usage of Possibility Measure for Fuzzy Events

Uemura paid attention to the Decision Making Problem Event in section 2-1 and showed how to use different measures for the possibility measure. When the information content of the Fuzzy Event is small, probability of a Fuzzy Event is used. On the other hand, when the information content is large the possibility measure of a Fuzzy Event is used [8] [9]. However, since the state identification problem addressed in this paper addresses the possibility of Fuzzy Event, we will address the possibility measure in Zadeh's Fuzzy Event. Next, in the Decision Making Problem in Section 2-2, the Indifferent Event is known and its information content is very large (see **Figure 1** and **Figure 3** for the height at the representative value of the Indifferent Event). Since there is a very high risk of making a wrong decision, we adopt the Max-Product method, which is suitable for the risk-tolerant problem proposed by Tanaka *et al.* [10].

3. Fuzzy Events with Indifferent Event

First, in the Decision Making Problem in section 2-1, the expected possibility measure, $E(D)$ for each action D_j is obtained by the operation (2) of the possibility distribution $\Pi_i(S)$ for event F_i and the utility function $\cup(S \mid D)$ for action D_j. The action with the largest expected possibility measure is the optimal action.

$$E_1(D_j) = \sum_i \max_S \min(\Pi_{Fi}(S), U_{Dj}(S)) \tag{2}$$

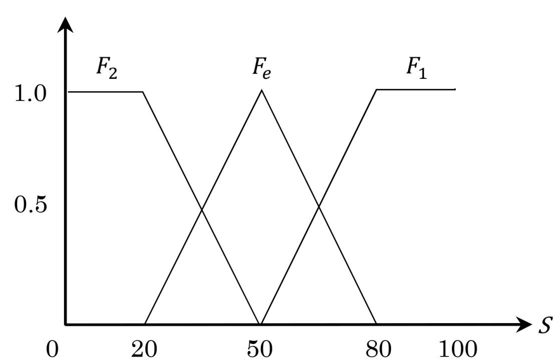

Figure 3. Indifferent events (No.2).

Next, in the Decision Making Problem in section 2-2, the expected possibility measure of each action D_j is obtained by the arithmetic equation (3), and the action with the largest expected possibility measure is the optimal action.

$$E_2(D_j) = \sum_i \max_S \Pi_{Fi}(S) \cdot U_{Dj}(S) \tag{3}$$

4. Multi-Step State Identification Method

D_3 When the pending decision is the optimal action, the decision is a multistage decision. To simplify the problem, we will use the example of the recommendation test in the previous section. The Decision Making Problem in section 2-1 and the Decision Making Problem in section 2-2, only the operation to derive the expected possibility measure is different. Note that the method for deriving the possibility information measure shown in Equation (4) is the same.

$$H_i = \max_S \Pi_{Fi}(S) \cdot \log \Pi_{Fi}(S) \tag{4}$$

Using this possibility-expectation possibility measure weighted by the possibility information content, we propose a decision-making rule as in Equations (5)-(7).

(Step 1) One-step identification method

When

$$H_1E(D_1) + H_2E(D_2) \geqq H_3E(D_3) \tag{5}$$

$$E(D_1) \geqq E(D_2) \rightarrow D^* = D_1$$

$$E(D_1) < E(D_2) \rightarrow D^* = D_2$$

End of identification

When

$$H_1E(D_1) + H_2E(D_2) < H_3E(D_3) \tag{6}$$

$$D^* = D_3$$

Go to Step 2.

(Step 2) Two-step identification method (pending decision)

$$H_1E(D_1) + H_2E(D_2) < H_3E(D_3) \tag{7}$$

When

1) $H_3 = \max_i H_i (i = 1,2,3)$

$$H_1' \triangleq H_3 - H_1$$

$$H_2' \triangleq H_3 - H_2$$

$$H_3' \triangleq \max(H_1', H_2')$$

Change from H_i to H_i' and go to step 1

2) Others

When $H_1E(D_1) \geqq H_2E(D_2)$, $D^* = D_1$

When $H_1E(D_1) < H_2E(D_2)$, $D^* = D_2$

End of identification.

5. Conclusions

In this paper, we proposed two state discrimination methods for two Fuzzy Events, dividing the cases into those in which Indifferent Event are pre-set and those in which they are automatically derived after the fact. First, the Max-Product method is adopted for the case where an Indifferent Event is pre-set, since it increases the amount of information on the possibility of an Indifferent Event, while Zadeh's Fuzzy Event possibility measure is adopted for the case where an Indifferent Event is automatically derived after the fact.

The Decision Making Problem in an Fuzzy Event derives the amount of possible information for each Fuzzy Event, weights and sums it with expected utility possibility as weights, and designates the action with the largest maximum as the optimal action according to their size relationship. Furthermore, for Decision Making Problem in which the first-stage optimal action is pending judgment, the two-stage possibility state identification method is mentioned. As a future issue, when an Indifferent Event is pre-set, the second-stage Indifferent Event with reduced possibility information may be indistinguishable from the first-stage Indifferent Event set after the fact. Therefore, we plan to focus on the change from before to after and model this transition with a possibility Markov decision process in Fuzzy Event.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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