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Determining the Charge-to-Mass Ratio of the Electron

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Abstract

The aim of this lab was to determine an experimental value for the charge-to-mass ratio e/m_e of the electron. In order to do this, an assembly consisting of Helmholtz coils and a helium-filled fine beam tube containing an electron gun was used. Electrons were accelerated from rest by the electron gun at a voltage of 201.3 V kept constant across trials. When the accelerated electrons collided with the helium atoms in the fine beam tube, the helium atoms entered an excited state and released energy as light. Since the Helmholtz coils put the electrons into centripetal motion, this resulted in a circular beam of light, the radius of which was measured by taking a picture and using photo analysis. This procedure was used to test currents through the Helmholtz coils ranging from 1.3 A to 1.7 A in increments of 0.1 A. Using a linearization of these data, the experimental value for the charge-to-mass ratio of the electron was found to be 1.850×10^{11} C/kg, bounded between 1.440×10^{11} C/kg and 2.465×10^{11} C/kg. This range of values includes the accepted value of 1.759×10^{11} C/kg, and yields a percent error of 5.17%. The rather low percent error is a testament to the accuracy of this procedure. During this experiment, the orientation of the ambient magnetic field due to the Earth at the center of the apparatus was not considered. In the future, it would be worthwhile to repeat this procedure, taking care to position the Helmholtz coils in such a way to negate the effects of the Earth's magnetic field on the centripetal motion of electrons.

Keywords

Helmholtz Coils, Charge-to-Mass Ratio, Electron, Magnetic Field

1. Introduction

The charge-to-mass ratio of the electron is an important constant due to its ability to measure mass effects using charge effects. Since electrons have very small mass, it requires extraordinary precision to measure the mass effects of an electron, but with a precise value of the charge-to-mass ratio, much more powerful and easily measured charge effects can be used to determine mass effects. J.J. Thompson was the first to determine an experimental value for the charge-to-mass ratio of the electron in 1897. Thompson's experiment used a similar method in which he measured the deflection of a cathode ray due to a magnetic field [1] [2]. The accepted value of the charge-to-mass ratio in the literature is 1.759×10^{11} C/kg [3].

In this experiment, an assembly consisting of Helmholtz coils and a fine beam tube filled with helium was used. An electron gun in the fine beam tube accelerated electrons from rest and the nearly uniform magnetic field produced by the Helmholtz coils put the electrons into centripetal motion. When the electrons collided with helium atoms in the fine beam tube, the helium atoms became excited and released blue light. The radius of the centripetal motion was measured twice for current values ranging from 1.3 A to 1.7 A in increments of 0.1 A. Using the experimental relationship between current and radius (see Appendix A.3), a linearized relationship between the electron path radius and the inverse of the current through the Helmholtz coils was determined, involving the charge to mass ratio in its slope. A linearized plot was then created, the slope of which was set equal to the theoretical slope, and an experimental value for the charge-to-mass ratio of the electron was determined (see Appendix A.4).

2. Methods

An apparatus consisting of a helium-filled fine beam tube with an electron gun placed at the center of Helmholtz Coils was used to perform this experiment. A vertical ruler was placed beside the bulb of the fine beam tube in order to measure the radius of the electron path while minimizing the effects of parallax, and a hood was placed around the coils to ensure the light of the electron beam was visible. The electron gun was then set to an accelerating voltage of 201.3 V which was kept constant across trials, and the Helmholtz coils were set to a current of 1.3 A. A picture of the electron beam was then taken, and, with the ruler as a scale, LoggerPro photo analysis was used to find the radius of the circular beam (see **Figure 1**). This process was performed for currents of 1.3 A, 1.4 A, 1.5 A, 1.6 A, and 1.7 A. Once all currents were tested, the process was repeated, starting again at 1.3 A, for a total of two trials per current at five currents. The average radius for each current was also recorded in order to reduce random error.

3. Results

A plot of average path radius vs. current (see **Figure 2**) and a linearized plot of average path radius vs. the inverse of current (see **Figure 3**) were created using the data in **Table 1** (for raw experimental data see **Table A1** in Appendix A.1). Using the slope of the linearization (see **Figure 3**), 5.986×10^{-2} A·m, it is found that $e/m_e = 1.850 \times 10^{11}\ \text{C/kg}$ (See Appendix A.4). The slope of the trendline was found to have an uncertainty of 7.995×10^{-3} A·m, which means that the experimental charge-to-mass ratio is bounded between 1.440×10^{11} C/kg and 2.465 $\times 10^{11}$ C/kg, which includes the accepted value of 1.759×10^{11} C/kg. The experimental value yields a percent error of 5.17% when compared to the accepted value (see Appendix A.5).

Table 1. The relationship between the current through the Helmholtz coils and the average electron path radius (and quantities plotted in linearization).

Current (A)	Inverse of current (A^{-1})	Average radius (m)	Radius standard deviation (m)
1.30	0.769	0.03958	0.00019
1.40	0.714	0.03373	0.00001
1.50	0.667	0.03293	0.00004
1.60	0.625	0.02973	0.00075
1.70	0.588	0.02820	0.00089

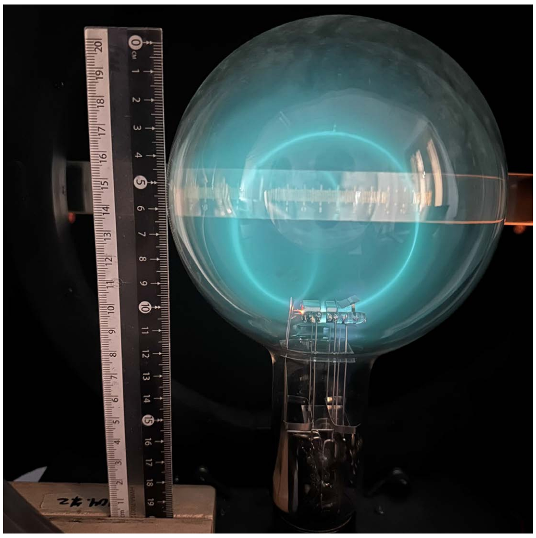

Figure 1. Experiment apparatus and electron beam for $I = 1.5$ A.

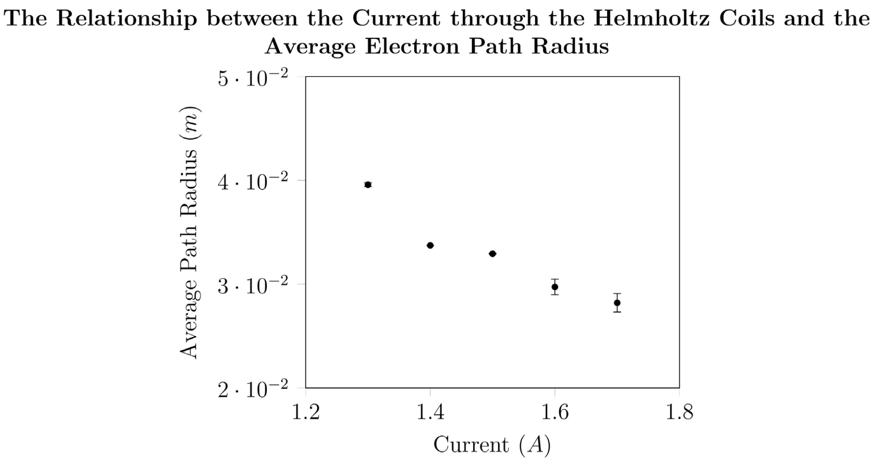

Figure 2. Current vs. average radius with standard deviation as error bars.

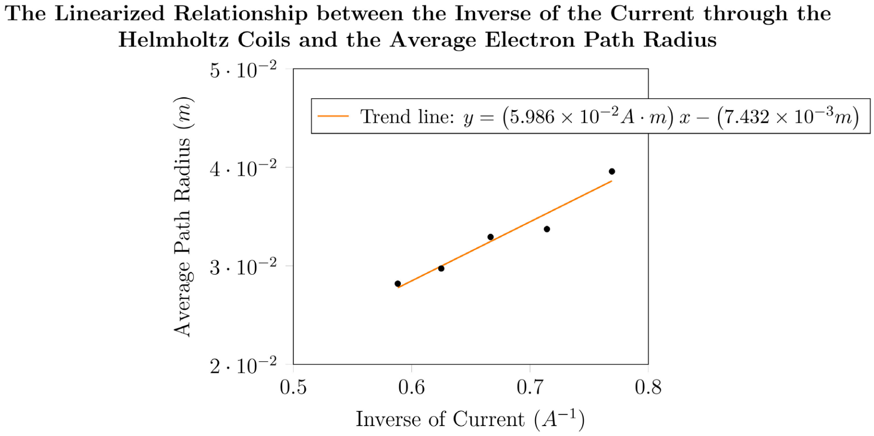

Figure 3. Inverse of current vs. average radius.

4. Discussion

According to the data obtained, the experimental value of e/m_e, the charge-to-mass ratio of the electron, was found to be 1.850×10^{11} C/kg, bounded between 1.440×10^{11} C/kg and 2.465×10^{11} C/kg. The accepted value lies within the range of the experimental value and gives a percent error of 5.17%.

When measuring the radius of the electron beam, photo analysis was used which yielded an uncertainty of least count measurement of 1×10^{-5} m. In order to minimize error, multiple measurements were conducted for each current value tested, and their averages were used in determining the experimental value. Although steps were taken to reduce error, the high energy conditions of this experiment are prone to both random and systematic errors. In order to reduce error propagation, first order quantities were used in linearization. If squares (r^2 and I^{-2}) were used in the linearization, these errors would be magnified and obfuscate the trend. To remedy this, first order quantities (r and I^{-1}) were plotted, and the square of the slope was used to find the experimental charge-to-mass ratio (see Appendix A.4). This contributed to the low percent error yielded by the experimental charge-to-mass ratio found

One likely source of error was the parallax from the glass bulb of the fine beam tube. Although a ruler was set up beside the bulb in order to reduce parallax, there are still optical effects affecting the radii measured inside the bulb. Since the bulb is effectively a convex lens, it projects the image of the the electron beams onto the bulb, making their radii appear greater. Since the lens is curved, Datapoints with greater path radii deviate from their theoretical values less than those with smaller radii, which leads to a smaller slope value. Since the charge-to-mass ratio is inversely proportional to the squared slope of the trendline, a smaller slope value will lead to a greater charge-to-mass ratio, which is observed in the greater-than-accepted experimental charge-to-mass ratio.

The negative y-intercept of this linearization suggests that as the magnetic field due to the Helmholtz coils decreases to zero, there is still motion of electrons in a path of nonzero radius. This is likely due to the ambient magnetic field from the Earth, which would oppose the field generated by the Helmholtz coils.

Interestingly, although the y-values of the regression should always be positive because they are radii, their signs indicate the direction of motion of electrons. In Appendix A.3, it can be seen that, when neglecting the ambient magnetic field due to the Earth, the velocity of the electron can be treated as a positive quantity. However, in reality the ambient magnetic field will switch the direction of motion when the field due to the Helmholtz coils is weak enough, which leads to the radius appearing negative.

5. Conclusions

One clear conclusion of this experiment is that this procedure was successful. There are many factors which lead to error in this experiment; when dealing with masses and charges as small as an electron's, small lacks of precision are magnified. Errors in excess of 10% were expected, so an error of 5.17% is a testament to this procedure. Regardless of this, there are steps which could be taken in future experiments to further reduce error.

When performing this experiment, there was no attempt to account for the ambient magnetic field. In the future, it would be worthwhile to measure the ambient magnetic field due to the Earth at the center of the apparatus, and align the apparatus so that the direction of the Earth's magnetic field lies in the plane of electron motion. This would ensure that the Earth's magnetic field does not exert any additional centripetal force on electrons.

An interesting extension of this method is in studying the effects of relativity on the data obtained. At lower voltages, electrons are accelerated to low enough speeds for relativistic effects to be negligible, but as the accelerating voltage increases, relativistic effects become less negligible. It would be interesting to test this same procedure on higher voltage and compare the efficacy of classical and relativistic considerations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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[2] Thomson, J.J. (1913) Bakerian Lecture:—Rays of Positive Electricity. *Proceedings of the Royal Society A*, **89**, 1-20. https://doi.org/10.1098/rspa.1913.0057

[3] (2018) Electron Charge to Mass Quotient. https://physics.nist.gov/cgi-bin/cuu/Value?esme

Appendix

A.1. Raw Data from Experiment

Table A1. Raw data from experiment.

Current (A)	Trial 1 radius (m)	Trial 2 radius (m)	Average radius (m)	Radius standard deviation (m)
1.30	0.03958	0.76923	0.00157	0.00019
1.40	0.03373	0.71429	0.00114	0.00001
1.50	0.03293	0.66667	0.00108	0.00004
1.60	0.02973	0.62500	0.00088	0.00075
1.70	0.02820	0.58824	0.00080	0.00089

A.2. Derivation of $\vec{B}$ Due to Helmholtz Coils

The magnetic field due to one loop of the Helmholtz Coil, the geometry of which is described in Figure A1 and Figure A2, $\vec{B}_0$ is found by the Biot-Savart law:

$$\mathrm{d}\vec{B}_0 = \frac{\mu_0}{4\pi} \cdot \frac{I\mathrm{d}\vec{\ell} \times \hat{r}}{r^2}$$

$$\mathrm{d}\vec{B}_0 = \mathrm{d}B_{0,x}\hat{i} = \frac{\mu_0}{4\pi} \cdot \frac{I\mathrm{d}\ell \cos\theta}{r^2}\hat{i}$$

Using $r = \sqrt{x^2 + R^2}$ and $\cos\theta = \frac{R}{\sqrt{x^2 + R^2}}$:

$$d\vec{B}_0 = \frac{\mu_0}{4\pi} \cdot \frac{Id\ell R}{\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

$$\vec{B}_0 = \int_{\text{coil}} \frac{\mu_0}{4\pi} \cdot \frac{Id\ell R}{\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

$$\vec{B}_0 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi RId\ell R}{\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

$$\vec{B}_0 = \frac{\mu_0 R^2 I}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

By superposition, the magnetic field due to a coil with N loops $\vec{B}_1$ is found to be

$$\vec{B}_1 = \frac{\mu_0 R^2 IN}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

Since the currents in both coils are travelling in the same direction, both coils will generate a $\vec{B}_1$ in the $+\hat{i}$ direction. The $\vec{B}$ field at the point halfway between the two coils ($x = R/2$) due to the complete Helmholtz coil assembly is:

$$\vec{B} = \frac{\mu_0 R^2 IN}{\left((R/2)^2 + R^2\right)^{3/2}}\hat{i}$$

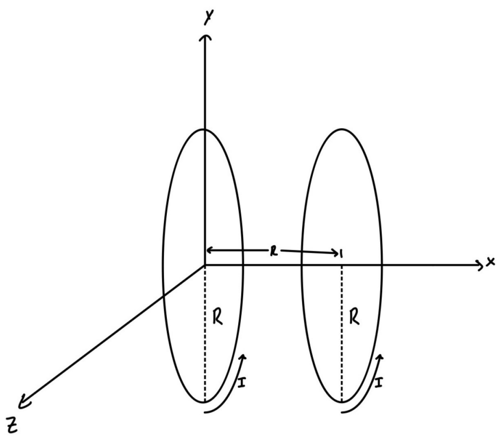

Figure A1. Diagram of assembly with relative distances.

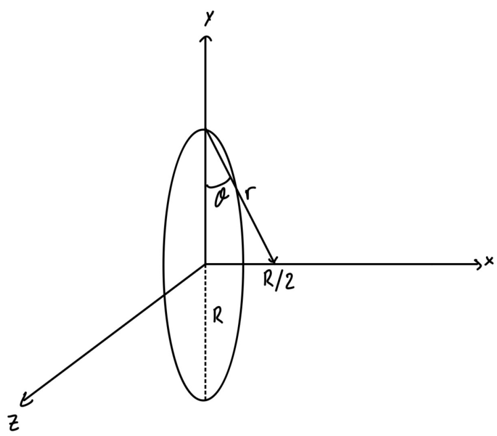

Figure A2. Coil.

$$\vec{B} = \frac{\mu_0 R^2 IN}{\left(5R^2/4\right)^{3/2}} \hat{i}$$

$$\vec{B} = \frac{8\mu_0 IN}{5\sqrt{5}R} \hat{i}$$

A.3. Expression for Charge-to-Mass Ratio

Since the electrons are being put into centripetal motion by the magnetic force, it is found that:

$$F_B = F_{ctr}$$

Since the $\vec{B}$ field is perpendicular to the plane of centripetal motion, there is no need for a cross product in computing F_B

$$evB = \frac{m_e v^2}{r}$$

$$\frac{e}{m_e} = \frac{v}{Br}$$

To find v, the conservation of energy is used. Electrons are accelerated from rest by a potential difference V by the electron gun. This gives

$$eV = \frac{1}{2} m_e v^2$$

$$v = \sqrt{\frac{2eV}{m_e}}$$

After substituting B and v into the expression for e/m_e, e/m_e can be isolated.

$$\frac{e}{m_e} = \frac{5\sqrt{5}R\sqrt{\frac{2eV}{m_e}}}{8\mu_0 INr}$$

$$\left(\frac{e}{m_e}\right)^2 = \frac{2V \cdot 125R}{64\left(\mu_0 INr\right)^2}\left(\frac{e}{m_e}\right)$$

$$\frac{e}{m_e} = \frac{125VR^2}{32\left(N\mu_0 Ir\right)^2}$$

A.4. Linearization and Calculation of Experimental Charge-to-Mass Ratio

To calculate the charge-to-mass ratio, r was plotted against I^{-1} to create a linearized plot, the slope of which was used to calculate the charge-to-mass ratio. The radius and inverse current were used rather than their squares to reduce error propagation.

$$r = \sqrt{\frac{125VR^2}{32\left(N\mu_0\right)^2}\left(\frac{e}{m_e}\right)^{-1}}\, I^{-1}$$

$$\text{slope} = \sqrt{\frac{125VR^2}{32\left(N\mu_0\right)^2}\left(\frac{e}{m_e}\right)^{-1}}$$

$$\text{slope}^2 = \frac{125VR^2}{32\left(N\mu_0\right)^2}\left(\frac{e}{m_e}\right)^{-1}$$

$$\frac{e}{m_e} = \frac{1}{\text{slope}^2} \cdot \frac{125VR^2}{32\left(N\mu_0\right)^2}$$

Using experimental values:

$$\frac{e}{m_e} = \frac{1}{\left(5.986\times10^{-2}\ \text{A}\cdot\text{m}\right)^2} \cdot \frac{125\times201.3\ \text{V}\times\left(0.15\ \text{m}\right)^2}{32\left(130\times\left(4\pi\times10^{-7}\ \text{H/m}\right)\right)^2}$$

$$\frac{e}{m_e} = 1.850\times10^{11}\ \text{C/kg}$$

A.5. Calculation of Percent Error

$$\text{Percent Error} = \frac{\left|\text{Accepted Value} - \text{Experimental Value}\right|}{\text{Accepted Value}} \cdot 100\%$$

$$\%\text{Error} = \frac{\left|1.759 \times 10^{11} - 1.850 \times 10^{11}\right|}{1.759 \times 10^{11}} \cdot 100\% = 5.17\%$$