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Three-Drug Therapies in Psychiatry in the Light of the Maximum Ordinality Principle and the Explicit Solution to the "Three-Body Problem"

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Abstract

The present paper aims at showing the possible adoption in Psychiatry of a *general methodology* finalized to prescribe the most appropriate Therapy based on the knowledge of its correlative effects *in advance*, instead of recognizing them *ex post*. The specific case here considered is the "bipolar disorder", in which the adoption of three different drugs is the most common practice, although with a possible differentiation between the prescription *in the morning* and *in the evening*, respectively. Thus, the proposed methodology will consider the *Ordinal Interactions* between the various drugs by evaluating their combined effects, which will result as being *not a simple additive* "sum", because they are evaluated on the basis of the Maximum Ordinality Principle (MOP) and, in addition, *in Adherence* to the Explicit Solution to the "Three-Body Problem". In this way the Methodology here proposed is able to suggest how to account for the *synergistic effects* of the various drugs, especially when the latter are characterized by *different concentrations* and, at the same time, by generally *different half-lives* respectively.

Keywords

Three-Drug Therapies, Bipolar Disorder, Psychiatric Therapies, Maximum Ordinality Principle (MOP), "Three-Body Problem"

1. Introduction

The Methodology here proposed, as indicated by the same title of the paper, is

based on the MOP [1], with specific reference to a *Three-Drug Therapy*, according to the "Solution to the Three-Body Problem" [2]. However, for the sake of simplicity and clarity in "Favor" of the Reader, both References [1] and [2] are synthetically illustrated in the Appendices A, A1, A2.

Such a Methodological Approach has already been proposed (although as a preliminary version) in the case of "Protein Folding, Molecular Docking, Drug Design" [3] and in the case of *Immune-targeted therapies* [4], when the targeted (oncological therapy) foresees the adoption of two or more molecules, theoretically designed to interact with the same selected target, according to a *pre-defined time sequence*.

In such a case, the approach based on the MOP is able to show that *the most appropriate sequence* of the considered entities (molecules or enzyme) can lead to a "*global efficacy*" which can be *even higher* than the corresponding efficacy when the latter is estimated by considering two or more *distinct and separated interactive processes*. This is because, as is well known, in Self-Organizing Systems "*The Whole is much more than the sum of its parts*".

More specifically, in this paper we want to consider another important field of pharmacological therapies: the adoption of "Tthree-Drug Therapies" *in Psychiatry*. In particular, we will consider the case of a "bipolar disorder", nowadays rather diffused, in which the three (generally) adopted drugs *are finalized to the inhibition of the Inositol*, because the latter is a catalyzer of several undesired reactions in the brain.

This is a particular field of Therapies that, at the same time, presents the *additional advantage* (with respect to oncological therapies) of a *shorter response time* (usually some weeks, with respect to one or two years in the other case). Such an aspect clearly represents a particular advantage as far as *the confirmation* of *the theoretical evaluations* is concerned [4].

The specific drugs usually (although not exclusively) adopted in the case of "bipolar disorder" are *Carbolithium*, *Depakin* and *Olanzapine*, at *different* specific doses *in the morning* with respect to their prescription *in the evening* of the same day.

2. The Rational of the Methodology

The Methodology consists in modeling both *the considered drugs*, *the Inositol* and *their resulting interaction compounds* as *Self-Organizing Systems*, all of them described in the light of *the Maximum Ordinality Principle*, widely illustrated in [1], and here synthetically reproposed in **Appendix A**.

In favor of the validity of the Methodology it is worth recalling that the latter is nothing but the transposition of the same methodology already suggested to "be adopted" in the case of Protein-Protein Interaction (PPI) [5], where the considered process was analogously modeled in the light of the MOP, and also in the case of the Research for a New Therapy for the Duchenne Muscular Dystrophy [6].

This is because any interaction process, when modeled in mere "functional" terms, is always characterized by *an intrinsic insolubility in explicit terms*, as a consequence of the famous "Three-body Problem" (H. Poincaré, 1889, more explicitly recalled in [2]).

The MOP, vice versa, *overcomes* the limitations associated to the "Three-body Problem", because, as clearly shown in [2], it offers the possibility of getting the correlative *explicit formal solution* even in the case of *any arbitrary N-body Interaction Problem*.

Consequently, when both the three drugs and the Inositol are modeled as Self-Organizing Systems in the light of the MOP, *the corresponding solutions* to the various different interaction processes involved can be *easily* obtained, in explicit terms, and in a *fast* and *reliable* way, because they correspond to the *formal solution to particular cases of an N-body Interaction Problem* [2].

In addition, the Methodology here proposed, even though with the specific reference to the therapy of a "bipolar disorder", it presents *some additional special characteristics* that facilitate *its transposition* to *other forms of Psychiatric Therapies* or, more in general, *to different other Pharmacological Therapies*.

3. *Ordinal* Reconfigurations of the Three Drugs and the Inositol

The modeling of the various Inter-Action Processes starts with the *Ordinal* Reconfigurations of both the three drugs and the Inositol. Where the expression "Ordinal Reconfigurations" means that each component is modeled as a *Self-Organizing System*, and thus it is characterized *by its specific Generativity*, which tends to structure the correlative component according to the MOP [1] (as synthetically shown in **Appendix A**).

This means that the various elements that compose both the three drugs and the Inositol are *internally related* to each other in terms of *Ordinal Relationships*, of *Generative Nature*, which, for their specific characteristics, can be termed as "Harmony Relationships" (ib.).

On the basis of such specific properties, the *Ordinal Structure* of *each one* of the four considered components can be obtained by simply defining the topology of *one sole arbitrary couple* of the basic elements that constitute the selected component, which is *assumed as reference* (as it is widely illustrated in [1] and also in **Appendix A**), together with some associated parameters which characterize the entire Structure of each one of the considered four components in the space.

Such a limited number of parameters represent the input to a *Simulator*, *termed as* EQS (Emerging Quality Simulator) [7], which is precisely based on the MOP and its associated "Harmony Relationships" (which are faithfully illustrated in Appendices A, A1, A2).

So that, the input to the Simulator, corresponding to each considered component 1) is thus represented by:

1) The *total number* of elements (N_i);

2) Three topological parameters $\left(\Sigma_{12},\Phi_{12},\Theta_{12}\right)_i$ that define, in polar coordinates, the reciprocal positions of two *arbitrary* elements (conventionally termed as "12"), understood as being *one sole* "*Ordinal*" *entity*. This is also the reason why the latter is topologically referred to its proper *internal reference system*;

3) Five additional parameters $\left(\varepsilon_1,\varepsilon_2,\lambda,\psi_1,\psi_2\right)_i$ which, together with those previously mentioned, complete the definition of the so-called internal *Relation Space* (RS) of the component analyzed. More specifically: $\varepsilon_{1,i}$ and $\varepsilon_{2,i}$ characterize the *spatial orientation* of the component *i* (understood as a Whole), with respect to its *internal reference* axes, while $\left(\psi_1,\psi_2,\lambda\right)_i$ define *the periodicities* (along the three basic axes) of the mathematical solutions which "*Emerge*" from the MOP.

Such solutions are thus precisely those that give the positions in the space of all the constitutive elements with respect to *the internal axes* of each considered component. In this way, the afore-mentioned solutions characterize *each* considered component as a *unique*, *specific* and *irreducible* entity.

This is the reason why any Component, precisely because modeled as a "Self-Organizing" System of *Ordinal Nature* ([1] [7]), is also characterized by its own specific *Self-organizing Capacity*, whose *Activity* can faithfully be represented by its associated "virtual work", defined (in polar coordinates) as

$$W_i=\sum_{j=2}^{N_i}\left\{\left(\rho_{1j}\right)+\left(\rho_{1j}\varphi_{1j}\right)+\left(\rho_{1j}\vartheta_{1j}\right)\right\}_i \tag{1},$$

where the subscripts 1*j* indicate the couples of the constitutive elements successively considered in the sum.

The corresponding *Ordinal Ri-Configurations*, given in terms of "*generative coordinates*" and their correlative "*Uniances*" (instead of *mere* "*distances*" (see par. 8, 9, 9.1, 9.2 of **Appendix A**)), are characterized by the following corresponding virtual works

$$W_C=0.97 \tag{2}$$

$$W_D=25.05 \tag{3}$$

$$W_O=40.80 \tag{4}$$

$$W_I=18.07 \tag{5}.$$

The abovementioned *Ordinal Ri-Configurations*, as already anticipated, are obtained by means of EQS Simulator. Consequently, *they do not represent a simple* "*reproduction*" of the Components as they are correspondently *available in Literature*. This is because such Re-Configurations are *adherently* obtained in the light of the MOP and its *explicit Emerging Solutions*, when the latter are *properly structured* in the form of *Harmony Relationships*. In addition, it has to be considered that the "Structure" (or better, the "*Relational Space*") of *any* component "*evolves*" in its *specific Proper Space* and *Proper Time* [8].

This means that the various elements of each Component *are not related to each other in terms of* "*functional relationships*", as usually described in Literature. In other terms, *they are not thought as being related in terms of* "*forces*"

(such as Coulomb forces, Van del Waals forces, Hydrogen bonds, etc.), but they are related to each other only in terms of *Ordinal Relationships*, always of *Generative Nature*, *because the MOP is assumed as being the Unique* Reference Principle [9].

This consequently means that *some* "*differences*" between the two respective "*topological*" *representations* of each Component (that is, the one given *in Ordinal terms* and that *usually* given in Literature, respectively) are mainly due to such a different *Gnoseological* Perspective.

In addition, it is worth pointing out, *once again*, that each Component, when modeled on the basis of the MOP and its associated Harmony Relationships, it is always "*Reconfigured*" (and also it *progressively* "*evolves*") *in its Proper Space of Relations*. Whereas, on the contrary, the corresponding structures usually *available in Literature* are generally represented *in a plane* or, at most, in Cartesian space and, more specifically, they are *always interpreted* in terms of *functional* relationships (ib.).

4. Reciprocal *Ordinal* Inter-Actions between the Three Drugs and the Inositol

The first step consists in evaluating the specific *Affinity* between each one of the three drugs with the Inositol, according to the concept of *Ordinal Inter-Action* (indicated by the symbol ®), where *the Affinity* is expressed by the following ratio

$$(\delta W)_{r,j} = \left\{W_{3,j} - \left(W_{1,j} + W_{2,j}\right)\right\} \Big/ \left(W_{1,j} + W_{2,j}\right) \quad (6),$$

that is: the difference between *the Virtual Work* $\left(W_{3,j}\right)$ of the final Compound understood as the "*Exit*" of the Inter-Action *j*, with respect to the sum $\left(W_{1,j} + W_{2,j}\right)$ of *the Virtual Works* of Components 1 and 2 of the same Interaction *j*, when the previous difference is referred to the latter sum.

The resulting "Exits" of the Inter-Actions between the three considered drugs and the Inositol are indicated here below, where, for the sake of brevity, they are expressed by the following synthetic symbology: Carbolithium = C, Depakin = D, Olanzapine = O, Inositol = I, where the symbol ® represents the Process of Inter-Action

$$\text{Interaction}\{\text{C} \circledR \text{I}\}: \text{ Affinity } = 54.59\% \quad (7)$$

$$\text{Interaction}\{\text{D} \circledR \text{I}\}: \text{ Affinity } = 0.27\% \quad (8)$$

$$\text{Interaction}\{\text{O} \circledR \text{I}\}: \text{ Affinity } = 0.88\% \quad (9).$$

Such "results" show that Carbolithium has a high *direct* Affinity with the Inositol while the other two Inter-Actions show a negligible Affinity.

However, it is also important to evaluate the Affinity between the three drugs *among themselves* considered, in the perspective of their *subsequent* specific Ordinal Inter-Actions with the Inositol, that is between the latter and the compounds previously obtained. The corresponding "results" are the following ones:

$$\text{Interaction}\{\text{C} \circledR \text{D}\}: \text{ Affinity } = 41.94\% \quad (10)$$

$$\text{Interaction}\{C \overset{\circledR}{} O\}: \text{ Affinity } = 17.56\% \quad (11)$$

$$\text{Interaction}\{D \overset{\circledR}{} O\}: \text{ Affinity } = 1.88\% \quad (12)$$

The successive step consists in evaluating the *Affinity* of the previous compounds so obtained with the Inositol, which represents the *main target* of the selected drugs. In fact, any Inter-Action with the Inositol, as previously said, is finalized to the "inhibition" of the latter, because it represents a basic catalyzer of undesired reactions in the brain.

The "results" pertaining to such *Ordinal* Inter-Actions are shown here below

$$\text{Interaction}\{\{C \overset{\circledR}{} D\} \overset{\circledR}{} I\}: \text{ Affinity } = 1.77\% \quad (13)$$

$$\text{Interaction}\{\{C \overset{\circledR}{} O\} \overset{\circledR}{} I\}: \text{ Affinity } = 5.76\% \quad (14)$$

$$\text{Interaction}\{\{D \overset{\circledR}{} O\} \overset{\circledR}{} I\}: \text{ Affinity } = 27.44\% \quad (15).$$

To complete the analysis of *all the possible Inter-Actions* it is worth *also* considering the following "*multiple*" Inter-Action

$$\{\{\{C \overset{\circledR}{} D\} \overset{\circledR}{} O\} \overset{\circledR}{} I\}: \text{ Affinity } = 0.21\% \quad (16)$$

whose value, as well as the values of all the other Inter-Action Processes, are *invariant* with respect *to the order* of the considered components represented in parentheses, as it is *correlatively*, and *correspondently*, shown by the same EQS Simulator.

The previous "results" show that Carbolithium has *a particular Affinity* with the Inositol (see Equation (7)). This is the reason why Carbolithium is considered as being *the* "*elective*" *drug* in the case of "bipolar disorder".

Carbolithium also has an appreciable affinity with Depakin (see Equation (10)). However, such a resulting compound has not a significant affinity with Inositol.

Depakin, on the contrary, even if it has not a good direct affinity with Inositol (see Equation (8)), after its Inter-Action with Olanzapine, even if at a reduced level of activity (Equation (12)), shows *an inhibition activity* on Inositol which is precisely *amplified* by the form of the latter "resulting" (or better, "*Emerging*") compound (see Equation (15)).

A similar effect is due to the Inter-Action between Carbolithium and Olanzapine, which have a significant direct affinity (Equation (11)), which reflects, although in a reduced form, in the subsequent *Ordinal Inter-Action* with the Inositol (Equation (14)).

Consequently, *the Inhibition Process* of Inositol:

- is mainly due to Carbolithium (Affinity 54.59%);

- then it is due to *a previous Inter-Action* between Depakin and Olanzapine (affinity 27.44%);

- finally, to a *previous* Inter-Action between Carbolithium and Olanzapine (Affinity 5.76%).

These "results", however, only represent *an* "*Ostensive*" *Example* of the methodological approach adopted, because the analysis refers to *one sole structure* of Inositol, *among its* 9 *possible isomeric forms*.

Analogously, the analysis supposes that Carbolithium interacts *in all its specific integrity*, that is as Li2CO3, with respect to its possible decomposition in Li2O and CO2.

At this stage, by considering a typical prescription *in the morning* (C = 150 mg, D = 500 mg, O = 5 mg), together with their respective *molecular weight* (C = 73.89, D = 144.21, O = 312.44, I = 180.16), and their *specific solubility* (13 g/l, 1.2 g/l, negligible), it is possible to evaluate the "*Global*" *Inhibition* "*Effect*" on the Inositol.

An analogous evaluation process can be performed in the case of *Two different Prescriptions*: one *in the morning*, the other *in the evening*.

In this case, however, in order to evaluate the "*Comprehensive*" (and not the *mere* "additive") "Effects" of the *two distinct* prescriptions, it is fundamental to account for *the specific* and *different half-life* of each drug, especially when the prescription lasts for some weeks or more, so as to reach *the desired* "*Stable Effects*" *at regime*.

In all cases, these evaluations are always *subsequent* to the fundamental steps of the methodological analysis previously presented, that is precisely those which concern the evaluations of *the various Affinities* (from (7) to (16)), which can be obtained on the basis of the MOP and its correlative EQS Simulator.

5. Informatics Advantages

The informatics advantages of the Methodology here proposed are directly referable to the fact that, any System, when modeled on the basis of the MOP, always presents an *Explicit Solution* in terms of the *Incipient Differential Calculus* (IDC) [10] [11].

This means that the present Methodology has the *capacity* of *predicting*, *in explicit formal terms*, *the* 3*D structure of the* "*Emerging*" *Resulting Compound of any Inter-Action*, essentially because such "*Compound*" is *always* understood, *in itself* and *by itself*, as a Self-Organizing System, whose description is *Intrinsically* "*Irreducible*" to f*unctional* relationships between its "parts" (ib).

This correlatively means that the EQS Simulator only requires *a reduced number of computations*, without the adoption of *special numerical methods* in order to get the corresponding "Solution" (7).

In addition, it is *particularly worth* underlining that the Explicit Solutions so obtained can be termed as "*Emerging Solutions*", because *they always show an Information Content that is much higher than the corresponding content of the initial formulation of the problem* [1] [2] (see the correlative synthetic exposition in **Appendix A**1). This is because the MOP is specifically finalized to describe "Self-Organizing" Systems according to a *Holistic Approach*, in which, as already said, "*The Whole is much more than the sum of its parts*" (ib.).

6. Conclusions

On the bases of the previous exposition, it is possible to assert that the Metho-

dology here proposed, although referred to the *specific* case of a "*bipolar disorder*", it is also applicable to *other* Psychiatric Therapies, even if these are based on *two* or even *one sole* drug, because the latter cases are evidently included in the former case considered and analyzed.

This aspect also enables us to point out, in addition, that the Methodology will always result as being applicable, at the same time, to *the majority of Therapies* based on *even more* General *Pharmacological* Approaches.

This, at the same time, also shows that the General Applicability of the Methodology specifically manifests *its validity* when it is considered in its more appropriate *General Context*, that is, in the light of *The Maximum Ordinality Principle*.

In fact, by adopting the MOP as the *Basic Reference Criterion*, together with its associated EQS Simulator, it is possible to models *all the Biological Systems*, with very significant related advantages, especially the one of knowing the "results" of any Ordinal Inter-Action *in advance*, instead of *ex post*, as already shown in [9]. This is because for the basic reasons widely presented in [12] and, consequently, the for the manifested possibility to do the same in the considered case of a Three-drug Therapy in Psychiatry.

In fact, in the latter case the Paper shows that, from *a theoretical* point of view, *any given Compound* manifests *its specific positive* "Emerging Effects" (as it happens, for instance, in the case of Carbolithium, which is considered as being the "elective" drug in psychiatric therapies), *not* specifically when it is by itself considered, but when it is considered as being *previously* "*combined*", in Ordinal Terms, *together with* all the other *initially co-present* Compounds foreseen by the Therapy (such as Depakin and Olanzapine, respectively, in the case of a "bipolar disorder").

On the basis of the previous presentation, it is also possible to analyze the basic differences with respect to the Traditional Scientific Approach, by considering, for instance, References [13] and [14], according to which the Analysis of a "Three-Drug System" is considered as being a *mere* "*mechanism*", and *not* a *Self-Organizing System*. Consequently, the various (and "multiple") correlative Fundamental Difference are *clearly illustrated* in the subsequent Appendices A, A1, A2 respectively and, *especially*, in their *correlative Ordinal* "*combinations*", understood as "*particular cases*" of a "Three-Body (Conceptual) System" of *Formal Aspects*.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. *Physical*, *Logical* and *Formal* Presuppositions for the Solution to the "Three-Drug Therapy": The Maximum Ordinality Principle, from the "Incipient" Derivative to EQS Simulator

Appendix A, articulated in three parts, presents a synthesis of the developments concerning the Maximum Ordinality Principle, with reference to a Self-Organizing System made up of *an arbitrary number N* of Bodies, *in any Field of Analysis*.

All the various developments, in adherence to what is indicated in the title, have been illustrated in the various papers presented at the Biennial Emergy Conferences (University of Florida) from 1999 to 2020.

In this respect, **Appendix A** will also present two aspects of particular Relevance:

1) The Process of Genesis of *The Harmony Relationships*;

2) The Process of Genesis of *The Ordinal Roots of Unity*.

The Maximum Ordinality Principle [1] is nothing but the reformulation of Odum's Maximum Em-Power Principle (1995), given however in a more general form by means of a new concept of derivative, *the* "*incipient*" *derivative*, whose mathematical definition introduced, for the first time, in [10] [11].

In this way, both Emergy and Transformity are replaced by the concept of *Ordinality*. This is the reason why the Principle was renamed as the Maximum Ordinality Principle.

Consequently, on the basis of the Mathematical Formulation of the Maximum Ordinality Principle [1] and, in particular, its adoption as "One Sole Reference Principle" [9], we can now present, in more details, the radically New Perspective that such a Principle offers to Modern Science. That is: "*Every System is a Self-Organizing System*".

In order to give a clear presentation of the fundamental differences between such a New Perspective with respect to the Traditional Scientific Approach, **Appendix A** will start from the consideration of a synoptic picture of the basic characteristics of the two mentioned Scientific Approaches (see **Table A1**), successively analyzed and compared, in more detail, in the context of **Appendix A**.

1. Fundamental Characteristics of the Two Scientific Approaches

The consequential faithful developments of Odum's approach have led us to the formulation of a unique general Principle, the Maximum Ordinality Principle (MOP) [1], which is able to describe, by itself, the behavior of any given System as a Self-Organizing System: both "*non-living*" Systems, "*living*" Systems and "*thinking*" Systems too (e.g. Human Systems).

Such a conclusion then results as being deeply different from that of Modern Science, which, from Newton on, is persistently orientated at describing any known system as it were a "mechanism".

Appendix A, after having synthetically recalled the formulation of the MOP and after having pointed out its corresponding descriptive advantages, will focus on the intrinsic new perspective offered by the MOP, especially *in thinking*, *de-*

cision making and acting, with respect to the Traditional Approach. In particular, with reference to any form of relationship between Man and his surrounding Environment, in any Field of Analysis.

In particular, and with reference to this fundamental aspect, the basic differences between the two afore-mentioned perspectives will be brought out by comparing, on the one hand, "*side effects*" (related to the Traditional Approach) and, on the other hand, the "*Emerging Exits*" (specifically pertaining to the New Approach).

Let us then consider first the Traditional Approach that characterizes Modern Science.

1.1. The Traditional Scientific Approach

Modern Science is characterized by a persistent and progressively ascendancy toward ever more general Physical Laws and Principles.

However, before any formulation of a single hypothesis or a physical theory, Modern Science (let us say, from Newton on) adopts three fundamental *presuppositions* (see Table A1): the *causality principle* (also termed as "efficient causality"), *classical logic* (also termed as "necessary logic"), and *functional relationships* (between the various parts of any System analyzed).

On the basis of such fundamental presuppositions, and only after having developed a strictly conform consequential *formal language* (that is the Traditional Differential Calculus (TDC)), Modern Science progressively ascends toward ever more general Physical Laws and Principles:

1) From Phenomenological Laws (e.g. Kepler's Laws); 2) to Physical Laws specific of each Discipline (e.g. Newton's Laws, Maxwell's Equations, etc.); 3) up to the three well-known Thermodynamic Principles.

Such a progressive development has given origin to a hierarchy of a multiplicity of *quantitative* Physical Laws and Principles, in particular as a consequence of the first basic presupposition: the *causality principle*. This Principle, in fact, has led Modern Science to introduce "different causes" in different Disciplines. The Principle of causality, in fact, tends to "sub-divide" the entire phenomenology (at present known) in different "branches", precisely because, on the basis of such a presupposition, it leads Scientists to research for the most "appropriate causes" pertaining each specific set of phenomena each time considered.

In this way, Modern Science persistently propends to show that: "*Every System is a mechanism*".

Such a conclusion, however, although confirmed by experimental results, can be considered as being valid *only* from an *operative* point of view, but not from an *absolute point of view*. This is because "necessary logic" (second basic presupposition) does not admit any form of "*perfect induction*" (see Popper's *Falsification Principle*).

In fact, as synthetically illustrated in Table A1, in the strict contest of "necessary logic":

1) After having formulated a single or more hypotheses (such as in the case of a Theory);

Table A1. Synoptic comparison between the basic presuppositions of the two *differential formal languages* and their main corresponding fundamental characteristics.

Basic Presuppositions	**"*Emerging Quality*" of Self-Organizing Systems**
• 1) Causality Principle (Efficient Causality)	• 1') Generative Causality
• 2) Classical Logic (Necessary Logic)	• 2') Adherent Logic (Emerging Conclusions)
• 3) Functional Relationships	• 3') Ordinal Relationships
	Development of an appropriate Language
d/*dt* is the corresponding formal translation	- L. Boltzmann, A. Lotka
f(*t*) represents a *functional relationship*	- H. T. Odum: Emergy Algebra and M. Em-P. P.
	- Further developments in transient conditions
	- Introduction of the "Incipient" derivative $\tilde{d}/\tilde{dt}$
- **Thermodynamic Principles (1st, 2nd, 3rd)**	
- **Physical Laws (specific for each Discipline)**	**The Maximum Ordinality Principle**
Every System is a "Mechanism"	**- is applicable to *any* Field of analysis: *non-living* Systems, *living* Systems, "*thinking*" Systems (e.g. Human Systems)**
Hypotheses	**- at *any space-time scale* and in *variable conditions***
↓	**- it also offers a *more appropriate* description of any given System and its surrounding habitat**
Mathematical Formalization	**Every System is a "Self-Organizing System"**
↓	
Conclusions	
↓	
Confirmation by experimental results	

2) After having formalized them in an appropriate formal language (faithfully conform to the three above-mentioned basic presuppositions);

3) After having drawn the consequential conclusions;

4) And after having also obtained experimental confirmations of the previous formal conclusions;

5) It is impossible, *in any case whatsoever*, to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* and *unique* hypotheses capable to explain those experimental results.

This is precisely because of the *absence*, in "*necessary*" logic, of any form of *perfect induction*.

In fact, only in the presence of a *perfect induction* it would be possible to assure the *uniqueness* of the *inverse* process and, thus, to transform the adopted hypotheses into an *absolute* perspective.

This means that Modern Science, precisely because based on *necessary logic*, should always be "open" to recognize that *there always exist* many other *possible* Approaches (in principle *infinitive*) capable to interpret the same experimental results.

At this stage, after having synthetically recalled the basic characteristics of Modern Science, we can analyze in more detail the fundamental properties of the New Perspective, synthetically indicated in parallel (for a better comparison) in the right side of Table A1.

1.2. "Emerging Quality" of Self-Organizing Systems and Adoption of New Mental Categories

After having synthetically recalled the basic characteristics of Modern Science

and its corresponding formal language, we can now analyze the fundamental properties of a New Scientific Perspective, which leads to the introduction of a new Formal Language, the *Incipient Differential Calculus* (IDC). As anticipated, the fundamental properties we are referring to are synthetically indicated in parallel (for a better comparison) in the right side in **Table A1**.

Such a New Scientific Perspective is based on the *phenomenological* "Emerging Quality" of Self-Organizing Systems [8]. This represents the fundamental aspect that leads to the adoption of the corresponding *new mental categories* (shown in **Table A1**).

The expression "*Emerging Quality of Self-Organizing Systems*" refers to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So that they usually say: "*The Whole is much more than its parts*".

Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*.

This evidently suggests a *radically new* gnoseological perspective, which corresponds to recognize that: "*There are processes*, *in Nature*, *which cannot be considered as being pure mechanisms*".

This also leads, *in adherence*, to the adoption of "*new mental categories*"[1] and, correspondently, to the development of a completely *new formal language*, so that the description of Self-Organizing Systems might result as being faithfully conform to their "Emerging Quality".

2. The Progressive Development of an Appropriate Formal Language

L. Boltzmann was the first who attempted at describing Self-Organizing Systems in more appropriate formal terms, by proposing the adoption of a new Thermodynamic Principle: The Principle of Maximum Exergy *Inflow* to the System [15].

Some years later, A. Lotka [16] reformulated such a Principle in the form of: The Principle of Maximum Exergy *Flow through* the System (Lotka [17] [18]).

Both such attempts were not perfectly successful, because still based on the concept of Exergy, which is a quantity that is strictly pertaining to Classical Thermodynamics. Consequently, it re-proposes, by itself, the concepts of *efficient causality*, *logical necessity*, *functional relationships*.

[1]These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are chosen only "a posteriori", on the basis of the "Emerging Quality" previously recognized. "*Generative Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Adherent Logic*", correspondently, refers to the capacity of our mind to draw "*emerging conclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, which will be illustrated in more details later on, with reference to any *Generative Process*.

A really *new formal language* only appears with H. T. Odum, with the genial introduction of Emergy (*Em*), defined as Exergy (*Ex*) by Transformity (*Tr*)

$$Em = Ex \cdot Tr \tag{2.1}$$

Equation (2.1) clearly shows that Emergy is *still* based on "Exergy". However:

1) *Quality Factor Tr* "*Transforms*" *Ex* into a *new physical quantity*: Emergy;

2) The latter in fact is not defined in "functional terms", but only by "*assignation Rules*";

3) This is precisely because *Tr* is expressed by means of a *non-conservative Algebra*;

4) Consequently, the output "excess" of the three Fundamental Process in Emergy Analysis (Co-Production, Inter-Action, Feed-Back) is always understood as being "irreducible" to its specific inputs in *mere functional terms*.

This means that Emergy is able to represent the "Emerging Quality" of Self-Organizing *Processes*. Consequently, the general enunciation of the *Maximum Em-Power Principle* (Odum [19] [20] [21]) can *equally be referred*, at a phenomenological level, to the *corresponding maximization tendency* of the "Emerging Quality" on behalf of *Self-Organizing Systems*.

The Maximum Em-Power Principle, however, had not a corresponding and specific formulation under *variable conditions*. On the other hand, such a formulation in *variable conditions* could not be given in terms of the Traditional Differential Calculus, because the traditional derivatives, as a consequence of their conceptual basic presuppositions (see **Table A1**), are not properly apt at representing the "generative" behavior of "Self-Organizing Systems", and consequently they tend to partially "filter" such a "generative" behavior.

This is why, in order to achieve an appropriate mathematical formulation of the Maximum Em-Power Principle, I introduced the concept of "*Incipient Derivative*" [10] [11] [12], defined as

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^{+}}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t)$$

for

$$\tilde{q} = \tilde{m}/\tilde{n} \tag{2.2}$$

a definition that will be illustrated in detail in the next paragraph.

However, it is already possible to anticipate that such a definition shows that the "*Incipient Derivative*" is not an "operator", like the traditional derivative (d/dt), but it could be termed as a "*generator*", because it describes a Process in its same act of being born [12].

The Mathematical Formulation of the M. Em-P. Principle in terms of *Incipient Derivatives* was given in its most appropriate form in a specific book co-financed by the Center for Environmental Policy [22].

During the successive eight years (2002-2010), such a mathematical formulation was adopted in several Disciplines, such as *Classical Mechanics*, *Quantum*

Mechanics, *General Relativity*, *Chemistry*, *Biology*, *Economics* and the corresponding results were reunited *in two books* (titled: "*Lightness of Quality*" [23] *and* "*Ascendency of Quality*" [24]).

At the end of this wide range of applications, I realized that it was possible to give a more general formulation of the Maximum Em-Power Principle, in the abovementioned form of the "*Maximum Ordinality Principle*" [1].

For the sake of clearness, the Rational of such a generalization process, articulated in a few logical steps, is recalled in the next sections.

3. The Incipient Derivative of Ordinality $\tilde{q}$

The "Incipient" Derivative of a given Ordinality $\tilde{q}$, whose definition previously introduced is here recalled for the sake of clarity

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^{+}}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t)$$

for

$$\tilde{q} = \tilde{m}/\tilde{n} \tag{3.1}$$

will be illustrated by considering first its *general properties* and, immediately after, its more *specific properties*.

To this purpose, it is worth preliminary pointing out that the concept of "Ordinality" refers to two "distinct" concepts, which however are considered as being *one sole entity*, that is as a *Whole*. These are: its "*cardinality*" and its "*ordinal genetic relationships*". This means that the Ordinality $\tilde{q}$, synthetically represented as $\tilde{q} = \tilde{m}/\tilde{n}$ (as in Equations (2.2) and (3.1)), in reality it has to be more properly understood as

$$\{\tilde{m}/\tilde{n}\} = \{k, (\tilde{m}/\tilde{n})\} \tag{3.2}$$

in which:

- k represents its *cardinality*;
- While $(\tilde{m}/\tilde{n})$ represents its *Ordinal Genetic Relationships*, where the *round brackets* expressly indicate that they represent *only a part* of the concept of Ordinality, understood as a Whole. In fact, the first member of Equation (3.2) is represented in *curly brackets*, precisely because this symbol is usually adopted to indicate the concept of a Whole.

The Ordinal Genetic Relationships $(\tilde{m}/\tilde{n})$ can also more synthetically termed as "Ordinal Relationship", not only because they are not "functional" Relationships, but especially because the adjective "Ordinal" also indicates that they are precisely those Relationships that give the most significant contribution to the definition of the general concept of Ordinality understood as a Whole.

3.1. General Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Definition (2.2) clearly shows what we have synthetically anticipated, that is: the "*Incipient Derivative*" is not an "operator", like the derivative (d/dt) in the Traditional Differential Calculus (TDC), but it could be termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being*

born [9]-[14]. In fact:

1) The sequence of the symbols is now interpreted according to the *direct priority* of the three elements that constitute its definition (*from left to right*). This is the reason why they acquire a completely new different meaning with respect to the traditional one;

2) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;

3) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "*threshold*" of that "*ideal window*" from which we observe and describe the considered phenomenon;

4) The symbol $\tilde{\Delta}t:0\to 0^{+}$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) *in its proper act of being born*, as a Generative Process;

5) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). So that the ratio which appears in Equation (3.1) indicates not only a quantitative variation in time, but both the variation in Quality and quantity;

6) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *Exit* of such a process will keep "memory" of its genetic origin. This is because, besides its quantity, it will result as being Ordinally structured (as shown at the next Paragraph 3.2.2) according to the indication of such an exponent. The latter in fact precisely expresses how each part of the output is *genetically Ordered* to the Whole and, at the same time, *how each part is related to all the others* in terms of *Ordinal Harmony Relationships* (illustrated at Paragraph 5.6);

7) In this way, the "incipient" derivative represents the *Generativity of the considered Process*, that is the output "excess" (per unit time) characterized by both its *Ordinal Genetic Relationships* and its related *cardinality*, while the sequence of the symbols in its definition can be interpreted as representing a *unique inter-action process* between the same;

8) The above-mentioned reasons clearly show why the "Incipient" Derivative, precisely because of such properties, is able to *unify* (and, at the same time, to specify) the description of the various Self-Organizing Processes of the surrounding World, when they are explicitly understood in terms of Quality.

3.2. Specific Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Let us start from considering first its *specific cardinality k*.

3.2.1. The "Incipient" Derivative of cardinality k

On the basis of Definition (3.1), the exit of the incipient derivative of Ordinality k is [12]

$$\frac{\tilde{d}^{k}}{\tilde{d}t^{k}}f(t)=\left(f'(t)\tilde{/}f(t)\right)^{k}\cdot f(t) \tag{3.2.1}$$

In fact, through successive formal passages, we have that

$$\left(\tilde{\delta}-1\right)f\left(t\right)=f\left(t+\tilde{\Delta}t\right)-f\left(t\right)=f\left(t\right)+f'\left(t\right)\tilde{\Delta}t-f\left(t\right)=f'\left(t\right)\cdot\tilde{\Delta}t \quad (3.2.2)$$

and, consequently

$$\left(\tilde{\delta}-1\right)\Big/\tilde{\Delta}t=f'\left(t\right)\Big/f\left(t\right) \quad (3.2.3)$$

Such an expression, when introduced in the Definition (3.1), gives

$$\tilde{\lim_{\tilde{\Delta}t:0\to 0^{+}}}\left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{k}\cdot f\left(t\right)=\left(f'\left(t\right)\tilde{\Big/}f\left(t\right)\right)^{k}\cdot f\left(t\right) \quad (3.2.4)$$

Such an explicit formal process shows that the definition of the "*Incipient*" Derivative of cardinality k is based on a concept of limit, which however is "*prior*" with respect to the considered function. In fact, it is specifically referred to the considered function only *after* the corresponding evaluation of the latter.

It is also worth adding that in Equations (3.2.2) and (3.2.3), we have adopted the simple notation $f'\left(t\right)$, which in reality is more typical of TDC. It is thus now particularly important to point out that, apart from the similarity of the symbol, the traditional derivative $f'\left(t\right)$ presents specific differences with respect to the "Incipient" Derivative $f'\left(t\right)$.

In fact, if we consider the "Incipient" Derivative of cardinality k of the exponential function, that is, if we assume that $f\left(t\right)=e^{\alpha(t)}$, on the basis of Equation (3.2.4), we get

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{k}e^{\alpha(t)}\overset{*}{=}e^{\alpha(t)}\cdot\left[\overset{\circ}{\alpha}\left(t\right)\right]^{k} \quad (3.2.5)$$

in which the specific symbology adopted $\overset{\circ}{\alpha}\left(t\right)$ is finalized to point out that, even if on the basis of Equation (3.2.4) the first order "Incipient" Derivative (now indicated with $\overset{\circ}{\alpha}\left(t\right)$) coincides with the traditional derivative $\alpha'\left(t\right)$, the *logical processes* that lead to such identical (quantitative) results are radically different. A difference which, in particular, is also pointed out by the adoption of the symbol $\overset{*}{=}$, which reminds us that any "Incipient" Derivative is always the *Exit* of a *Generative Logical Process* and not of a *necessary* logical process.

Equation (3.2.5) can thus preferentially be adopted as the *general definition* of the "Incipient" Derivative of cardinality k. This is because any function $f\left(t\right)$ can always be written in the form $f\left(t\right)=e^{\ln f(t)}=e^{\alpha(t)}$.

Such a formal representation, in fact, leads to the same result as that of Equation (3.2.5). However, such a formal representation will reveal the "Ostensive" Valence of the "Incipient" Derivative of cardinality k when, in the next paragraphs, we will introduce the general definition of Relational Space and, even more, when we will deal with the explicit solution to the Maximum Ordinality Principle.

At the same time, such a definition is also particularly apt at showing the deep differences between the cardinal values of the "Incipient" Derivatives and those pertaining to the traditional derivatives.

In fact, if we compare the traditional derivative of order n of the function $e^{\alpha(t)}$, evaluated according to Faà di Bruno's formula

$$\left(\frac{d}{dt}\right)^n e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1!k_2!\cdots k_n!} \cdot \left(\frac{\dot{\alpha}}{1!}\right)^{k_1} \left(\frac{\ddot{\alpha}}{2!}\right)^{k_2} \cdots \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \tag{3.2.6}$$

with the "Incipient" Derivative of the corresponding cardinality n

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^n e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot \left[\overset{\circ}{\alpha}(t)\right]^n \tag{3.2.7},$$

we can easily recognize that they are *deeply different*. And, even if in some cases the two derivatives of the same order k coincide (for instance when $\alpha(t)$ is linear), such a coincidence has always to be seen in the light of the symbol $\overset{*}{=}$ in Equation (3.2.7), which reminds us that any "Incipient" Derivative is always the *Exit* of a *Generative Logical Process* and not of a *necessary* logical process. A concept that is contextually and specifically underlined in Equation (3.2.7) by the explicit adoption of the "notation" $\left[\overset{\circ}{\alpha}(t)\right]^n$.

3.2.2. The Ordinal Genetic Relationships $(\tilde{m}/\tilde{n})$ of the "Incipient" Derivative of Ordinality $\tilde{q}$

As already anticipated, beside its proper cardinality k, the "Incipient" Derivative of Ordinality $\tilde{q}$, according to Equation (3.1), is characterized by the genesis of its corresponding *Ordinal Genetic Relationships*, whose specific indication is represented by $(\tilde{m}/\tilde{n})$.

In this respect, it is worth pointing out that the symbol $f(t)$ does not represent anymore a simple "function", such as in the case of TDC, but it represents a *Physical Entity*, of *Generative Nature*. Consequently, a more appropriate symbol should be $\tilde{f}(t)$, where the "tilde" notation specifically reminds us its *Generative Nature*.

More specifically, in the general context of Self-Organized Systems, the symbol $\tilde{f}(t)$ will be more properly understood as being representing the *Relational Space* of a given System, as it will be shown in the next paragraphs.

After these due premises, we can assert that the "Incipient" Derivative of Ordinality $\{\tilde{q}\} = \{k, (\tilde{m}/\tilde{n})\}$ describes a *Generative Process* which, with reference to a given System, is characterized by both its *cardinal* and "*internal genetic properties*", and it can be represented as follows

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^{\{k,(\tilde{m}/\tilde{n})\}} e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot \left\{ \begin{pmatrix} \left[\overset{\circ}{\alpha}_{11}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{21}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{m1}(t)\right]^k \end{pmatrix}, \begin{pmatrix} \left[\overset{\circ}{\alpha}_{12}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{22}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{m2}(t)\right]^k \end{pmatrix} \begin{pmatrix} \cdots \\ \cdots \\ \vdots \\ \cdots \end{pmatrix} \begin{pmatrix} \left[\overset{\circ}{\alpha}_{1n}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{2n}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{mn}(t)\right]^k \end{pmatrix} \right\} \tag{3.2.8}$$

where:

- k represents the *cardinality* of the "Incipient" Derivative;

- $\overset{\circ}{\alpha}_{ij}(t)$ are the *genetic characteristics* of the considered system, which are highlighted by the *Generative Process* described by the "Incipient" Derivative. For this reason, they should more properly be represented as being characterized by a "tilde" notation. However, for the sake of a simpler notation, the "tilde" notation has been omitted, and thus it is simply understood;

- Such genetic characteristics $\overset{\circ}{\alpha}_{ij}(t)$ are generally referred to the specific properties of the *Relational Space* $\alpha(t)$ and are evidently characterized by the initial and boundary conditions of the System;

- At the same time, the "matrix" which appears in the second member of Equation (3.2.8) *is not* a traditional matrix. In fact, it is an "*Ordinal*" Matrix, whose various elements are related between them through Ordinal Relationships, of Genetic Nature, in the form N Co-Generated genetic properties (vertical columns), further related between them in the form of N Interaction Ordinal Relationships (parallel sequence of the N column). The "*Ordinal*" Matrix thus represents an *Ordinal Cooperation* of N Co-Productions and their associated N Inter-actions.

In this way, the various elements form *One Sole Entity*, faithfully represented by the abovementioned *Ordinal Matrix*. A concept that is explicitly pointed out, also in this case, by the adoption of *curly brackets*.

In addition, in order to distinguish such an *Ordinal Matrix* from a traditional matrix, from now on, for the sake of brevity, it will be simply termed by means of the single term "*Matrioska*".

The structure of the "Incipient" Derivative (3.2.8) is then able to Ostend even more clearly the concepts previously anticipated. That is:

- The symbol $(\tilde{m}/\tilde{n})$ represents the *Ordinal Genetic Relationships* that characterize the "Incipient" Derivative, where the round brackets expressly indicate that they represent only *a part* of the concept of Ordinality, which vice versa is understood, by itself, as a Whole;

- In fact, for this reason, in Equation (3.2.8) the latter concept is represented by means of the adoption of *curly brackets*;

- The *Ordinal Genetic Relationships* can also more synthetically be termed as "*Ordinal Relationship*", both because they are not, in themselves, "functional" Relationships, but especially because the adjective "Ordinal" clearly indicates that they are precisely those that give the most significant contribution to the definition of the general concept of Ordinality;

- In addition, Equation (3.2.8) allows us to point out that, when we preliminary introduced the concept of *cardinal* "Incipient Derivative", this was represented as a simple and proper *mathematical concept*, which, in this sense, has some similarities with that of a traditional derivative. This is why it was possible to continue to adopt the term "function" and the correlative symbol $f(t)$, even if it was well clear the profound difference between the correlative Logical Process adopted;

- Vice versa, when we consider the "Incipient" Derivative of Ordinality $\tilde{q}$, its

meaning, when considered in the descriptive context of Self-Organizing Systems, is more properly referable as the description of a *Generative Process*;

- Consequently, in such a case it is more appropriate to consider Equation (3.2.8) as representative of a *Generative Process*, which highlights the Genetic Properties of a *Physical Entity* that, in the case of a Self-Organizing System, it is usually represented by the proper *Relational Space* of the System;

- So that, to take into account the abovementioned different aspects between the two considered Derivatives, in general it is preferable to adopt the synthetic tilde notation $\tilde{f}(t)$, in order to more specifically indicate, in addition, that the considered System is already the Exit of *a previous* Generative Process.

3.2.3. Specific Properties of the "Incipient" Derivative When Understood of *Higher Ordinality*

The Ordinality of the "Incipient" Derivative, as previously defined (see Equation (3.2)), represents the most frequent form of Ordinality of the Self-Organizing Systems usually considered.

However, in particularly cases (especially in "Living" Systems), it may be characterized by a more "articulated" structure. For example, its cardinality can directly be associated to a correlative Ordinality $\tilde{2}/\tilde{2}$, corresponding to an "*additional*" Coproduction-Interaction Process.

In such a case, the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \left\{k \uparrow \tilde{2}/\tilde{2}; (\tilde{m}/\tilde{n})\right\} \tag{3.2.9}$$

in order to have, in such a way, a more adherent representation of the Internal Generativity of the System under consideration.

In this respect, however, some examples of more articulated forms of "Incipient" Derivative, with reference to *particularly complex* "Living" System, are illustrated in [8].

4. Mathematical Formulation of the Maximum Ordinality Principle

The Maximum Ordinality Principle (MOP), whose verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", is formulated by means of two fundamental equations, which are so *strictly related to each other*, so as to form *a Whole* [13] [16] [21].

4.1. The First Fundamental Equation of the Maximum Ordinality Principle

On the basis of the previous concept of "Incipient" Derivative, the First Fundamental Equation is formulated as follows

$$\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}} \{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{4.1}$$

$$(\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\} \tag{4.1.1}$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration (see Paragraph 5.1), while $\{\tilde{m}/\tilde{n}\} = \{k, (\tilde{m}/\tilde{n})\}$ represents its corresponding Ordinality, while $(\tilde{m}/\tilde{n})$ indicates the *Ordinal Genetic Relationships* characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, and *the Maximum Ordi-*

nality is reached when $(\tilde{m}/\tilde{n})$ equals $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\}$ (as indicated in Equation (4.1.1)).

In this respect, it is worth noting that:

1) The *underlined* symbol $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s$ explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*. This is because it is precisely that which gives origin to its Self-Organization as a Whole;

2) The symbol "$\overset{[\rightarrow}{=}\{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact, it now represents, at the same time:

- The specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (4.1);

- While the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its *Generative Evolution*, is persistently "adherent" to its "origin and habitat" conditions.

4.2. The Second Fundamental Equation of the Maximum Ordinality Principle

It is formulated as follows

$$\left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})}\left\{\{\tilde{r}\}\circledR\left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\right\}\overset{[\rightarrow}{=}\{\tilde{0}\} \tag{4.2}$$

and it can be considered as representing a *global Feed-Back Process* of *Ordinal Nature*, which is *internal* to the same System. Equation (4.2), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® (defined at Paragraph 5.1) with *its proper Generative Capacity* $\left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive Generative Capacity*, which *at any time*, is always adherent to the origin and habitat conditions of the Second Fundamental Equation.

This is an aspect which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being part of its proper habitat.

The Maximum Ordinality Principle, in its two fundamental equations, *always* presents an *explicit solution*.

The latter will now be presented:

1) By preliminarily illustrating its basic elements;

2) Then, by formulating the correlative solution in explicit terms;

3) Finally, at the end of **Appendix A**, the general explicit solution to the MOP will also be presented and structured in a corresponding *operative form*, so that it may result as being more directly and easily adopted in analyzing any System under consideration.

5. Explicit Solution to the Mathematical Formulation of the Maximum Ordinality Principle

In order to show the explicit solution to the Maximum Ordinality Principle, it is worth recalling the fundamental concepts pertaining to the *Relational Space* of

a System.

5.1. The Relational Space of a System (and the "Proper Space" of the System)

In this respect, it is fundamental to recall that the symbol $\{\tilde{r}\}$ in Equation (4.1) represents the *Relational Space* of the System, which obviously depends on the Nature of the System analyzed.

We can then start from the consideration of a System whose *Relational* Space is characterized, for example, by the following three topological coordinates $\{\tilde{\sigma},\tilde{\varphi},\tilde{\vartheta}\}$.

Such a hypothesis is surely valid in the case of a "*non-Living*" System. Nonetheless, it is also valid in the case of a "*Living*" System too. Whereas, in the case of "Conscious" Systems, the three coordinate will surely be different.

For example, in the case of the Economic Analysis of European Community, with its 27 States, the variables could be (K, L, N), that is *Kapital*, *Labour* and *Natural Resources*, as shown in [25].

In all cases whatsoever, the three topological coordinates $\{\tilde{\sigma},\tilde{\varphi},\tilde{\vartheta}\}$ are always considered as *the Exit of a Generative Process* (this is the reason for the tilde notation), and we always have that

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} \tag{5.1.1}$$

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\begin{aligned}\{\tilde{r}\}_s &= \left\{\tilde{\rho}\circledR\tilde{i}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\right\} = \left\{e^{\tilde{\sigma}\circledR\tilde{i}}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\right\} \\ &= e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} = e^{\tilde{\alpha}(t)}\end{aligned} \tag{5.1.2}$$

where the traditional versors $\vec{i},\vec{j},\vec{k}$ are now replaced by three unit *spinors* $\tilde{i},\tilde{j},\tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i}\circledR\tilde{i} = \oplus 1 \quad \tilde{i}\circledR\tilde{j} = \tilde{j} \quad \tilde{i}\circledR\tilde{k} = \tilde{k} \tag{5.1.3}$$

$$\tilde{j}\circledR\tilde{i} = \tilde{j} \quad \tilde{j}\circledR\tilde{j} = \Theta 1 \quad \tilde{j}\circledR\tilde{k} = \tilde{k} \tag{5.1.4}$$

$$\tilde{k}\circledR\tilde{i} = \tilde{k} \quad \tilde{k}\circledR\tilde{j} = \tilde{k} \quad \tilde{k}\circledR\tilde{k} = \Theta 1 \tag{5.1.5}$$

where the symbols $\oplus$ and $\circledR$ express more intimate relationships between the same spinors: both in terms of sum $\oplus$ and in terms of (relational) product $\circledR$ with respect to the case of traditional versors $\vec{i},\vec{j},\vec{k}$.

So that representation (5.1.1) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one real unit ($\tilde{i}$) and two imaginary units ($\tilde{j}$ and $\tilde{k}$).

5.2. The Generative Capacity of the System

As already anticipated, the incipient derivative $\left(\underline{\tilde{d}/\tilde{dt}}\right)_s^{\{\tilde{m}/\tilde{n}\}}$, when it is *underlined*, explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*.

This is precisely because under these conditions it represents the Self-Organi-

zation of the System as a *Whole*.

At the same time, this is also the reason why, differently from the traditional "incipient" derivative, in our case the "Incipient" Derivative is directly *referred to the exponent* of the Relational Space, that is

$$e^{\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}} \tag{5.2.1}$$

In addition, it is also important to underline that such an exponent, according to the same symbols adopted, is understood as a *Whole* (see the curly brackets, together with the symbols $\oplus$ and $\circledR$).

This means that the corresponding derivative have to be taken with reference to such a *Whole*. Otherwise, its corresponding value will be generally underestimated.

If now, for the sake of clarity, we synthetically indicate $\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}=\tilde{\alpha}(t)$, the explicit solution to Equation (4.1) will result in the form (5.2.2), when it is given in terms of an External Representation. That is, when the coordinates of the various elements of the System are referred to a Reference System of coordinates whose origin is *external* to the System under consideration.

$$\left\{\tilde{r}\right\}_s=e^{\begin{Bmatrix}\tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1n}(t)\\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \cdots & \tilde{\alpha}_{2n}(t)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\alpha}_{n1}(t) & \tilde{\alpha}_{n2}(t) & \cdots & \tilde{\alpha}_{nn}(t)\end{Bmatrix}} \tag{5.2.2}$$

The "Matrioska" in Equation (5.2.2), however, as consequence of the Internal Generativity of the System $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$ and the correlative Self-Organization Process, *when the System reaches its Maximum Ordinality*, *transforms* its initial internal structure $\left(\tilde{m}/\tilde{n}\right)$ *into the form* $\left(\tilde{n}/\tilde{n}\right)$. In such a case, the corresponding $\tilde{\alpha}_{ij}(t)$ will always depend on the initial and boundary conditions, and in the next paragraphs, we will show how it is possible to find their explicit expressions.

5.3. Explicit Expression of the Internal Generativity $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$

Let assume that, under the conditions previously described, the explicit expression of the Ordinality $\left\{\tilde{m}/\tilde{n}\right\}$, in Equation (3.2), equals

$$\left\{\tilde{N}/\tilde{N}\right\}=\left\{k,\left(\tilde{N}/\tilde{N}\right)\right\} \tag{5.3.1}$$

Equation (4.1) then becomes

$$\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{N}/\tilde{N}\}}e^{\{\tilde{\alpha}(t)\}}=e^{\left\{\begin{pmatrix}\left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{11}(t)\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{21}(t)\\ \vdots\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{N1}(t)\end{pmatrix},\begin{pmatrix}\left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{12}(t)\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{22}(t)\\ \vdots\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\ \cdots\\ \vdots\\ \cdots\end{pmatrix}\begin{pmatrix}\left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{1N}(t)\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{2N}(t)\\ \vdots\\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k\tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}}\overset{[\rightarrow}{=}\left\{\tilde{0}\right\} \tag{5.3.2}$$

where the symbol "$\overset{[\rightarrow}{=}\left\{\tilde{0}\right\}$", as previously anticipated, represents, at the same time:

- The specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (4.1);
- While the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its *Generative Evolu-*

tion, is persistently "adherent" to its "origin and habitat" conditions.

5.4. The Initial and Boundary Conditions

Given the particular structure of Equation (5.3.2), it is possible to directly explicit the term $\overset{[\rightarrow}{=}\left\{\tilde{0}\right\}$ in exponential form, so that it can be written as follows

$$e^{\left\{\begin{pmatrix}\left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{11}(t)\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{21}(t)\\ \vdots\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{N1}(t)\end{pmatrix},\begin{pmatrix}\left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{12}(t)\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{22}(t)\\ \vdots\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\ \cdots\\ \vdots\\ \cdots\end{pmatrix}\begin{pmatrix}\left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{1N}(t)\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{2N}(t)\\ \vdots\\ \left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}}\overset{[\rightarrow}{=}e^{\left\{\begin{pmatrix}\tilde{\beta}_{11}(t)\\ \tilde{\beta}_{21}(t)\\ \vdots\\ \tilde{\beta}_{N1}(t)\end{pmatrix},\begin{pmatrix}\tilde{\beta}_{12}(t)\\ \tilde{\beta}_{22}(t)\\ \vdots\\ \tilde{\beta}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\ \cdots\\ \vdots\\ \cdots\end{pmatrix}\begin{pmatrix}\tilde{\beta}_{1N}(t)\\ \tilde{\beta}_{2N}(t)\\ \vdots\\ \tilde{\beta}_{NN}(t)\end{pmatrix}\right\}} \tag{5.4.1}$$

which shows that, *in principle*, its explicit solution can be obtained by solving $N \times N$ corresponding differential equations of the form

$$\left(\tilde{d}/\tilde{dt}\right)^k\tilde{\alpha}_{ij}(t)=\tilde{\beta}_{ij}(t) \tag{5.4.2}$$

In reality, as we will see, from an "*operative point of view*", it *is not strictly necessary* to integrate Equations (5.4.2), because it *is sufficient the integration* of the *sole* "*couple of reference*", generally indicated as $\tilde{\alpha}_{12}(t)$. This is because, as shown later on, all the other couples $\tilde{\alpha}_{ij}(t)$ are related to the *reference couple* $\tilde{\alpha}_{12}(t)$ in the form of "*Assignation*" conditions, according to the *Harmony Relationships* (Paragraph 5.6), this means that the next paragraph, which concerns the explicit solution in terms of Equations (5.4.2), in reality is here presented only for *generality* of exposition.

In fact, by considering the "Emerging Property" of the *Harmony Relationships*, the Solution can strictly be formulated in terms of the *sole integration* of the *condition* concerning the couple $\tilde{\alpha}_{12}(t)$.

5.5. Explicit Solution to Equation (5.4.1), Understood in Terms of External Description

Equation (5.4.1), considered *from a general point of view*, always presents an *explicit solution*. This is because in the majority of the most frequent Self-Organizing Systems (both "*non-Living*", "*Living*" and "*Conscious*" Systems), the general structure of the initial conditions can be assumed as being equal to

$$\beta_{ij}(t)=\left(a_{ij}+b_{ij}\cdot t\right)^p \tag{5.5.1}$$

in which p can also be a fractional number.

Such initial conditions always lead to the explicit solution of any unknown $\tilde{\alpha}_{ij}(t)$ that appears in Equation (5.4.1). This is because by considering the general definition of the cardinal "Incipient Derivative" (3.2.1), we have that

$$\frac{\tilde{d}^k}{\tilde{dt}^k}f(t)=\left(\frac{\tilde{f'}(t)}{f(t)}\right)^k\cdot f(t)=\beta(t) \tag{5.5.2}$$

in which $\beta(t)$ now represents the initial condition for the generic function $f(t)$.

Consequently, through successive formal passages, we have

$$f(t)^{1-k}\cdot f'(t)^k=\beta(t) \tag{5.5.3}$$

from which

$$f(t)^{\frac{1-k}{k}} \cdot f'(t) = \beta(t)^{\frac{1}{k}} \tag{5.5.4}$$

whose integral

$$\int_0^t f(t)^{\frac{1-k}{k}} \cdot \tilde{f}'(t) \cdot dt = \int_0^t \left(\beta(t)\right)^{\frac{1}{k}} \cdot dt \tag{5.5.5}$$

leads to

$$f(t)^{1/k} \cdot k = \int_0^t \left(\beta(t)\right)^{\frac{1}{k}} \cdot dt \tag{5.5.6}$$

and, consequently, we have

$$f(t) = 1/k \cdot \left\{ \int_0^t \left(\beta(t)\right)^{\frac{1}{k}} \cdot dt \right\}^k \tag{5.5.7}$$

where $f(t)$ now represents any $\alpha_{ij}(t)$, while $\beta(t)$ represents the corresponding associated initial condition $\beta_{ij}(t)$.

The explicit solution of the generic $\alpha_{ij}(t)$ is then given by

$$\begin{aligned} \alpha_{ij}(t) &= \frac{1}{k} \cdot \left\{ \int_0^t \left(\beta_{ij}(t)\right)^{\frac{1}{k}} \cdot dt \right\}^k = \frac{1}{k} \cdot \left\{ \int_0^t \left(\left(a_{ij} + b_{ij} \cdot t\right)^p \right)^{\frac{1}{k}} \cdot dt \right\}^k \\ &= \frac{1}{k \cdot b_{ij}} \cdot \left(\frac{k}{p+k} \cdot \left(a_{ij} + b_{ij} \cdot t\right)^{\frac{p}{k}+1} \right)^k \end{aligned} \tag{5.5.8}$$

However, as already anticipated, such a *formal procedure it is not specifically required* to evaluate all the couples $\alpha_{ij}(t)$ that characterize the explicit solution to the First Fundamental Equation (4.1). This is because *The Explicit Solution does not end up at this level*.

In fact, the General Solution to Equation (4.1) is characterized by an *additional contribution*. That is, the contribution of the *Harmony Relationships*, which represent an "Emerging Solution" which, correspondently, shows an "*Emerging Property*" of the Self-Organizing Systems: that is, the *Diffusive Generativity*, among the various elements of the same System, which in reality represents the *proper origin* of the *Harmony Relationships*.

5.6. The Harmony Relationships, as the Exit of a *Diffusive Generativity* of the System

The *Process of Genesis* of the *Harmony Relationships* can be shown by adopting two different *descriptive modalities*, that is: by adopting an *External Representation* or, alternatively, an *Internal Representation*.

The two Representations are substantially equivalent between them. However, the adoption of an *Internal Representation* is able to Ostend much more clearly the abovementioned "Excess of Quality" on behalf of the System analyzed.

This is because, as already anticipated, an External Representation is the one in which each element of the System is referred to a system of coordinates characterized by an origin which is external to the System analyzed. Whereas, in the case of an Internal Representation, the various elements of the System are re-

ferred to a system of coordinates which is internal to the same System analyzed.

In the latter case, each element $\tilde{\alpha}_{ij}(t)$ of the System, at its Maximum Ordinality, is preferably referred to the corresponding element of the main diagonal belonging the same row *i*, and, this leads to the following Representation

$$\left\{\tilde{r}\right\}_s = e^{\begin{Bmatrix} 0 & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1N}(t) \\ \tilde{\alpha}_{21}(t) & 0 & \cdots & \tilde{\alpha}_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}_{N1}(t) & \tilde{\alpha}_{N2}(t) & \cdots & 0 \end{Bmatrix}} \tag{5.6.1}$$

in which all the elements of the main diagonal are evidently equal to zero, whereas all the other elements $\tilde{\alpha}_{ij}(t)$ assume a binary-duet structure, and thus satisfy the following *Specularity* Relationships

$$\left\{\tilde{\alpha}_{ij}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{\alpha}_{ji}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \tag{5.6.2}$$

which represent a much more profound concept with respect to the traditional symmetry (the symbol "$\overset{*}{=}$", in fact, does not represent an equality, but a simple *assignation condition*).

Such a Representation then allows us to show the *Generative Process* that leads the System to its Maximum Ordinality and, at the same time, to its Maximum Stability conditions, because it *restructures the internal relationships* between the various elements in such way as these show an additional "*Emerging*" *Property*, which is *initially* based on the following "topological" Relationships:

$$\tilde{\lambda}_{12} \oplus \tilde{\alpha}_{12}(t) \overset{*}{=} \tilde{\lambda}_{1j} \oplus \tilde{\alpha}_{1j}(t)$$

for

$$j = 3, \cdots, N \tag{5.6.3}$$

together with all their associated *incipient* derivatives, up to the order $N-1$

$$\left\{\overset{\circ}{\tilde{\lambda}}_{12} \oplus \overset{\circ}{\tilde{\alpha}}_{12}(t)\right\}^{\tilde{k}} \overset{*}{=} \left\{\overset{\circ}{\tilde{\lambda}}_{1j} \oplus \overset{\circ}{\tilde{\alpha}}_{1j}(t)\right\}^{\tilde{k}}$$

for

$$k = 1, \cdots, N-1 \tag{5.6.4}$$

where $\tilde{\lambda}_{ij}$ represent their corresponding *internal reciprocal* Correlating Factors, which are clearly distinct from the values of the initial conditions, because the latter are already included in the correlative expressions $\tilde{\alpha}_{ij}(t)$.

Such properties represent *the bases* of the previously mentioned Property of *Diffusive Generativity*, which is faithfully represented by the following *Harmony Relationships*

$$\left\{\tilde{\alpha}_{1,j+1}(t) \oplus \tilde{\lambda}_{1,j+1}(t)\right\} \overset{*}{=} \left(\sqrt[N-1]{\left\{\tilde{1}\right\}}\right)_j \circledR \left\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\right\}$$

per

$$j = 1, 2, 3, \cdots, N-1 \tag{5.6.5}$$

whose explicit *Process of Genesis* is illustrated in **Appendix A1**, while the asso-

ciated Ordinal Roots of Unity $\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_j$ are illustrated in **Appendix A2**.

If we now take into account the Harmony Relationships (5.6.5), together with their *specific structure* and the *correlative symbology* adopted, the Solution to the First Fundamental Equation pertaining to the System analyzed can be represented as follows

$$\{\tilde{r}\} = e^{\{\tilde{a}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_1,\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_2,\cdots,\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_{N-1}\right\}} \tag{5.6.6}$$

which reflects *the Self-Organization* of the Systems in terms of "*couples*", according to an *Internal Description*.

At the same time, it shows that the basic "*topological*" structure in terms of the reference couple "12" (see Equation (5.6.3)) has been correspondently "*transformed*" and, at the same time, "*updated*", as a consequence of the Diffusive Generative Process which leads to the Harmony Relationships that, as anticipated, result as being substantially structured in terms of *the sole reference couple* "12".

6. Explicit Solution to the Two Fundamental Equations of the M.O.P, Understood as *a Whole*

The MOP, considered in its two Fundamental Equations understood *as a Whole*, differently from the problems formulated in TDC, *always* presents an *explicit solution*. This is especially due to IDC and, in particular, both to the solution to the First Fundamental Equation in the form of Matrioska and the associated Harmony Relationships, which allow us to represent the System in the form of "couples", by assuming *one arbitrary couple* of elements as a reference.

So that, precisely because of such specific characteristics, the M. O. P. enabled us to reconsider and explicitly solve some "particular" problems, generally dealt with in literature in terms of TDC, which are generally considered as being "*unsolvable*", "*intractable*", "*with a drift*". The solutions of which ended up by showing that the Maximum Ordinality Principle has an extremely general validity [16] [21].

The *Explicit Solution* to the Two Fundamental Equations of the M.O.P, understood as a Whole, can be obtained by introducing the solution to the First Fundamental Equation (4.1), previously shown,

$$\{\tilde{r}\} = e^{\{\tilde{a}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_1,\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_2,\cdots,\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_{N-1}\right\}} \tag{5.6.6}$$

into the Global Feed-Back Process represented by the Second Fundamental Equation (4.2). The latter consequently transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{a}(t)\}} = e^{\{\tilde{B}(t)\}\circ\left\{\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_{13},\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_{14},\cdots,\left({}^{N-1}\sqrt{\{\tilde{1}\}}\right)_{1N}\right\}} \tag{6.1}$$

where

$$\tilde{B}(t) = \left\{\begin{pmatrix}\oplus\tilde{A}(t)\\ \Theta\tilde{A}(t)\end{pmatrix},\begin{pmatrix}\Theta\tilde{A}(t)\\ \oplus\tilde{A}(t)\end{pmatrix}\right\} \tag{6.2}$$

and

$$\tilde{A}(t)=\left\{\left\{\tilde{\alpha}_{12}(0)\right\}^{\{\tilde{2}/\tilde{2}\}}\oplus\left\{\tilde{\lambda}_{12}(0)\right\}^{\{\tilde{2}/\tilde{2}\}}\right\}\circ\left\{\left(\sqrt[N-1]{\{\tilde{1}\}}\right)^{\uparrow\{\tilde{N}/\tilde{N}\}}\right\}^{(\tilde{2}/\tilde{2})}\oplus\ln\left(\tilde{c}_1\oplus\{\tilde{c}_2,t\}\right)\quad(6.3)$$

where the term $\ln\left(\tilde{c}_1\oplus\{\tilde{c}_2,t\}\right)$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.

This latter contribution, as already anticipated, is particularly important for *the System stability*, especially when the System interacts with another System of its surrounding Habitat.

Equation (6.1), together with Equations (6.2) and (6.3), then represents *the Explicit* "*Emerging Solution*" *to the Maximum Ordinality Principle*, formulated in two "Incipient" Differential Equations (4.1) and (4.2), when the latter are properly considered *as being a Whole*.

7. General Validity of the Explicit Solution to the Maximum Ordinality Principle

Equation (6.1), considered with the associated Equations (6.2) and (6.3), has a *general validity* because, at the same time, it is *valid* not only for *non-Living* Systems, but also for *Living* Systems and *Human* Systems too.

What's more, the same fact that Solution (6.1) is *always an Explicit Solution* represents *a very general property* that evidently has a huge relevance from an *operative* point of view.

In addition, Solution (6.1) introduces some further fundamental novelties of *gnoseological nature*, which enabled us to clearly assert that "The 'Emerging Quality' of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (MOP), offers a *Radically New Perspective to Modern Science*" [21]. This is exactly what also suggested a possible of reformulation of such a Solution into a corresponding version in *operative terms*.

8. Explicit Solution to the MOP Reformulated *in Operative Terms* by Means an EQS Simulator

In order to have an explicit solution that may result much easier to be programmed on a computer and, in particular, on a simple PC, the previous Explicit Solution can be restructured in *operative terms*, in order to realize an "Emerging Quality Simulator" (EQS), which, however, is not "equivalent", by itself, to a traditional computer program. This is because, even if conceived for *operative finalities*, EQS always *keeps memory* of the genetic Ordinality of the Processes analyzed. So that the various forms of Ordinality, although considered in operative terms, will always be accounted for in terms of their "correlative associated cardinalities".

If we then suppose for example that the *Relational Space* of the System is represented by the following three generative coordinates $\left\{\tilde{\sigma},\tilde{\varphi},\tilde{\vartheta}\right\}$, characteristic of a "non-Living" System, the fundamental Relationships of EQS are

i. $$\tilde{\rho}_{1j}(t)=A\cdot e^{\tilde{S}_j(t)}\quad(7.1)$$

$$\tilde{S}_l(t) = \psi_{1,1} \cdot E_{l,1} \cdot \left[B_l \cdot \tilde{\Sigma}_0(t) - C_l \cdot \left(\tilde{\Phi}_0(t) + \tilde{\Theta}_0(t) \right) \right] \tag{7.1.1}$$

ii. $$\tilde{\varphi}_{1j}(t) = \psi_{1,2} \cdot E_{l,2} \cdot \left[B_l \cdot \tilde{\Phi}_0(t) + C_l \cdot \tilde{\Sigma}_0(t) \right] \tag{7.2}$$

iii. $$\theta_{1j}(t) = \psi_{1,3} \cdot E_{l,3} \cdot \left[B_l \cdot \tilde{\Theta}_0(t) + C_l \cdot \tilde{\Sigma}_0(t) + C_l \left(\tilde{\Phi}_0(t) + \tilde{\Theta}_0(t) \right) \right] \tag{7.3}$$

$$E_{l,i} = \frac{\varepsilon_{l,i} + 4\pi \cdot l}{N-1} \quad B_l = \cos\left(\sqrt{2} \cdot \psi_l\right) \quad C_l = D_l = \frac{1}{\sqrt{2}} \sin\left(\sqrt{2} \cdot \psi_l\right) \tag{7.4}$$

and

$$\psi_l = \psi_2 \cdot \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \tag{7.5}$$

in which:

1) $\tilde{\Sigma}_0, \tilde{\Phi}_0, \tilde{\Theta}_0$ synthetically represent the Ordinal coordinates of the *reference couple*, generally termed as "*couple 12*", which, on the other hand, can be arbitrarily chosen. So that the symbols $\Sigma_0, \Phi_0, \Theta_0$ stand for $\left\{\tilde{\sigma}_{12}, \tilde{\varphi}_{12}, \tilde{\vartheta}_{12}\right\}$.

Such coordinates, however, considered *in transient conditions*, will correspond to the solution to Equation (5.6.5), with reference to *the sole couple* "12".

Consequently, in adherence to the symbology previously adopted, those coordinates can be represented as $\Sigma_0(t), \Phi_0(t), \Theta_0(t)$;

2) The Ordinal factors $\psi_{1,i} \cdot E_{l,i}$ originate from the assumption that the Harmony Relationships, here reproduced for the sake of clearness

$$\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left(\sqrt[N-1]{\{\tilde{1}\}} \right)_j \circledR \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$$

for

$$j = 1, 2, 3, \cdots, N-1 \tag{5.6.5}$$

are modulated by the correlative Ordinal terms $\left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, which, apart from specific cases of given Habitat conditions, can be considered "null", because the *initial* topological "assignation" of the Correlative Factors is "Transfigured" by the *Diffusive Generative Process*.

In this respect, the terms $\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, after a previous *reduction of the Ordinality* $\left\{\tilde{2}/\tilde{2}\right\} \to 1$, are characterized by three different periodicities $E_{l,i} = \frac{\varepsilon_{1i} + 4\pi l}{N-1}$, each one *specific for each coordinate*, which originate from the explicit expression of the Ordinal Roots of *Unity* and, at the same, are also characterized by the specific factors $\psi_{1,i}$;

3) In fact, after having rewritten the Ordinal Relationships in the following form

$$Exp\left\{\tilde{\sigma}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\right\} \overset{*}{=} Exp\left[\left(\sqrt[N-1]{\tilde{1}} \right)_l \circledR \left\{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\right\} \right] \tag{7.6}$$

4) And after having assumed the explicit expression of the *Ordinal Roots of Unity*, illustrated in **Appendix A2** (Equations (A2.5) and (A2.6)), here explicitly

recalled for the sake of clarity

$$\mathrm{gbt}\left(\sqrt[N-1]{\tilde{1}}\right)_l^* = Exp\left\{\tilde{\alpha} \circledR \tilde{i} \oplus \tilde{\beta} \circledR \tilde{j} \oplus \tilde{\gamma} \circledR \tilde{k}\right\} \tag{A2.5}$$

where

$$\alpha = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \quad \beta = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \quad \gamma = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1} \tag{A2.6}$$

the expansion series of Equation (A2.5), together with the contextual adoption of the Rules of the Ordinal Product (5.1.3), (5.1.4), (5.1.5), leads to the Ordinal Relationships (7.1), (7.1.1), (7.2), (7.3), initially introduced, with the associated coefficients given by Equations (7.4), (7.5).

For the sake of completeness, it is worth adding that:

- The symbol $\left\{\tilde{1}\right\}$ represents *the Unity of the System* (understood as *a Whole*) by means the representation of the *Unity* of its *Proper Space of Relations*;

- $\varepsilon_1, \varepsilon_2, \varepsilon_3$ characterize the spatial orientation of the System as a Whole, with reference to its Ordinal Proper Space and, more specifically, with respect to the Reference "Couple 12";

- The "periodicity" of the "spinor" $\tilde{i}$ is assumed equal to 4π, because it is expressed in steradians;

- While the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians, because these spinors are always "orthogonal", both between them and with respect to the spinor $\tilde{i}$. An "orthogonality" that can be seen as a form of reciprocal "irreducibility" (as also indicated by the same Relational Products);

- While the Factor "$\tilde{A}$" represents an *Internal Ordinal Factor* according to which all the *radial* "*Uniances*" of the various Couples are appropriately referred to the *radial* "*Uniance*" of the Reference Couple "12". This latter concept will clearly be illustrated in a next section specifically titled "*Distance and Uniance*".

At the same time, by means of the *Internal Ordinal Factor* "$\tilde{A}$", the cardinalities "associated" to the various "*Uniances*" are all expressed in terms of a desired scale of measure.

9. General Considerations on the Explicit Solution Reformulated in *Operative Terms* by EQS

From the previous exposition, it should result as being evident that the *Harmony Relationships* (further illustrated in **Appendix A1**) represent an "Irreducible Excess". That is an "Exceeding" manifestation of the *Generativity of the System*, where the latter is at the same time *Self-Organizing*, of *Ordinal Nature*, and understood as *a Whole*.

This means that *the same Explicit Solution* reformulated in *operative terms*, precisely because obtained through an Ordinal Deductive Process from the Harmony Relationships and the Ordinal Roots of Unity (further illustrated in **Appendix A2**), represents an "*Emerging Solution*" from the Maximum Ordinality Principle.

Consequently, even if the single Relationships refer to each single couple "*1j*", and thus to the three "distinct" variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, the latter do not represent

a simple traditional "vector", but an "*Ordinal vector*". That is a *unique and sole* Relational Entity, which is usually represented in *curly brackets*, such as $\left\{\tilde{\rho}_{1j},\tilde{\varphi}_{1j},\tilde{\vartheta}_{1j}\right\}$, precisely because it is understood as *a Whole*.

This means that the three variables $\tilde{\rho}_{1j},\tilde{\varphi}_{1j},\tilde{\vartheta}_{1j}$, although recognizable as being "distinct", they are *not conceptually* "*separable*" *between them*.

Such an assertion is also even truer (and in particular way) with reference to the *various triples of variables* pertaining to *all the couples which compose* the System, which *a fortiori* are not conceptually "separable" between them precisely because the System is understood *as a Whole*.

In other words, the Fundamental Relations pertaining to EQS previously shown do not only furnish the $N-1$ single *Ordinal vectors* $\left\{\tilde{\rho}_{1j}(t),\tilde{\varphi}_{1j}(t),\tilde{\vartheta}_{1j}(t)\right\}$ that characterize each single couple of the System, but they also represent, even more, a *Unified Ordinal Description* of the System understood as Whole.

In other terms, the coordinates furnished by the *Operative Solution* are not conceptually "separable" between them, *neither with reference to each single couple*, *nor* with reference to *all the various couples* of the System as a Whole.

This leads us to point out another important aspect always in the Light of the Maximum Ordinality Principle.

9.1. Distance and "Uniance"

A direct and correlative consequence is that, even if at a "preliminary and intuitive" interpretation, such Ordinal Relationships could be thought as giving the "distances" between the various couples of the System analyzed, in reality, in adherence to the M.O.P, such an interpretation (and the corresponding "terminology"), should be substantially modified. In particular, by adopting a more appropriate term, such as "*Uniance*", instead of that of "distance".

This is because, as already anticipated, the concept of "*distance*" tends more to *divide*, than to *unify*. In fact, the same *etymology* of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

Consequently, in an Ordinal Perspective the term "distance" should preferably be replaced by a different term, possibly able to indicate the concept of "*union*" of two elements, more than their "distance".

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" perspective represents a "*dis-tance*", in an Ordinal Perspective indicates a "*uni-ance*". That is, it indicates that the two elements form "*one sole thing*" of *Ordinal Nature*, precisely because they are the Exit of the same Generative Process. So that the term "*Uniance*" expresses an *Ordinal concept*, and not a mere cardinal concept, such as that of "distance". Any "*Uniance*", in fact, is characterized by *its own Ordinality*.

As a simple example, let us think of a couple of elements $\tilde{\alpha}_{ij}(t)$ whose Relationship is characterized by a *Binary-Duet* Ordinality $\left\{\tilde{\alpha}_{ij}(t)\right\}^{\{2/2\}}$. Such a specific and proper *Ordinality* is precisely that which represents *the Ordinal* "*Unity*"

between two elements of the System. While, at the same time, its "associated cardinality" only indicates their topological distribution in the Relational Space of the System.

Consequently, when all the various "*Uniances*" are considered in the context of the Harmony Relationships, they reveal that the System is a *Whole of Ordinal Nature*, in perfect adherence to the Maximum Ordinality Principle.

In addition, such an assertion also has an *even more general sense*, that is: it is precisely the *Generativity* of the Self-Organizing System the one which, with its proper *Diffusivity*, characterizes all the elements of the System in terms of "Ordinal Relationships". In that sense, such Ordinal Relationships are all of *genetic nature*, like in the case of "brothers".

"Brothers", in fact, are termed as such not because of their "direct reciprocal relationships", but because of their *direct reference* to *the same genetic principle*: their father (or their mother or both).

Consequently, in perfect "*Adherence*", the term "*Uniance*" *synthesizes* the concept of an *Ordinal Unity of Genetic Nature*.

9.2. *Proper Space* and *Proper Time* of a Self-Organizing System

Another important aspect that has to be underlined, always in the Light of the Maximum Ordinality Principle, is precisely that synthetically indicated in the title.

The Maximum Ordinality Principle, in fact, shows that Each Self-Organizing System, precisely because characterized by its own "Emerging Quality", evolves in a "*time*" and a "*space*" which are *exclusive and specific* of each System analyzed. Consequently, the latter can be more faithfully termed as "*Proper Time*" and "*Proper Space*" of the System [25].

This is an aspect that is radically different from the case of the Traditional Scientific Approach, in which *time* and *space* are assumed as being *absolute*.

Such a difference, however, *does not represent a real* "*obstacle*" with reference to the interpretation of the output of EQS Simulator. What is important, in fact, is to know that such a "difference" *exists* and, at the same time, to be aware of their correlative *different Nature*. In this case, in fact, such a "difference" can always be dealt with in perfect analogy with the "reduction" of the *Uniances*, when the latter have to be compared with the correlative *distances*.

In addition, such a "difference" is so specific and characteristic of the Self-Orgnizing Systems, that it cannot even be "reduced" to the *space-time* conception of General Relativity.

General Relativity, in fact, introduces the concept of "*space contraction*" and "*time dilatation*" between two reference systems in a reciprocal movement, according to the following relationships [8]

$$\Delta x' = \Delta x \cdot \sqrt{1 - V^2/c^2} \tag{9.2.1}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}} \tag{9.2.2}$$

It is then possible to show that Einstein's "*space-time conception*" represents a *particular modality* at introducing the concept of the *second order* "*incipient*" *derivative* [25]. Such a particular modality, however, manifests itself at *a simple cardinal level*, corresponding to a "reduction" process of the *Proper Space* and *Proper Time* of a given System.

This means that Einstein's *space-time conception* in reality corresponds to the introduction of the *second order* "*incipient*" *derivative*, considered, however, at its mere "*cardinal level*" [ib.].

10. Two "*Com-Possible*" Scientific Approaches, Albeit "not Equivalent" between Them

The two above mentioned Scientific Approaches, with their corresponding formal languages, TDC and IDC, respectively, when considered with reference to their corresponding "presuppositions" (that is the subjacent "way of thinking") result as being two different descriptive modalities which are always "*com-possible*". In the sense, that they *do not exclude each other*. They simply *co-exist*.

This is because, as already anticipated, the Traditional Scientific Approach, which leads to TDC, *cannot exclude* (in principle) the adoption of different mental categories and their corresponding formal language (e.g. IDC), because of the *absence* in its presuppositions (especially "necessary" logic) of any form of *perfect induction*.

On the other hand, the same happens in the case of the adoption of IDC, precisely because of the *same reason*, although IDC is based on mental categories characterized by a different form of Logic (e.g. the "Generative" Logic).

Consequently, the two formal languages, TDC and IDC, can *always* be adopted independently from one another. Although this "com-possibility" does not mean that they are "equi-valent" between them (as it is clearly shown by the case of the "Three-body Problem").

Their "in-equivalence", in fact, can easily be shown by comparing the different *consequences* of their respective adoption, when such consequences are obviously considered in the light of their corresponding "mental categories".

In fact, *beside* the Traditional Scientific Approach, which affirms that "Every System is a *mechanism*" (at a phenomenological level), there is also the possibility of a different Approach, according to which "*Every System is a Self-Organizing System*" (always at a phenomenological level). This is the fundamental reason why they lead to the adoption of *two corresponding different formal languages*, with some associated important consequences.

In the first case, in fact, the adoption of TDC leads to:

1) *Unsolvable Problems* in explicit formal terms (as in the case of the "Three-body Problem");

2) *Intractable Problems* even by adopting the most advanced computers (as in the case of Protein Folding);

3) *Problems characterized by experimental* "*drifts*", which always represent an indication of possible "side effects";

4) In addition, it is worth pointing out that TDC can present some "side ef-

fects" even in the case of accurate experimental confirmations. Such "side effects", in fact, can result as being "*masked*" by the same fact that all the experimental confirmations are always based on the adoption of *methods*, *instrumentation and measurements* that are conceived (and designed) in a perfect conformity with the fundamental presuppositions of TDC [21].

Vice versa, the adoption of IDC does not present such problems, whereas, in turn, it presents several advantages.

In fact, as already anticipated, the adoption of IDC is finalized to describe the "Emerging Quality" of "Self-Organizing Systems". This leads to the formulation of the MOP, which is able to offer *a radically New Perspective* to Modern Science. That is: "*Every System is a Self-Organizing System*" (see **Table A1**).

This is because IDC results as being the most appropriate language able to describe the fundamental characteristics of "Self-Organizing Systems". In fact, the "Incipient Differential Calculus" (IDC):

1) is able to represent, in appropriate formal terms, the "Emerging Quality" of Self-Organizing Systems as an "*Irreducible Excess*";

2) In this way, IDC enabled us to formulate a very general Principle, the Maximum Ordinality Principle (MOP), which can be understood as "*One Sole Reference*" Principle [16];

3) The latter in fact results as being valid *in any field* of analysis (from *non-living* Systems, to *living* Systems and *human social* Systems too);

4) In addition, the adoption of IDC *always* leads to *explicit formal solutions* (such as in the case of the "*Three-body Problem*");

5) At *any topological scale* (e.g. from atoms (*Quantum Mechanics*) to Galaxies (*Celestial Mechanics*));

6) Both *under steady state and variable conditions*;

7) What's more, the corresponding Solution to *any* mathematical model based on the MOP (and thus formulated in terms of IDC) always results as being an "*Emerging Solution*". That is, a Solution whose *Ordinal Information content* is always *much higher* than the Ordinal content corresponding to the initial formulation of the problem;

8) As a direct consequence, this leads to the fact that any "Emerging Solution" *can never be reduced* to mere "functional relationships" (as previously shown in the case of the "Three-body Problem");

9) This is also means that the adoption of IDC *does not require any* specific reference to the traditional Physical Laws or to the well-known Thermodynamic Principles (precisely because the latter are always understood as "functional relationships"). In this respect, see also previous Paragraph 8.1, concerning *the Relationship* between "*Forces*" and "*Diffusive Generativity*";

10) Finally, the adoption of IDC never leads to "side effects". This is because, even when an "Emerging Solution" might manifest some related "Emerging Exits" [21], the latter can always be interpreted as being corresponding "Extra Benefits", initially not recognized as such. This leads to point out another fundamental aspect, always in the Light of the Maximum Ordinality Principle and the

correlative maximization of the Ordinality of its associated Habitat.

11. More General *In-Equivalence* between the Two Scientific Approaches, Especially with Reference to the Relationships between Man and the Environment, in Any Field of Analysis

Although from a general point of view the *in-equivalence* between the two formal languages can preliminarily be recognized at the level of "Thinking", such an in-equivalence is even much more marked at the level of "Decision Making and Acting". Especially when considering, as a basic reference criterion, the corresponding different concepts of "inter-relationships" *between Man and the Environment* [21].

This is because the adoption of TDC always "reflects" the general idea that "every system is a *mechanism*", while the "com-possible" formal language IDC is always orientated at describing any system as a "*Self-Organizing System*". This is the fundamental reason for the adoption of the three *new mental categories* (shown in **Table A1**), which are radically different from the three basic presuppositions of the former.

This easily leads to recognize that the most profound "in-equivalence" between TDC and IDC situates at the level of Decision Making and Acting, in particular with respect to *the Environment*. In fact:

1) At the level of "Decision Making", the two formal languages will evidently lead to make decisions (that will become consequential future *actions*) in a perfect *conformity* with their respectively different way of thinking: TDC, in conformity with its "*aprioristic*" presuppositions; IDC, vice versa, in conformity with the *new mental categories* that, on the contrary, are adopted "*a posteriori*".

Consequently, in both cases the two formal languages will suggest "decisions" in perfect *conformity* with their corresponding concepts of "*surrounding habitat*": understood as a "*set of mechanisms*", in the case of TDC or, respectively, as "*a unique Self-Organizing System*" in the case of IDC [21].

2) At the level of Action, however, it is exactly *where* it is possible to recognize the most marked differences between the two Scientific Approaches. This is because, in such a case, the specific different *origin* of each formal language, together with the associated *powerful expressive capacity that any formal language is able to manifest*, represent the fundamental aspects that systematically "guide" (sometimes even "force") the research for specific *practical solutions* to the various problems and their subsequent actual implementation, in particular with respect to the Environment.

In other terms, the profound differences between the two Scientific Approaches, characterized by their corresponding formal languages, TDC and IDC, respectively, become particularly evident at the "level of Action", because the corresponding formal solutions *become consequential facts*.

In this respect, the Ostensive Examples previously considered in the various Biennial Emergy Conferences (from 2010 to 2020), are sufficiently clear to show the profound differences that may result, *in practice*, when adopting the one or

the other descriptive formal language.

In addition, *an ulterior* and more *radical form* of *in-equivalence* will be analyzed in the next paragraph.

12. Radical *In-Equivalence* between Falsification and Relaunce

Another aspect that points out even more clearly the *in-equivalence* between the Traditional Approach and the Ordinal Approach is the fact that the first one is characterized by "*confirmation*/*falsification*" processes whereas the second one is characterized by "*Emerging Exits*".

The "confirmation" processes, in fact, are strictly necessary in the case of Traditional Theories, which are adopted "*a priori*", and are specifically based on those mental categories previously recalled. In particular, *necessary logic*.

At the same time, *the absence of experimental confirmations* of the corresponding conclusion of Traditional Theories represents a valid argumentation for their "*falsification*" (according to Popper's Falsification Principle).

On the contrary, the Ordinal Approach based on the "Emerging Quality" of Self-Organizing Systems, strictly speaking *does not require* correlative "confirmation processes" in order to be accepted as being a "valid" Approach.

This is because the Ordinal Approach is adopted "*a posteriori*", that is downstream the recognition of the *Manifestation* of Quality as an "Irreducible Excess", and consequential adoption of the new correlative Mental Categories.

So that, the research for the "*Maximum Adherence*" of the correlative Over-Deductions (in Generative Logic) to experimental results, does not represent, properly speaking, the research for a "confirmation". But, paradoxically, it represents the "confirmation" of a "*denial*". Or better, "a confirmation" that can be termed as being "*not less than*".

In fact, it is exactly such circumstance the one that properly generates the concept of *Relaunce*.

The latter in fact consists in recognizing that the description of the "Emerging Quality", as performed at a preliminary given stage, if characterized by "*Emerging Exits*", can be recognized as being "not less than". That is, the description can be re-proposed at a *Higher Level of Ordinality* with respect to the one initially supposed and assumed to describe the Process (or Phenomenon) analyzed.

At this stage, the profound "*in-equivalence*" previously shown between the two formal languages, which mainly and clearly manifests at the level of "facts", may suggest, as a possible conclusion, the consideration of an extremely important question: "where are we going", as a consequence of the adoption of *one* or *the other* descriptive formal language: TDC or IDC?

13. Conclusion: Where Are We Going?

The afore-mentioned differences between the two Scientific Approaches and their correlative formal languages, TDC and IDC, which can preliminarily be recognized at a gnoseological level and, even more, at the level of their respective *practical* consequences, enable us to draw some general conclusions that can be synthetically summarized as follows.

From a general point of view, in fact, it is possible to delineate three possible answers to the previous question:

1) Modern Science is so radically rooted in TDC (and in its corresponding presuppositions) that it is extremely improbable to hypothesize, in spite of the afore-mentioned intrinsic *limitations* of such a formal language, a rapid change of the corresponding paradigm (as, for example, the same case of the "Three-body Problem" would suggest).

In this sense, we have to expect a generalized persistence in the adoption of the traditional formal approach (TDC);

2) This fact, however, does not prevent from thinking that some Scientists, with reference to some specific problems (for instance related to the "Three-body Problem"), will decide to *preferentially* adopt the innovative IDC approach;

3) Even if, more probably, because of the afore-mentioned "*com-possibility*" between TDC and IDC, it may be expected the adoption of both formal approaches *at the same time*, so as to choose the optimal *operative* solutions on the basis of the corresponding experimental results.

By always taking into account, however, that TDC translates, in formal terms, a "*self-referential*" gnoseological approach, while IDC represents, always in formal terms, a "*hetero-referential*" gnoseological approach (as previously illustrated and synthetically summarized in Table A1).

Appendix A1. Process of Genesis of the Harmony Relationships from a Diffusive Generativity

Appendix A1 points out, in more explicit terms, what synthetically previously asserted, that is: the Harmony Relationships represent, by themselves, an "*Emerging Solution*" which, in addition, is also "*Exceeding*" with respect to the Solution to the First Fundamental Equation.

In fact, what we presented at Paragraph 5.6 of **Appendix A** are nothing but the *basic presuppositions* for the formulation of the Harmony Relationships, which, however, do not represent a "*necessary consequence*" of those presuppositions, because they manifest an "*Extra*", or better, an "*Irreducible Excess*" with respect to them.

Let us thus recall the basic elements that will enable us to show that the Harmony Relationships precisely represent an "Emerging Extra" of *Generative Nature*.

We have seen in fact that the Emerging Solution to the First Fundamental Equation allows us to write the following *topological* "*Assignation* Relationships"

$$\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{\alpha}_{1_j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1_j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$$

for

$$j = 3, 4, \cdots, N \tag{A1.1}$$

and, at the same time, their corresponding *topological* "Assignation Relationships", written in terms of "Incipient" Derivatives in the form

$$\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}\overset{*}{=}\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}$$

for

$$k=1,2,\cdots,N-1 \tag{A1.2}$$

in which, for simplicity of notation, the Ordinalities $\{\tilde{2}/\tilde{2}\}$, which appear in Equation (A1.1), are considered as being included in the symbols of the quantities to which they refer to.

More specifically, Equations (A1.2) cannot be interpreted as a "necessary consequence" of Equations (A1.1), because the latter are obtained on the basis of "Incipient" Derivatives. Consequently, they are all of *Generative Nature*.

In fact, if rewritten in the following form

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}}\overset{*}{=}\tilde{1}$$

for

$$k=1,2,\cdots,N-1 \tag{A1.3}$$

they allow to assert that the considered System is already characterized by a proper and specific "*Interior Unit*", of *Generative Nature*, formally represented by the symbol "$\tilde{1}$".

Such a "Unity", however, is still in the form of "*Not Less Than*". This is because:

- In a Generative Contest, they are certainly not *the parts* that, through the Relationships "*between*" them, give "Origin" to the "Excess of Unity".

- Because it is exactly true *the opposite*: in fact, it is the *Generative Unit* of the System that, with its *proper* "*Excess*", *Qualifies* the Relationships "*between*" *the parts*.

Consequently, the most Adherent Formulation of the Self-Organizing Generative Process is that which can be obtained by re-proposing Equations (A1.3) in the form

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}}\overset{*}{=}\{\tilde{1}\}\quad\text{for}\quad\forall k \tag{A1.4}$$

or better, even more properly, as follows

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}}\overset{*}{=}\{\tilde{1}\}^{\frac{1}{(\tilde{N}-1)}}\quad\text{for}\quad j=2,\cdots,N \tag{A1.5}$$

in which the symbol $\{\tilde{1}\}$ now formally represents the *Generative Whole*, which,

at the same time, is *Self-Organizing* and of *Ordinal Nature*. While its *unique* and *sole* exponent $1/(\tilde{N}-1)$ explicitly represents the fundamental concept previously anticipated, that is: it is the "Whole", with its *proper* Generative "*Excess*", the one that properly "*Qualifies*" the Relationships "*Between*" *the parts*.

This is obviously true not certainly in the sense of Relationships understood "two by two", but as the specific Reflex of an *Ordinal Unit*, which, in any case, represents an "Irreducible Excess" with respect to the simple "composition" of the single "parts".

Consequently, Relation (A1.5), can also be written in the form

$$\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{*}=\left\{\tilde{1}\right\}^{\frac{1}{\{\tilde{N}-1\}}}\circ\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}\quad\text{for}\quad j=2,\cdots,N \tag{A1.6}$$

which, reinterpreted in terms of "*Progenitor Relationships*", finally leads to the formal expression of the Harmony Relationships. The latter, written in the form

$$\left\{\tilde{\alpha}_{1,j+1}(t)\oplus\tilde{\lambda}_{1,j+1}(t)\right\}^{*}=\left(\sqrt[\tilde{N}-1]{\left\{\tilde{1}\right\}}\right)_{j}\circledR\left\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\right\}$$

for

$$j=1,2,\cdots,N-1 \tag{A1.7}$$

clearly show that the *Diffusive Generativity of the System* "updates", *by Assignation*, *all the couples* at the first member, and, contextually, the *same reference couple* "12".

Equations (A1.7) then clearly show that all the elements of the Ordinal Matrix (5.6.1) can be obtained on the basis of *one sole couple* $\tilde{\alpha}_{ij}(t)$ assumed as reference and their associated Correlating Factors.

In this respect, it is also worth noting that condition (A1.2) is properly the one that represents the *fundamental presupposition* of what could be termed as an *Intensive Whole*, precisely because of the "*consonance*" between all the generative derivatives up to the order $N-1$, that are due to the "*Diffusive Generativity*" of the Self-Organizing System.

This is the specific reason why, by means of the M. O. P., and its correlative Harmony Relationships, it was possible to reconsider some "particular" problems that, in the Traditional Scientific Literature, are generally known as being "*unsolvable*" (such as, for example, the "Three-body Problem"), or "*intractable*", or "*with a drift*". Whose solutions ended up by showing that the Maximum Ordinality Principle has an extremely general validity [16]?

Appendix A2. The Ordinal Roots of Unity $\left\{\tilde{1}\right\}$ and Their Role in the Harmony Relationships

In this respect, it is worth observing that previous Relationships (A1.7) are written in such a form only for reasons of clarity and exposition simplicity. In such a form, however, it could seem that the various elements that characterize the System are "still" related, "between" them, according to Relationships of the type

"two by two".

In reality, if one makes explicit the term $\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j$ according to its more specific meaning, that is as $\left\{\tilde{1}\right\}^{\frac{1}{\{\tilde{N-1}\}}} \equiv \left\{\tilde{1}\right\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}}$, in which $N-1$ refers to the cardinality, while $\left(\tilde{N-1}\right)$ refers to the Internal Ordinal $(N-1)$-ary Relationship, it is possible to more appropriately write (by pointing out the Ordinalities $\left\{\tilde{2},\tilde{2}\right\}$, previously underwritten)

$$\left\{\tilde{\alpha}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{1}\right\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{(A2.1)}$$

that is, even more explicitly, in the form

$$\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \begin{pmatrix} \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_1 \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_2 \\ \vdots \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_{N-1} \end{pmatrix} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{(A2.2)}$$

from which it is possible to recognize that the single "cardinal" values that in Equation (A1.7) appear as they were "distinct", and, in addition, as being "separated", in reality they are the *Reflex of an Ordinal Unit* that transcends them, and relates them in the form of an $(N-1)$-*ary* Relationship.

This is the aspect that (more than others) clearly manifests that *the Harmony Relationships* represent an "Excess" with respect the initial Assignation Relationships (5.6.3) and (5.6.4).

In addition, as far as the "explicit" meaning of the Ordinal Routs of Unity is concerned, previously synthetically indicated in the form

$$\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j \quad \text{per} \quad j=1,2,3,\cdots,N-1 \quad \text{(A2.3)}$$

it is worth expressly pointing out that the symbol $\left\{\tilde{1}\right\}$ represents the *Unity of the System* (understood as a *Whole*), with specific reference to the *Unity of its Proper Space* (as well as its *Relational Space*).

Such a Fundamental Unit can be then expressed by the following Relationship

$$\left\{\tilde{1}\right\} = e^{\left\{\alpha \circledast \tilde{i} \oplus \beta \circledast \tilde{j} \oplus \gamma \circledast \tilde{k}\right\}} \quad \text{(A2.4)}$$

Consequently, the Ordinal Roots $\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_l$ will be represented in the following form

$$\left\{\tilde{1}\right\}_l = e^{\frac{\left\{\alpha \circledast \tilde{i} \oplus \beta \circledast \tilde{j} \oplus \gamma \circledast \tilde{k}\right\}}{N-1}} \quad \text{(A2.5)}$$

where:

- $\tilde{i},\tilde{j},\tilde{k}$ are the fundamental spinors of the Relational Space, understood in

their more general sense, that is, as the specific foundation of any given System;

- α, β, γ are respectively equal to

$$\alpha = \varepsilon_1 + \frac{4\pi \cdot l}{N-1} \quad \beta = \varepsilon_2 + \frac{2\pi \cdot l}{N-1} \quad \gamma = \varepsilon_3 + \frac{2\pi \cdot l}{N-1}; \tag{A2.6}$$

- Where the "periodicity" of the "spinor" $\tilde{i}$, as we already know, is equal to 4π, because expressed in *steradians*;

- While the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians (each), because these spinors are always "orthogonal", both between them, and with respect to the spinor $\tilde{i}$ (an orthogonality that can be understood, inter alia, as a form of reciprocal "irreducibility");

- The quantities ε_1 ε_2 ε_3 represent specific "parameters" of the *Relational Space* each time considered, with specific reference to the "couple 12".

Sometimes (for example in the case of Protein Folding), for an easier "topological" representation, Equations (A2.6) can also represented as

$$\frac{\alpha}{N-1} = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \quad \frac{\beta}{N-1} = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \quad \frac{\gamma}{N-1} = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1} \tag{A2.7}$$

which however can always re-proposed in the previous form (A2.6) through an appropriate choice of the parameters ε_1, ε_2, ε_3.

On the basis of the previous exposition, it should be clearer that *The Ordinal Roots of Unity* $\{\tilde{1}\}$ contribute to manifest that the associated *Harmony Relationships* represent an "*Irreducible Excess*", that is an "*Exceeding*" Manifestation of a *Generative System*, which, at the same time, is *Self-Organizing*, of *Ordinal Nature*, and, above all, it is understood as *a Whole from the very beginning*, and not vice versa.