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A Comparative Analysis of the New $-3(-n) - 1$ Remer Conjecture and a Proof of the $3n + 1$ Collatz Conjecture

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Abstract

This scientific paper is a comparative analysis of two mathematical conjectures. The newly proposed $-3(-n) - 1$ Remer conjecture and how it is related to and a proof of the more well known $3n + 1$ Collatz conjecture. An overview of both conjectures and their respective iterative processes will be presented. Showcasing their unique properties and behavior to each other. Through a detailed comparison, we highlight the similarities and differences between these two conjectures and discuss their significance in the field of mathematics. And how they prove each other to be true.

Keywords

$-3(-n) - 1$ Remer Conjecture, $3n + 1$ Collatz Conjecture, Comparative Analysis, Proof, Natural Numbers, Integer Sequences, Factorial Processes, Partial Differential Equations, Bounded Values, Collatz Conjecture, Collatz Algorithm, Collatz Operator, Collatz Compliance, And Mathematical Conjectures

1. Introduction

The $-3(-n) - 1$ Remer Conjecture, newly proposed by the author in 2023 is a mathematical sequence which involves iterating an algorithm on negative integers. In contrast, the $3n + 1$ Collatz Conjecture, proposed by Lothar Collatz in 1937, involves iterating an algorithm on positive integers. In this paper, we present a comparative analysis of these two conjectures, examining their properties, behaviors, and how they prove each other to be true [1] [2] [3].

2. Overview of the $3n + 1$ Conjecture

The $3n + 1$ conjecture, also known as the Collatz Conjecture, is defined by an iterative process on positive integers. Given a positive integer N, the algorithm involves two steps. If N is odd, multiply it by 3 and add 1, if N is even, divide it by 2. Repeat this process iteratively until N becomes 1 (**Table 1**).

The conjecture states that for any positive integer N, this iterative process will always reach the value of 1 eventually [1]. Despite extensive computational evidence supporting the conjecture, a proof for its validity has remained elusive to date [2].

2.1. Properties and Behavior of the 3*n* + 1 Conjecture

The $3n + 1$ conjecture exhibits several interesting properties and behaviors. Empirical evidence suggests that the conjecture holds true for a vast majority of positive integers, with the iterative process eventually reaching the value of 1.

2.2. Previous Research on the 3*n* + 1 Collatz Conjecture

The $3n + 1$ Collatz Conjecture has been the subject of extensive research [1] [2] [3]. Numerous others have been employed to study the conjecture. A conclusive proof or disproof of the Collatz Conjecture has not been found to date. However, within these efforts and efforts of many others, they have provided a means for alternative solutions to be discovered to prove the Collatz conjecture. They verify each in their own way that there is a solution that just has not been discovered previously to date.

3. Overview of the New −3(−*n*) − 1 Remer Conjecture

The $-3(-n) - 1$ conjecture, a new mathematical sequence, involves iterating a formula on negative integers. Given a negative integer N, the algorithm involves applying the formula $-3(-n) - 1$ iteratively to generate a sequence of numbers [3]. For example, if we start with $n = -1$, the sequence will start as $-3(-1) - 1 = 2$. Change the positive result to a negative integer, −2. Then the next term in the sequence would be −2. Rewrite and calculate as $-3(-2) - 1= 5$. Change the positive result to a negative integer, −5. The next term in the sequence would be −5. Rewrite and calculate as $-3(-5) - 1 = 14$ and so on. The result will continue to INCREASE to infinity(k). 2, 5, 14 … If you start at a different beginning number say −7. $-3(-7) - 1$ would result in a positive 20. $-3(-20) - 1 = 59$, 7, 20, 59 … Start at $-3(-3) - 1 = 8$ then $-3(-8) - 1 = 23$, $-3(-23) - 1 = 68$, $-3(-68) - 1 = 203$ … to infinity (**Table 2**).

Table 1. Example for Collatz conjecture starting at n = 10.

$n = 10 \div 2 = 5$
$n = 5 \times 3 = 15 + 1 = 16$
$n = 16 \div 2 = 8$
$n = 8 \div 2 = 4$
$n = 4 \div 2 = 2$
$n = 2 \div 1 = 1$
$n = 1$

Table 2. Example for Remer Conjecture starting at $n = (-1)$.

$-3(-1) - 1 = 2$
$-3(-2) - 1 = 5$
$-3(-5) - 1 = 14$
$-3(-14) - 1 = 41$
$-3(-41) - 1 = 122$
$-3(-122) - 1 = 365$

4. Comparative Relation of Conjectures to Each Other with Comparison Table

The behavior of the $-3(-n) - 1$ conjecture is newly proposed and to be the proof for the Collatz Conjecture [1]. "If a similar, general relationship was to be discovered such that each positive integer n becomes a number less than itself, one can prove the Collatz conjecture." One interesting property of the sequence within the Remer Conjecture is that it starts with a number less than itself (less than the result) and is always increasing, which can be proven by performing the operations within the sequence [1]. If we assume that the sequence is increasing for all values of N to infinity, therefore we can show that it is also increasing. The Collatz Conjecture and The Remer Conjecture are related to each other. One compliments the other in its sequences and form in reverse. The $3n + 1$ conjecture deals with positive integers, whereas the $-3(-n) - 1$ sequence deals with negative integers. Additionally, the $3n + 1$ conjecture involves an iterative process that attempts to reach the value of 1, whereas the $-3(-n) - 1$ sequence attempts to reach an infinite positive value. To gain a better understanding of the behavior of both sequences, we can analyze them in more detail. For the $3n + 1$ conjecture, it has been shown that for all starting values of N up to 2^{60}, the process eventually reaches the value of 1. However, despite extensive research, a proof for the conjecture has not been previously found to date. The behavior of the $-3(-n) - 1$ conjecture is newly proposed to be the proof for the Collatz Conjecture [1] [2] [3]. Proportionally to the beginning value of N, the Collatz Conjecture and The Remer Conjecture are related to each other. One compliments the other in its sequences and form of operations in opposite ways. As if we say $3 \times 9 = 27$, we can check that by, $27/9 = 3$ and or $27/3 = 9$. They are proofs of each other [1] [2] [3]. In terms of similarities therefore, there is a connection between the $3N + 1$ Collatz Conjecture sequence and the $-3(-n) - 1$ Remer Conjecture sequence. The $-3(-N) - 1$ Remer Conjecture states that $-3(-n) - 1$ Take any negative number and insert as $-N$. calculate. problem and answer is positive. Take the positive answer and change to a negative and reinsert as $-N$. Then continue calculation. Answer will be positive. Again, take positive answer change to negative and reinsert as $-N$ and further calculate. The result will be positive and will always INCREASE as a positive number to infinity. This property is connected to the Collatz Conjecture, and we show that the behavior of the Remer Conjecture provides the proof of the Collatz Conjecture sequence (**Table 3**).

Table 3. Side by side process Collatz Conjecture starting at $n = 10$ and Remer Conjecture starting at $n = (-1)$.

Collatz	Remer
$n = 10/2 = 5$	$-3(-1) - 1 = 2$
$n = 5 \times 3 = 15 + 1 = 16$	$-3(-2) - 1 = 5$
$n = 16/2 = 8$	$-3(-5) - 1 = 14$
$n = 8/2 = 4$	$-3(-14) - 1 = 41$
$n = 4/2 = 2$	$-3(-41) - 1 = 122$
$n = 2/1 = 1$	$-3(-122) - 1 = 365$
$n = 1$	$n = 365$

Our proof is based on the dynamics of the function of $-3(-n) - 1$ and the conjectures ability to always create an INCREASING positive result (**Table 2**), which is in direct contrast to the Collatz Conjecture sequence [1]. But is a proof within itself for each other. Incorporating a novel use of combinatorial techniques *i.e.*, changing the positive result into a negative and reinserting as −N. which allows the main result. The Collatz Conjecture will always find its way back to 1 using positive numbers as shown by **Table 1**. The Remer Conjecture will always produce a positive result using only negative numbers to infinity(k), as shown by **Table 3**. These findings have implications for the study of dynamical systems, sequences, relationships between them, and number theory. More broadly it is the hope that this result will stimulate further research in this direction and contribute to a deeper understanding of the behavior of integer sequences [1] [2] [3]. These conjectures prove each other to be true.

5. Conclusions

In conclusion, $-3(-n) - 1$ Remer Conjecture and the $3n + 1$ Collatz Conjecture are intriguing mathematical problems that involve iterative processes on positive and negative integers, respectively. While the $3n + 1$ Collatz Conjecture remained an open problem in mathematics with no conclusive proof or disproof, the $-3(-n) - 1$ Remer Conjecture presented in this paper is the proof for the Collatz Conjecture. The proof showcases the unique properties and behavior of the $-3(-n) - 1$ conjecture, including its ever-increasing pattern and the relationship within the sequence. Through the comparative analysis, we have highlighted the similarities and differences between the $3n + 1$ Collatz Conjecture and the $-3(-n) - 1$ Remer Conjecture. Both conjectures involve iterative processes, but with different opposing goals. The $-3(-n) - 1$ Remer Conjecture, though newly proposed when compared against the operating sequencing properties of the $3n + 1$ Collatz Conjecture, presents an interesting mathematical sequence with its own set of properties and behavior. The proof presented in this paper adds to the body of mathematical knowledge and contributes to our understanding of positive and negative integer sequences and how they are used to prove each other. Further research can be conducted to explore other properties and behaviors of the $-3(-n) - 1$ Remer conjecture, as well as potential connections or implications of this conjecture in other areas of mathematics.

Furthermore, the comparative analysis and proof presented in this paper sheds light on the unique characteristics and behaviors of the $3n + 1$ Collatz Conjecture and the $-3(-n) - 1$ Remer Conjecture. Further research and exploration of these conjectures together can contribute to our understanding of number theory and computational mathematics, and may have potential applications in other fields of science and technology.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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