Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2023, 11, 2121-2136
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Nottingham Heating, Inversion Temperature and Joule Heating

Zahra Behboodi

Departamento de Fsica, FCEE, Universidade da Madeira, Funchal, Portugal
Email: Behboodi.z@yahoo.com

How to cite this paper: Behboodi, Z. (2023) Nottingham Heating, Inversion Temperature and Joule Heating. *Journal of Applied Mathematics and Physics*, **11**, 2121-2136.
https://doi.org/10.4236/jamp.2023.117134

Received: June 27, 2023
Accepted: July 25, 2023
Published: July 28, 2023

Abstract

Long-term research has been done on the unstable behaviors and electron emission from microprotrusions, but the whole reason is still not clear. It is difficult to study instabilities experimentally since vacuum breakdown can happen. In this model, we show the factors that lead to thermal instability during field emission. After the Nottingham flux inversion, we see a considerable rise in temperature above a threshold electric field, followed by a thermal runaway. Cathode spots experience unexpected thermal defects and breakdowns, which is a phenomenon known as the Nottingham Inversion Instability. Although the idea of micro protrusions is frequently used in modeling studies, this study concentrates on the thermal effects during field emission from a planar cathode without taking the existence of such protrusions into account. The study reveals how Nottingham's heating effect changes from heating to cooling. In our study, we have investigated the interaction between Nottingham, Joule heating, and effective work function. The results also imply that faster reaching critical temperature is associated with larger maximum beta values. These discoveries have significance for the design and improvement of high-voltage systems and help to understand vacuum breakdown. The possibility of cathode spot ignition and subsequent vacuum breakdown is predicted by our model, which would make it possible to create a self-consistent model for that.

Keywords

Nottingham Heating, Joule Heating, Field Emission, Vacuum Breakdown

1. Introduction

Field emission is significantly influenced by the Nottingham heat, often referred to as the inversion temperature, which is proportional to the strength of the electric field [1] [2] [3]. Field emission refers to the emission of electrons from a solid surface under the influence of a high electric field. A theory such as the Fowler-Nordheim equation has been developed to describe the field emission process [4] [5]. Factors affecting field emission include the work function of the emitter material, electric field strength, surface roughness, and temperature [2] [6] [7] [8] [9]. A fit formula is suggested to characterize the cooling mechanism at higher temperatures in order to accurately reflect the Nottingham effect and get over the restrictions of the Fowler-Nordheim approximation [2]. By multiplying the applied electric field by the field enhancement factor β, the Fowler-Nordheim formula is used to fit the experimental current-voltage characteristics of field emission from cold electrodes in a vacuum [7] [10] [11] [12].

A research study at CERN seeks to understand better the vacuum breakdown mechanism, especially for the operation of the next linear colliders. The discovery of the field enhancement factor (β) was crucial since it showed that the local breakdown field is mainly determined by the electrode material and remains constant. The value provides an accurate estimate of the macroscopic breakdown field and characterizes the electrode's surface condition. Independent of gap size, the local breakdown field for copper electrodes was found to be around $10.8 \times 10^9\ \text{V}\cdot\text{m}^{-1}$ [13]. The interaction of the local electric field, enhancement factor (β), and the Nottingham heating, affects the thermal behavior in field emission processes. Material evaporation, surface destruction, and erosion are only a few of the negative effects of Nottingham heating, which is characterized by large changes in surface temperature [1] [14] [15] [16] [17] [18]. The exact cause of this instability is not fully understood and remains an active area of research.

When the cathode temperature in field emission increases, electron emission becomes easier and can potentially lead to breakdown through scenarios such as vapor release [1]. This self-heating phenomenon, driven by Joule heating and the Nottingham effect, which balances emitted and replaced electrons, results in heat flux at the metal/vacuum interface. Micro/nano-protrusions on the cathode surface, which are controlled by surface roughness, play a crucial role in breakdown initiation. Heat is emitted from the emission surface when the electric field strength exceeds a certain threshold, leading to self-heating and the Nottingham effect. Interestingly, the direction of heat flux can reverse depending on the temperature [1] [2] [10] [17] [18].

To better comprehend and minimize the impacts of Nottingham heating, researchers have carried out experimental experiments, created theoretical models, and run numerical simulations [1] [14] [15] [16] [17] [18].

Understanding and controlling thermal effects during field emission processes are crucial for efficient electron source development. Analytical studies complement experimental efforts by providing insights into the self-heating process and improving the design and performance of field electron sources and high-voltage vacuum insulation capabilities. Thermal instability, which can result in material deterioration and failure, is the uncontrolled rise in temperature at the emission site during field emission [14] [17] [18]. Engineers and designers may improve the design of high-voltage systems by understanding these aspects and making sure that the proper cooling methods and material choices are in place to minimize thermal instability and increase system efficiency and longevity. This leads to cost reduction, efficiency improvement, and enhanced safety. On the other hand, the Nottingham Inversion Instability is a phenomenon in which localized heating results in an unexpected electric field reversal close to the emission source. This may cause the vacuum insulation to break down, which might result in high-voltage systems failing drastically. Investigating thermal instabilities during field emission can enhance understanding of plasma-wall interactions and develop strategies to mitigate adverse effects, such as cathode erosion and plasma disruptions [2] [4].

Existing 1D models may not fully capture the Nottingham effect, necessitating further research to develop more comprehensive and accurate models for thermal stability in field emission processes [18]. Researchers looked into the interaction between heating mechanisms and the critical point of vaporization by examining the self-heating evolution during electron emission from spots with various radii on a cathode [8] [14] [15] [17] [18]. Cathode spots are localized regions of high current density and elevated temperature on the cathode surface in high-current vacuum arcs. Cathode spot ignition refers to the initiation and development of these spots, which can lead to various phenomena such as vaporization, melting, and thermal explosion. The ignition of a cathode spot is influenced by factors such as the presence of an enhanced electric field, current transfer mechanisms, plasma interaction, and Joule heating. Understanding the ignition process is important for comprehending the thermal behavior and overall performance of the cathode and the vacuum arc system [19] [20].

It is tricky to research since it is difficult to understand the enhancement process. Additionally, the occurrence of a wide variety of length scales represents a fundamental problem in addition to the unpredictability and volatility of the dominating physical mechanisms. The crater radius is often a few micrometers or larger, while microprotrusions modeled in [21] have a tip radius of 3 nm and a total height of 93 nm. Microprotrusions are small surface irregularities or protrusions on the electrode material, typically at the micron scale. They can arise from various manufacturing processes or material characteristics. Microprotrusions can significantly impact field emission phenomena by altering the electric field distribution and enhancing electron emission at localized regions. They can contribute to forming and developing cathode spots and influence the system's overall stability and breakdown behavior.

Some research also indicates that in order to account for the observed field emission currents adequately, it is essential to postulate that the microprotrusions have a thin, needle-like shape. These protrusions on electrode surfaces are typically undetectable under normal conditions [9] [12]. It is guessed in the study by [13] that the diameter of the emission area in a circular geometry typically ranges from 20 to 80 nm. However, neither does the report explain whether the measurement relates to the planar cathode or the microprotrusion's tip size nor does it include accurate references or visual representations of the emission region or crater. [17] [18] essays on Nottingham heating are equally impressive. The simulations carried out in these works take into account microprotrusions; according to their work, 1D simulation is not applicable for radii smaller than 20 nm. It is also important to remember that the circumstances used in studies are different, resulting in some variances in the simulation and experiment results. Nevertheless, most experimental findings and simulations described in the literature have spot sizes in the micrometer range [17] [18] [22] [23] [24].

In conclusion, Nottingham heating demands a thorough study to control thermal effects, maximize effectiveness, and enhance emission procedures. Current research attempts to reduce the effects of Nottingham heating, improve device performance, and investigate novel opportunities for various applications. It uses experimental, theoretical, and numerical methods. Comprehensive models that faithfully represent the Nottingham effect are required to improve thermal stability in field emission processes. Particle accelerators, high-voltage vacuum equipment, and other high-voltage applications can all benefit from solutions to Nottingham Heating's problems [1] [4] [14] [15] [16] [17] [18] [25] [26]. While significant progress has been made in understanding field emission and Nottingham Inversion Instability, specific gaps and unresolved issues persist: The exact mechanisms causing Nottingham Inversion Instability need further investigation. More comprehensive experimental and theoretical studies are required to elucidate the underlying processes. The influence of environmental factors, such as humidity and gas composition, on the stability of high-voltage systems needs to be further explored. Developing accurate predictive models for both field emission and Nottingham Inversion Instability remains a challenge. More research is needed to understand the interplay between surface effects, electron emission, and breakdown phenomena.

It would be interesting to use this process to simulate the advanced stages of vacuum breakdown, which would include a change from field to thermo-field to thermionic emission. In the current study, the microprotrusion from the model geometry will be eliminated, and a planar electrode will be taken into consideration. Also, it concentrates on the range of micrometers for the spatial distribution of the electric field.

The primary objective of this paper is to provide a comprehensive characterization of the Nottingham effect by investigating the influence of electric fields, enhancement factor, and effective work function. The study is divided into two parts. The first part focuses on the 2D modeling of different radii to survey the time of rising in cathode temperature to the critical temperature of the copper. The second part investigates the evolution of Joule heating, Nottingham heating, and effective work function with increasing the temperature, considering the limits of the effective work function contribution to the emitted current. This simulation is carried out using the COMSOL Multiphysics software's finiteelement technique, which enables the temporal evolution of temperature inside the cathode.

2. The Model

In this investigation, the simulation method is based on previously published models of stationary cathode spots in vacuum arcs. The model considers the calculation domain and boundary conditions of those earlier studies [22] [24] [27]. So in this work, the simulation is performed for copper electrodes; the equations are solved using axial symmetry in cylindrical coordinates (r,z). The cathode is planar and serves as an axisymmetric region with a $h = 20\ \text{mm}$ radius.

The simulation method solves the time-dependent heat conduction by considering Joule heating and current continuity equations for the domain:

$$\rho c_\rho \frac{\partial T}{\partial t} = \nabla\cdot(\kappa \nabla T) + \sigma(\nabla\varphi)^2 \tag{1}$$

$$\nabla\cdot(\sigma(\nabla\varphi)) = 0 \tag{2}$$

where ρ is the the mass density of the metal, c_p, κ, and σ are, respectively, the specific heat, and the thermal and electrical conductivities of the metal (known functions of the temperature T), and φ is the electric potential, $\boldsymbol{j} = -\sigma\nabla\varphi$ is the density of electric current in the cathode, the entire cathode is the calculation domain for Equations (1) and (2) (all these parameters havebeen taken from previous work [24]). The parameters in this model's framework that control the temperature evolution within the cathode are a function of the product βE rather than β or E alone [12] [13] [28] and the local temperature of the cathode surface. Both the Nottingham heat flux and emission current density are imposed as Neumann boundary conditions at the top surface of the cathode, which represents the interface between the vacuum and the cathode, in order to solve the temporal evolution of the heat and current equation.

$$-\kappa \left.\frac{\partial T}{\partial n}\right|_h + q_{\text{rad}_h} = q(\beta E, T) = q_{em} \tag{3}$$

$$q_{em} = \frac{j_{em}}{e}\left(2k_B T + A_{\text{eff}}\right) \tag{4}$$

$$q_{\text{rad}_h} = \varepsilon\sigma_{SB}\left[\left(T|_{z=h}\right)^4 - \left(T_{\text{amb}}\right)^4\right] \tag{5}$$

$$\sigma \left.\frac{\partial\varphi}{\partial n}\right|_h = -j(\beta E, T) = j_{em} \tag{6}$$

where n is the unit vector normal to the cathode surface and directed outward, A_{eff} is an effective work function that considers the Nottingham effect by considering inversion temperature [2], e is the electron charge, k_B is the Boltzmann constant, the Stefan-Boltzmann constant is σ_{SB}, and the copper electrode surface's emissivity is ε. q_{rad_h} represents the radiation from the top of the surface. So an effective work function $A_{\text{eff}}(\beta E, T)$ was used in the energy flux q_{em} is implemented to the model by a table taken from a Python code [28]. Field emission acts as the main method for current transfer $J(\beta E,T)$ to the electrode, as determined by the Murphy and Good formalism which is numerically computed by using a code that has already been created (ELEM code) [28] [29]. This numerical simulation incorporates a Gaussian spatial distribution for enhancement factor (β):

$$\beta(r)=1+90\times\exp-\left(\frac{r-0}{a}\right)^2,$$

where a is a given parameter characterizing the spatial extension of the electric field. The value of the electric field have considered equal to $1.2 \times 10^8\ \text{V}\cdot\text{m}^{-1}$, so the product of the electric field and enhancement factor (β) would be $10.8 \times 10^9\ \text{V}\cdot\text{m}^{-1}$ [13]. Assuming an asymptotic condition, Dirichlet boundary conditions are applied at the bottom and wall surface of the cathode, which are far from the spot [22] [24] [30].

$$T_0 = T_{col} - r\times T_r - z\times T_z \tag{7}$$

$$\varphi_0 = -r\times\varphi_r - z\times\varphi_z \tag{8}$$

T_{col} is a given input parameter, in this work is 300 K, the variations of temperature and electric potential components along the r and z directions are T_r, T_z, φ_r and φ_z respectively.

A zone of interest of 30 μm has been considered on the cathode surface for applying energy flux and current density sources. The mesh has been appropriately refined to ensure accurate observation of changes within this region. Figure 1 depicts a visual representation of all the boundary conditions.

3. Results

3.1. Influence of Radius on Field Emission Possibility

The maximum temperature evolution inside the cathode for different radii is depicted in Figure 2. It is evident that for radii of 0.05 μm and 0.5 μm, the

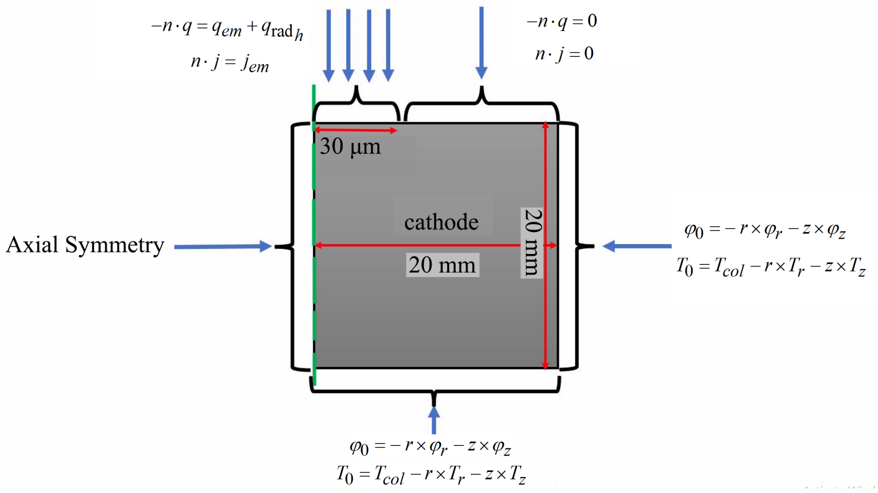

Figure 1. Geometery and boundary conditions on simulation domain not at scale.

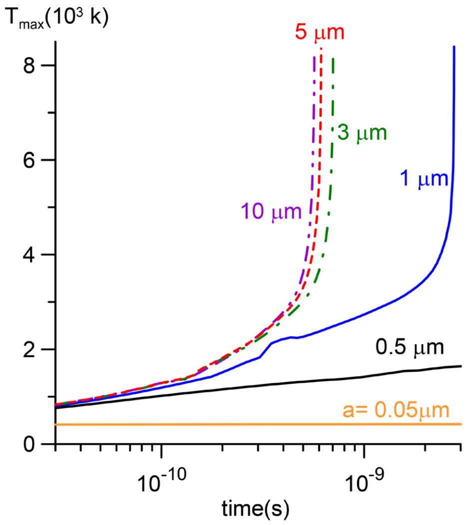

Figure 2. Temporal evolution of the maximum temperature inside the cathode for different radii.

temperature does not reach a level sufficient to be able ignite a spot, It reaches 420 K for the first one up to 1 μs, and roughly 1900 K for the latter. However, for a radius of 1 μm, the temperature reaches the critical temperature of copper cathode around 8300 K at around 3 ns. For radii of 3 μm, 5 μm, and 10 μm, the time taken to reach it is approximately the same, around 7×10^{-10} s, 6×10^{-10} s, and 5.7×10^{-10} s, respectively.

The area of concentrated electron emission on the cathode surface, also known as the spot radius, has a specific size. It significantly influences the breakdown behavior, emission current density, and field enhancement. Experimental findings and computer simulations show that the spot radius considerably impacts the possibility of field emission. Spot radius and field emission potential have a complicated relationship that depends on a number of variables, including the qualities of the material, the state of the surface, and the electric field that is being used. Surface imperfections and flaws can also change the ideal spot radius for effective electron emission and alter the field emission behavior [12].

So regarding the definition of enhancement factor (β) in this work, it plays a crucial role in determining the system's behavior, particularly the rapid temperature increase. As the radius increases, so does the corresponding beta value. Consequently, higher beta values accelerate the heating process, resulting in faster attainment of the desired temperature threshold. Thus, radii deviating significantly from the central value will experience variations in beta, leading to altered heating characteristics and affecting the time required to reach critical temperature and the possibility of the formation of the cathode spot. Formation of it is the basic process for igniting a vacuum break down. These findings provide valuable insights into the interplay between radius, beta, and temperature evolution within the system.

Understanding this relationship enables more precise control and prediction of the heating process, facilitating the optimization of experimental conditions or system design. For the best performance of electron emission devices and effective field emission, it is essential to comprehend the impact of spot radius. It is essential to acknowledge that further investigation and validation are necessary to confirm the consistency and reliability of these findings. Sensitivity analyses and comparisons with experimental data can help evaluate the robustness of the observed relationship and provide additional insights into the underlying physics governing the system.

Figure 3 shows 2D evolution of the maximum temperature distribution in the planar cathode for a = 5 μm. The aim of choosing this radius is the presence of previously published studies that have examined distinct mechanisms for initiating the cathode spot in the same conditions [22] [24], which is an important phenomenon causing vacuum breakdown. Therefore, the opportunity to compare the findings with these works adds further captivation and value to this study. The shown figures clearly show that the dominance of Joule heating over Nottingham heating becomes apparent when the temperature rises constantly, reaching about 3000 K. This observation supports recent research that identified the electric field threshold at which Joule heating surpasses Nottingham heating in 1D simulations and samples with microprotrusions [12] [28], in other words, thermal runaway occurs. As a result, this study supports the existence of a similar phenomenon in planar cathodes as well.

The occurrence of thermal runaway, which has been previously reported in relevant investigations, is shown in the provided figures. Joule heating, in which electrical energy is changed into heat and causes an increase in temperature, is the primary source of thermal runaway in metals. This positive feedback loop may cause the temperature to rise quickly and eventually explode in the heat [17] [18] [24]. Notably, the amount of time needed to achieve the critical temperature after reaching 3000 K seems to be greatly reduced in all cases (**Figure 1**). Moreover, it can be seen from the presented figures that the heated region caused by the applied enhancement electric field is less than the 5 μm considered radius. This is in contrast to earlier research that reported a bigger heated area of roughly 5 μm under comparable circumstances (Figure 4 in [24]). The observed variation in the extent of the heated region was also evident in the depth. The underlying factors influencing this difference, such as the short time required to reach the critical temperature or the inversion of the Nottingham effect, remain uncertain and warrant further investigation. Also, one possibility could be the definition of the enhancement factor (β) in the present work, even though the mechanisms and terminology linked to current density and flux energy are different between the two works. The enhancement factor (β) increases the exponential component by constant coefficients. This alters the function's total amplitude without changing its shape or spread. Similar to the one have used in [24], the spatial distribution for the external source is Gaussian: $f_1(r) = \exp - (r/a)^2$. In both cases, the radius of influence is constrained by the exponential component's decay. However, in the present work, one can say this

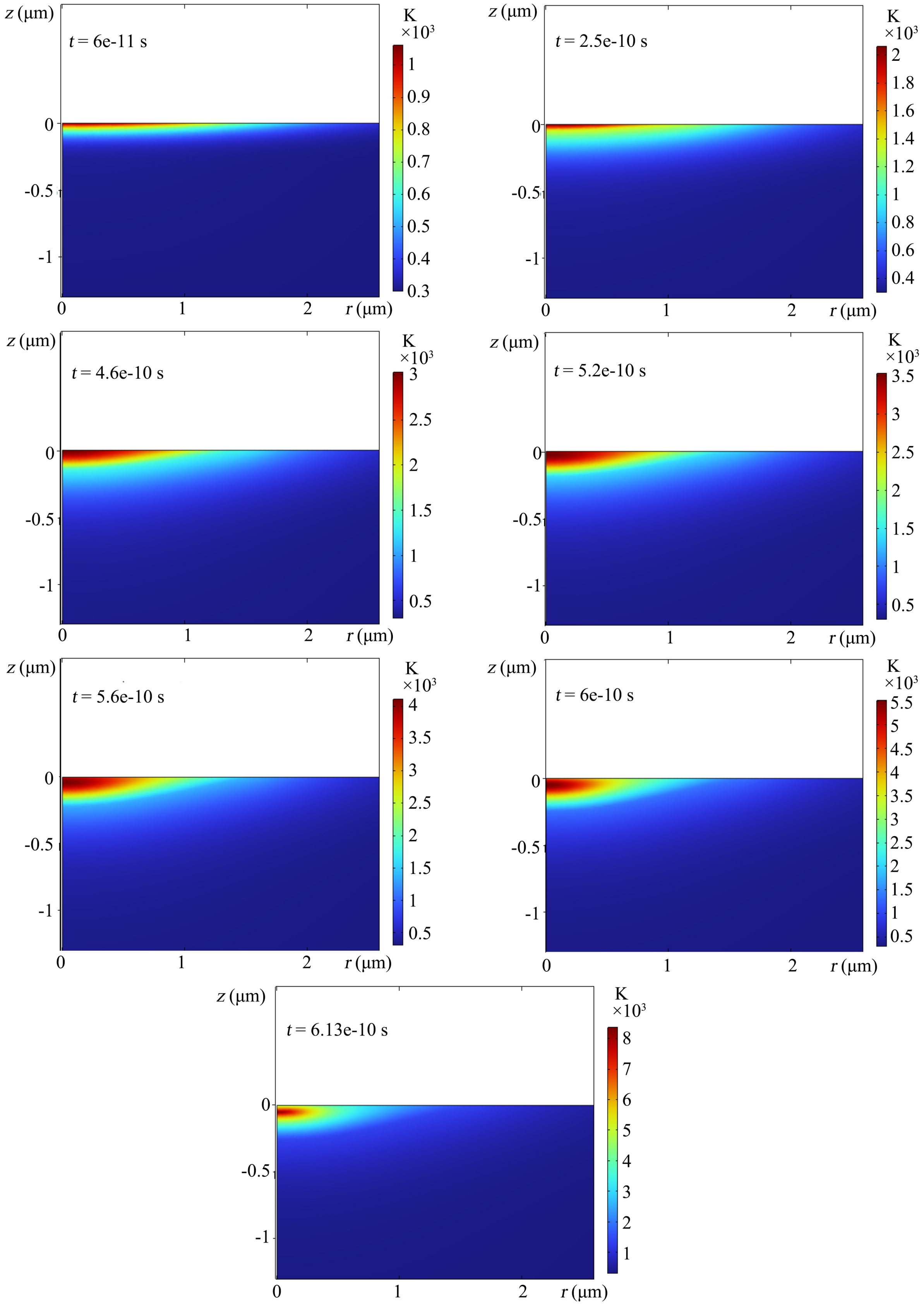

Figure 3. Evolution of temperature distribution in the planar cathode. $a = 5$ μm.

parameter has a higher peak value due to the added those constants. This results in a larger influence of current density within a smaller area, leading to a smaller heated region compared to the work mentioned.

3.2. Correlation between Effective Work Function, Nottingham Heating and Joule Heating

Figure 4 presents the evolution of Nottingham energy flux and Joule heating as a function of temperature. It is observed that around 3000 K, the magnitude of Nottingham heating begins to decrease, eventually becoming negative. This transition signifies a shift in its role from a heating mechanism to a cooling mechanism. Concurrently, as depicted in the figure, Joule heating demonstrates a noticeable increase during this period. Additionally, referring to **Figure 1**, it can be observed that in the majority of cases, the cathode temperature reaches 8300 K, indicating the significant contribution of Joule heating in sustaining and further elevating the cathode temperature. This Joule heating has a significant effect on the ignition of the spot as it has already been mentioned in some studies [10] [17] [18]. **Figure 5** provides insights into the variation of the effective work function with increasing temperature. Notably, throughout the entire temperature range, the effective work function remains negative. However, around 3000 K, it exhibits a more negative value, resulting in a negative Nottingham energy flux and introducing a cooling mechanism. This correlation among the three parameters is clearly demonstrated in the figure.

The effective work function is a key factor influencing electron escape from a material's surface and is influenced by various variables, including material characteristics and applied electric field. Nottingham heating is affected by the effective work function. When the effective work function has negative values, indicating increased electron emission and Nottingham heating. As temperature increases, Joule heating becomes dominant over Nottingham heating. Joule

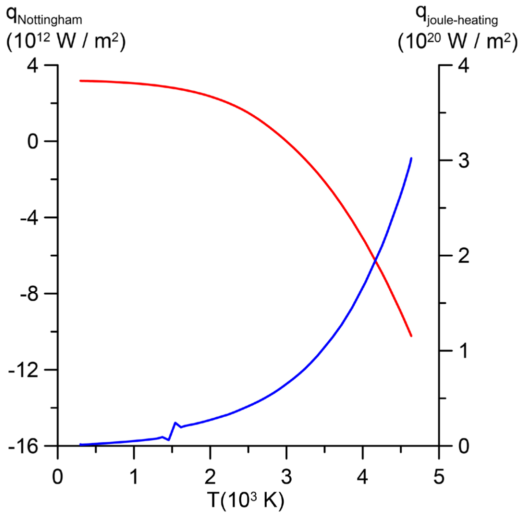

Figure 4. A comparison of Joule heating and changing Nottingham energy flux with temperature for $a = 5$ μm.

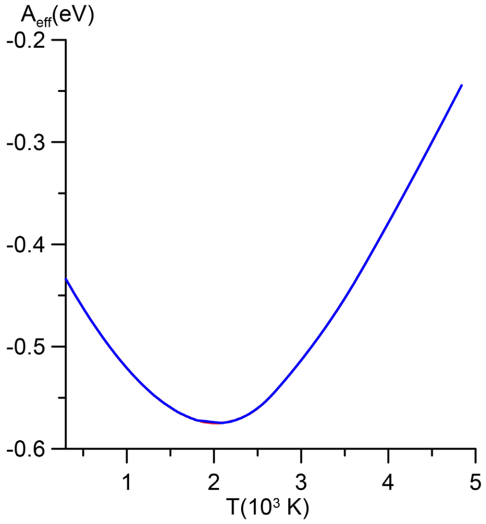

Figure 5. The evolution effective work function with temperature.

heating occurs due to the resistive properties of the material as electrons pass through it. The interplay between these heating mechanisms is crucial in vacuum breakdown and can be influenced by changes in the effective work function. Both processes contribute to the cathode's temperature rise, with Joule heating becoming significant at high temperatures and current densities. These findings highlight the significant role of Joule heating in maintaining and elevating the cathode temperature, regardless of the radius, potentially leading to spot ignition and a transition from field to thermionic emission.

3.3. Enhancement Factor (β)

Figure 6 illustrates the evolution of the temperature distribution, for different maximum β values for a fixed radius of 5 μm and a constant electric field. It is evident that as the maximum beta value increases, the time required to reach the critical temperature decreases. This observation suggests that the possibility of initiating an arc significantly affected by this parameter.

In **Figure 7**, the evolution of the temperature distribution are shown for various electric field and enhancement factor (β) configurations, chosen to have a product of approximately 10.8×10^9 V·m^{-1}. Notably, irrespective of the different configurations, the time needed to reach the critical temperature remains nearly constant. This intriguing consistency about the time for reaching the critical temperature among the diverse electric field and enhancement factor configurations further strengthens and validates the findings presented in [13].

For the radius of 5 μm and the same applied electric field, **Figure 8** shows the link between the maximum beta value and the effective work function during increasing of the temperature. The effective work function shows a quicker transition to positive values at lower temperatures as the maximum beta value lowers, pointing to the existence of a cooling mechanism. In contrast, when the beta value is high, it changes with temperature more slowly but has a chance to turn more negative due to the high electric field, which would result in a negative energy flow and promote cooling. There is a certain level of complexity to this occurrence. It is possible to identify the precise beta value that can produce an energy flux sufficient to cause vacuum breakdown by analyzing the Nottingham energy flux during these changes. The enhancement factor (β) states the impact of surface conditions and material attributes on the emission process and quantifies the departure from the idealized field emission behavior. A greater beta value denotes a more significant amplification of the local electric field and could have an impact on the breakdown properties and emission current density.

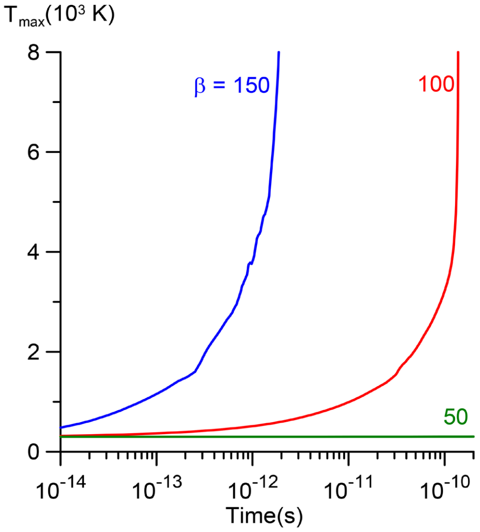

Figure 6. Evolution of the temperature distribution for different maximum values of β for $a = 5\ \mu\text{m}$ and $E = 1.2 \times 10^8$ V/m.

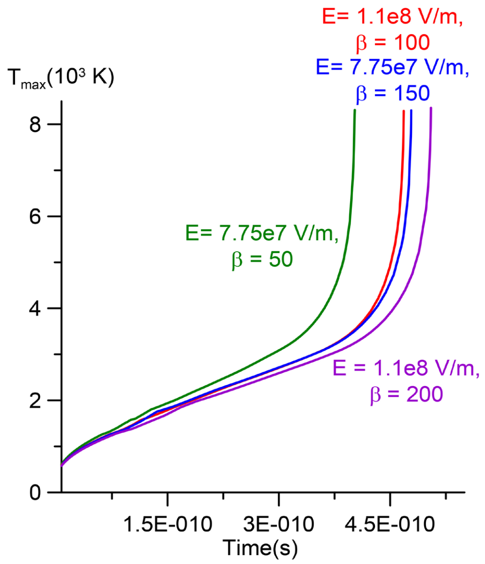

Figure 7. Evolution of the temperature distribution for various electric field (E) and enhancement factor (β) configurations.

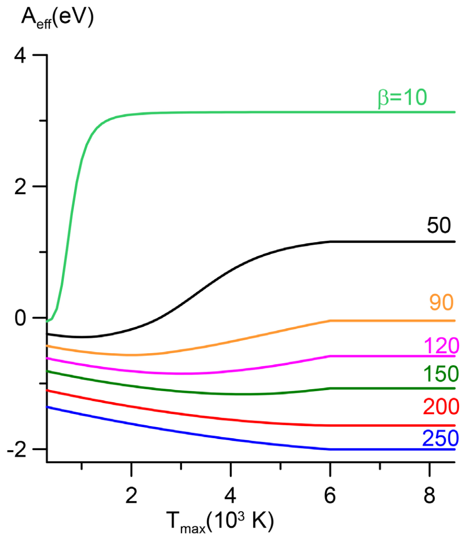

Figure 8. The effective work function for different maximum value of β for a = 5 μm and E = 1.2 × 10^8 V/m.

4. Conclusions

This work has provided valuable insights into the behavior of effective work function, Nottingham heating, and Joule heating, and their influence on thermal instabilities. By focusing on a copper planar cathode in an axisymmetric coordinate system and applying an enhanced electric field, the study aimed to investigate the impact of thermal runaway and critical temperatures on vacuum breakdown ignition.

The analysis revealed important findings regarding temperature evolution and other mechanisms. Nottingham heating decreases and becomes negative around 3000 K, indicating a transition from heating to a cooling mechanism. Simultaneously, Joule heating significantly increases, playing a crucial role in maintaining and further elevating the cathode temperature. Thermal runaway occurred, due to Joule heating. This instability is largely caused by heat reflux from the high-temperature domain towards the emission surface. This phenomenon involves the rapid increase in temperature due to the conversion of electrical energy into heat, leading to a thermal explosion. The interplay between Nottingham and Joule heating contributes to understanding thermal dynamics during vacuum breakdown. The study also observed negative variations of the effective work function with temperature throughout the range, with a more negative effective work function after around 3000 K leading to a negative Nottingham energy flux. This intriguing phenomenon highlights the existence of a cooling mechanism and demonstrates the intricate correlation between effective work function, Nottingham energy flux, and temperature. Additionally, consistent ignition times were observed across different electric field and enhancement factor configurations. Furthermore, it is discovered that the heated zone produced by the applied enhanced electric field is lower than the previously estimated size of around 5 μm [24]. The current work's definition of the enhancement factor produces a higher peak value and a greater effect within a smaller region. This discrepancy is observed in some other works already [19] [23] [31].

These findings emphasize the importance of considering multiple parameters, including Nottingham heating, Joule heating, effective work function, and the multiplication of electric field and enhancement factor, in understanding and predicting vacuum breakdown dynamics. While this study has provided valuable insights, further experimental validations and investigations are required to deepen our understanding of the underlying mechanisms. Continued research in this field will lead to the development of advanced strategies for arc ignition control, enhancing the reliability and efficiency of high-voltage systems in various applications.

Acknowledgements

I would like to express our sincere gratitude to the staff of the Physics Department of Madeira University for their valuable contributions to this research project. Their expertise, guidance, and support have been instrumental in shaping my understanding of field emission and thermal instability phenomena. I am deeply appreciative of their time, insightful discussions, and collaborative efforts, which have significantly enriched the quality and depth of this study. Their commitment to excellence and dedication to advancing scientific knowledge has been truly inspiring. I extend our heartfelt thanks to each of them for their invaluable contributions.

Specially Dr. Nelson Almeida, for making available the code simulating the initial stage of high-voltage vacuum breakdown in the framework of the phenomenological approach.

Conflicts of Interest

There is no conflict of interest in this work.

References

[1] Utsumi, T. (1991) Vacuum Microelectronics: What's New and Exciting. *IEEE Transactions on Electron Devices*, **38**, 2276-2283. https://doi.org/10.1109/16.88510

[2] Paulini, J., Klein, T. and Simon, G. (1993) Thermo-Field Emission and the Nottingham Effect. *Journal of Physics D: Applied Physics*, **26**, 1310-1315. https://doi.org/10.1088/0022-3727/26/8/024

[3] Charbonnier, F.M., Strayer, R.W., Swanson, L.W. and Martin, E.E. (1964) Nottingham Effect in Field and *T-F* Emission: Heating and Cooling Domains, and Inversion Temperature. *Physical Review Letters*, **13**, 397-401. https://doi.org/10.1103/PhysRevLett.13.397

[4] Fridman, A. and Kennedy, L.A. (2011) Plasma Physics and Engineering. 2nd Edition, CRC Press, Boca Raton.

[5] Fowler, R.H. and Nordheim, L. (1928) Electron Emission in Intense Electric Fields. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, **119**, 173-181. https://doi.org/10.1098/rspa.1928.0091

[6] Chatterton, P.A. (1966) A Theoretical Study of Field Emission Initiated Vacuum Breakdown. *Proceedings of the Physical Society*, **88**, 231-245. https://doi.org/10.1088/0370-1328/88/1/326

[7] Latham, R.V. and Xu, N.S. (1991) 'Electron Pin-Holes': The Limiting Defect for Insulating High Voltages by Vacuum, a Basis for New Cold Cathode Electron Sources. *Vacuum*, **42**, 1173-1181. https://doi.org/10.1016/0042-207X(91)90127-5

[8] Descoeudres, A, Ramsvik, T, Calatroni, S, Taborelli, M. and Wuensch, W. (2009) DC Breakdown Conditioning and Breakdown Rate of Metals and Metallic Alloys under Ultra-High Vacuum. *Physical Review Accelerators and Beams*, **12**, Article ID: 032001. https://doi.org/10.1103/PhysRevSTAB.12.032001

[9] Zadin, V., Pohjonen, A., Aabloo, A., Nordlund, K. and Djurabekova, F. (2014) Electrostatic-Elastoplastic Simulations of Copper Surface under High Electric Fields. *Physical Review Special Topics-Accelerators and Beams*, **17**, Article ID: 103501. https://doi.org/10.1103/PhysRevSTAB.17.103501

[10] Rossetti, P., Paganucci, F. and Andrenucci, M. (2002) Numerical Model of Thermoelectric Phenomena Leading to Cathode-Spot Ignition. *IEEE Transactions on Plasma Science*, **30**, 1561-1567. https://doi.org/10.1109/TPS.2002.804165

[11] Latham, R.V. (1995) High Voltage Vacuum Insulation: Basic Concepts and Technological Practice. Academic Press, Cambridge.

[12] Kaufmann, H.T.C., Almeida, N.A. and Benilov, M.S. (2022) Phenomenological Description of Vacuum Breakdown. 10*th International Workshop on Mechanisms of Vacuum Arcs*, Chania, 19 September 2022.

[13] Descoeudres, A., Levinsen, Y., Calatroni, S., Taborelli, M. and Wuensch, W. (2009) Investigation of the Dc Vacuum Breakdown Mechanism. *Physical Review Special Topics-Accelerators and Beams*, **119**, Article ID: 092001. https://doi.org/10.1103/PhysRevSTAB.12.092001

[14] Haase, J.R. and Go, D.B. (2016) Analysis of Thermionic and Thermo-Field Emission in Microscale Gas Discharges. *Journal of Physics D: Applied Physics*, **49**, Article ID: 055206. https://doi.org/10.1088/0022-3727/49/5/055206

[15] Vibrans, G.E. (1964) Vacuum Voltage Breakdown as a Thermal Instability of the Emitting Protrusion. *Journal of Applied Physics*, **35**, 2855-2857. https://doi.org/10.1063/1.1713118

[16] Bocharov, G.S. and Eletskii, A.V. (2007) Thermal Instability of Field Emission from Carbon Nanotubes. *Technical Physics*, **52**, 498-503. https://doi.org/10.1134/S1063784207040160

[17] Mofakhami, D., Seznec, B., Minea, T., Teste, P., Landfried, R. and Dessante, P. (2021) Thermal Effects in Field Electron Emission from Idealized Arrangements of Independent and Interacting Microprotrusions. *Journal of Physics D: Applied Physics*, **54**, Article ID: 235305. https://doi.org/10.1088/1361-6463/abd9e9

[18] Mofakhami, D., Seznec, B., Minea, T., Landfried, R., Testé, P. and Dessante, P. (2021) Unveiling the Nottingham Inversion Instability during the Thermo-Field Emission from Refractory Metal Micro-Protrusions. *Scientific Reports*, **11**, Article No. 15182. https://doi.org/10.1038/s41598-021-94443-7

[19] Jüttner, B. (1997) Properties of Arc Cathode Spots. *Journal de Physique IV*, **7**, C4-31-C4-45. https://doi.org/10.1051/jp4:1997404

[20] Yang, H., Shen, S., Xu, R., Zhou, M., Yan, J. and Wang, Z. (2023) Molecular Dynamics Simulation of Cathode Crater Formation in the Cathode Spot of Vacuum Arcs. *Journal of Physics D: Applied Physics*, **56**, Article ID: 375203. https://doi.org/10.1088/1361-6463/acdadf

[21] Kyritsakis, A., Veske, M., Eimre, K., Zadin, V. and Djurabekova, F. (2018) Thermal Runaway of Metal Nano-Tips during Intense Electron Emission. *Journal of Physics D: Applied Physics*, **51**, Article ID: 225203. https://doi.org/10.1088/1361-6463/aac03b

[22] Kaufmann, H.T.C., Cunha, M.D., Benilov, M.S., Hartmann, W. and Wenzel, N. (2017) Detailed Numerical Simulation of Cathode Spots in Vacuum Arcs: Interplay of Different Mechanisms and Ejection of Droplets. *Journal of Applied Physics*, **122**, Article ID: 163303. https://doi.org/10.1063/1.4995368

[23] Schwirzke, F.R. (1991) Vacuum Breakdown on Metal Surfaces. *IEEE Transactions on Plasma Science*, **19**, 690-696. https://doi.org/10.1109/27.108400

[24] Cunha, M.D., Kaufmann, H.T.C., Benilov, M.S., Hartmann, W. and Wenzel, N. (2017) Detailed Numerical Simulation of Cathode Spots in Vacuum Arcs—I. *IEEE Transactions on Plasma Science*, **45**, 2060-2069. https://doi.org/10.1109/TPS.2017.2697005

[25] Timko, H., Ness Sjobak, K., Mether, L., Calatroni, S., Djurabekova, F., Matyash, K., Nordlund, K., Schneider, R. and Wuensch, W. (2015) From Field Emission to Vacuum Arc Ignition: A New Tool for Simulating Copper Vacuum Arcs. *Contributions to Plasma Physics*, **55**, 299-314. https://doi.org/10.1002/ctpp.201400069

[26] Venkattraman, A. (2014) Generalized Criterion for Thermo-Field Emission Driven Electrical Breakdown of Gases. *Applied Physics Letters*, **104**, Article ID: 194101. https://doi.org/10.1063/1.4876606

[27] Almeida, N.A., Benilov, M.S., Benilova, L.G., Hartmann, W. and Wenzel, N. (2013) Near-Cathode Plasma Layer on CuCr Contacts of Vacuum Arcs. *IEEE Transactions on Plasma Science*, **41**, 1938-1949. https://doi.org/10.1109/TPS.2013.2260832

[28] Kaufmann, H.T.C., Almeida, N.A. and Benilov, M.S. (2022) Numerical Simulation of Vacuum Breakdown II: Thermal Instability Mechanism. 10*th International Workshop on Mechanisms of Vacuum Arcs*, Chania, 19 September 2022.

[29] Benilov, M.S. and Benilova, L.G. (2013) Field to Thermo-Field to Thermionic Electron Emission: A Practical Guide to Evaluation and Electron Emission from Arc Cathodes. *Journal of Applied Physics*, **114**, Article ID: 063307. https://doi.org/10.1063/1.4818325

[30] Benilov, M.S. and Cunha, M.D. (2003) Heating of Refractory Cathodes by High-Pressure Arc Plasmas: II. *Journal of Physics D: Applied Physics*, **36**, 603-614. https://doi.org/10.1088/0022-3727/36/6/301

[31] Timko, H., Matyash, K., Schneider, R., Djurabekova, F., Nordlund, K., Hansen, A., Descoeudres, A., Kovermann, J., Grudiev, A., Wuensch, W., Calatroni, S. and Taborelli, M. (2011) A One-Dimensional Particle-in-Cell Model of Plasma Build-up in Vacuum Arcs. *Contributions to Plasma Physics*, **51**, 5-21. https://doi.org/10.1002/ctpp.201000504