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Finite-Time H_∞ Control of Switched Nonlinear Systems under State-Dependent Switching

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Abstract

This paper investigates the finite-time H_∞ control problem of switched nonlinear systems via state-dependent switching and state feedback control. Unlike the existing approach based on time-dependent switching strategy, in which the switching instants must be given in advance, the state-dependent switching strategy is used to design switching signals. Based on multiple Lyapunov-like functions method, several criteria for switched nonlinear systems to be finite-time H_∞ control are derived. Finally, a numerical example with simulation results is provided to show the validity of the conclusions.

Keywords

Finite-Time H_∞ Control, Switched Nonlinear Systems, Multiple Lyapunov-Like Functions, State-Dependent Switching

1. Introduction

Switched systems consist of a series of continuous or discrete subsystems and a switching signal that coordinates the switching between these subsystems. As a special class of hybrid systems, switched systems have been widely used in the traffic control, network control, automotive roll dynamics control and mobile robot control in the past years [1] [2] [3] [4]. It should be pointed out that the dynamic behavior of switching systems become complicated due to the existence of switching signals, which has attracted much attention. Recently, there have been a lot of important work on switched systems, such as stability, stabilization, sliding mode control and so on [5] [6] [7] [8]. Specifically, the stability of switching systems is not equivalent to the stability of subsystems, as switching signals play a crucial role. Thus, appropriate controllers are often needed to better achieve system performance. Generally speaking, from the perspective of the switching instant, the switching signal can be divided into two types: time-dependent [9] [10] [11] [12] and state-dependent [13] [14] [15] [16] (sometimes also combinations of the above types). For the former type, the switching instant must be known in advance, and the switching between subsystems occurs at a fixed time. However, the switching instant is usually a priori unknown, and state-dependent switching that based on the current state of the system is needed under the circumstance, as is shown in **Figure 1**. Because the switching instant does not need to be predetermined, state-dependent switching strategies have been widely used in various fields [13] [17], such as power control, chemical system and aircraft control [18]-[24].

In recent years, most of the existing literature related to the stability of switched systems has focused on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in many practical applications, it always needs to concern the behavior of the system over a finite time, where large values of the state are unacceptable. In addition, a dynamical system may show poor performances in a finite time interval with short working time although it eventually satisfies asymptotic stability. Therefore, it is necessary to study the transient performance of systems. In this case, the finite-time stability as a concept of short time stability is introduced, which is an independent concept comparing with Lyapunov asymptotic stability and can be used to estimate the boundedness of states within a prescribed bound in a fixed time interval if a bound on the initial condition is given. Recently, some results have been obtained about transient performance of systems in a finite time interval. For example, in [6], authors studied the finite-time stability of switched systems based on time-dependent switching. Furthermore, some achievements have also been made about the transient performances of switched systems in the framework of state-dependent switching strategy. In [25], authors investigated the finite-time stabilization and boundedness problems of switched linear systems. Several sufficient criteria were given to guarantee the finite-time stabilization of switched nonlinear systems in [26].

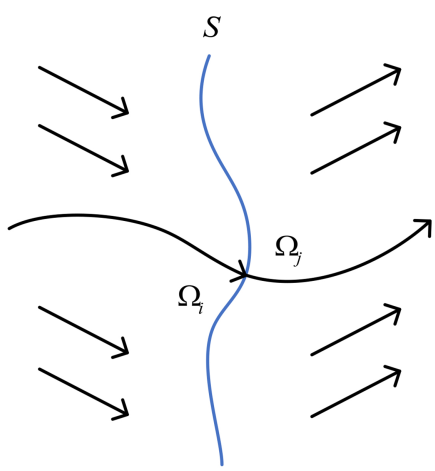

Figure 1. State-dependent switching.

In many practical systems, the inevitable external disturbance may lead to system instability. In this case, we hope that not only the influence of external disturbance on output variables can be controlled, but also the expected performance of the system can be obtained. H_∞ control is an effective method to deal with these issues. The advantage is that the output has an upper bound by limiting external disturbances, while ensuring internal stability. Many literatures on H_∞ control have been reported [27] [28] [29] [30] [31]. In [27] [28], authors investigated the finite-time H_∞ control of switched systems under time-dependent switching. Under state-dependent switching, the finite-time H_∞ dynamic output feedback control for nonlinear impulsive switched systems has been proposed by [29]. In [31], authors studied asynchronous H_∞ control for discrete-time switched systems under state-dependent switching with dwell time constraint. It should be noted that the finite-time H_∞ state feedback control for switched nonlinear systems has not been fully studied, especially under state-dependent switching. This motivated the research of this paper.

This paper aims to investigate the finite-time H_∞ control problem of switched nonlinear systems under state-dependent switching. The main contributions of this paper can be highlighted as follows: 1) different from time-dependent switching, the state-dependent switching strategy studied in this paper is based on the state of the system without giving the switching instant in advance; 2) using the multiple Lyapunov-like functions method, the sufficient conditions to guarantee the finite-time H_∞ control for switched nonlinear systems are proposed, and sliding motion is also considered; 3) what is different from the previous method is that the largest region function strategy is adopted in this paper.

The remainder of this paper is organized as follows. Section 2 introduces some necessary preliminary knowledge. In Section 3, several sufficient criteria for switched nonlinear systems to be finite-time H_∞ control are proposed. A numerical simulation is given to demonstrate the validity of our methods in Section 4. Section 5 describes the conclusions of this paper.

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ represent the n-dimensional real space equipped with the Euclidean norm $\|\cdot\|$, $\mathbb{Z}_+$ be the set of positive integers. For a matrix A, $A>0$ ($A<0$, $A\geq 0$, $A\leq 0$) denotes that A is a positive definite (negative definite, positive semi-definite, negative semi-definite) matrix, $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) denotes the maximum (minimum) eigenvalue of A, A^{T} means the transpose of A and A^{-1} represents the inverse of A. Unless otherwise specified, I stands for the identity matrix with appropriate dimensions, $\star$ is the symmetric block in a symmetric matrix and $\Lambda=\{1,2,\cdots,n\}, n\in\mathbb{Z}_+$ is an index set.

2. Preliminaries

Consider the following switched nonlinear system

$$\begin{cases} \dot{x}(t)=A_\sigma x(t)+B_\sigma u(t)+C_\sigma f\left(x(t)\right)+G_\sigma\omega(t), \\ y(t)=D_\sigma x(t)+E_\sigma\omega(t), \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the neural state vector, $f(x(\cdot))$ represents the neuron activation function; $u(t)\in\mathbb{R}^l$ is the control input, $\omega(t)\in\mathbb{R}^q$ is the exogenous disturbance, and $y(t)\in\mathbb{R}^m$ is the controlled output. $A_\sigma, B_\sigma, C_\sigma, D_\sigma, E_\sigma$, and G_σ are constant real matrices. $\sigma:=\sigma(t)$ which takes values in the finite set $I_N=\{1,2,\cdots,N\}$, $N\in\mathbb{Z}_+$ is the switching signal characterized by the switching sequence $(i_0,t_0),(i_1,t_1),\cdots,(i_j,t_j)$ and $\sigma=i$ means that the *i*th subsystem is activated.

The state feedback controller is given by

$$u(t)=K_\sigma x(t). \tag{2}$$

Substituting Equation (2) into system (1), we can get the following closed-loop system

$$\begin{cases}\dot{x}(t)=\tilde{A}_i x(t)+C_i f(x(t))+G_i\omega(t),\\ y(t)=D_i x(t)+E_i\omega(t),\end{cases} \tag{3}$$

where $\tilde{A}_i=A_i+B_iK_i$.

For further discussion, the following assumptions are made.

(**H**$_1$) For the continuously bounded neuron activation functions $f_j(\cdot), j\in\Lambda$, there exist some real constants l_j^-, l_j^+ such that

$$l_j^-\le\frac{f_j(v_1)-f_j(v_2)}{v_1-v_2}\le l_j^+,\ v_1,v_2\in\mathbb{R},\ v_1\ne v_2.$$

Define $L_1=\operatorname{diag}\{l_1^-l_1^+,\cdots,l_n^-l_n^+\}$, $L_2=\operatorname{diag}\left\{\frac{l_1^-+l_1^+}{2},\cdots,\frac{l_n^-+l_n^+}{2}\right\}$.

(**H**$_2$) Given a constant *T*, the exogenous disturbance $\omega(t)$ satisfies

$$\int_0^T\omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}s\le d,\ d>0.$$

To identify a Lyapunov-like function for one of the switched subsystems, the whole state space $\mathbb{R}^n$ should be divided into N pieces, denoted by Ω_i cover the whole state space, *i.e.*, the following covering property holds

$$\bigcup_{i=1}^N\Omega_i=\mathbb{R}^n.$$

For simplicity, we assume that each region Ω_i has the following quadratic representation

$$\Omega_i=\{x\in\mathbb{R}^n\mid x^{\mathrm{T}}Q_ix\ge0\},\ i\in I_N,$$

where $Q_i\in\mathbb{R}^{n\times n}, i\in I$ is symmetric matrix.

The following lemma gives a sufficient condition for the covering property.

Lemma 1 ([32]). *If for every* $x\in\mathbb{R}^n$,

$$\sum_{i=1}^N\theta_i x^{\mathrm{T}}Q_ix\ge0,$$

where $\theta_i\ge0, i\in I_N$, *then* $\Omega_1\cup\Omega_2\cup\cdots\cup\Omega_N=\mathbb{R}^n$.

Define the switching law according to the following largest region function

$$\sigma(t) = \arg\max_{i\in I}\left\{x^{\mathrm{T}} Q_i x\right\},$$

and define Ω_{ij} as follows

$$\Omega_{ij} = \left\{x \in \mathbb{R}^n \mid x^{\mathrm{T}} Q_i x = x^{\mathrm{T}} Q_j x\right\}, \ \ i, j \in I_N.$$

To have a well-defined switched system, these regions have to satisfy two properties, that is, covering property and switching property $\Omega_{ij} \subseteq cl\Omega_i \cap cl\Omega_j$, where cl denotes the closure of a set.

Throughout this paper, the following definitions are needed.

Definition 1. (*Finite-Time Stable, FTS* [33]) *Given a positive definite matrix R, three positive constants* c_1, c_2, *T, with* $c_1 < c_2$, *and a switching signal* σ, *the continuous-time switched nonlinear system* (1) *with* $u(t) \equiv 0$ *and* $\omega(t) \equiv 0$ *is said to be finite-time stable with respect to* (c_1, c_2, T, R, σ), *if* $x_0^{\mathrm{T}} R x_0 \le c_1 \Rightarrow x^{\mathrm{T}}(t) R x(t) < c_2$, $\forall t \in [0, T]$.

Definition 2. (*Finite-Time Bounded, FTB* [33]) *Given a positive definite matrix R, three positive constants* c_1, c_2, *T, with* $c_1 < c_2$, *and a switching signal* σ, *the system* (3) *is said to be finite-time bounded with respect to* $(c_1, c_2, d, T, R, \sigma)$, *if* $x_0^{\mathrm{T}} R x_0 \le c_1 \Rightarrow x^{\mathrm{T}}(t) R x(t) < c_2$, $\forall t \in [0, T]$, $\forall \omega(t) : \int_0^T \omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t \le d$.

Definition 3. (*Finite-Time H_∞ contro* [28]) *The switched nonlinear system* (1) *is said to be finite-time stabilizable with H_∞ disturbance attenuation level* γ, *if there exists a control input* $u(t)$ *and a switching signal* σ, $\forall t \in [0, T]$ *such that*

1) *The corresponding closed-loop system is finite-time bounded*;
2) *Under the zero-initial condition, the controlled output* $y(t)$ *satisfies*

$$\int_0^T y^{\mathrm{T}}(t) y(t)\mathrm{d}t \le \gamma^2 \int_0^T \omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t,$$

where $\gamma > 0$ *is a prescribed scalar and* $\omega(t)$ *satisfies* (H_2).

3. Main Results

Theorem 1. *Suppose that* (H_1) *holds. If there exist matrices* $P_i > 0$, $Q_i = Q_i^{\mathrm{T}}$, X_i, M_i, *diagonal matrices* $W_i > 0$ *with compatible dimensions and constants* $\theta_i \ge 0$, $\eta_{ij} > 0$, $\alpha > 0$, $\beta > 0$ *such that*

$$\begin{pmatrix} \Pi_{11} & \Pi_{12} & P_i G_i \\ \star & -W_i & 0 \\ \star & \star & -\beta I \end{pmatrix} < 0, \tag{4}$$

$$\sum_{i=1}^{N} \theta_i Q_i \ge 0, \tag{5}$$

$$P_i - P_j + \eta_{ij}\left(Q_i - Q_j\right) = 0, \ i \ne j, \tag{6}$$

$$P_i B_i = B_i M_i, \tag{7}$$

$$\mathrm{e}^{\alpha T}\left(\lambda_1 c_1 + \beta d\right) < \lambda_2 c_2, \tag{8}$$

where

$$\Pi_{11} = A_i^{\mathrm{T}} P_i + P_i A_i + X_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i X_i - L_1 W_i - \alpha P_i, \quad \Pi_{12} = P_i C_i + L_2 W_i,$$

$$\bar{P}_i = R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}, \quad \lambda_1 = \max_{i \in I_N} \left(\lambda_{\max} \left(\bar{P}_i \right) \right), \quad \lambda_2 = \min_{i \in I_N} \left(\lambda_{\min} \left(\bar{P}_i \right) \right),$$

then switched nonlinear system (3) *is FTB with respect to* $(c_1, c_2, d, T, R, \sigma)$ *under the switching signal* $\sigma(t) = \arg\max_{i \in I_N} \left\{ x^{\mathrm{T}} Q_i x \right\}$. *Moreover, the feedback control gains can be designed by* $K_i = M_i^{-1} X_i$.

Proof. Choose the Lyapunov function $V_i(t) = x^{\mathrm{T}}(t) P_i x(t)$, which is used to measure the energy in the region Ω_i.

Case 1: No sliding motion occurs. Assume $\sigma\left(x\left(t_k^-\right)\right) = j$ and $\sigma\left(x\left(t_k\right)\right) = i$. when $t \in [t_k, t_{k+1})$, the derivative of $V_i(t)$ along the trajectories of subsystem i yields

$$\dot{V}_i(t) = x^{\mathrm{T}}(t) \left[\tilde{A}_i^{\mathrm{T}} P_i + P_i \tilde{A}_i \right] x(t) + 2x^{\mathrm{T}}(t) P_i C_i f\left(x(t)\right) + 2x^{\mathrm{T}}(t) P_i G_i \omega(t). \quad (9)$$

From (7), we obtain

$$\begin{aligned} \dot{V}_i(t) = {} & x^{\mathrm{T}}(t) \left[A_i^{\mathrm{T}} P_i + P_i A_i + X_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i X_i \right] x(t) + 2x^{\mathrm{T}}(t) P_i C_i f\left(x(t)\right) \\ & + 2x^{\mathrm{T}}(t) P_i G_i \omega(t). \end{aligned}$$

It follows from assumption (H_2) that

$$\left(f_j\left(x(t)\right) - l_j^- x(t) \right)\left(f_j\left(x(t)\right) - l_j^+ x(t) \right) \le 0, \; j \in \Lambda,$$

i.e.

$$\begin{pmatrix} x(t) \\ f\left(x(t)\right) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} l_j^- l_j^+ e_j e_j^{\mathrm{T}} & -\dfrac{l_j^- + l_j^+}{2} e_j e_j^{\mathrm{T}} \\ \star & e_j e_j^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} x(t) \\ f\left(x(t)\right) \end{pmatrix} \le 0,$$

for every $j \in \Lambda$, where e_j indicates the column vector with *j*-th element to be 1 and others to be 0. Then, for any diagonal matrices $W_i > 0$, the following inequalities hold.

$$\begin{pmatrix} x(t) \\ f\left(x(t)\right) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} L_1 W_i & -L_2 W_i \\ \star & W_i \end{pmatrix} \begin{pmatrix} x(t) \\ f\left(x(t)\right) \end{pmatrix} \le 0. \quad (10)$$

Noting that no sliding motion occurs, we have $\hat{\beta} = \beta$. It can be deduced from (4), (9) and (10) that

$$\begin{aligned} \dot{V}_i(t) \le {} & x^{\mathrm{T}}(t) \left[A_i^{\mathrm{T}} P_i + P_i A_i + X_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i X_i - L_1 W_i \right] x(t) \\ & + 2x^{\mathrm{T}}(t) \left[P_i C_i + L_2 W_i \right] f\left(x(t)\right) + 2x^{\mathrm{T}}(t) P_i G_i \omega(t) - f^{\mathrm{T}}\left(x(t)\right) W_i f\left(x(t)\right) \\ = {} & \begin{pmatrix} x(t) \\ f\left(x(t)\right) \\ \omega(t) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} A_i^{\mathrm{T}} P_i + P_i A_i + X_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i X_i - L_1 W_i & P_i C_i + L_2 W_i & P_i G_i \\ \star & -W_i & 0 \\ \star & \star & 0 \end{pmatrix} \begin{pmatrix} x(t) \\ f\left(x(t)\right) \\ \omega(t) \end{pmatrix} \\ < {} & \alpha V_i(t) + \beta \omega^{\mathrm{T}}(t) \omega(t). \end{aligned} \quad (11)$$

Integrating (11) from t_k to t gives

$$V_i(t) < \mathrm{e}^{\alpha(t-t_k)} V_i(t_k) + \beta \int_{t_k}^{t} \mathrm{e}^{\alpha(t-s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s. \tag{12}$$

When the systems switch from Ω_j into Ω_i at $t = t_k$, it follows from the switching rule that $x^{\mathrm{T}} Q_i x > x^{\mathrm{T}} Q_j x$ at $t = t_k$. Then by (6), we have

$$x^{\mathrm{T}} P_i x - x^{\mathrm{T}} P_i x = -\eta_{ij} x^{\mathrm{T}} \left(Q_i - Q_j\right) x < 0,$$

which implies that $V_i(t_k) < V_j(t_k^-)$. Thus,

$$\begin{aligned}
V_i(t) &< \mathrm{e}^{\alpha(t-t_k)} V_j(t_k^-) + \beta \int_{t_k}^{t} \mathrm{e}^{\alpha(t-s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \\
&< \mathrm{e}^{\alpha(t-t_k)} \left[\mathrm{e}^{\alpha(t_k - t_{k-1})} V_j(t_{k-1}) + \beta \int_{t_{k-1}}^{t_k} \mathrm{e}^{\alpha(t_k - s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \right] \\
&\quad + \beta \int_{t_k}^{t} \mathrm{e}^{\alpha(t-s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \\
&< \mathrm{e}^{\alpha(t-t_{k-1})} V_j(t_{k-1}) + \beta \int_{t_{k-1}}^{t} \mathrm{e}^{\alpha(t-s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \\
&< \mathrm{e}^{\alpha t} V_{\sigma(0)}(0) + \beta \int_{0}^{t} \mathrm{e}^{\alpha(t-s)} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \\
&\le \mathrm{e}^{\alpha T} V_{\sigma(0)}(0) + \beta \mathrm{e}^{\alpha T} \int_{0}^{T} \omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s \\
&\le \mathrm{e}^{\alpha T} \left(V_{\sigma(0)}(0) + \beta d \right).
\end{aligned} \tag{13}$$

Note that

$$V_i(t) = x^{\mathrm{T}}(t) P_i x(t) = x^{\mathrm{T}}(t) R^{\frac{1}{2}} \bar{P}_i R^{\frac{1}{2}} x(t) \ge \lambda_2 x^{\mathrm{T}}(t) R x(t), \tag{14}$$

$$V_{\sigma(0)}(0) = x^{\mathrm{T}}(0) P_{\sigma(x(0))} x(0) = x^{\mathrm{T}}(0) R^{\frac{1}{2}} \bar{P}_{\sigma(x(0))} R^{\frac{1}{2}} x(0) \le \lambda_1 x^{\mathrm{T}}(0) R x(0). \tag{15}$$

According (8) and (13) - (15), we get

$$x^{\mathrm{T}}(t) R x(t) < \frac{\mathrm{e}^{\alpha T}}{\lambda_2} \left(\lambda_1 c_1 + \beta d \right) < c_2.$$

Therefore, the switched system (3) is finite-time bounded.

Case 2: When sliding motion occurs, it will occur at states satisfying $\max_{i \in I_N} x^{\mathrm{T}} Q_i x = \max_{i \in I_N} x^{\mathrm{T}} Q_j x$, which are states where the subsystem changes occur. A sliding motion may occur at surface Ω_{ij}, *i.e.* $x^{\mathrm{T}} Q_i x = x^{\mathrm{T}} Q_j x \ge 0$. When the sliding motion occurs along the hyper-surface Ω_{ij}, it implies that

$$\begin{aligned}
& x^{\mathrm{T}}(t) \left[\tilde{A}_i^{\mathrm{T}} \left(Q_i - Q_j\right) + \left(Q_i - Q_j\right) \tilde{A}_i \right] x(t) + 2x^{\mathrm{T}}(t) \left[\left(Q_i - Q_j\right) C_i \right] f\left(x(t)\right) \\
& + 2x^{\mathrm{T}}(t) \left[\left(Q_i - Q_j\right) G_i \right] \omega(t) < 0, \\
& x^{\mathrm{T}}(t) \left[\tilde{A}_j^{\mathrm{T}} \left(Q_i - Q_j\right) + \left(Q_i - Q_j\right) \tilde{A}_j \right] x(t) + 2x^{\mathrm{T}}(t) \left[\left(Q_i - Q_j\right) C_j \right] f\left(x(t)\right) \\
& + 2x^{\mathrm{T}}(t) \left[\left(Q_i - Q_j\right) G_j \right] \omega(t) > 0.
\end{aligned}$$

The system dynamic on the sliding surface can be expressed as

$$\begin{aligned}
\dot{x}(t) &= \rho \left[\tilde{A}_i x(t) + C_i f\left(x(t)\right) + G_i \omega(t) \right] \\
&\quad + (1-\rho) \left[\tilde{A}_j x(t) + C_j f\left(x(t)\right) + G_j \omega(t) \right], \ \rho \in [0,1].
\end{aligned} \tag{16}$$

From (6) and $\eta_{ij} > 0$, we have

$$x^{\mathrm{T}}(t)\left[\tilde{A}_i^{\mathrm{T}}(P_j-P_i)+(P_j-P_i)\tilde{A}_i\right]x(t)+2x^{\mathrm{T}}(t)\left[(P_j-P_i)C_i\right]f(x(t))$$
$$+2x^{\mathrm{T}}(t)\left[(P_j-P_i)G_i\right]\omega(t)<0,$$
$$x^{\mathrm{T}}(t)\left[\tilde{A}_j^{\mathrm{T}}(P_j-P_i)+(P_j-P_i)\tilde{A}_j\right]x(t)+2x^{\mathrm{T}}(t)\left[(P_j-P_i)C_j\right]f(x(t))$$
$$+2x^{\mathrm{T}}(t)\left[(P_j-P_i)G_j\right]\omega(t)>0.$$

It follows from Case 1 that

$$x^{\mathrm{T}}(t)\left[\tilde{A}_i^{\mathrm{T}}P_j+P_j\tilde{A}_i\right]x(t)+2x^{\mathrm{T}}(t)P_jC_if(x(t))+2x^{\mathrm{T}}(t)P_jG_i\omega(t)$$
$$<x^{\mathrm{T}}(t)\left[\tilde{A}_i^{\mathrm{T}}P_i+P_i\tilde{A}_i\right]x(t)+2x^{\mathrm{T}}(t)P_iC_if(x(t))+2x^{\mathrm{T}}(t)P_iG_i\omega(t)$$
$$<\alpha x^{\mathrm{T}}(t)P_ix(t)+\beta\omega^{\mathrm{T}}(t)\omega(t),$$
$$x^{\mathrm{T}}(t)\left[\tilde{A}_j^{\mathrm{T}}P_i+P_i\tilde{A}_j\right]x(t)+2x^{\mathrm{T}}(t)P_iC_jf(x(t))+2x^{\mathrm{T}}(t)P_iG_j\omega(t)$$
$$<x^{\mathrm{T}}(t)\left[\tilde{A}_j^{\mathrm{T}}P_j+P_j\tilde{A}_j\right]x(t)+2x^{\mathrm{T}}(t)P_jC_jf(x(t))+2x^{\mathrm{T}}(t)P_jG_j\omega(t)$$
$$<\left[\alpha x^{\mathrm{T}}(t)P_jx(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)\right].$$

Then the derivatives of $V_i(t)$ and $V_j(t)$ along the trajectories of system (16) yield

$$\begin{aligned}\dot{V}_i(t)=&\,x^{\mathrm{T}}(t)\left[\rho\left(\tilde{A}_i^{\mathrm{T}}P_i+P_i\tilde{A}_i\right)+(1-\rho)\left(\tilde{A}_j^{\mathrm{T}}P_i+P_i\tilde{A}_j\right)\right]x(t)\\&+2x^{\mathrm{T}}(t)\left[\rho P_iC_i+(1-\rho)P_iC_j\right]f(x(t))\\&+2x^{\mathrm{T}}(t)\left[\rho P_iG_i+(1-\rho)P_iG_j\right]\omega(t)\\<&\,\rho\left[\alpha x^{\mathrm{T}}(t)P_ix(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)\right]\\&+(1-\rho)\left[\alpha x^{\mathrm{T}}(t)P_jx(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)\right]\\=&\,\alpha x^{\mathrm{T}}(t)P_ix(t)+\beta\omega^{\mathrm{T}}(t)\omega(t),\end{aligned}$$

$$\begin{aligned}\dot{V}_j(t)=&\,x^{\mathrm{T}}(t)\left[\rho\left(\tilde{A}_j^{\mathrm{T}}P_j+P_j\tilde{A}_j\right)+(1-\rho)\left(\tilde{A}_i^{\mathrm{T}}P_j+P_j\tilde{A}_i\right)\right]x(t)\\&+2x^{\mathrm{T}}(t)\left[\rho P_jC_j+(1-\rho)P_jC_i\right]f(x(t))\\&+2x^{\mathrm{T}}(t)\left[\rho P_jG_j+(1-\rho)P_jG_i\right]\omega(t)\\<&\,\rho\left[\alpha x^{\mathrm{T}}(t)P_jx(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)\right]\\&+(1-\rho)\left[\alpha x^{\mathrm{T}}(t)P_ix(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)\right]\\=&\,\alpha x^{\mathrm{T}}(t)P_jx(t)+\beta\omega^{\mathrm{T}}(t)\omega(t).\end{aligned}$$

Then the rest of proof is the same as Case 1. This completes the proof. □

Remark 1. *In* [34] [35] [36], *authors studied the FTB of switched systems, but most of them are based on time-dependent switching, that is, the switching instant needs to be given in advance. There are few results using state-dependent switching strategy, especially for nonlinear systems. Moreover, the system state may generate sliding mode motion on the switching surface under the switching signal* $\sigma(t)$. *However, with the help of the multiple Lyapunov method and LMIs technique, the FTB of the nonlinear switched system* (3) *can be ensured by Theorem* 1 *whether the sliding mode appears or not under the state-dependent switching signal we designed.*

Theorem 2. *Suppose that* (H1) *and* (H2) *hold. Given constant* $\gamma > 0$, *if there exist matrices* $P_i > 0$, $Q_i = Q_i^{\mathrm{T}}$, X_i, M_i, *diagonal matrices* $W_i > 0$ *with compatible dimensions and constants* $\theta_i \ge 0$, $\eta_{ij} > 0$, $\alpha > 0$, $\beta > 0$ *such that*

$$\begin{pmatrix} \Pi_{11} & \Pi_{12} & P_i G_i & D_i^{\mathrm{T}} \\ \star & -W_i & 0 & 0 \\ \star & \star & -\beta I & E_i^{\mathrm{T}} \\ \star & \star & \star & -\delta I \end{pmatrix} < 0, \tag{17}$$

$$\sum_{i=1}^{N} \theta_i Q_i \ge 0, \tag{18}$$

$$P_i - P_j + \eta_{ij}\left(Q_i - Q_j\right) = 0,\ i \ne j, \tag{19}$$

$$P_i B_i = B_i M_i, \tag{20}$$

$$\mathrm{e}^{\alpha T}\left(\lambda_1 c_1 + \beta d\right) < \lambda_2 c_2, \tag{21}$$

where $\delta = \dfrac{\gamma^2 \mathrm{e}^{-\alpha T}}{\beta}$, $\Pi_{11}, \Pi_{12}, \lambda_1, \lambda_2$ *are the same as in Theorem 1, then switched nonlinear system* (*1*) *is finite-time stabilizable with* H_∞ *disturbance attenuation level* γ *with respect to* $\left(c_1, c_2, d, T, R, \sigma\right)$ *under the switching law* $\sigma(t) = \arg\max_{i \in I_N}\left\{x^{\mathrm{T}} Q_i x\right\}$. *Moreover, the feedback control gains can be designed by* $K_i = M_i^{-1} X_i$.

Proof. Choose the Lyapunov function $V_i(t) = x^{\mathrm{T}}(t) P_i x(t)$. The FTB can be easily proved following the proof procedure of Theorem 1. Moreover, by schur complement lemma, condition (17) is equivalent to

$$\begin{pmatrix} \tilde{\Pi}_{11} & \Pi_{12} & \tilde{\Pi}_{13} \\ \star & -W_i & 0 \\ \star & \star & \tilde{\Pi}_{33} \end{pmatrix} < 0$$

where

$$\tilde{\Pi}_{11} = A_i^{\mathrm{T}} P_i + P_i A_i + X_i^{\mathrm{T}} B_i + B_i X_i + \frac{1}{\delta} D_i^{\mathrm{T}} D_i - L_1 W_i - \alpha P_i,$$

$$\tilde{\Pi}_{13} = P_i G_i + \frac{1}{\delta} D_i^{\mathrm{T}} E_i,\quad \tilde{\Pi}_{33} = \frac{1}{\delta} E_i^{\mathrm{T}} E_i - \beta I.$$

Then, we have

$$\begin{aligned} \dot{V}_i(t) < {} & \alpha V_i(t) - \frac{1}{\delta}\Big(x^{\mathrm{T}}(t) D_i^{\mathrm{T}} D_i x(t) + w^{\mathrm{T}}(t) E_i^{\mathrm{T}} D_i x(t) + x^{\mathrm{T}}(t) D_i^{\mathrm{T}} E_i w(t) \\ & + w^{\mathrm{T}}(t) E_i^{\mathrm{T}} E_i w(t)\Big) + \beta \omega^{\mathrm{T}}(t)\omega(t) \\ = {} & \alpha V_i(t) - \frac{1}{\delta} y^{\mathrm{T}}(t) y(t) + \beta \omega^{\mathrm{T}}(t)\omega(t). \end{aligned}$$

It can be derived that

$$V_i(t) < \mathrm{e}^{\alpha t} V_{\sigma(0)}(0) + \beta \int_0^t \mathrm{e}^{\alpha(t-s)}\left[-\frac{\mathrm{e}^{\alpha T}}{\gamma^2} y^{\mathrm{T}}(s) y(s) + \omega^{\mathrm{T}}(s)\omega(s)\right]\mathrm{d}s.$$

Note that $V_i(t) > 0$ and $V_{\sigma(0)}(0) = 0$ with the given conditions in Theorem 2. It follows that

$$\int_0^T e^{\alpha(t-s)}\left[-\frac{e^{\alpha T}}{\gamma^2} y^{\mathrm{T}}(s)y(s) + \omega^{\mathrm{T}}(s)\omega(s)\right]ds > 0,$$

which implies

$$\int_0^T y^{\mathrm{T}}(s)y(s)ds < \gamma^2 e^{-\alpha T}\int_0^T e^{\alpha(t-s)}\omega^{\mathrm{T}}(s)\omega(s)ds.$$

Therefore,

$$\int_0^T y^{\mathrm{T}}(s)y(s)ds < \gamma^2\int_0^T \omega^{\mathrm{T}}(s)\omega(s)ds.$$

The switched nonlinear system (1) is finite-time stabilizable with H_∞ performance γ. This completes the proof. □

Remark 2. *In [28], based on time-dependent switching, authors investigated the finite time H_∞ control for switched systems. In [36], authors studied the FTB problems of switched systems under time-dependent switching. Different from the time-dependent switching, the switching instant of the state-dependent switching is unknown, which is more practical. The finite-time stabilization and FTB of switched systems were considered based on state-dependent switching in [25]. Several sufficient conditions for the finite-time stability and L_2-gain analysis of switched linear systems were derived in [33]. However, the systems they considered are linear and the resultes do not involve the investigation of the finite-time H_∞ control. Moreover, the output only depended on the states of the system. By designing the appropriate switching law and considering the output related to both the system states and the external disturbance, some sufficient conditions are provided to guarantee the finite-time H_∞ controllability of the nonlinear switched system (1) in Theorem 2, which extends and enriches the results in [25] [33].*

Corollary 1. *Suppose that* (H_1) *holds. If there exist matrices* $P_i > 0$, $Q_i = Q_i^{\mathrm{T}}$, *diagonal matrices* $W_i > 0$ *with compatible dimensions and constants* $\theta_i \ge 0$, $\eta_{ij} > 0$, $\alpha > 0$ *such that*

$$\begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \star & -W_i \end{pmatrix} < 0, \tag{22}$$

$$\sum_{i=1}^{N}\theta_i Q_i \ge 0, \tag{23}$$

$$P_i - P_j + \eta_{ij}(Q_i - Q_j) = 0,\ i \ne j, \tag{24}$$

$$e^{\alpha T}\lambda_1 c_1 < \lambda_2 c_2, \tag{25}$$

where

$$\Pi_{11} = A_i^{\mathrm{T}}P_i + P_iA_i - L_1W_i - \alpha P_i,\quad \Pi_{12} = P_iC_i + L_2W_i,$$

$$\bar{P}_i = R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}},\quad \lambda_1 = \max_{i\in I_N}\left(\lambda_{\max}\left(\bar{P}_i\right)\right),\quad \lambda_2 = \min_{i\in I_N}\left(\lambda_{\min}\left(\bar{P}_i\right)\right),$$

then switched nonlinear system (1) with $u(t)=0$ *and* $\omega(t)=0$ *is FTS with respect to* (c_1,c_2,T,R,σ) *under the switching law* $\sigma(t)=\arg\left(\max_{i\in I_N} x^{\mathrm{T}}Q_i x\right)$.

4. Example

In this section, an example is given to illustrate the effectiveness of the proposed methods.

Two subsystems are considered, *i.e.*, $\sigma(t)\in I_N=\{1,2\}$. Consider the switched nonlinear system (1) with parameters as follows

$$A_1=\begin{pmatrix}0.37 & 0.47\\ 1.8 & 0.79\end{pmatrix},\ B_1=\begin{pmatrix}1.15\\ 0.64\end{pmatrix},\ C_1=\begin{pmatrix}1.15 & 0.21\\ 0.59 & 0.2\end{pmatrix},$$

$$G_1=\begin{pmatrix}0.59 & 0.15\\ 0.15 & 0.62\end{pmatrix},\ D_1=\begin{pmatrix}1.05 & 0.45\end{pmatrix},\ E_1=\begin{pmatrix}0.16 & 0.59\end{pmatrix},$$

$$A_2=\begin{pmatrix}0.29 & 0.97\\ 1 & 0.9\end{pmatrix},\ B_2=\begin{pmatrix}0.27\\ 0.56\end{pmatrix},\ C_2=\begin{pmatrix}1.2 & 0.64\\ 0.45 & 1.12\end{pmatrix},$$

$$G_2=\begin{pmatrix}0.62 & 0.1\\ 0.16 & 0.58\end{pmatrix},\ D_2=\begin{pmatrix}1.05 & 0.95\end{pmatrix},\ E_2=\begin{pmatrix}0.26 & 0.29\end{pmatrix},$$

$$\omega(t)=\begin{pmatrix}-0.2\cos(2.7t+1)\\ 0.2\sin(2t)\end{pmatrix},\quad f(s)=f_r(s)=\tanh(0.4s).$$

It is easy to get that $L_1=\mathrm{diag}\{0,0\}$, $L_2=\mathrm{diag}\{0.2,0.2\}$. We choose $\alpha=0.01$, $\beta=0.65$, $c_1=0.015$, $c_2=0.6$, $\theta_1=\theta_2=1$, $\eta_{ij}=0.9$, $\gamma=0.9$, $R=I$, $T=10$. Then solve the inequality in Theorem 2 by using Matlab LMI toolbox, one may find the following feasible solutions

$$Q_1=\begin{pmatrix}5.5107 & -0.1112\\ -0.1112 & 6.9027\end{pmatrix},\ Q_2=\begin{pmatrix}6.5242 & 0.1112\\ 0.1112 & 5.1322\end{pmatrix},$$

$$P_1=\begin{pmatrix}1.4711 & 0.2079\\ 0.2079 & 0.7740\end{pmatrix},\ P_2=\begin{pmatrix}0.5589 & 0.0077\\ 0.0077 & 2.3675\end{pmatrix},$$

$$W_1=\begin{pmatrix}3.2795 & 0\\ 0 & 3.2795\end{pmatrix},\ W_2=\begin{pmatrix}5.5623 & 0\\ 0 & 5.5623\end{pmatrix},$$

$$X_1=\begin{pmatrix}-7.8213 & -5.2151\end{pmatrix},\ X_2=\begin{pmatrix}-9.1366 & -19.7584\end{pmatrix},$$

$$K_1=\begin{pmatrix}-5.2742 & -3.5168\end{pmatrix},\ K_2=\begin{pmatrix}-4.4955 & -9.7217\end{pmatrix},$$

$$M_1=1.4829,\ M_2=2.0324.$$

As we can see, the switched nolinear system (1) is unbound without control in **Figure 2**. **Figure 3** shows that the trajectories of the closed-loop system (3) with the state feedback controller (2). Moreover, we can clearly see that the system (3) is FTB with respect to $(0.015,0.6,0.41,10,I,\sigma)$ from **Figure 4**. And **Figure 5** illustrates the switching signal $\sigma(t)$. Furthermore, the system (1) has H_∞ disturbance attenuation level $\gamma=0.9$, which is shown in **Figure 6**.

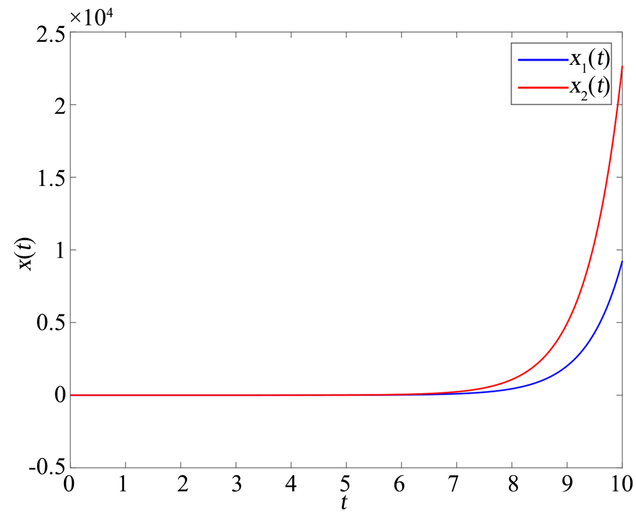

Figure 2. The state trajectories of x_1 and x_2 without control.

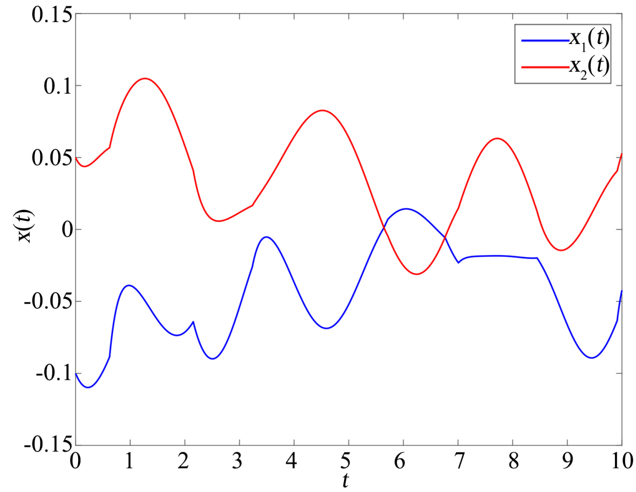

Figure 3. The state trajectories of x_1 and x_2 with the state feedback controller (2).

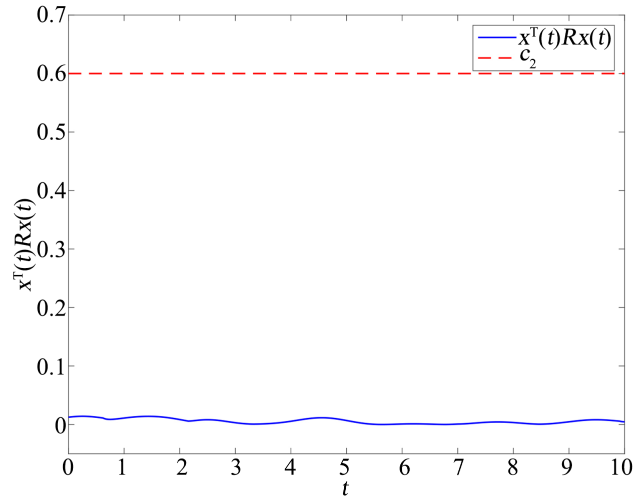

Figure 4. The trajectories of $x^{\mathrm{T}}(t)Rx(t)$ with the state feedback controller (2).

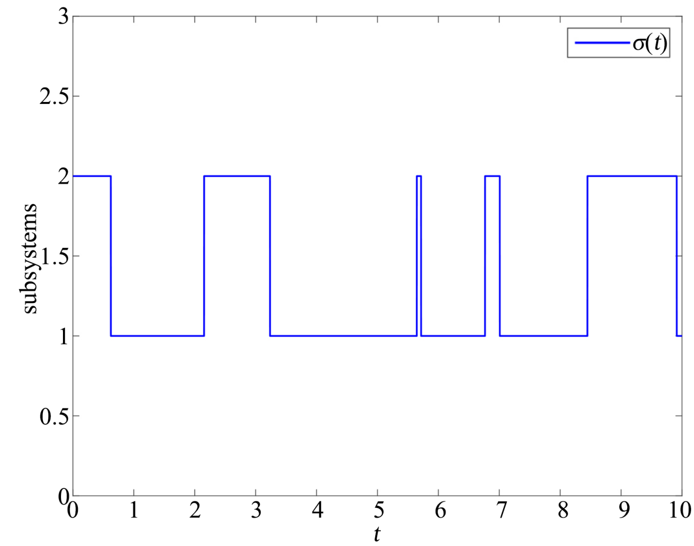

Figure 5. The switching signal $\sigma(t)$.

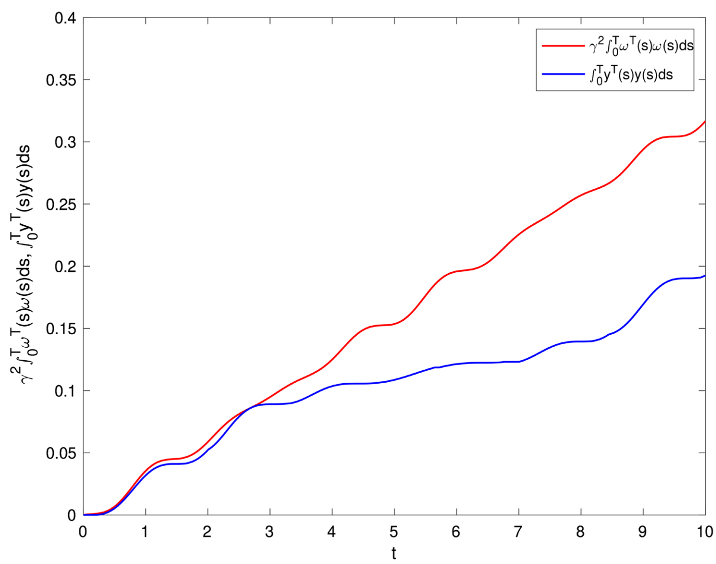

Figure 6. The H_∞ disturbance attenuation level $\gamma = 0.9$.

5. Conclusion

In this paper, the problem of finite-time H_∞ control for switched nonlinear systems with a state-dependent switching signal has been investigated, where the output considered is related to both the current states and the external disturbance. By using the multiple Lyapunov method and the free weight matrix technique, several sufficient conditions for finite-time H_∞ control of the system with and without sliding mode motion are proposed respectively. Finally, the validity of the conclusion is verified by numerical simulation.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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