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On the Expression of Composite Natural Numbers as Tensorial Products of Prime Natural Powers Vectors

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Abstract

This study describes how one can construct sets of composite natural numbers as tensorial products of the vectors created with the natural powers of prime numbers.

Keywords

Composite Natural Numbers, Prime Numbers, Prime Numbers Powers, Vectors of Prime Numbers Powers, Tensors Holding the Composite Natural Numbers

1. Introduction

In a recent paper [1], one of us described a mathematical framework where natural numbers were attached to a geometrical structure. One has based this procedure on simple ideas associated with normed vector spaces defined over natural numbers.

In this previous study, composite natural numbers resulted from geometrical norms attached to vectors, whose structure was associated with the natural powers of prime numbers. Prime number's natural powers acted as scalars providing homotheties of the canonical basis set.

Such simple mathematical construction permits the connection between an infinite-dimensional vector space with his associate canonical basis set and a finite-dimensional vector space, the so-called inner space, where the canonical coordinates correspond to the prime number's natural powers.

Here, a similar but completely different idea starts from the prime natural power sets, which one might easily consider as infinite-dimensional vectors associated with every prime number.

Such an infinite set of infinite-dimensional vectors can be the subject of appropriate tensorial products, yielding an infinite set of infinite-dimensional arbitrary rank tensors. The elements of such tensors correspond to subsets of the set of composite natural numbers.

One can reach the practical computational side of such a possible construction by considering finite-dimensional prime power vectors, whose tensorial products produce finite-dimensional tensors of arbitrary rank, containing as elements partial sets of the composite natural numbers.

To describe this prospect, the present paper will start with the basic notation and the description of prime number power vectors, which one will first use to construct an example made by composite natural number tensors of rank two. Then, one will describe the general structure of arbitrary rank tensors for completeness. Finally, one will present a practical finite-dimensional, computationally adapted algorithm.

2. Basic Notation and Prime Number Powers Vectors

Although this trivial section is easy to grasp, it becomes essential when considering the infinite-dimensional structure of the vectors needed to build the tensors associated with composite numbers. It describes the notation used here from now on.

2.1. Ordered Prime Number Set

Let us note in the following way the ordered set of prime numbers, $\mathbf{P}$, as a subset of the natural number set $\mathbb{N}$:

$$\mathbf{P} \subset \mathbb{N} .$$

That is, one can think of the ordered prime number set as the sequence:

$$\mathbf{P} = \{2, 3, 5, 7, 11, \cdots, \boldsymbol{p}_I, \cdots\},$$

where: p_I denotes the I-th prime number. Then this means that one can also write:

$$\boldsymbol{p}_1 = 2; \boldsymbol{p}_2 = 3; \boldsymbol{p}_3 = 5; \cdots$$

Of course, the cardinalities of both primes ad whole natural numbers sets are related in general by:

$$Card(\mathbf{P}) \leq Card(\mathbb{N}) .$$

2.2. Vectors of Prime Number Natural Powers

Next, let us adopt Dirac's bra-ket notation for row-column natural number vectors and define the infinite-dimensional row vector of the sequence of natural powers attached with the I-th prime number as:

$$\left\langle \mathbf{p}_I^{\mathbb{N}} \right| = \left(\boldsymbol{p}_I^1; \boldsymbol{p}_I^2; \boldsymbol{p}_I^3; \cdots; \boldsymbol{p}_I^N; \cdots \right) .$$

Then, according to these preliminary considerations, the transpose of $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$

is an infinite-dimensional column vector:

$$\left\langle \mathbf{p}_I^{\mathbb{N}} \right|^{\mathrm{T}} = \left| \mathbf{p}_I^{\mathbb{N}} \right\rangle .$$

Thus, one can suppose the dual vector pair attached to each prime number: $\forall \boldsymbol{p}_I \in \mathbf{P} : \exists \left\{ \left\langle \mathbf{p}_I^{\mathbb{N}} \right| ; \left| \mathbf{p}_I^{\mathbb{N}} \right\rangle \right\}$.

One can consider any vector $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$ and its dual as tensors of rank one.

At the same time, because of the tuple structure, by the symbol $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$ one can understand, if necessary, both a vector and a set of prime powers.

One can highlight the character of the set of prime numbers' natural powers vectors defining an infinite-dimensional vector space, defined in turn over the natural number set, where these vectors belong:

$$\forall \boldsymbol{p}_I \in \mathbf{P} : \exists \left\langle \mathbf{p}_I^{\mathbb{N}} \right| \in \mathrm{V}_\infty \left(\mathbb{N} \right).$$

From this point of view, one can construct every prime power vector $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$ through the canonical basis set defined as:

$$\forall I \in \mathbb{N} : \left\langle \mathbf{e}_I \right| = \left\{ e_{IJ} = \delta_{IJ} \left| \forall J \in \mathbb{N} \right. \right\} \Rightarrow \left\langle \mathbf{e}_I \right| \in \mathrm{V}_\infty \left(\mathbb{N} \right).$$

Then, one can rewrite the vectors $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$ in terms of the infinite collection of canonical basis vectors as follows:

$$\forall \boldsymbol{p}_I \in \mathbf{P} : \exists \left\langle \mathbf{p}_I^{\mathbb{N}} \right| = \sum_{J \in \mathbb{N}} \left(\boldsymbol{p}_I \right)^J \left\langle \mathbf{e}_J \right|.$$

As one can consider the prime power vectors constructed with ordered non-zero elements, they are *perfect* vectors in a sense provided by the reference [2].

2.3. The Role of the Natural Numbers Set with Zero Added

Also, it is interesting to notice that one could write the prime natural powers vectors as a homothecy of the powers of the natural numbers set with zero added: $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$, where the homothetic factor could be the corresponding *I*-th prime number $\boldsymbol{p}_I$. That is, one can write the following alternative form:

$$\left\langle \mathbf{p}_I^{\mathbb{N}} \right| = \boldsymbol{p}_I \left\langle \mathbf{p}_I^{\mathbb{N}_0} \right| = \boldsymbol{p}_I \left(1; \boldsymbol{p}_I; \boldsymbol{p}_I^2; \cdots; \boldsymbol{p}_I^{N-1}; \cdots \right),$$

where one emphasizes the character of the *I*-th prime number as a homothetic parameter.

2.4. An Apparent Paradox about Cardinalities of Natural Numbers and Powers of Prime Numbers

One can now be aware that each natural bra vector of prime powers $\left\langle \mathbf{p}_I^{\mathbb{N}} \right|$, considered the set of natural powers of the *I*-th prime number, bears the same cardinality as the natural number set $\mathbb{N}$.

The set of prime numbers, even infinite, as a subset of the natural numbers, apparently seems to bear a cardinality less than the natural set $\mathbb{N}$; see, for example, references [3] [4] [5] [6]. However, prime set infinity precludes the existence of an infinite set of prime natural powers vectors, constructed as one has commented in section 2.2.

One can write these considerations and practically admit one can suppose the bra vectors as set representations of the natural powers of each unique prime number.

Then, the elements of the union of all of the prime power vectors, which can be symbolized by $\mathbf{P}^{\mathbb{N}}$, is just a subset of the natural numbers:

$$\mathbf{P}^{\mathbb{N}} = \bigcup_{I \in \mathbb{N}} \left\langle \mathbf{p}_I^{\mathbb{N}} \right| \to \mathbf{P}^{\mathbb{N}} \subset \mathbb{N},$$

yielding the inclusion as the composite numbers made by two or more prime power products are not contained in the prime powers' union set $\mathbf{P}^{\mathbb{N}}$.

In the same way that one can assume the cardinality of the prime set is less than the one attached to the whole natural number set, as discussed before in section 2.1, it seems that one can also write in general:

$$Card\left(\mathbf{P}^{\mathbb{N}}\right) \le Card\left(\mathbb{N}\right).$$

However, there a paradox might develop. Because it seems evident that while:

$$\forall \boldsymbol{p}_I \in \mathbf{P} : Card\left(\left\langle \mathbf{p}_I^{\mathbb{N}} \right|\right) = Card\left(\mathbb{N}\right),$$

One might perhaps compare the cardinality of the union of prime powers with the prime and natural number cardinalities as:

$$Card\left(\mathbf{P}^{\mathbb{N}}\right) = Card\left(\mathbf{P}\right) \times Card\left(\mathbb{N}\right) \ge Card\left(\mathbb{N}\right).$$

One can solve the paradox by realizing that the cardinality of the prime numbers set is the same as the natural number one and that the product of two equal cardinalities yields the factor cardinality.

Such unequal cardinality relationships, though, might be valid in the discrete dimension description in section 5. below, where one discusses the creation of prime vector powers and tensors. Then, infinite cardinality algebra should transform into a standard natural number algebra of the chosen natural vector space dimensions; for example, see reference [5] [6].

2.5. Linear Independence of Prime Natural Powers Vectors

One can represent the set of the prime natural powers' vectors like:

$$\mathbf{P}^{\mathbb{N}} = \left\{ \left\langle \mathbf{p}_I^{\mathbb{N}} \right| \middle| I \in \mathbb{N}; \boldsymbol{p}_I \in \mathbf{P} \right\},$$

and the associated dual set of column vectors, which one can construct so that no vector possesses a shared element with the rest. Thus, in the same way as the canonical vector set, the set $\mathbf{P}^{\mathbb{N}}$ is linearly independent, that is, a set of natural numbers:

$$\mathrm{A} = \left\{ a_I \middle| I \in \mathbb{N} \right\} \equiv \left| \mathbf{a} \right\rangle \leftrightarrow \left\langle \mathbf{a} \right| \in \mathrm{V}_\infty\left(\mathbb{N}\right),$$

used as unknown values in the following equation implies that the vector $\left\langle \mathbf{a} \right|$ is null:

$$\sum_{I \in \mathbb{N}} a_I \left\langle \mathbf{p}_I^{\mathbb{N}} \right| = \left\langle \mathbf{0} \right| \Rightarrow \left\langle \mathbf{a} \right| = \left\langle \mathbf{0} \right|.$$

Suppose one accepts the usual vector operations in the natural vector space. In that case, one must also consider that the natural set necessarily includes zero as the null natural element, consistent with the discussion in the above section 2.3.

To show the linear independence of the vector set $\mathrm{P}^{\mathbb{N}}$, it is only necessary to use the reduction to absurd, a customary procedure in these cases, and realize that one cannot describe a power *vector* of a prime number by a linear combination of the power vectors of different prime numbers.

2.6. Definition of the Natural Powers of Prime Numbers Functions

Connected with this linear independency property of the elements of the set $\mathbf{P}^{\mathbb{N}}$, one can also define a set of natural discrete functions using a simple algorithm:

$$\forall I, N \in \mathbb{N} \wedge \boldsymbol{p}_I \in \mathbf{P} : \boldsymbol{p}_I(N) = \boldsymbol{p}_I^N .$$

That is, for example, one can see that if: $I = 4 \wedge N = 6 : \boldsymbol{p}_4 = 7 \wedge \boldsymbol{p}_4(6) \equiv 7^6$.

Then the set of natural power functions, which one can collect in the set:

$$\mathbf{P}^{\mathbb{N}} = \left\{ \boldsymbol{p}_I(N) \middle| I, N \in \mathbb{N}; \boldsymbol{p}_I \in \mathbf{P} \right\},$$

Corresponds to a set of linearly independent functions.

2.7. Prime Natural Powers as Unique Elements

As a consequence of these previous definitions and properties, one can admit as an impossible occurrence that a sum of powers of different prime numbers yields a power of a prime number. That is:

$$\forall N > 1 \in \mathbb{N} \wedge \boldsymbol{p}_I \in \mathbf{P} : \sum_{I \in \mathbb{N}} \boldsymbol{p}_I^N \notin \mathbf{P} .$$

Such a property implies that, when considering the nature of the elements of the sums involved in Fermat's theorem of any dimension, they cannot be in any case all of them prime numbers; see, for example, reference [7], to peruse the definition and extension of the theorem.

For instance, when considering the Pythagorean triples, only the ones that one can call *true* Fermat triples comprise two or one prime components, resulting in the triples without primes being homothetic from a true Fermat triple.

3. Composite Natural Number Tensors of Rank Two

One can manipulate the vectors constructed with elements of natural powers of prime numbers subject to the usual vector space operations; for example, see references [2] [8].

The absence of a field structure on the natural number set $\mathbb{N}$, a semiring, limits everything. Even under this restriction, one can consider tensor operations available on these natural vectors as another useful geometrical tool.

One can envisage the construction of all the composite natural numbers bearing *two* prime natural powers as the rank two tensorial product, yielding an infinite-dimensional array structure.

The result can be written, as usual, by the equivalent notations:

$$\left|\mathbf{p}_I^{\mathbb{N}}\right\rangle\left\langle\mathbf{p}_J^{\mathbb{N}}\right| \equiv \left|\mathbf{p}_I^{\mathbb{N}}\right\rangle \otimes \left|\mathbf{p}_J^{\mathbb{N}}\right\rangle = \mathbf{C}_{IJ}^{\mathbb{N}\mathbb{N}} .$$

Thus, one can take the elements of the above rank two tensor as those of an alternative infinite-dimensional square symmetric matrix, whose elements one can express as:

$$\forall\left\{\boldsymbol{p}_I;\boldsymbol{p}_J\right\} \subset \mathbf{P} : \mathbf{C}_{IJ}^{\mathbb{N}\mathbb{N}} = \left\{\boldsymbol{p}_I^A \boldsymbol{p}_J^B \middle| \forall\left(A,B\right) \in \mathbb{N}\right\} .$$

According to the possible homothecy point of view of the vectors $\left\langle\mathbf{p}_I^{\mathbb{N}}\right|$, as described at the end of section 2.2 above, one can also consider the following expression of the rank two tensors:

$$\mathbf{C}_{IJ}^{\mathbb{N}\mathbb{N}} = \left(\boldsymbol{p}_I \boldsymbol{p}_J\right)\left|\mathbf{p}_I^{\mathbb{N}_0}\right\rangle\left\langle\mathbf{p}_J^{\mathbb{N}_0}\right| \equiv \left(\boldsymbol{p}_I \boldsymbol{p}_J\right)\left|\mathbf{p}_I^{\mathbb{N}_0}\right\rangle \otimes \left|\mathbf{p}_J^{\mathbb{N}_0}\right\rangle .$$

One can note now the fact that the prime number pairs entering the construction of the rank two tensors of composite natural numbers can be chosen being:

$$I < J \leftrightarrow \boldsymbol{p}_I < \boldsymbol{p}_J .$$

The reverse relationship between prime numbers does not provide new information about calculating the elements of the related tensors, as these tensor elements are the same.

4. The General Composite Natural Number Tensors of Arbitrary Rank

From the rank two tensorial constructs, as presented in the section above, one can write tensors of any arbitrary superior rank whose elements are composite numbers bearing at least N distinct prime number natural powers.

Yet, they possess the characteristics of an infinite-dimensional tensorial product that one might generally write as:

$$\mathbf{C}_{\left(I_K|K=1,N\right)}^{\mathbb{N}^N} = \bigotimes_{K=1}^{N}\left|\mathbf{p}_{I_K}^{\mathbb{N}}\right\rangle = \left\{\prod_{K=1}^{N} \boldsymbol{p}_{I_K}^{A_K} \middle| \left(A_K \middle| K=1,N\right) \subset \mathbb{N}\right\} .$$

In the same way as in the rank two tensors, one can write an equivalent expression of the arbitrary rank tensor above like:

$$\mathbf{C}_{\left(I_K|K=1,N\right)}^{\mathbb{N}^N} = \left(\prod_{K=1}^{N} \boldsymbol{p}_{I_K}\right) \bigotimes_{K=1}^{N}\left|\mathbf{p}_{I_K}^{\mathbb{N}_0}\right\rangle .$$

As in the previous rank two tensors, one can suppose that the arbitrary rank tensors are made using an ordered set of prime numbers, like:

$$\boldsymbol{p}_{I_1} < \boldsymbol{p}_{I_2} < \boldsymbol{p}_{I_3} < \cdots < \boldsymbol{p}_{I_K} < \cdots < \boldsymbol{p}_{I_N} .$$

Other possible ordering choices will generate tensors with reordered equivalent composite natural number elements. Unless one needs the corresponding tensor elements reorganization associated with a permutation of the involved prime numbers, one can consider the prime numbers involved in constructing the tensor ordered as in the prime number set. As discussed earlier, such a possibility appears to be in close connection with the structure of vector spaces [2].

5. The Practical Expression of the Composite Natural Number Tensors

Of course, from the practical point of view, one can transform the infinite-dimensional tensors of section 3 into a collection of finite-dimensional ones. One needs to consider the prime vector powers up to some natural number N and rewrite the involved power vector as a N-dimensional one, like:

$$\left\langle \mathbf{p}_I^N \right| = \left(p_I^1; p_I^2; p_I^3; \cdots; p_I^N \right),$$

Which one can also express as a homothecy:

$$\left\langle \mathbf{p}_I^N \right| = p_I \left(1; p_I^1; p_I^2; \cdots; p_I^{N-1}\right) = p_I \left\langle \mathbf{p}_I^{N-1} \right|.$$

5.1. Finite-Dimensional Rank Two Tensors

Therefore, one can construct the tensors of rank two by designing an algorithm like:

$$\forall (M,N) \subset \mathbb{N} : \mathbf{C}_{IJ}^{MN} = \left| \mathbf{p}_I^M \right\rangle \left\langle \mathbf{p}_J^N \right| = \left| \mathbf{p}_I^M \right\rangle \otimes \left| \mathbf{p}_J^N \right\rangle = \left\{ p_I^A p_J^B \middle| A = 1, M; B = 1, N \right\},$$

or instead, like:

$$\forall (M,N) \subset \mathbb{N} : \mathbf{C}_{IJ}^{MN} = \left(p_I p_J \right) \left| \mathbf{p}_I^{M-1} \right\rangle \otimes \left| \mathbf{p}_J^{N-1} \right\rangle$$

Then alternatively, one can consider the corresponding tensors as $(M \times N)$-dimensional matrices.

For example, one can write one of the simplest of these tensors as the (2×2) matrix:

$$\mathbf{C}_{12}^{22} = \begin{pmatrix} 2 \times 3 & 2 \times 3^2 \\ 2^2 \times 3 & 2^2 \times 3^2 \end{pmatrix} = \begin{pmatrix} 6 & 18 \\ 12 & 36 \end{pmatrix} \equiv (2 \times 3) \begin{pmatrix} 1 & 3 \\ 2 & 2 \times 3 \end{pmatrix}.$$

Choosing the corresponding prime numbers in reverse order will generate the corresponding tensor connected with an attached transpose matrix.

5.2. Finite-Dimensional Arbitrary Rank Tensors of Composite Numbers

One can transform the infinite-dimensional tensors of section 4, into finite-dimensional tensors associated with the composite numbers holding N prime powers. One can build these finite-dimensional tensors even using different finite dimensions for each vector. For instance, according to the set: $\left\{ N_K \middle| K = 1, N \right\} \subset \mathbb{N}$.

Therefore, one can write a finite-dimensional tensor of arbitrary rank N as an array like:

$$\mathbf{C}_{(I_K | K=1,N)}^{(N_K | K=1,N)} = \bigotimes_{K=1}^{N} \left| \mathbf{p}_{I_K}^{N_K} \right\rangle = \left\{ \prod_{K=1}^{N} p_{I_K}^{A_K} \middle| \left(A_K \middle| K = 1, N_K \right) \subset \mathbb{N} \right\}.$$

Then, one can see every tensor element made with products of N prime powers. Therefore, one can also consider every tensor of this kind as an array containing a subset of natural composite numbers.

One can also write the alternative expression in the homothecy formalism as:

$$\mathbf{C}_{(I_K|K=1,N)}^{(N_K|K=1,N)}=\left(\prod_{K=1}^{N}\boldsymbol{p}_{I_k}\right)\bigotimes_{K=1}^{N}\left|\mathbf{p}_{I_K}^{N_K-1}\right\rangle .$$

The same remark about ordering the involved prime numbers as before in the infinite-dimensional case applies here. The structure of the tensor will vary; however, the information about the contained composite natural number elements will be the same as in the ordered construction.

When the dimension of the involved prime number powers is uniformly the same for all the used primes, one can observe the magnitude of the generated composite natural numbers.

Suppose, then, that:

$$N_1=N_2=N_3=\cdots=N_N=M ,$$

thus, the number of composite number elements will be: N^M .

So, the number of computable composite numbers quickly becomes astronomically large. Suppose one chooses ten prime numbers and assumes their discrete power vectors of dimension 11. One can obtain a vast number of composite natural numbers: $M=11\wedge N=10\rightarrow N^M=10^{11}$. A quantity that roughly corresponds to the average number of stars in a galaxy the size of the Milky Way.

Combinatorics can produce an infinite set of sets of this kind; think about the freedom to choose among the known ones (see, for example, references [2] [7] [9]) 10^6 different heaps of 10 prime numbers natural power vectors of dimension 11. Then the number of composite natural numbers in this set will be 10^{17} elements.

6. Conclusions

This study's simple arguments show the straightforward construction of composite natural numbers as feasible, defined through tensor products of the vectors constructed with prime powers indefinite sequences.

Thus, using the fact that all composite numbers are expressible from products of powers of primes and from what one has discussed so far, natural numbers can be well-understood on the one hand from prime numbers and their natural power set vectors, and on the other with the tensorial products of these prime power vectors, containing the prospect to generate all the composite numbers.

Thus, from the point of view of the present study, composite numbers might appear as an infinite set of tensors, starting from the prime power vectors and the tensor product of two prime powers elements, driving to the tensor product of an arbitrary number of vectors.

Then, one can imagine the whole set of composite numbers as tensor elements made of an infinite (or quasi-infinite, if the computational or practical side is preferred) number of tensorial products of prime powers vectors.

Finally, one can think about the composite numbers in terms of infinite rank tensors made by infinite tensorial products of infinite-dimensional prime powers vectors.

Consequently, one can add another mathematical thought to understand natural numbers' using geometrical connections.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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