Journal of Applied Mathematics and Physics, 2023, 11, 1634-1655
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

A Study on Stochastic Differential Equation Using Fractional Power of Operator in the Semigroup Theory

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How to cite this paper: Hagenimana, E., Uwilingiyimana, C. and Clarisse, U. (2023) A Study on Stochastic Differential Equation Using Fractional Power of Operator in the Semigroup Theory. *Journal of Applied Mathematics and Physics*, **11**, 1634-1655.
https://doi.org/10.4236/jamp.2023.116107

Received: May 23, 2023
Accepted: June 26, 2023
Published: June 29, 2023

Abstract

Stochastic differential equation (SDE) is an ordinary differential equation with a stochastic process that can model the unpredictable real-life behavior of any continuous systems. It is the combination of differential equations, probability theory, and stochastic processes. Stochastic differential equations arise in modeling a variety of random dynamic phenomena in physical, biological and social process. The SDE theory is traditionally used in physical science and financial mathematics. Recently, more researchers have been conducted in the application of SDE theory to various areas of engineering. This dissertation is mainly concerned with the existence of mild solutions for impulsive neutral stochastic differential equations with nonlocal conditions in Hilbert spaces. The results are obtained by using fractional powers of operator in the semigroup theory and Sadovskii fixed point theorem.

Keywords

Stochastic, Impulsive Stochastic, Neutral Functional, Mild Solution, Wiener Process, Brownian Motion, Banach Space

1. Introduction

A stochastic differential equation is an equation in which one or more terms is stochastic process. The solution of stochastic differential equation is also a stochastic process. In mathematics, an equation of de form $\mathrm{d}x_t = b(t, x_t)\mathrm{d}t + a(t, x_t)\mathrm{d}\beta_t$ is called a stochastic differential equation, where x_t denotes a stochastic process and $b(t, x_t)$ and $a(t, x_t)$ are function of t and x_t, β_t denotes the winner process or standard brownian motion. The Wiener process is nondifferentiable and requires its own rules of calculus. Thus the interpretation of the SDE expression requires additional background of mathematics, which is to be introduced in the following sections. The SDE theory is traditionally used in physical science and financial mathematics. Recently, more researchers have been conducted in the application of SDE theory to various areas of engineering. A stochastic differential equations has been successfully needed to model and examine K-distributed electromagnetic scattering, a first order stochastic autoregressive model for a flat stationary wireless channel based on stochastic differential equation theory and Stochastic channel models based on SDEs for cellular networks. The theory of stochastic differential equations set down by [1] and independently established by [2] with [3], together with the previous mathematical works of Wiener and Levy on Brownian motion has provided the basic tools making the more fertile approach of constructing sample paths feasible. Applications of stochastic differential equations are found in such areas as economics, biology, finance, ecology and other sciences by [4] and [5]. Some of the typical applications of nonlinear stochastic differential equations are vibrations of tall buildings and bridges under the action of wind or earth quack loads, vehicles moving on rough roads, ships and offshore oil platforms subjected to wind and ocean waves, aerospace vehicles due to atmospheric turbulence, price processes in financial markets as well as electronic circuits subjected to thermal noise. Brownian motion have been named after the botanist Robert Brown and referred to either the random movement of particles expelled in a fluid or the mathematical model used to describe such random movements, often called a Wiener process. Brownian motion is among the simplest continuous-time stochastic processes, and it limits both simpler and more complicated stochastic processes. This universality is nearly connected to the universality to the normal distribution. In both cases, it is often mathematical convenience rather than model accuracy that motivates their use. In mathematics, the Wiener process is a continuous-time stochastic process named in honour of Norbert Wiener, an American theoretical and applied mathematician. He occurred as initiator in the study of stochastic and noise processes, promoting work relevant to electronic engineering, electronic communication, and control systems. The Wiener process performs an significant role both in pure and applied mathematics. The concept of dynamic process operating under multi-time scales in sciences and engineering, a mathematical model described by a system of multi-time scale stochastic differential equations is formulated by [6]. The non-instantaneous impulsive stochastic differential equations generated by mixed fractional Brownian motion with poisson jump in real separable Hilbert space is also discussed by [7] [8] and [9]. Specifically, it plays a vital role in stochastic calculus, diffusion processes, and even potential theory. In applied mathematics, the Wiener process is used to represent the integral of a white noise process, and so is useful as a model of noise in electronics engineering, instrument errors in filtering theory, and mathematical factor which is not known forces in control theory. For more details reader may refer to [10] established.

2. Frame of Stochastic Differential Equations

Consider the vector ordinary differential equations defined by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a(x,t) \tag{1}$$

Now, we suppose that the system has random components and η is added on it such as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a(x,t) + b(x,t)\eta(t) \tag{2}$$

The given solution to this random differential equation is problematic due to the presence to the randomness prevents of the system from having bounded measures. The outcome is that the derivative does not exist. One way to understand the equations such as (2), is to look at them in differential form,

$$\mathrm{d}x = a(x,t)\mathrm{d}t + b(x,t)\mathrm{d}\eta(t) \tag{3}$$

or

$$\mathrm{d}x = a(x,t)\mathrm{d}t + b(x,t)\mathrm{d}W(t) \tag{4}$$

The solution to (2) or equivalently (3) or (4) can be regarded as the result of performing the integration,

$$x(t) = x(0) + \int_0^t a(x,s)\mathrm{d}s + \int_0^t b(x,s)\mathrm{d}\eta(s) \tag{5}$$

$$x(t) = x(0) + \int_0^t a(x,s)\mathrm{d}s + \int_0^t b(x,s)\mathrm{d}W(s) \tag{6}$$

This is known as the solution to the stochastic differential Equation (4).

3. Impulsive Differential Systems

In nature, various evolution process under goes abrupt changes of their state at certain moments of time between intervals of continuous evolution. In mathematical modeling of such process, it is reasonable to ignore the duration of these abrupt changes compared to the total duration of the process and to assume that the process changes its state instantaneously, that is in the form of impulses. These processes can be modeled more suitably by impulsive differential equations and the existence of Stepanov-like pseudo almost periodic in distribution mild solutions for impulsive partial stochastic functional differential equations under non-Lipschitz conditions discussed by [11] [12] and [13]. The theory of impulsive differential equations has wide applications in many real world phenomena in which impulses occur. For example, mechanical systems with impact, biological systems involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, frequency modulated systems, blood flows, population dynamics, chemical technology, pharmacokinetics and ecology exhibit impulsive effects. Iimpulsive differential equation is detailed by three components: A continuous-time differential equation, which rules the state to the system between impulses; impulse equation, which designs an impulsive jump specified by a jump function at the instant impulsetake pace; and a jump criterion, which defines a set of jump events in which the impulse equation is active. The mathematical model of an impulsive differential equation takes the form,

$$x'(t) = f(t, x(t)), t \neq t_k, t \in J$$

$$\Delta x(t_k) = I_k(x(t_k)), k = 1, 2, 3, \cdots, m$$

where J is any real interval such that $f : J \times R^n \to R^n$ is a given function and $I_k : R^n \to R^n, k = 1, 2, 3, \cdots, m$, and $\Delta x(t_k) = x(t_k^+) - x(t_k^-), k = 1, 2, 3, \cdots, m$. The numbers t_k are called instant or moments and I_k represent the jump of the state at each t_k, $x(t_k^+)$ and $x(t_k^-)$ represent the right and left hand limits respectively of the state at $t = t_k$. The solution of (1.1) is a piecewise continuous function that has discontinuous of the first kind at $t = t_k$. satisfying jump function $\Delta x(t_k) = I_k(x(t_k))$. The moments of impulse may be fixed or depend on the state of the system. In our study, only fixed moments will be considered. The theory of impulsive differential equations is greater than to the corresponded theory of differential equations without impulse effects. Due to the difficulties caused by the specific properties of the impulsive equations such as beating, bifurcation, merging, and loss of property of autonomy of the solutions, the theory of impulsive differential equations is appearing as an important domain of investigation. Moreover, such equations represent a natural framework for mathematical modeling of several real world phenomena. The more details about this theory and its applications we allude to the monographs of [14] [15] [16] [17] and [18]. In many process including physical, chemical, political, economic, biological and control systems, time delays are an important factor. The rate of change of the state $x'(t)$, may depend on historical values of the state at times $t + s$, where $s \leq 0$, as well as present state values. These processes tend to be modeled by differential equations with delay. In practical, many process exhibit both impulse and time delay. So, we focus on impulsive differential systems in our study. The nonlocal controllability of Hilfer fractional stochastic differential equations via almost sectorial operators also discussed in [19].

Methods

Semigroup Theory: The theory of semigroups of bounded linear operators may be a part of functional analysis. This theory developed quite rapidly since the invention of the generation theorem by Hille and Yosida in 1948. By now, it's an in depth mathematical subject with substantial applications to several fields of study. [20] discussed the existence and uniqueness of mild, classical and powerful solutions of evolution equations using semigroup theory and glued point theorems.

Fixed Point Technique: The fixed point technique is one among the useful methods mainly applied within the existence and uniqueness of solutions of differential equations and therefore the controllability of differential equations. The Banach fixed point theorem is a crucial source of proving existence and uniqueness in several branches of study. In 1967, Sadovskii gave a hard and fast point result more general than Darbos theorem using the concept of condensing map. Thus, the fixed point theory for condensing mappings allows us to get a relationship between the 2 theories. In this paper, we use Sadovskii fixed point theorem in [21] to prove the existence results for impulsive neutral integrodifferential systems. The fixed point technique, and measures of noncompactness have been disscussed in this research work as it is mentionned in [22]. Among the foremost fundamental qualitative properties of differential systems, my paper is especially concerned into existence of solutions for neutral impulsive stochastic differential systems where some researchers have been introduced this theory such as [23]. The paper is organized as follows.

In section 2 of this paper, we will recall some basic definitions and necessary preliminaries.

In section 3, we discuss the existence results for neutral functional differential equations by using fractional power of operators and Sadovskii fixed point theorem.

In section 4, we establish the existence results for stochastic impulsive neutral functional differential systems using Sadovskii fixed point theorem.

Preliminaries 1

In this case, some basic definitions, lemmas and theorems which are used to prove our main theorems.

Definition 2.1

A complete normed linear space is known as **Banach Space**.

Definition 2.2

Let (x,d) be a metric space and $T: X \to X$ be a mapping which maps X onto X. The mapping T is called a **contraction mapping** of X or simply T is a **contraction** if and only if there is a constant $0 \le \alpha < 1$ satisfying the Lipschitz condition.

$$d(Tx,Ty) \le \alpha d(x,y), \forall x, y \in X$$

Definition 2.3

Let Y be the non-empty subset of a metric space X. The subspace Y is said to **relaticely compact** if and only if $\bar{Y}$ is known as compact.

Definition 2.4

Let X and Y be two metric spaces and f be a family of functions from X to Y defined by $F: X \to Y$, the family F is **equicontinuous** at a point $x_0 \in X$, if for every $s > 0$. There exist a $\delta > 0$ such that $d(f(x_0), f(x)) < s, \forall f \in F$ and $\forall x \in X$ such that $d(x_0, x) < \delta$. The family is **equicontinuous** if it is equicontinuous at each point of X.

Definition 2.5

A mapping $A: D \to X$ is said to be **completely continuous** if it is both continuous and compact.

Definition 2.6

Let $\Gamma(t)=\exp(At)$ be strongly continuous. One parameter Semigroup on a Banach space $(X,\|.\|)$ with infinitesimal generator *A*. Γ is said to be an **Analytic Semi-group** if

(i) for some $0<\theta<\frac{\pi}{2}$, the continuous linear operator defined by $\exp(At):X\to X$ can be extended to $t=\Delta_\theta$ where $\Delta_\theta=\theta\cup t\in C:\left|\arg(t)\right|<\theta$ and the usual semigroup conditions hold for $s,t=\Delta_{\theta:}$ $\exp(A0)=id$; $\exp(A(t+s))=\exp(At)\cdot\exp(As)$, and for each $x\in X$, $\exp(At)x$ is continuous in *t*.

(ii) For all $t=\Delta_\theta/0$, $\exp(At)$ is analytique in *t* in the sense of uniform operator topology.

Definition 2.7

Let X be any space and *f* be a map of X or of a subset of X onto *X*. A point $x\in X$ is called **fixed point** for *f* if $x=f(x)$.

Definition 2.8

Let X be a Banach space. A one parameter family $T(t),0\le t<\infty$ of bounded linear operator from X onto X is a **Semigroup** of bounded linear operator on *X*, if

(i) $T(0)=I$, where I is the identity operator on *X*.

(ii) $T(t+s)=T(t)T(s)$ for every $t,s\ge 0$ is the semigroup property.

A semigroup of bounded linear operators $T(t)$ is uniformly continuous if

$$\lim_{t\to\theta}\left\|T(t)-I\right\|=0.$$

The linear operator A defined by

$$D(A)=\left\{x\in X:\lim_{t\to 0}\frac{T(t)x-x}{t}\text{ exists}\right\}$$

and

$$Ax=\lim_{t\to 0}\frac{T(t)x-x}{t}=\left.\frac{\mathrm{d}^+T(t)x-x}{\mathrm{d}t}\right|_{t=0}$$

for $x\in D(A)$ is the infinitesimal generator of the semigroup $T(t)$ where $D(A)$ is the domain of *A*.

Definition 2.9

Let X be a locally convex space and M be a subset of *X*. A mapping $T:M\to X$ is called **condensing** if for each bounded but not relatively compact subset A of M we have

Theorem 1 (Arzela-Scoli Theorem) Let X be a compact metric space, then a non-empty subset of $C_k(X)$ is relatively compact, if and only if it is bounded and equicontinuous on *X*.

Theorem 2 (Contraction Mapping Principle) If X is a Banach space and $T:X\to X$ is a contraction mapping, then T has a unique fixed point.

Theorem 3 (Sadovskii Fixed Point Theorem) Let P be a condensing operator on a Banach space, that is, P is a continuous and takes bounded sets into bounded sets and let $\alpha(P(B)) \le \alpha(B)$ for every bounded set B of X with $\alpha(B) > 0$ of $P(H) \subseteq H$ for a convex, closed and bounded set H of X, then P has a fixed point in H where denotes Kuratowski's measure of nonCompactness.

Existence results for neutral functional differential equations

Neutral differential equations arise in many areas of applied mathematics and such equations have received much attention in recent years [24] and [25]. In this section, we establish the existence of solutions for semilinear neutral functional differential evolution equations with nonlocal conditions of the form as in [26] where it has been specifically discussed by [27].

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)+F\left(t,x(t),x\left(b_1(t)\right),\cdots,x\left(b_m(t)\right)\right)\right]+Ax(t) \\ &=G\left(t,x(t),x\left(a_1(t)\right),\cdots,x\left(a_n(t)\right)\right) \end{aligned} \tag{7}$$

$$x(0)+g(x)=x_0 \tag{8}$$

Where the linear operators -A generates an analytic semigroup and F, G and g are given functions to be specified later.

Preliminaries 2

Throught this section, X will be a Banach space with norm $\|.\|$ and $-A:D(A)\to X$ will be the infinitesimal generator of a compact of uniformly bounded linear operators $T(t)$. Let $0\in\rho(A)$. Then it is possible to define the fractional power A^α, for $0<\alpha\le 1$, as closed linear operator on its domain $D(A^\alpha)$. Furthermore, the subspace $D(A^\alpha)$ is dense in X and the expression.

$$\|x\|_\alpha = \left\|A^\alpha x\right\|, x\in D\left(A^\alpha\right)$$

define a norm on $D(A^\alpha)$. Here after we denote by X_α the Banach space $D(A^\alpha)$ normed with $\|x\|_\alpha$. then for each $0<\alpha\le 1$, X_α is a Banach Space, and $X_\alpha \to X_\beta$ for $0<\beta<\alpha\le 1$ and the imbedding is compact whenever the resolvent operator of A is compact and this can be seen in [28].

For a semigroup $\left\{T(t)_{t\ge 0}\right\}$ the following properties will be used:

(a) there is $M\ge 1$ such that $\|T(t)\|\le M$, for all $0\le t\le a$

(b) for any $a\ge 0$, there exists a positive constant C_α such that

$$\left\|A^\alpha T(t)\right\| \le \frac{C_\alpha}{T^\alpha}, 0<t\le a \tag{9}$$

The following assumptions need to be taken into consideration:

(H_1) $F:[0,a]\times X^{m+1}\to X$ is a continuous function and there exists a $\beta\in(0,1)$ and $L, L_1>0$ such that the function $A^\beta F$ satisfies the Lipschitz conditions.

$$\left\|A^\beta F\left(s_1,x_0,x_1,\cdots,x_m\right)-A^\beta F\left(s_1,\overline{x}_0,\overline{x}_1,\overline{x}_m\right)\right\| \le L\left(\left|s_1-s_2\right|+\max_{i=0,\cdots,m}\left\|x_i-\overline{x}_i\right\|\right)$$

For any $0\le s_1,s_2\le a$, $x_i,\overline{x}_i\in X, I=0,1,\cdots,m$ and the inequality

$$\left\|A^\beta F\left(t,x_0,x_1,\cdots,m\right)\right\| \le L_1\left(\max\left\{\left\|x_i\right\|;i=0,1,\cdots,m\right\}+1\right)$$

holds for any $(t, x_0, x_1, \cdots, x_m) \in [0, a] \times X^{m+1}$.

(H₂) The function $G : [0, a] \times X^{m+1} \to X$ satisfies the following conditions.

(i) For each $t \in [0, a]$, the function $G(t, .) : X^{n+1} \to X$ is continuous and for each $x_0, x_1, \cdots, x_n \in X^{n+1}$, the function $G(., x_0, x_1, \cdots, x_n) : [0, a] \to X$ is strongly measurable.

(ii) For each positive number $k \in N$, there is a positive function $g_k \in L'([0, a])$ such that

$$\sup_{\|x_0\| \cdots \|x_n\| \le k} \left\| G(t, x_0, x_1, \cdots, x_n) \right\| \le g_k(t)$$

and

$$\liminf_{k \to \infty} \frac{1}{k} \int_0^a g_k(s) \mathrm{d}s = \gamma < \infty$$

(H_3) $a_i, b_j \in C([0, a]; [0, b]), i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$.

$g \in C(E; X)$, here and hereafter $E = C([0, a]; X)$ and g satisfies that

(i) There exist a positive constant L_2' and L_2 such that

$$\|g(x)\| \le L_2 \|x\| + L_2', \forall x \in E$$

(ii) g is a completely continuous map.

Definition: A continuous function $x(.) : [0, a] \to X$ is said to be **mild solution** of the non-local Cauchy problem (0.7)-(0.8), if the function

$$AT(t - s) F\left(s, x(s), x(b_1(s)), \cdots, x(b_m(s))\right), s \in [0, a)$$

is integrable on $[0, a)$ and the following integral equation is verified.

$$\begin{aligned} x(t) = T(t) \Big[& x_0 + F\left(0, x(0), x(b_1(0)), \cdots, x(b_m(0))\right) - g(x) \Big) \\ & - F\left(t, x(t), x(b_1(t)), \cdots, x(b_m(t))\right) \Big) \\ & + \int_0^t T(t - s) F\left(s, x(s), x(b_1), \cdots, x(b_m(s))\right) \mathrm{d}s \\ & + \int_0^t T(t - s) G\left(s, x(s), x(a_1(s)), \cdots, x(a_n(s))\right) \mathrm{d}s. \end{aligned}$$

Existence Results

Theorem 4 If assumptions (H₁)-(H₃) are satisfied and $x_0 \in X$, then the non-local Cauchy problem (0.7)-(0.8) has a mild solution provided that

$$L_0 = L\left[(M + 1) M_0 + \frac{1}{\beta} C_{1-\beta} a^\beta \right] \le 1 \tag{10}$$

and

$$(M_0 L_1 + L_2 + \gamma) M + M_0 L_1 + \frac{1}{\beta} C_{1-\beta} a^\beta L_1 \le 1 \tag{11}$$

where $M_0 = \left\| A^{-\beta} \right\|$

Proof:

For the sake of brevity, we rewrite that

$$\left(t, x(t), x(b_1(t)), \cdots, x(b_m(t))\right) = (t, v(t))$$

and

$$\left(t,x(t),x\left(a_1(t)\right),\cdots,x\left(a_n(t)\right)\right)=\left(t,u(t)\right)$$

Define the operator P on E, by the formula

$$(Px)(t)=T(t)\left[x_0+F\left(0,v(0)-g(x)\right)\right]-F\left(t,v(t)\right)+\int_0^t AT(t-s)F\left(s,v(s)\right)\mathrm{d}s$$

For each positive integer k, let

$$B_k=\left\{x\in E:\left\|x(t)\right\|\le k,0\le t\le a\right\}$$

Then for each k, B_k is clearly a bounded closed convex set in E. Since by 0.7 and (H_1), the following relation holds:

$$\left\|AT(t-s)F\left(s,v(s)\right)\right\|\le\left\|A^{1-\beta}T(t-s)A^{\beta}F\left(s,v(s)\right)\right\|\le\frac{C_{1-\beta}}{(t-s)^{1-\beta}}L_1(k-1)$$

Then from Bocher's, theorem, it follows that $AT(t-s)F\left(s,v(s)\right)$ is integrable on $[0,a]$, so P is well defined on B_k. We claim that there exists a positive integer k such that $PB_k\subseteq B_k$, but $Px_k\notin B_k$, that is $\left\|(Px_k)(t)\right\|>k$ for some $t_k\in[0,a]$ where $t(k)$ denotes t is independent of k. However, on the other hand we have

$$\begin{aligned}
k&<\left\|(Px_k)(t)\right\|=\left\|T(t)\left[x_0-g(x_k)+F\left(0,v_k(0)\right)\right]-F\left(t,v_k(t)\right)\right\|\\
&\quad+\left\|\int_0^t AT(t-s)F\left(s,v_k(s)\right)\mathrm{d}s+\int_0^t T(t-s)G\left(s,u_k(s)\right)\mathrm{d}s\right\|\\
&\le\left\|T(t)\left[x_0-g(x_k)+F\left(0,v_k(0)\right)\right]\right\|\left\|A^{-\beta}A^{\beta}F\left(t,v_k(t)\right)\right\|\\
&\quad+\left\|\int_0^t AT(t-s)F\left(s,v_k(s)\right)\mathrm{d}s\right\|+\left\|\int_0^t T(t-s)G\left(s,u_k(s)\right)\mathrm{d}s\right\|\\
&\le\left\|T(t)\left[x_0-g(x_k)+F\left(0,v_k(0)\right)\right]\right\|\left\|F\left(t,v_k(t)\right)\right\|\\
&\quad+\left\|\int_0^t A^{1-\beta}T(t-s)A^{\beta}F\left(s,v_k(s)\right)\mathrm{d}s\right\|+\left\|\int_0^t T(t-s)G\left(s,u_k(s)\right)\mathrm{d}s\right\|\\
&\le\left\|T(t)\right\|\left\{\left\|x_0\right\|+\left\|g(x_k)\right\|\left\|F\left(0,v_k(0)\right)\right\|\right\}+\left\|A^{-\beta}\right\|\left\|A^{\beta}F\left(t,v_k(t)\right)\right\|\\
&\quad+\int_0^t\left\|A^{1-\beta}T(t-s)A^{\beta}F\left(s,v_k(s)\right)\mathrm{d}s\right\|+\int_0^t\left\|T(t-s)\right\|\left\|G\left(s,u_k(s)\right)\mathrm{d}s\right\|\\
&\le M\left\{\left\|x_0\right\|+L_2k+L_2'+M_0L_1(k+1)\right\}+M_0L_1(k+1)\\
&\quad+\int_0^t\frac{C_{1-\beta}}{(t-s)^{1-\beta}}L_1(k-1)\mathrm{d}s+M\int_0^t g_k(s)\mathrm{d}s
\end{aligned}$$

By dividing both sides by k, we get

$$\begin{aligned}
1&\le M\left\{\frac{1}{k}\left\|x_0\right\|+\frac{1}{k}\left(L_2k+L_2'\right)+\frac{1}{k}M_0L_1(k+1)\right\}+\frac{1}{k}M_0L_1(k+1)\\
&\quad+\frac{1}{k}\int_0^t\frac{C_{1-\beta}}{(t-s)^{1-\beta}}L_1(k+1)\mathrm{d}s+\frac{1}{k}M\int_0^t g_k(s)\mathrm{d}s
\end{aligned}$$

Taking the lower limit as $k\to\infty$, we get

$$\left(M_0L_1+L_2+\gamma\right)M+M_0L_1+\frac{1}{\beta}C_1-\beta a^{\beta}L_1.$$

We will show that the operator P has a fixed point on B_k which implies equation (0.10)-(0.11) has a mild solution. To this end, we decompose P as $P = P_1 + P_2$, where the operators P_1, P_2 are defined on B_k respectively by

$$\left(P_1 x\right)(t) = T(t)F\left(0, v(0)\right) - F\left(t, v(t)\right) + \int_0^t AT(t-s)F\left(s, v(s)\right)\mathrm{d}s$$

and

$$\left(P_2 x\right)(t) = T(t)\left[x_0 - g(x)\right] + \int_0^t T(t-s)G\left(s, v(s)\right)\mathrm{d}s$$

For $0 \le t \le a$, we will verify that P_1 is a contraction while P_2 is a compact operator.

To prove that P_1 is a contraction, we take $x_1, x_2 \in B_k$. Then for each $t \in [0, a]$ and by condition (H$_1$) and (0.10), we have

$$\begin{aligned}
&\left\|\left(P_1 x_1\right)(t) - \left(P_2 x_2\right)(t)\right\| \\
&= \left\|T(t)F\left(0, v_1(0)\right) - F\left(t, v_1(t)\right) + \int_0^t AT(t-s)F\left(s, v_1(s)\right)\mathrm{d}s\right\| \\
&\quad - \left\|T(t)F\left(0, v_2(0)\right) - F\left(t, v_2(t)\right) + \int_0^t AT(t-s)F\left(s, v_2(s)\right)\mathrm{d}s\right\| \\
&\le \left\|T(t)\left[F\left(0, v_1(0)\right) - F\left(0, v_2(0)\right)\right]\right\| + \left\|F\left(t, v_1(t)\right) - F\left(t, v_2(t)\right)\right\| \\
&\quad + \left\|\int_0^t AT(t-s)F\left(s, v_1(s)\right)\mathrm{d}s\right\| - \left\|\int_0^t AT(t-s)F\left(s, v_2(s)\right)\mathrm{d}s\right\| \\
&\le \left\|T(t)F\left(0, v_1(0)\right) - F\left(0, V_2(0)\right)\right\| + \left\|F\left(t, v_1(t)\right) - F\left(t, v_2(t)\right)\right\| \\
&\quad + \left\|\int_0^t AT(t-s)\left[F\left(s, v(s)\right) - F\left(s, v_2(s)\right)\right]\mathrm{d}s\right\| \\
&\le \left\|T(t)\right\|\left\|A^{-\beta}A^{\beta}F\left(0, v_1(0)\right) - A^{\beta}F\left(0, v_2(0)\right)\right\| \\
&\quad + \left\|A^{-\beta}\left(A^{\beta}F\left(t, v_1(t)\right) - A^{\beta}F\left(t, v_2(t)\right)\right)\right\| \\
&\quad + \left\|\int_0^t A^{1-\beta}T(t-s)\left[A^{\beta}F\left(s, v_1(s)\right) - A^{\beta}F\left(t, v_2(s)\right)\right]\mathrm{d}s\right\| \\
&\le \left\|T(t)\right\|\left\|A^{-\beta}\right\|\left\|A^{\beta}F\left(0, v_1(0)\right) - A^{\beta}F\left(0, v_2(0)\right)\right\| \\
&\quad + \left\|A^{-\beta}A^{\beta}(t)F\left(t, v_1(t)\right) - A^{-\beta}A^{\beta}(t)F\left(t, v_2(t)\right)\right\| \\
&\quad + \int_0^t \left\|A^{1-\beta}T(t-s)\right\|\left\|A^{\beta}F\left(s, v_1(s)\right) - A^{\beta}F\left(s, v_2(s)\right)\right\|\mathrm{d}s \\
&\le MM_0 L \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| + M_0 L \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| \\
&\quad + \int_0^t \frac{C_{1-\beta}}{(t-s)^{1-\beta}} L\mathrm{d}s \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| \\
&\le (M+1)M_0 L \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| + L\frac{1}{\beta}C_{1-\beta}a^{\beta} \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| \\
&\le L\left[(M+1)M_0 + \frac{1}{\beta}C_{1-\beta}a^{\beta}\right]\sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\| \\
&\le L_0 \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\|
\end{aligned}$$

$$\left\|P_1\left(x_1(t)\right) - P_1\left(x_2(t)\right)\right\| \le L_0 \sup_{0\le s\le a}\left\|x_1(s) - x_2(s)\right\|$$

Thus,

$$\|P_1x_1 - P_1x_2\| \le L_0\|x_1 - x_2\|$$

By assumption $0 < L_0 < 1$, we obseve that P_1 is a contaction. To prove that P_2 is compact, first we prove that P_2 is continuous on B_k. Let $\{x_n\} \subseteq B_k$ will be $x_n \to x$ in B_k, then by (H2), we have $G(s,u_n(s)) \to G(s,u(s)), n \to \infty$ Since

$$\|G(s,u_n(s)) - G(s,u(s))\| \le 2g_k(s)$$

Therefore by dominated convergence theorem, we have.

$$\begin{aligned}
&\|P_2x_n - P_2x\| \\
&= \sup_{0\le t\le a}\left\|T(t)\left[x_0 - g(x_n)\right]\right\| + \int_0^t T(t-s)G(s,u_n(s))\mathrm{d}s \\
&\quad - T(t)\left[x_0 - g(x)\right] - \int_0^t T(t-s)G(s,u(s))\mathrm{d}s \\
&= \sup_{0\le t\le a}\left\|T(t)\left[g(x) - g(x_n)\right]\right\| + \int_0^t \|T(t-s)\|\left\|G(s,u_n(s)) - G(s,u(s))\right\|\mathrm{d}s \\
&\to 0 \text{ as } n \to \infty
\end{aligned}$$

That is P_2 is continuous.

Next, we have to prove that $\{P_2x : x \in B_k\}$ is a family of equicontinuous functions. To check this, we have to fix $t_2 > 0$ and we let $t_2 > t_1$ as $\varepsilon > 0$ be enough small. Then

$$\begin{aligned}
&\|(P_2x)(t_2) - (P_2x)(t_1)\| \\
&\le \left\|\{T(t_2) - T(t_1)\}\left[x_0 - g(x)\right]\right\| + \left\|\int_0^t \{T(t_2 - s) - T(t_1 - s)\}G(s,u(s))\mathrm{d}s\right\| \\
&\quad + \left\|\int_{t_1}^{t_2} T(t_2 - s)G(s,u(s))\mathrm{d}s\right\| \\
&\le \|T(t_2) - T(t_1)\|\|x_0 - g(x)\| + \left\|\int_0^{t_1-\varepsilon} T(t_2 - s) - T(t_1 - s)G(s,u(s))\mathrm{d}s\right\| \\
&\quad + \left\|\int_{t_1-\varepsilon}^{t_1} T(t_2 - s) - T(t_1 - s)G(s,u(s))\mathrm{d}s\right\| + \left\|\int_{t_1}^{t_2} T(t_2 - s)G(s,u(s))\mathrm{d}s\right\| \\
&\le \|T(t_2) - T(t_1)\|\|x_0 - g(x)\| + \int_0^{t_1-\varepsilon}\|T(t_2 - s) - T(t_1 - s)\|\|G(s,u(s))\|\mathrm{d}s \\
&\quad + \int_{t_1-\varepsilon}^{t_1}\|T(t_2 - s) - T(t_1 - s)\|\|G(s,u(s))\|\mathrm{d}s + \int_{t_1}^{t_2}\|T(t_2 - s)\|\|G(s,u(s))\|\mathrm{d}s
\end{aligned}$$

Noting that $\|G(s,u(s))\| \le g_k(s)$ and $g_k(s) \in L^1$, and we see that $\|(P_2x)(t_2) - (P_2x)(t_1)\| \to 0$ independently for $x \in B_k$ as $t_2 - t_1 \to 0$.

Since the compactness of $T(t), t > 0$ in t in the set of uniform operators topology. We can prove that the functions $P_2x, x \in B_k$ are equicontinuous.

It remains to prove that

$$V(t) = \{(P_2x)(t) : x \in B_k\}$$

is relatively compact in *X*. Obviously by assumption (H3), $V(0)$ is relatively compact in *X*.

Let $0 < t \le a$ be a fixed point and $0 < \varepsilon < t$. For $x \in B_k$, we define

$$\begin{aligned}
(P_{2,\varepsilon}x)(t) &= T(t)\left[x_0 - g(x)\right] + \int_0^{t-\varepsilon} T(t-s)G(s,u(s))\mathrm{d}s \\
&= T(t)\left[x_0 - g(x)\right] + T(\varepsilon) + \int_0^{t-\varepsilon} T(t-\varepsilon-s)G(s,u(s))\mathrm{d}s
\end{aligned}$$

Then from the compactness of $T(\varepsilon), \varepsilon > 0$, we obtain $V_\varepsilon(t) = \{(P_{2,\varepsilon}x)(t) : x \in B_k\}$ is relatively compact in X for every ε, $0 < \varepsilon < t$. Moreover, for every $x \in B_k$, we have

$$\begin{aligned}
&\left\|(P_2x)(t) - (P_{2,\varepsilon}x)(t)\right\| \\
&= \Big\| T(t)[x_0 - g(x)] + \int_0^t T(t-s)G(s,u(s))\mathrm{d}s \\
&\quad - T(t)[x_0 - g(x)] - \int_0^{t-\varepsilon} T(t-s)G(s,u(s))\mathrm{d}s \Big\| \\
&\left\|(P_2x)(t) - (P_{2,\varepsilon}x)(t)\right\| \\
&= \Big\| \int_0^{t-\varepsilon} T(t-s)G(s,u(s))\mathrm{d}s + \int_{t-\varepsilon}^t T(t-s)G(s,u(s))\mathrm{d}s \\
&\quad - \int_0^{t-\varepsilon} T(t-s)G(s,u(s))\mathrm{d}s \Big\| \\
&= \left\| \int_{t-\varepsilon}^t T(t-s)G(s,u(s))\mathrm{d}s \right\| \\
&= M\int_{t-\varepsilon}^t g_k(s)
\end{aligned}$$

Therefore, there are relatively compact sets arbitrary closed to the set $V(t)$.

Hence the set $V(t)$ is also compact in X.

Therefore, by Arzela-Ascoli theorem, P_2 is compact operator. Those argument enable us to conclude that $P = P_1 + P_2$ is a condensing map B_k and by the fixed point $x(.)$ for P on B_k.

Therefore, the Cauchy problem 0.7 - 0.8 has a mild solution and the proof is completed.

Existence Results for impulsive Stochastic Neutral Differential Equations

Recently, Stochastic differential systems with impulsive conditions have been studied by different authors such as [29] [30] [31] [32] [33]. Therefore, it seems interesting to study the impulsive stochastic differential equations with nonlocal conditions. studied the existence of solutions for semilinear neutral stochastic functional differential equations with nonlocal conditions by using the fractional powers of operator and Sadovskii's fixed point theorem, whereas proved the existence of mild solutions for a class of impulsive neutral stochastic functional integro-differential systems with nonlocal conditions in Hilbert spaces, and [34] established the existence of solutions of impulsive neutral differential and integro-differential equations with nonlocal conditions via fractional operators and Sadvoskii's fixed point theorem. Motivated by the above mentioned, in this chapter, we are interested in studying the existence of solutions of the following impulsive neutral stochastic differential equation with nonlocal conditions.

$$\begin{aligned}
&\mathrm{d}\left[x(t) + f_1(t, x(t), x(a_1(t)), \cdots, x(a_m(t)))\right] \\
&= \left[Ax(t) + f_2(t, x(t), x(b_1(t)), \cdots, x(b_m(t)))\right]\mathrm{d}t \\
&\quad + G(t, x(t), x(c_1(t)), \cdots, x(c_m(t)))\mathrm{d}w(t), t \in J := [0,b], t \notin, k = 1,2,3,\cdots,m
\end{aligned} \tag{12}$$

$$x(0) = x_0 + g(x) \tag{13}$$

where, A is the infinitesimal generator of analytic semigroup of bounded linear operators $\{T(t), t \le 0\}$ on a separable Hilbert space with inner product $(.,.)$ and norm $\|.\|$. Let K be the another separable Hilbert space with inner product $(.,.)_k$ and norm $\|.\|_k$.

Suppose that $\{w(t)\}_{t\le 0}$ is a given K-valued Brownian motion or Wiener process with a finite trace nuclear Covarience operator and $Q \le 0$ defined on a filtered complete space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\ge 0}, P)$. The function f_1, f_2, G and g are the given functions to be defined later.

Preliminaries 3

In this section, we recall a few stochastic results, Lemmas and notations which are needed to establish our main results. Throughout this paper $(H, \|.\|)$ and $(K, \|.\|)$ denotes the two real separable Hilbert space. Let $\mathfrak{L}(K, H)$ be the set of all inner product bounded operator from K into H equiped with the usual norm operator $\|.\|$. Let $(\Omega, \mathcal{F}, P, H)$ be the complete probability space furnished with a complete family of right continuous increasing σ-algebra $\{\mathcal{F}_t, t \in \mathcal{F}\}$ satisfying $\mathcal{F}_t \subset \mathcal{F}$.

An H-valued random variable is an $\mathcal{F}$-measurable function $x(t): \Omega \to H$ and a collection of random variable $S = \{x(t,\omega): \Omega \to H, t \in J\}$ is called Stochastic process. Usually we write $x(t)$ instead of $x(t,\omega)$ and $x(t): J \to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K.

Suppose that $\{\omega(t): t \ge 0\}$ is a cylindrical K-valued Wiener process with a finite trace nuclear Covariance operator $Q \ge 0$, denote $Tr(Q) = \sum_{i=0}^{\infty} \lambda_i = \lambda \le \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So actually, $\omega(t) = \sum_i^{\infty} \sqrt{\lambda_i \omega_i(t) e_i}$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one dimensional standard Wiener process. We assume that $\mathcal{F}_t = \sigma\{\omega(s): 0 \le s \le\}$ is the σ-algebra generated by ω and $\mathcal{F}_t = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K, H)$ and define $\|\Psi\|_Q^2 = Tr(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \left\|\sqrt{\lambda_n} \Psi e_n\right\|^2$

If $\|\Psi\|_Q \le \infty$, then Ψ is called a Q-Hilbert Schmidt Operator. Let $\mathcal{L}_Q(K, H)$ denotes the space of of all Q-Hilbert Schmidt Operator $\Psi: K \to H$. The completion $\mathcal{L}_Q(K, H)$ of $\mathcal{L}(K, H)$ with respect to the topology induced by the norm $\|.\|_Q$ where $\|Q\|_Q^2 = \langle \Psi, \Psi \rangle$ is Hilbert space with the above norm topology.

Let A be the infinitesimal generator of an analytic semigroup $T(t)$ in H. Suppose that $0 \in \rho(A)$ where $\rho(A)$ denote the resolvent set of A and that semi-group $T(.)$ is uniformily bounded that is to say $\|T(t)\| \le M$ for some constant $M_T \ge 1$ and for every $t \ge 0$. Then for $\alpha \in (0,1]$, it is possible to define the fractional operator $\left((-A)^{\alpha}\right)$ as a closed linear invertible operator on its domain $\left((-A)^{\alpha}\right)$. Furthermore, the subspace $\left((-A)^{\alpha}\right)$ is dense in H and the expression

$$\|x\|_{\alpha} = \left\|(-A)^{\alpha} x\right\|, x \in D\left((-a)^{\alpha}\right)$$

Define the norm on $H_{\alpha} = D\left((-A)^{\alpha}\right)$. Furthermore of fractional power of operator and semigroup refer (16). Then the following property is well known

16. Suppose that the following properties are satisfied.

Let $0 \le \alpha \le 1$. Then H_α is a Banach space.

If $0 \le \beta\alpha \le 1$, then $H_\alpha \subset H_\beta$ and the imbedding is compact whenever the resolvent operator of A is compact. For every $0 \le \alpha \le 1$, there exists a positive constant $M_\alpha > 0$ such that

$$\left\|(-A)^\alpha T(t)\right\| \le \frac{M_\alpha}{t^\alpha}, \tag{14}$$

for all $0 < t \le b$.

The collection of all strongly measurable, square integrable H-valued random variables, denoted by $\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P, H)) = \{x : J \to L_2 : x(t)\}$ is a continuous every where except for some t_k at which $x(t_k^-)$ and $x(t_k^+)$ exists and $x(t_k^-) = x(t_k), k = 1,2,3,\cdots,m$ is the Banach space of piecewise continuous maps from J into $L_2(\Omega, \mathcal{F}, P, H)$ satisfying the condition $\sup_{t\in J} E\|x(t)\|^2 < \infty$.

Let $\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P, H))$ be the closed subspace of $\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P, H))$ consisting of measurable $\mathcal{F}$, adapted and H-valued processes $x(t)$. Then, $\mathcal{PC}(J, L_2(\Omega, \mathcal{F}, P, H))$ is a Banach space endowed with the norm. $\|x\|_{\mathcal{PC}}^2 = \sup_{t\in J}\left\{E\|x(t)\|^2 < \infty : x \in \mathcal{PC}(J, L_2(\Omega, H))\right\}$ The existence of solution for the system (0.7)-(0.8) is studied with the following assumptions: (H_4) There exist constant $\beta \in (0,1)$ such that $f_i : [0,b] \times H^{m+1} \to H$ is a continuous function, and $M_{f_i}, \bar{M}_{f_i} > 0$ such that $(-A^\beta) f_i$ satisfies the Lipschitz conditions.

$$\begin{aligned}&\left\|(-A)^\beta f_i(s_1, x_0, x_1, x_2, \cdots, x_m) - (-A)^\beta f_j(s_2, y_0, y_1, y_2, \cdots, y_m)\right\| \\ &\le M_{f_j}\left(|s_1 - s_2| + \sup_{i=0,1,\cdots,m}\|x_i - y_i\|\right)\end{aligned}$$

For any $0 \le s_1, s_2 \le b, x_i, y_i \in H, i = 0,1,2,\cdots,m$. However, the inequality

$$\left\|(-A)^\beta f_j(t, x_0, x_1, \cdots, x_m)\right\| \le \bar{m}_{f_j}\left(\max\left\{\|x_i\| : i = 0,1,2,\cdots,m\right\} + 1\right) \tag{15}$$

For every $(t, x_0, x_1, \cdots, x_m) \in J \times H^{m+1}, j = 1,2,\cdots$

(H_5) The function $G : [0,b] \times H^{n+1} \to L(K,H)$ satisfies the following

(i) for each $t \in [0,b]$, the function $G(t,.) : H^{n+1} \to L(K,H)$ is continuous and for each $(x_0, x_1, \cdots, x_n) : J \to L(K,H)$ is $\mathcal{F}_t$-measurable.

(ii) For each positive number $l \in N$, there is a positive function $h_t \in L^2(J)$ such that

$$\sup_{\|x_0\|^2,\cdots,\|x_n\|^2 < l} E\left\|G(t, x_0, x_1, \cdots, x_n)\right\|^2 \le h_l(t)$$

and

$$\liminf_{l\to\infty} \frac{\int_0^b h(s)\,\mathrm{d}s}{l} = \mu < +\infty$$

(H_6) $a_k, b_k, c_k \in C([0,b],[0,b],[0,b]), k = 1,2,3,\cdots,m$. $g : \mathcal{PC} \to L_2^0(\Omega, \mathcal{PC})$ satisfies that

(i) There exists a positive constants M_g and $\bar{M}_g$ such that

$$\left\|g(x)\right\| \le M_g \left\|x\right\|_{\mathcal{PC}} + \bar{M}_g \quad \forall x \in \mathcal{PC}.$$

(ii) g is a completely continuous.

Our main results are based upon the following fixed point theorem (17) (Sadovskii's fixed point theorem).

Let Φ be a condensing operator on Banach space, that is Φ is continuous and takes bounded sets into bounded sets, and let $\alpha(\Phi(B)) \le \alpha(B)$ for every bounded set B of H with $\alpha(B) > 0$ of $\Phi(\Omega) \subset \Omega$ for a convex,closed and bounded set Ω of *H,* then Φ has a fixed point in *H.* Where $\alpha(.)$ denotes Kuratorawski's measure of non-compactness.

Existence Results

In this section we state and prove our main results, now we define the mild solutions of system (0.12)-(0.13).

Theorem 5. An $\mathcal{F}_t$-adapted stochastic process $x(t): J \to H$ function $x \in \Omega$ is said to be mild solution of the system (0.12)-(0.13) if the following conditions are satisfied

(i) $x_0 \in L_2^0(\Omega, H), g(x) \in L_2^0(\Omega, \mathcal{PC})$

$$\begin{aligned} x(t) = {} & T(t)\left[x_0, g(x) + f_1\left(0, x(0), x\left(a_1(0)\right), \cdots, x\left(a_m(0)\right)\right)\right] \\ & - f_1\left(t, x(t), x\left(a_1(t)\right), \cdots, x\left(a_m(t)\right)\right) \\ & + \int_0^t T(t-s) f_2\left(s, x(s), s\left(b_1(s)\right), \cdots, x\left(b_m(s)\right)\right) \mathrm{d}s \\ & - \int_0^t AT(t-s) f_1\left(s, x(s), x\left(a_1(s)\right), \cdots, x\left(a_m(s)\right)\right) \mathrm{d}s \\ & + \int_0^t T(t-s) G\left(s, x(s), x\left(c_1(s)\right), \cdots, x\left(c_m(s)\right)\right) \mathrm{d}w(s), t \in J \end{aligned}$$

Assume that the conditions (H$_1$)-(H$_5$) are satisfied and $x(0) \in L_2^0(\Omega, H)$, then the non-local Cauchy problem (0.12)-(0.13) has a mild solutions provided that

$$L_0 = M_{f_i}^2 \left\{ M_0^2 \left(M_T^2 + 1 \right) + \frac{M_1 - \beta^b}{2\beta - 1} \right\} \le 1 \tag{16}$$

and

$$\begin{aligned} & 36\left\{ M_T^2 \left[6^2 \left(\left(M_g^2 + \left(M_0 M f_1 \right)^2 \left(2bM_0 \bar{M} f_2 \right)^2 + Tr(Q) M + \sum_{k=1}^m \lambda_k \right. \right. \right. \right. \\ & \left. + \left(2M_0 \bar{M} f_1 \right)^2 + \frac{1}{2\beta - 1} \left(2M_{1-\beta} \bar{M}_{f_1} b^\beta \right)^2 \right\} < 1 \end{aligned} \tag{17}$$

where $M_0 = \left\| (-A)^{-\beta} \right\|$ and $M_{1-\beta}$ is defined in (0.12)

Proof: For the sake of brevity, we write that

$$\left(t, x(t), x\left(a_1(t)\right), \cdots, x\left(a_m(t)\right)\right) = \left(t, u(t)\right)$$

$$\left(t, x(t), x\left(a_1(t)\right), \cdots, x\left(b_m(t)\right)\right) = \left(t, v(t)\right)$$

$$\left(t, x(t), x\left(p_1(t)\right), \cdots, x\left(c_m(t)\right)\right) = \left(t, p(t)\right)$$

Consider the operator Φ on $\mathcal{PC}$ defined by

$$
\begin{aligned}
&(\Phi x)(t)\\
&=T(t)\left[x_0+g(x)+f_1(0,u(0))\right]-f_1(t,u(t))+\int_0^t T(t-s)f_2(s,v(s))\mathrm{d}s\\
&\quad-\int_0^t AT(t-s)f_2(s,u(s))\mathrm{d}s+\int_0^t T(t-s)G(s,p(s))\mathrm{d}s,t\in J\\
&=T(t)[x_0+f_1(0,u(0))+g(x)-f_1(t,u(t))-I_{f_1}^{u}(t)+I_{f_2}^{v}(t)+I_G^{P}(t),t\in J
\end{aligned}
$$

We shall show that the operator Φ has a fixed point which is a solution of the system (0.12)-(0.13). For each positive integer *l*, let

$$B_l=\left\{x\in\mathcal{PC}:E\left\|x(t)\right\|_{\mathcal{PC}}^2\le l,0\le t\le b\right\}$$

It is clear that for each *l*, $B_l\subseteq\mathcal{PC}$ is clearly a bounded closed convex set in $\mathcal{PC}$. In addition to the familiar Young Holder and Minkowskii the inequalities of the form $\left(\sum_{i=1}^{n}a_i\right)^m\le n^m\sum_{i=1}^{n}a_i^m$ where a_i are non-negative constants $i=1,2,3,\cdots,n$ and $m,n\in N$ is helpful in establishing various estimates, from (0.14) and (0.15) together with Holder inequality, yields the following relation:

$$
\begin{aligned}
\left\|I_{f_1}^{u}(t)\right\|^2&=\left\|\int_0^t(-A)T(t-s)f_1(s,u(s))\mathrm{d}s\right\|^2\\
&=\left\|\int_0^t(-A)^{1-\beta}T(t-s)(-A)^{\beta}f_1(s,u(s))\mathrm{d}s\right\|^2\\
&=4\int_0^t\frac{M_{1-\beta}^2}{(t-s)^{2(1-\beta)}}\bar{M}_{f_1}^2\left(\max\left\{E\left\|\int x_i\right\|^2:i=0,1,2,\cdots,m\right\}+1\right)\mathrm{d}s
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
E\left\|I_{f_2}^{v}(t)\right\|^2&=\left\|\int_0^t T(t-s)f_2(s,v(s))\mathrm{d}s\right\|^2\\
&=4bM_0^2M_T^2\bar{M}_{f_2}^2\left(\max\left\{E\left\|x_i\right\|^2:i=0,1,2,\cdots,m\right\}+1\right)
\end{aligned}
\tag{19}
$$

It follows that $(-A)T(t-s)f_1(s,u(s))\mathrm{d}s$ and $T(t-s)f_2(s,v(s))$ is integrable on *J*, so Φ is well defined on B_q. Similarly, from (H_2) (ii) and together with the ito's formula, a compution can be performed to obtain the following:

$$
\begin{aligned}
E\left\|I_G^{P}(t)\right\|^2&=E\left\|\int_0^t T(t-s)G(s,P(s))\mathrm{d}w(s)\right\|^2\\
&\le Tr(Q)M_T^2\int_0^t\left\|G(s,P(s))\right\|_Q^2\mathrm{d}s\\
&\le Tr(Q)M_T^2\int_0^t h_t(s)\mathrm{d}s
\end{aligned}
\tag{20}
$$

Step1: We claim that there exists a positive number l such that $\Phi B_l\subseteq B_l$. If it is not true, then for each positive number *l*, there is a function $x_l(.)\in B_l$ and $\Phi x_l(.)\in B_l$, but $\left\|\Phi_l(t)\right\|>l$ for some $t(l)\in J$, where $t(l)$ denotes that *t* is independent of *l*. However on the other hand we have,

$$l<\left\|\Phi_l x(t)\right\|^2$$

$$
\begin{aligned}
&\Big\|\Phi T(t)\left[x_0+g(x)+f_1(0,u(0))\right]-f_1(t,u(t))+\int_0^t T(t-s)f_2(s,u(s))\mathrm{d}s\\
&-\int_0^t AT(t-s)f_1(s,u(s))\mathrm{d}s+\int_0^t T(t-s)G(s,P(s))\mathrm{d}w(s)\Big\|^2
\end{aligned}
$$

$$\le 36(3M_T)^2\left[\|x_0\|_D^2+4\left(M_g^2 l+\bar{M}_g^2\right)+\left(2M_0\bar{M}_{f_1}\right)^2(l+1)\right]\left(2M_0\bar{M}_{f_1}\right)^2(l+1)$$

$$+b\left(2M_0M_T\bar{M}_{f_1}\right)^2(l+1)+\frac{1}{2\beta-1}\left(2M_{1-\beta}\bar{M}_{f_1}b^\beta\right)^2(l+1)+Tr(Q)M_T^2\int_0^t h_t(s)\mathrm{d}s$$

$$M^*+36\left(\left(M_T\right)^2\left[M_g^2 l+\left(M_0\bar{M}_{f_1}\right)^2(l)\right]+\left(2M_0\bar{M}_{f_1}\right)^2(l)+b\left(M_0M_t\bar{M}_{f_2}\right)^2(l)\right.$$

$$\left.+\frac{1}{2\beta-1}\left(2M_{1-\beta}\bar{M}_{f_1}b^\beta\right)^2(l)+Tr(Q)M_T^2 l\frac{1}{l}\int_0^t h_t(s)\mathrm{d}s\right)$$

where

$$M^*=36\left\{3M_T^2\left[\|x_0\|^2+4M_g^2+2M_0\bar{M}_g\right]+b\left(2M_0M_T\bar{M}_{f_2}\right)^2\right.$$
$$\left.+\frac{1}{2\beta-1}\left(2M_{1-\beta}\bar{M}_{f_1}b^\beta\right)^2\right\}$$

By dividing both side by l and taking the lower limit as $t\to\infty$, we get

$$36\left\{\left(M_T\right)^2\left[6^2\left(M_g^2+\left(M_0\bar{M}_{f_1}\right)^2\right)+b\left(2M_0\bar{M}_{f_2}\right)^2+Tr(Q)M\right]\right.$$
$$\left.+b\left(2M_0\bar{M}_{f_2}\right)^2+\frac{1}{2\beta-1}\left(2M_{1-\beta}\bar{M}_{f_1}b^\beta\right)^2\right\}\ge 1.$$

This is a contracts to (0.17). Hence for a positive integer l, $\Phi B_l\subseteq B_l$. Steps2: Next we will show that the operator Φ has a fixed point on B_l. Now we decompose $\Phi=\Phi_1+\Phi_2$ is condensing where Φ_1 is contraction and Φ_2 is compact.

The operator Φ_1,Φ_2 are defined on B_l respectively by

$$(\Phi_1x)(t)=T(t)f_1(0,u(0))-f_1(t,u(t))-\int_0^t AT(t-s)f_1(s,u(s))\mathrm{d}s$$

$$(\Phi_2x)(t)=T(t)\left[x_0+g(x)\right]+\int_0^t T(t-s)f_2(s,v(s))\mathrm{d}s$$
$$+\int_0^t T(t-s)G(s,P(s))\mathrm{d}w(s),t\in J$$

We would like to verify that Φ_1 is a contraction while Φ_2 is a completely continuous operator.

To prove that Φ_1 is a contraction, we take $x_1,x_2\in B_l$ arbitrarily. Then for each $t\in J$ and by condition (H_1) and (0.16) we have

$$E\left\|(\Phi_1x_1)(t)-(\Phi_1x_2)(t)\right\|^2$$
$$\le E\left\|f_1(t,u_1(t))-f_1(t,u_2(t))+T(t)\left[f_1(0,u_1(0))-f_1(0,u_2(0))\right]\right.$$
$$\left.+\int_0^t(-A)T(t-s)\left[f_1(s,u_1(s))-f_2(s,u_2(s))\right]\mathrm{d}s\right\|^2$$
$$\le 9\left(E\left\|(-A)^{-\beta}T(t-s)\left[(-A)^\beta f_1(s,u_1(t))-(-A)^\beta f_1(s,u_2(t))\right]\right\|^2\right.$$
$$+E\left\|(-A)^{-\beta}\left[(-A)^{-\beta}f_1(0,u_1(0))-f_1(0,u_2(0))\right]\right\|^2$$
$$\left.+E\left\|\int_0^t(-A)^{1-\beta}T(t-s)\left[(-A)^\beta f_1(s,u_1(s))-(-A)^\beta f_1(s,u_2(s))\right]\right\|^2\right)$$

$$\le 9\left(M_0^2 M_T^2 M_{f_1}^2 \sup_{0\le s\le b} E\left\|x_1(s)-x_2(s)\right\|^2 + M_0^2 M_{f_1}^2 \sup_{0\le s\le b} E\left\|x_1(s)-x_2(s)\right\|^2\right.$$

$$\left. + b\int_0^t \frac{M_{1-\beta}^2}{(t-s)^{2(1-\beta)}} E\left\|x_1(s)-x_2(s)\right\|^2\right)$$

Hence,

$$\left\|(\Phi_1 x_1)(t)-(\Phi_2 x_2)(t)\right\|^2 \le M_{f_1}^2\left[M_0^2\left(M_T^2+1\right)+\frac{\left(M_{1-\beta}b^\beta\right)^2}{2\beta-1}\right]\sup_{0\le s\le b}\left\|x_1(s)-x_2(s)\right\|^2$$

Thus,

$$\left\|(\Phi_1 x_1)-(\Phi_1 x_2)\right\|^2 \le L_0\left\|x_1-x_2\right\|^2$$

By the assumption $0\le L_0<0$, we see that Φ_1 is a contraction.

To prove that Φ_2 is compact, first we prove that Phi_2 is a contraction on B_l. Let $\{x_n\}_{n=0}^{\infty}\subseteq B_l$ with $x_n\to x\in B_l$, then (H$_2$) (i) and (H$_4$)

(i) $I_k, k=1,2,3,\cdots,m$ is continuous.

(ii) $G(s,P_n(s))\to G(s,P(s)), n\to\infty$.

Since

$$E\left\|G(S,P_n(s))-G(s,P(s))\right\|\le 2g_l(s)$$

Therefore, by dominated convergence theorem, we have

$$\begin{aligned}
&\left\|\Phi_2 x_n-\Phi_2 x\right\|^2\\
&=\sup_{0\le t\le b} E\left\|T(t)\left[g(x_n)-g(x)\right]+\int_0^t T(t-s)\left[f_2(s,v_n(s))-f_2(s,v(s))\right]\mathrm{d}s\right.\\
&\quad\left.+\int_0^t T(t-s)\left[G(s,P_n(s))-G(s,P(s))\right]\mathrm{d}w(s)\right\|^2\\
&\le M_T^2\left\|g(x_n)-g(x)\right\|+M_T^2\int_0^b\left\|f_2(s,v_n(s))-f_2(s,v(s))\right\|^2\mathrm{d}s\\
&\quad+M_T^2\int_0^t\left\|G(s,P_n(s))-G(s,P(s))\right\|^2\mathrm{d}w(s),\to 0,\text{ as } n\to\infty
\end{aligned}$$

Thus, Φ_2 is continuous.

Next, we prove that $\{\Phi_x : x\in B_t\}$ is a family of equicontinuous functions.

Let $x\in B_l$ and $\tau_1,\tau_2\in J$.

Thus if $0<\tau_1<\tau<b$ and $\varphi\in N_2(x)$, then for each $t\in J$, we have

$$\begin{aligned}
&\left\|\Phi_2 x(\tau_2)-\Phi_2 x(\tau_1)\right\|^2\\
&\le\left\|T(\tau_2)-T(\tau_1)\right\|^2\left\|x_0-g(x)\right\|^2+\int_0^{\tau_1-\varepsilon}\left\|T(\tau_2-s)-T(\tau_1-s)\right\|^2\left\|f_2(s,v(s))\right\|^2\mathrm{d}s\\
&\quad+\int_{\tau_1-\varepsilon}^{\tau_1}\left\|T(\tau_2-s)-T(\tau_1-s)\right\|^2\left\|f_2(s,v(s))\right\|^2\mathrm{d}s+\int_{\tau_1}^{\tau_2}\left\|T(\tau_2-s)\right\|^2\left\|f_2(s,v(s))\right\|^2\mathrm{d}s\\
&\quad+\int_0^{\tau_1-\varepsilon}\left\|T(\tau_2-s)-T(\tau_1-s)\right\|^2\left\|G(s,P(s))\right\|^2\mathrm{d}w(s)\\
&\quad+\int_{\tau_2-\varepsilon}^{\tau_1}\left\|T(\tau_2-s)-T(\tau_1-s)\right\|^2\left\|G(s,P(s))\right\|^2\mathrm{d}w(s)\\
&\quad+\int_{\tau_1}^{\tau_2}\left\|T(\tau_2-s)\right\|^2\left\|G(s,P(s))\right\|^2\mathrm{d}w(s)
\end{aligned}$$

The right hand side is independent of $x \in B_l$ and thend to zero as $\tau_2 - \tau_2 \to 0$, since the compactness of $\{T(t)\}_{t \geq 0}$ implies the continuity in the uniform operator topology. Similarly, using the compactness of the set $g(B_l)$ we can prove that the functions $\Phi x, x \in B_l$ are equicontinuous functions.

It remains to prove that $\Phi_2 x B_l(t)$ is relatively compact for each $t \in J$, where $V(t) = \{(\Phi_2)(t) : x \in B_l, t \in J\}$. Obviously, by conditions (H$_3$), $v(0)$ is relatively compact in B_l, we have

$$\begin{aligned}(\Phi_2^{\varepsilon})(t) &= T(t)\left[x_0 - g(x)\right] + \int_0^{t-\varepsilon} T(t-s-\varepsilon) f_2(s, v(s)) \mathrm{d}s \\ &\quad + T(\varepsilon)\int_0^{t-\varepsilon} T(t-s-\varepsilon) G(s, p(s)) \mathrm{d}w(s)\end{aligned}$$

Since $\{T(t)\}_{t \geq 0}$ is compact, the set $V_\varepsilon(t) = \{\Phi_2^{\varepsilon}(t) : x \in B_l\}$ is relatively compact in H for every $0 < \varepsilon < t$. Moreover, for every $x \in B_l$.

$$\begin{aligned}&\left\|(\Phi_2)(t) - \Phi_2^{\varepsilon} x(t)\right\| \\ &\leq \int_{t-\varepsilon}^{t} \left\|T(t-s) f_2(s, v(s))\right\|^2 \mathrm{d}s + \int_{t-\varepsilon}^{t} \left\|T(t-s) G(s, p(s))\right\|^2 \mathrm{d}w(s) \\ &\leq M_T^2 \int_{t-\varepsilon}^{t} h_t(s) \mathrm{d}w(s) + M_T^t \int_{t-\varepsilon}^{t} f_2(s, v(s)) \mathrm{d}s\end{aligned}$$

Therefore, letting $\varepsilon \to 0$, we see that, there are relatively compact sets arbitrarly close to the set $V(t) = \{\Phi_2(t) : x \in B_l\}$

Hence, the set $V(t)$ is relatively compact in B_l. A consequence of the above steps and the Arzela-Ascoli theorem, we can conclude that Φ_2 is a compact operator. These arguments enable us to conclude that $\Phi = \Phi_1 + \Phi_2$ is condensing map on B_l, and by the fixed point theorem of Sadovskii there exists a fixed point $x(.)$ for Φ on B_l.

Therefore the non-local system (0.12)-(0.13) has a mild solution which has studied by [35].

Hence the proof is completed.

4. Conclusion

In this paper, we presented the existence results for impulsive stochastic neutral differential systems through fractional power operators. We proved the results using semigroup theory and fixed point technique. Therefore, by Sadovskii fixed point theorem, it was possible to prove the existence for stochastic impulsive neutral differential system.

Acknowledgements

Sincere thanks to the members of JAMP for their professional performance, and special thanks to managing editor *Hellen XU* for a rare attitude of high quality.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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