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The Self-Deployment of Worlds on the Other Side of the Big Bang

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Abstract

String theory and quantum loop gravity have given us a lot of hope and have achieved some success in trying to build a quantum theory of gravity. However, we still don't have definitive answers. In this paper, we substantiate the idea of the existence of a Primary Parent Particle, constituents of the medium of primary matter before the Big Bang and describe its structure, consisting of three Beginnings united in the form of Borromeo rings. We derived a generalization of the Bohr-Sommerfeld quantization rules on the multidimensional case and considered problems of the embedding spaces into the each other based on the Nash theorems. Assuming that at the distances occupied by the triad we can consider the space Euclidean flat, we introduce quantum mechanical equations describing them. The problems of constructing the theory of quantum gravity and Dark matter radiation are considered separately. As part of our approach, we describe the acceleration of the Universe created by Dark Energy during expansion. We criticize principle of dualism, which can be used only for linear theories, but is not applicable to nonlinear theories. We investigated various distributions of dark matter densities and the gravitational potentials induced by them in galaxies and velocities for rotation curves. The main goal in the article was to show self-deployment of the Universe and its origin before Big Bang.

Keywords

Worlds, Upper and Lower Worlds, Materiality, the Principle of Three Beginnings, Borromeo Rings, The Primary Parent Particle, Self-Deployment of the Universe

1. Introduction

In this work, we develop the provisions set out in our article [1] in more details and explanations. We start from the description of the O_{SP}-Space and generalization of the Bohr-Sommerfeld quantization rule [2] on the high dimensional case. It is important for the more deep understanding the nature of the Higher and Lower[1] cosmic worlds and self-deployment of the Universe. This rule demonstrates us, that the structures of the objects in these worlds should have rather different geometric and physical properties. For the first time, Paul Ehrenfest pointed out the dependence of the physical properties of structures existing in space on its dimension [3]. Proceeding from the formulation of the Principle of the three Beginnings, we come to the existence of the Primary space O_{SP} in the form of two-phase model with the ground state in the form of a condensate of Primary Parent Particles[2] $O_{\{N,A,P\}}\big|_{BR}$, consisting from the three Borromeo rings and another phase in the form of bubbles in this condensate. The distribution of these bubbles in the Primary Parent Particles condensate can be arbitrary and they are places where form future universes, one of which is ours. On the boundaries between these two phases occurs evaporation of the $O_{\{N,A,P\}}\big|_{BR}$ particles and decays in the bubble interior on the following constituents [1]:

- O_N, O_A, O_P -3 particles, Beginnings ("packed"), which forms a triads.
- $"0"_0$, $"0"_+$, $"0"_-$, S_0, S_+, S_- -6 particles, 3 loops and 3 strings, "unpacked".
- Binary interactions, arising between these 6 particles give us additional 12 particles, carriers of binary fields, taking into the account that $"0"_0$, $"0"_+$ and $"0"_-$ particles do not interact.

All these particles form separate worlds due to their compositions and energies. These particles in the bubble interior are material for the formation of the clots—future universes with some core formed from the triads in the centre of the bubble [1]. Briefly we also can underline that the Principle of the three Beginnings (Forces, Energies) [4] states that any Phenomenon is always the result of the interaction of three Forces (Energies): active ("+"), passive ("−") and neutral ("0"). At the same time, the neutral force ("0") is the force of the Upper World, penetrating into the dual particles ("+") and ("−") of the Lower World and interacting with them, creates a phenomenon that is perceived as a Phenomenon in the Lower World. Indeed, the nested world is determined by its internal forces, between the *i*-th and *k*-th particles. The total momentum of its constituent particles $\boldsymbol{P} = \sum_i \boldsymbol{P}_i$:

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \sum_i \left\{ \boldsymbol{F}_i^{(ext)} + \sum_{k,k\neq i} \boldsymbol{F}_i^{(int)} \right\} \to \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \sum_i \boldsymbol{F}_i^{(ext)} = \boldsymbol{F}^{(ext)}, \tag{1}$$

where $\sum_i \sum_{k,k\neq i} \boldsymbol{F}_i^{(int)} = 0$. Here $\boldsymbol{F}_i^{(ext)}$ is an external force, acting from the Upper

[1]There is a hierarchy of embedded cosmic worlds, the Lower World is embedded in the Outer World, in the form of a matryoshka doll. The external world, we call the Higher World, in relation to the world embedded in it.

[2]This idea is not new. The hypothesis of the "Primary Atom" was expressed by Lemaitre back in 1950 and consisted in the suggestion that the entire Universe we observe was formed from fragments of the decay of this Primary atom.

World, on the i-th particle of the Lower World. We will call the $\boldsymbol{F}^{(ext)}$ Neutral force acting from the region of the Upper World, in which the Lower World is embedded. At the same time, Newton's third law does not work when the interaction is determined by the source of force in the Outside World $\boldsymbol{F}_{ik}^{(ext)} + \boldsymbol{F}_{ki}^{(ext)} \neq 0$, where the i-th particle belongs to the Outside World, and the k-th particle belongs to the Lower World. The action of a particle of the Higher World on a particle from the Lower World is not compensated by its reverse action, since the nature of their interaction is different. Other example of such kind of interaction is influence of the inertial forces, since there is no reverse compensating effect on them. The structure of presented paper is organized by the following way. In the Chapter 1 we have made a brief introduction where we formulate the motivation of the our research—justification of the approach formulated in [1] for the description processes before BB and their evolution.

In the Chapter 2, we derive Bohr-Sommerfeld rule of quantization on the multidimensional case. In the Chapter 3, we investigate properties of O_{SP}. Chapter 4 is devoted to the properties of the Primary Parent Particle and contribution of its influence in the multidimensional cases through out of the tension in the string formed from the decay of the Borromeo rings. In Chapter 5, we investigate Dark Matter (DM) properties and derive profile functions for the density, potential and velocities for DM. In the Chapter 6, we investigate the problem of the DM radiation and their possible interaction with electromagnetic forces. To the description of triads quantum states devoted Chapter 7. In the Chapter 8, we discuss the problems of quantum gravity and possible formulations of new approaches to the resolution the quantum gravity problem. At the end of the article we give a conclusion of the obtained results and list of the citing literature.

2. O_{SP}-Space and Generalization of Bohr-Sommerfeld Quantization Rule

According to our approach [1] let's define the dimensionality of O_{SP}-Space, since its geometry and physical properties determine the structures that can exist in it. The Bohr-Sommerfeld quantization rule [2], a generalization to the multidimensional case, determines the number of bound states, depending on the spatial dimension:

$$\oint\limits_{D\text{-dim}} \boldsymbol{p}\mathrm{d}\boldsymbol{q} \sim \pi \cdot \hbar \cdot n_D \tag{2}$$

Here, the integration is carried out in a multidimensional phase space of states of dimension D, n_D -the number of quantum states of the system, $\boldsymbol{p} = (p_1, p_2, \cdots, p_D)$ and $\boldsymbol{q} = (q_1, q_2, \cdots, q_D)$, are the generalized momentum and coordinate of the particle.

Changing the dimension of space leads to phase transitions in matter[3]. The energy released due to the destruction of the binding energy of the structures of the "higher" world can be enormous and, in principle, such a phase transition,

[3]This is a phase transition of the 1st kind, since the phase change of the state occurs in a jump.

with a change in the dimension of space, can serve as a possible mechanism for generating BB energy.

To estimate the number of bound states, in the case of a multidimensional flat Euclidean space, we divide the phase space of states in O_{SP}-Space into discrete quantum cells $\mathrm{d}p_i\mathrm{d}q_i \sim 2\pi\hbar$, where the index i runs through the values $1 \le i \le D$, and D-is the dimension of the O_{SP}-Space. Then, the volume of the phase space cell is:

$$\Gamma = \prod_{i=1}^{D}\left(\mathrm{d}p_i\mathrm{d}q_i\right) = \left(2\pi\hbar\right)^{DN} \tag{3}$$

If there is a minimum cell step size for one state l_D, then the number of quantum states n_D for N particles, taking into account their equivalence[4] and spin s, is equal to:

$$n_D = \frac{\Gamma\left(2s+1\right)}{l_D^{DN} \cdot N!} \approx \sqrt{2\pi N}\left(2s+1\right)\left(\frac{2\pi\hbar}{l_D}\right)^{DN}\left(\frac{N}{e}\right)^{N} \tag{4}$$

During the transition of states from the world of higher dimension to the states of the world with a lower spatial dimension, the released energy of connections is transformed into movement—the internal dynamics of structures, as a result of such a transition. It is easy to see that the number of subspaces with different nature, with dimensions $k \le D$, such as, that $O_{SP}^{(k)} \subset O_{SP}^{(D)}$, is proportional to 2^D. But then how the material structures of the Lower World themselves formed? If the complete energy of the system forming such a material structure is less than zero, then new bound states of the Lower World are formed, which cannot exist in the Higher World where energies are much higher than in the Lower World.

The generalization of the Bohr-Sommerfeld quantization rule follows from the Feynman hypothesis [5], which states that the amplitude of probabilities along the path $\{x\}$ is $A\{x\} = \mathrm{e}^{\frac{i}{\hbar}S\{x\}}$. Then, the amplitude of the transition $\{x\} \to \{x'\}$ probability will be expressed as:

$$A\left(x',x\right) = \int \mathrm{d}\{x\}\,\mathrm{e}^{\frac{i}{\hbar}S\{x\}}, \tag{5}$$

and the decomposing the action $S\left(x\right)$ in the vicinity of the $\{x\}$ into a Taylor series:

$$S\left(x'\right) = S\left(x\right) + \frac{\partial S}{\partial x}\mathrm{d}x + \frac{1}{2!}\frac{\partial^2 S}{\partial x^2}\mathrm{d}x^2 + \cdots, \tag{6}$$

we obtain, omitting the terms of the second order of smallness:

$$A\left(x',x\right) = \int \mathrm{d}\{x\}\,\mathrm{e}^{\frac{i}{\hbar}S\{x\}} = \int \mathrm{d}\{x\}\,\mathrm{e}^{\frac{i}{\hbar}\left[S(x)+\frac{\partial S}{\partial x}\mathrm{d}x+\cdots\right]} \approx \int \mathrm{d}\{x\}\,\mathrm{e}^{\frac{i}{\hbar}S(x)} \cdot \mathrm{e}^{\frac{i}{\hbar}\int\frac{\partial S}{\partial x}\mathrm{d}x} \tag{7}$$

From the last relation follows the condition of immutability of the amplitude of probabilities:

[4]That is, with permutation symmetry of particles between cells.

$$e^{\frac{i}{\hbar}\int\frac{\partial S}{\partial x}\mathrm{d}x} = 1 \to \frac{1}{\hbar}\int\frac{\partial S}{\partial \boldsymbol{x}}\mathrm{d}\boldsymbol{x} = \pi \cdot N_D, \tag{8}$$

where integration is carried out in the space of dimension *D*. Here, $\boldsymbol{p}_D = \frac{\partial S}{\partial \boldsymbol{x}}$ is a multidimensional momentum, $\boldsymbol{x} = (x_1, \cdots, x_D)$ is a multidimensional vector, N_D is the number of bound states in a multidimensional space, from which follows the proof of the relation (2).

The change in the dimension of space, due to exponential degeneracy with "time", can be described in cosmological models, such as Kazner model and its generalizations [6]. At the same time, the space of the highest dimension has an uncountable number of the embedded spaces of the lower dimension, with a rank below:

$$V_n = \bigcup_{i=1}^{\infty} V_{n-1}^{(i)} = \bigcup_{i=1}^{\infty}\bigcup_{k=1}^{\infty} V_{n-2}^{(i,k)} = \cdots \tag{9}$$

That is, that any space of high dimension can be represented as the union of an uncountable number of its subspaces, which have a dimension of a rank lower and the space of the higher world is inexhaustible, with the spaces of the lower worlds embedded in it.

3. About Geometric Nature and Dimension of O_{SP}-Space

The Upper world-the carrier of neutral particles ("0"), that is, particles of matter of "pure gravity"-characterizes a field that penetrates into any dimensions and has the maximum possible dimension in nature[5]. The standard description of gravity, within the framework of the classical Einstein equations, considers it as a 4-dimensional Riemannian manifold. However, there are multidimensional generalizations of Einstein's equations. And in this sense, gravity in different places is different, despite the fact that the embedding of one space into another is reflected in these equations and their solutions [7] [8].

Suppose we have a dimension space $n \to P^{(n)}$, then what should be the dimension space $N \to P^{(N)}$ external to it so that the space with $n < N$ can be "completely" embedded in a space of a larger dimension: $P^{(n)} \subset P^{(N)}$?.. This is a general statement of the problem and the solution is determined by the nature of both the external and the internal space invested in it. For spaces of Euclidean type, the solution is most likely simple.

Indeed, let the space $E^{(n)}$ be "completely" embedded in the space $E^{(N)}$. Then all possible continuous movements of space $E^{(n)}$ should leave it inside $E^{(N)}$. If we consider any movement as the displacement and rotation of space $E^{(n)}$ in space $E^{(N)}$, then it is necessary that:

$$N \geq \underbrace{n}_{\text{Translation}} + \underbrace{C_n^2}_{\text{Rotation}} = n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2} \tag{10}$$

Thus, the space of dimensions *n* can be embedded in the space of dimensions

[5]Since we describe gravity as a deformation of space and time.

$N \geq \frac{n(n+1)}{2}$, and for a 4-dimensional Euclidean space, the outer space necessary for its embedding must have dimension $N \geq 10$.

In general, when a curved Riemannian manifold is embedded in an external flat Euclidean space, the answer is given by Nash's theorem on regular embeddings [9]: any Riemannian manifold admits a smooth embedding in a sufficiently large Euclidean space[6]. We are talking about the isometric immersion of a Riemannian manifold (R^m, g) with dimension *m* and metrics *g*, into Euclidean space E^N with dimension *N*. Note that for the volume of a multidimensional sphere of unit radius in a Euclidean space of dimension *n*: $V_n = \pi^{\frac{n}{2}} \Big/ \Gamma\left(\frac{n}{2}+1\right)$, where Γ is the Euler gamma function. Assuming, for simplicity, that $n = 2k$, we get:

$$V_{2k} \underset{k\to\infty}{\to} \frac{\pi^k}{\sqrt{2\pi k}}\left(\frac{k}{e}\right)^{-k} \underset{k\to\infty}{\to} \frac{1}{\sqrt{2\pi k}}\left(\frac{\pi e}{k}\right)^{k} \to 0 \tag{11}$$

This means that the volume of a multidimensional unit ball $V_{2k} \to 0$, with an increase in the dimension of space. The maximum volume is achieved for space with $n = 6,7$, this fact may have important applications, since with a finite radius of the Universe, its volume may be small, for large dimensions *n*, *i.e.* multidimensional spaces are "point-like", in the sense that their volume tends to zero. Then, it can be assumed that the Universe could have been formed from a multidimensional point-like region, with the "collapse" of its dimensions to $n = 4$. Is it possible? If we consider the dimension of the space of "pure gravity" as a dynamic structure, then you can answer in the affirmative. In the case of $n = 4$ obvious degeneracy of the dimension of the original space:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{2p_1}\mathrm{d}x^2 - t^{2p_2}\mathrm{d}y^2 - t^{2p_3}\mathrm{d}z^2, \tag{12}$$

where $p_1 + p_2 + p_3 = 1$ and $p_1^2 + p_2^2 + p_3^2 = 1$, from which it easily follows that: $p_1 \cdot p_2 + p_2 \cdot p_3 + p_3 \cdot p_2 = 0$, *i.e.* one of the values: p_1, p_2, p_3 is negative and with growth of time *t*, and a 3-dimensional space defined by quantities (x, y, z) degenerates into a 2-dimensional one.

4. A Primary Parent Particle $O_{\{N,A,P\}}\Big|_{BR}$

The physical properties of O_{SP}-Space are determined by its geometric structure and the nature of its constituent Primary Parent Particles. Existence of the O_{SP}-Space was deduced by us from the existence of "worlds" and the Principle of 3 Beginnings, which determines the birth of the Phenomenon of BB and the formation of the observable Universe. At the same time, the action of the 3rd Force (energy) always occurs as the influence of the Higher World on the phenomena in the Lower worlds. Then, it is logical to assume, giving this principle a

[6]The best estimate for such an embedding obtained in [10] states that for a Riemannian manifold of dimensions *m*, it is equal to: $N \geq m^2 + 16m + 3$.

universal character, that in the Upper world, where the 3rd force "has nowhere" to come, it acts in this world, being soldered into one with two other Beginnings, forming a triad of all Beginnings in the Primary Parent Particle, which is the Beginning of everything. And here, the key question arises, and how are these Beginnings connected inside this unique particle? If we assume that additional internal connections must exist for them, then the uniqueness and self-sufficiency of these Beginnings is lost, since they cannot form a Primary Parent Particle without additional connections within it. How to resolve this paradox? A natural solution is to assume that they themselves provide connections in the Primary Parent Particle. But is it possible to come up with an appropriate geometric image for this idea? In our opinion, the most appropriate way is to connect these 3 Beginnings in the form of Borromeo rings. It is amazing that, being bound in Borromeo rings, each Beginning has its own colossal dynamics, rotating around its own axis and obeying its own vibrations, as in is shown in **Figure 1**.

Then we can estimate the internal energy of the Primary Parent Particle as:

$$E_O = \sum_{k=1}^{3}\left\{\frac{1}{2}M_k\left(R_k\omega_k\right)^2 + \sum_{n=1}^{\infty}\left(\frac{\hbar\cdot c}{R_k}\right)n_k\right\} + \delta E_{\text{Binding}}\,, \tag{13}$$

where M_k is the mass of the *k*-th ring, R_k is its radius, ω_k is the cyclic frequency of rotation, n_k is the number of waves that fit on the circumference of the *k*-th Borromeo ring and determine its vibration frequency, $\delta E_{\text{Binding}}$ is the binding energy required to break one Borromeo ring. If, in the ultra relativistic limit, we assume that the rest masses of the rings are zero, the energy contained in such a particle will still be colossal:

$$E_O = \sum_{k=1}^{3}\sum_{n=1}^{\infty}\left(\frac{\hbar\cdot c}{R_k}\right)n_k + \delta E_{\text{Binding}} \tag{14}$$

Assuming that the size of the ring is comparable to the Planck length $l_P = \sqrt{\frac{\hbar\cdot\gamma}{c^3}} \approx 1.6\times10^{-35}\,\text{m}$, for low frequency, dipole vibrations of the rings, we obtain $E_O \approx 3.75\times10^{22}\,\text{MeV}$. This energy lies beyond the capabilities of modern accelerators and, for our world this particle appears to be a point, despite the dynamics characterizing its internal motion.

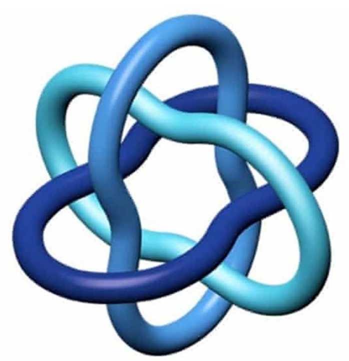

Figure 1. Dynamics of rotation and vibrations of Borromeo rings.

The Borromeo ring break condition can be estimated as:

$$\omega \geq \omega_{cr} = \frac{r_{ST}}{R_{ST}} \sqrt{\frac{\pi \cdot \sigma_{ST}}{\rho_{ST}}}, \tag{15}$$

where ω_{cr} is critical angular velocity of rotation, r_{ST}, R_{ST} are the transverse size of the Borromeo ring and its radius respectively, σ_{ST} is the tension inside the ring.

All this seems to be quite important. If we consider a multidimensional space, such as Gauss-Bonnet, then the contribution to the generalized equations of the gravitational field is possible at dimensions $n \geq 5$ and the value of tension inside the ring (loops, strings) becomes important, since the contribution to the field equations from higher dimensions is proportional to the multiplier: $(n-3)(n-4) \cdot \alpha$, where α -is the inverse of the string tension [11]. How the equation of state of matter for higher dimensions and the stability of the cosmological solution will change remains the subject of further research.

Regarding the nature of the Primary Parent Particle consisting of 3 Borromeo rings, it can be added that each ring can represent topologically different structures: these can be ribbon-like structures; or structures, in the form of a Mobius sheet or, instead of toroidal rings, ordinary rings. The last remark is important in the sense that there are approaches, that represent fundamental particles in the form of complex topological structures [12] [13]. There is no paradox of mass in our model, which is typical for preon models and string theory.

5. To the Profile Functions of Density, Potential and Velocity of Dark Matter

In this section we introduce several assumptions regarding the halo DM around galaxies, based on the fact that the profile rotation curves, as follows from observations, do not strongly depend on distance. Consider DM as a collisionless gas of particles, due to the fact that DM weakly interacts with baryonic matter, manifesting itself only gravitationally and, let's assume that the halo substance is in equilibrium, since the centrifugal force is balanced by the force of attraction. This allows us to write, for the case of a spherically symmetric distribution, that:

$$V(R) = \sqrt{\gamma \frac{M(R)}{R}} \approx const, \tag{16}$$

where $V(R)$ is the speed, in the dependence of distance *R*, and mass accordingly

$$M(R) = 4\pi \int_0^R r^2 \rho(r) \mathrm{d}r, \tag{17}$$

from where we finally write down that the ratio is true in the case:

$$\int_0^R r^2 \rho(r) \mathrm{d}r = \frac{C_0}{4\pi\gamma} R \to \rho(r) = \frac{C_0}{4\pi\gamma \cdot r^2} \tag{18}$$

The last expression for the distribution density has a singularity in the center, which, in principle, corresponds to the case when there is a black hole in the center of the galaxy with a singularity that cannot be removed, within the framework of Einstein's classical theory of gravity. Then, for the potential, from the Poisson equation, we get that:

$$\Delta\phi(r) = 4\pi\gamma \cdot \rho(r) \to \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = 4\pi\gamma \cdot \rho(r), \tag{19}$$

here, we took the Laplace operator in a spherical coordinate system and then, to determine the potential $\phi(r)$, we come to the following equation:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = C_0 \to \phi(\kappa) = C_0 \ln r \tag{20}$$

Note that when integrating the Poisson equation, we put all the integration constants equal to zero, based on the asymptotics of the expressions obtained at $r \to \infty$. Noticing that the magnitude $C_0 = \langle V(r)\rangle = \frac{1}{|a-b|}\int_a^b V(r)\mathrm{d}r$, where the values *a* and *b*, are the distances on the plateau of the rotation curves of the galaxy, we finally get their behavior, as it is shown in **Figure 2**.

The result we obtained corresponds to the simplest distribution of galactic matter, including DM and BM, obtained from a simple equilibrium condition between centrifugal forces and attractive forces. However, there are also more complex distributions in the literature, selected by quoting to observational data [14] [15]. If we take more complex distributions for halo DM [16], then we can, for example, for a distribution like:

$$\rho(r) = \frac{\rho_0}{1+\left(\frac{r}{r_c}\right)^2}, \tag{21}$$

where ρ_0 is the value of the DM density in the center of the galaxy, and r_c is the radius of its crust, then the nature of the distribution of the gravitational potential changes, but it is more difficult to obtain it in an analytical form. At the

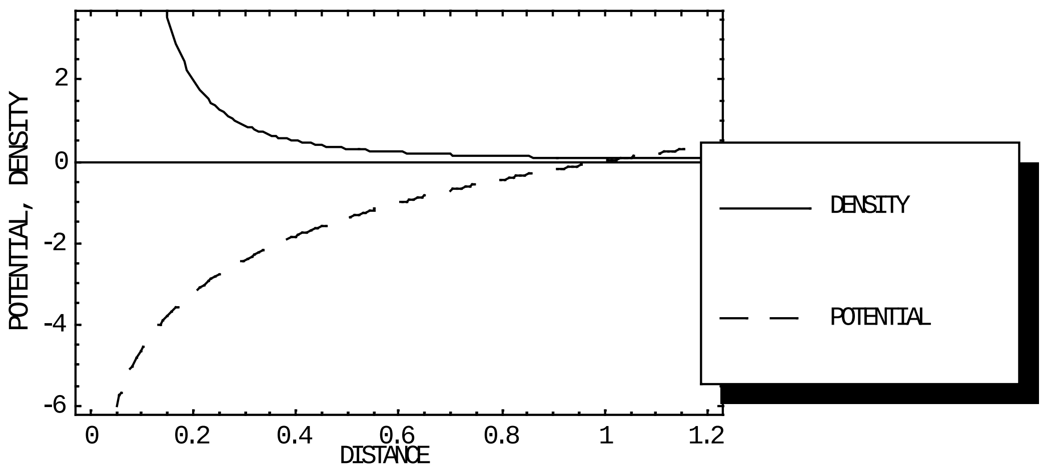

Figure 2. Profile functions of potential and density in the region of the plateau of rotational velocities at $a = 0.05$ and $b = 3.5$.

same time, it is easy to see that with $r \gg r_c$ the ratio (21), it passes, up to multipliers, into the ratio (18) that we obtained. Then, similarly, as when solving the Poisson equation in (19), we obtain for this density distribution, the following solutions of the distribution of the gravitational potential, depending on the radius:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \phi}{\partial r}\right)=4\pi\gamma\cdot\rho_0\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-1}, \tag{22}$$

from where it is easy to obtain:

$$\eta^2\frac{\partial \phi}{\partial \eta}=4\pi\gamma\cdot\rho_0\cdot r_c^2\int\frac{\eta^2 \mathrm{d}\eta}{1+\eta^2}+C_1, \tag{23}$$

where we introduced the dimensionless parameter $\eta=\frac{r}{r_c}$ and, integrating the right part, we write:

$$\frac{\partial \phi}{\partial \eta}=4\pi\gamma\cdot\rho_0\cdot r_c^2\left[-\frac{2}{\eta}+\frac{C_1}{\eta^2}+2\frac{\operatorname{arctg}(\eta)}{\eta^2}+\frac{\ln\left(1+\eta^2\right)}{\eta}\right], \tag{24}$$

where C_1 is the integration constant. The integration of (24), cannot be completely obtained in elementary functions, but the result, nevertheless, is presented in an analytical form, using a special function $\mathrm{PolyLog}\left[2,-\eta^2\right]$, as:

$$\begin{aligned}\phi(\eta)=&-\frac{4\pi\gamma\cdot\rho_0\cdot r_c^2}{\eta}C_1+4\pi\gamma\cdot\rho_0\cdot r_c^2\left\{-4\eta+\frac{4\eta-3}{\eta}\operatorname{arctg}(\eta)\right.\\&\left.+\left(2\eta-\frac{1}{4}\right)\ln\left(1+\eta^2\right)-\ln(\eta)-\frac{1}{2}\mathrm{PolyLog}\left[2,-\eta^2\right]\right\}\end{aligned} \tag{25}$$

Graphically, the distribution of density and potential, depending on the dimensionless Parameter η, looks like in **Figure 3**:

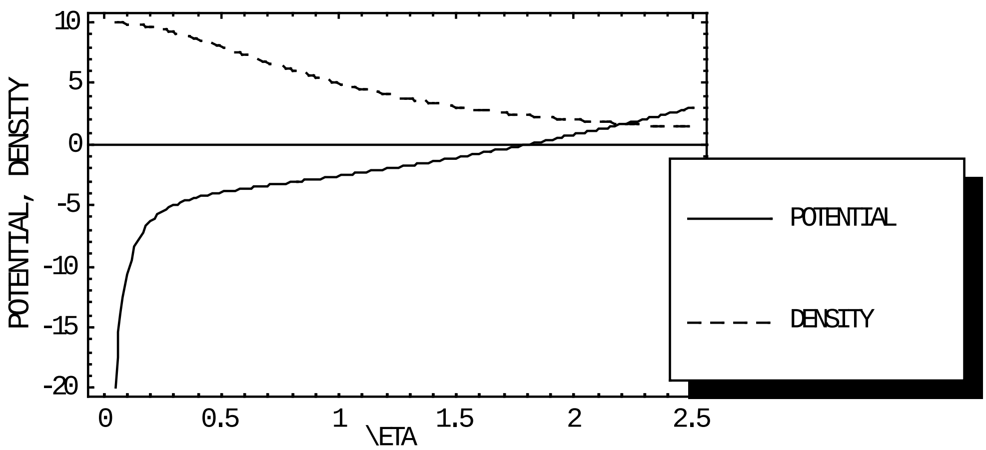

Figure 3. Profile distribution of normalized density $\frac{\rho}{\rho_0}$ and potential $\phi(\eta)$, functions created in the vicinity of the galaxy, taking into account DM, where we put that $4\pi\gamma\cdot r_c^2=1$, in the plateau region of rotation speeds $\eta_a=0.005$ and $\eta_b=2.5$.

6. Can Dark Matter Radiate?

Let's follow to the [1], where we introduced on the base of the existence Primary Parent Particle, that can exist 6 types of the DM, formed from the internal rearrangement of the triads structure (1, 2, 3) = {1(2, 3); 1(3, 2); 2(1, 3); 2(3, 1); 3(1, 2); 3(2, 1)}. Here we want to present these structures by the wave function in the following form: $\Psi_{\{i(j,k)\}}\left(\boldsymbol{r}_{iC},\boldsymbol{r}_{jk}\right)$, where i, j, k are a different permutations of the 1, 2, 3 numbers and $\boldsymbol{r}_{jk}$ is a radius vector between *j* and *k* Beginnings in the triad and $\boldsymbol{r}_{iC}$ is a radius vector between Beginning *i* and centre mass of the *j* and *k* Beginnings. Then we can suggest that rearrangement of the internal structure of the DM particle can lead to the radiation in the electromagnetic region and our problem is to establish the spectrum of such radiation, if it is in principle possible. In this case we immediately come to the necessity for the determination interaction potentials between constituents of the $\{i(j,k)\}$ structure. Yes, up to now we don't have an established theory for the solution such kind of problem and here let's only make a sketch of possible approach[7]. In the frame of the perturbation theory for the non-stationary case we can write, that:

$$i\hbar\frac{\partial\psi}{\partial t}=\left(\hat{H}_0+\hat{H}(t)\right)\psi\,, \tag{26}$$

where $\hat{H}(t)$ is a small perturbation of the considering quantum system[8]. Let's also suggest that wave functions of the unperturbed operator of interaction $\hat{H}_0$ for the non-stationary Schrödinger equation have the form:

$$\psi^{(0)}=\sum_K a_K^{(0)}\cdot\varphi_K^{(0)}\cdot \mathrm{e}^{-\frac{i}{\hbar}E_K^{(0)}t}\,, \tag{27}$$

here $a_K^{(0)}$ are constant coefficients of decomposition, $\varphi_K^{(0)}$ are wave functions of the stationary Schrödinger equation for the unperturbed operator $\hat{H}_0$. Then the solution of the equation (26) can be presented as:

$$\psi=\sum_n a_n^{(0)}(t)\cdot\varphi_n^{(0)}\cdot \mathrm{e}^{-\frac{i}{\hbar}E_n^{(0)}t}\,, \tag{28}$$

here $a_n^{(0)}(t)$ are coefficients defining the function ψ. Substituting this function into the (26) and taking into account the orthonormality of the wave functions of the unperturbed operator we can obtain for eigenfunctions $\varphi_n^{(0)}$ with according eigenvalues $E_n^{(0)}$ the system of differential equations in the form:

$$\frac{\mathrm{d}a_n(t)}{\mathrm{d}t}=-\frac{i}{\hbar}\sum_n a_n(t)\cdot H_{Sn}\cdot \mathrm{e}^{\frac{i}{\hbar}\left(E_S^{(0)}-E_n^{(0)}\right)t}\,, \tag{29}$$

where the matrix element $H_{Sn}=\int\varphi_S^{*(0)}\hat{H}\varphi_n^{(0)}\mathrm{d}\tau$. For the solution we can simply use an iteration method and a criterion for the correctness of the method will be its convergence. For the 1-st approximation we take $a_n^{(0)}(t=0)\equiv a_n^{(0)}$ and for it

[7]Fore correct and complete calculations we needs a regular and well established theory which on the moment we don't have.

[8]Under the smallness of the perturbation $\hat{H}(t)$, we assume that the perturbation of the system does not change its state much, compared to the initial undisturbed state determined by the operator $\hat{H}_0$.

the solution will looks as:

$$a_S(t) = a_S(0) - \frac{i}{\hbar}\sum_n a_n(t)\int_0^t H_{Sn}\cdot \mathrm{e}^{\frac{i}{\hbar}\left(E_S^{(0)}-E_n^{(0)}\right)t}\mathrm{d}t \tag{30}$$

Note that in the general case, the distribution over states in the quantum system under consideration is determined by the square of the coefficient $\left|a_S(t)\right|^2$. Then it is easy to see that if the system at the initial moment is in one of the occupied states, for example n, then the action of the disturbance translates it into a state S (see **Figure 4**), and the amplitude of the probability of such a transition is determined by the following expression:

$$a_{n\to S}(t) = -\frac{i}{\hbar}\int_0^t H_{Sn}\cdot \mathrm{e}^{\frac{i}{\hbar}\left(E_S^{(0)}-E_n^{(0)}\right)t}\mathrm{d}t \tag{31}$$

Here we took into account the fact that if the system is in a quantum state n at the initial moment, then the probability amplitude of this state $a_n(0)=1$.

Let's now put for the matrix element $H_{Sn} = \int \varphi_S^{*(0)}\hat{H}\varphi_n^{(0)}\mathrm{d}\tau$, that it is independent from the time and after simple calculations we come to the expression for the transition amplitude, written as:

$$a_{n\to S}(t) = \frac{H_{Sn}}{E_S^{(0)}-E_n^{(0)}}\left(1-\mathrm{e}^{\frac{i}{\hbar}\left(E_S^{(0)}-E_n^{(0)}\right)t}\right), \tag{32}$$

Then probability of such transition can be presented in the form:

$$P_{n\to S}(t) = \left|a_{n\to S}(t)\right|^2 = 4H_{Sn}^2\frac{\sin^2\left(\frac{\left(E_S^{(0)}-E_n^{(0)}\right)t}{2\hbar}\right)}{\left(E_S^{(0)}-E_n^{(0)}\right)^2} \tag{33}$$

It is obvious that the intensity of transitions is determined by the density of settlement of the corresponding levels.

6.1. On the Interaction of Electromagnetic Waves with a Quantum System

Let's assume that the external perturbation of a quantum system is an electromagnetic wave, formally defined by the relation: $\hat{H}(t) = H(\tau)\cos[\omega\cdot t]$, where τ -is some function of the coordinates, depending of the spatial structure of the considering quantum system. Taking it into account and using expression (31), we come to the relationship:

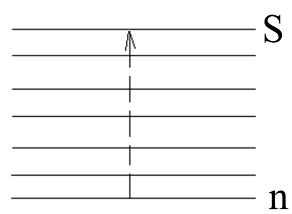

Figure 4. Transition $n\to S$, described by $\left|a_{n\to S}(t)\right|^2$.

$$a_{n\to S}(t) = -\frac{i}{\hbar} H_{Sn} \int_0^t \cos[\omega \cdot t] \mathrm{e}^{\frac{i}{\hbar}\left(E_S^{(0)} - E_n^{(0)}\right)t} \mathrm{d}t, \tag{34}$$

where $H_{Sn} = \int \varphi_S^{*(0)} H(\tau) \varphi_n^{(0)} \mathrm{d}\tau$ and by entering the transition frequency as $\omega(n \to S) = \frac{E_S^{(0)} - E_n^{(0)}}{\hbar}$, finally we come to:

$$P_{n\to S}(t) = \left|a_{n\to S}(t)\right|^2 = \frac{H_{n\to S}^2}{\hbar^2} \frac{\sin^2\left(\frac{(\omega_{n\to S} - \omega)t}{2}\right)}{(\omega_{n\to S} - \omega)^2} \tag{35}$$

The latter relation shows that the transition probabilities (and hence the intensity of the transition lines) depend on the natural frequency spectrum of the system, on its structure and on the frequency of external radiation. At the same time, it is easy to see that the transition maximum is achieved when the frequency of external radiation coincides with the natural oscillation frequencies of the system. In this case, we can talk about resonant radiation or absorption. Thus, we clearly see the need to calculate the structural functions of the system being irradiated, which is expressed in the calculation of the matrix element $H_{Sn} \equiv H_{n\to S} = \int \varphi_S^{*(0)} H(\tau) \varphi_n^{(0)} \mathrm{d}\tau$. These matrix elements depend on the wave functions of the stationary system.

6.2. About Origin of the Universe Expansion with Acceleration

It is well known, that our Universe acceleration [17] is connected with Dark Energy nature of which up to present days is unknown. Some of the explanation is connected with Λ term introduced by Einstein into the equations of the gravitation field. Other approach is quintessence, but the main characteristic of the DE is that it is constant in any point of the Universe. But how to explain it on the alternative way? Let's imagine our Universe inside "nut shell" with some mass of this shell equal to M_{nut}. From observation we can conclude, that our Universe is flat and in this case we can easily calculate gravitation potential in the shell interior as:

$$U = -\gamma \frac{M_{nut}}{R(t)}, \tag{36}$$

where $R(t) = R_0 + r_0 \cdot \sin(w \cdot t)$, R_0 is a constant radius of the shell, r_0 is an amplitude of the radial oscillations and w is cyclic oscillation frequency of the "nut shell". Introducing an operator, defined as:

$$\nabla^{(4)} = \frac{1}{c}\frac{\partial}{\partial t} - \nabla^{(3)}, \tag{37}$$

we can calculate, that in any point in the "nut shell" interior:

$$\boldsymbol{a} = \gamma \frac{M_{nut}}{R_0}\left[\frac{r_0}{R_0} w \cdot \cos(w \cdot t) - 2\left(\frac{r_0}{R_0}\right)^2 \cos^2(w \cdot t) + \cdots\right], \tag{38}$$

or $\boldsymbol{a} \approx \gamma \frac{M_{nut}}{R_0}\left(\frac{r_0}{R_0}\right) w \cdot \cos(w \cdot t)$ is constant due to the $w \cdot t \to 0$. Therefore such a simple model, which is in an agreement with our approach explain a constant acceleration in the any point of the interior of the "nut shell" defined by the mass M_{nut}.

7. To the Quantum Description of Triads

Above we considered the interaction of the external electromagnetic field with arbitrary quantum system. As an example such systems can be triads, introduced by us. Let us now turn to the question concerning the quantum mechanical description of triads and their interaction with electromagnetic waves and the response of the triads on such interactions. Of course, here we should consider the problem of describing such a system within the framework of a general relativistic generalization of the Faddeev equations [18], when a system from the three bound particles is in a strong gravitational field of densely packed triads. The construction of such a generalization of the Faddeev equations is a matter of the future. The most direct solution to this problem is to write these equations on the set of a Riemannian 4-dimensional manifold, appropriately redefining in it all the necessary mathematical operations such as differentiation operators, in the Hamiltonian of the system and correctly determine the integration on such a manifold. Mathematically, this does not present so big difficulties [19], however, the correctness of such a generalization may be far from the physical essence of the problem and mathematical equations may be inadequate to the physical situation. Therefore, it is desirable to directly derive such generalized equations in a curved Riemannian 4-dimensional manifold, based not on heuristic considerations, but on stricter grounds. Relativistic quantum mechanics is based on the invariant Poincare group, unlike conventional quantum mechanics, which is based on the Galilean invariant group. General relativistic quantum mechanics should be based on an invariant group of isometric transformations [20]. As a first step, we can assume that works the approximation when the space of the Universe before Big Bang, on the scales of the area occupied by the triads, is locally Euclidean flat. Then, the wave function of the triad can be written in the framework of the translationally invariant shell model in Dirac notation as [21]:

$$\begin{aligned}
&\left\langle \overline{x}^1, \overline{x}^2, \overline{x}^3 \middle| n_1 l_1, \Lambda_{23} M_{23}, \Lambda_{1(23)} M_{1(23)} \right\rangle \\
&= \left[\Psi_{n_1 l_1}\left(\overline{x}^1\right) \Psi_{n_2 l_2}\left(\overline{x}^2\right) \Psi_{n_3 l_3}\left(\overline{x}^3\right) \right]_{\Lambda_{1(23)} M_{1(23)}} \\
&= \sum_{m_1 M_{23}} C^{\Lambda_{1(23)} M_{1(23)}}_{l_1 \Lambda_{23} m_1 M_{23}} \Psi_{n_1 l_1 m_1}\left(\overline{x}^1\right) \left\langle \overline{x}^2 \overline{x}^3 \middle| n_2 l_2, n_3 l_3, \Lambda_{23} M_{23} \right\rangle,
\end{aligned} \tag{39}$$

where

$$\left\langle \overline{x}^2 \overline{x}^3 \middle| n_2 l_2, n_3 l_3, \Lambda_{23} M_{23} \right\rangle = \sum_{m_2 m_3} C^{\Lambda_{23} M_{23}}_{l_1 l_3 m_2 m_3} \Psi_{n_2 l_2 m_2}\left(\overline{x}^2\right) \Psi_{n_3 l_3 m_3}\left(\overline{x}^3\right), \tag{40}$$

here $\overline{x}^1, \overline{x}^2, \overline{x}^3$ are spatial coordinates[9] of the 1, 2, 3 particles, and (n_i, l_i, m_i) are their quantum numbers (main, orbital and spin), $C^{\Lambda_{23} M_{23}}_{l_1 l_3 m_2 m_3}$ -Clebsh-Gordon coef-

[9]For example $\overline{x}^1 = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)} \right)$ and so on...

ficients.

Naturally, to solve the problem of the quantum mechanical description of the triad, it is necessary to solve the Schrödinger equation for this system. Note that this is only in the case of the approximation we have noted[10]. For the first approximation of interaction potential between triad constituents, we can take a potential of Yukawa type:

$$V_{ij} = -g_{(ij)} \frac{\exp\left[-k_{ij} \cdot r_{ij}\right]}{r_{ij}} \to -g_{(ij)} \frac{1 - 2\pi\left(\frac{m_{ij}c^2}{\hbar c}\right) r_{ij} + \cdots}{r_{ij}}, \tag{41}$$

at $k_{ij} \cdot r_{ij} \to 0$, and then interaction potential can be described it the form of Coulomb potential, where $V_{ij} \approx -\frac{g_{(ij)}}{r_{ij}} + 2\pi\left(\frac{m_{ij}c^2}{\hbar c}\right)$, here $g_{(ij)}$ are constants of interaction intensity between *i*-th and *j*-th constituents in triads, r_{ij} are distance between them, values $k_{ij} = \frac{2\pi}{\lambda_{ij}}$, where $\lambda_{ij} = \frac{\hbar}{m_{ij} \cdot c}$ are a Compton wave lengths of "brace", binding *i*-th and *j*-th constituents inside triad, m_{ij}-mass of "brace", *c*-speed of light and $\hbar$-Planck constant. Then the Hamiltonian of the described system, in the approximation that the region occupied by the triad of "swaddled" particles is locally flat, will be written as $\hat{H} = \hat{K} + \hat{V}$, where:

$$\hat{K} = -\frac{1}{2}\hbar^2\left(\frac{\Delta_{\bar{x}^1}}{m_1} + \frac{\Delta_{\bar{x}^2}}{m_2} + \frac{\Delta_{\bar{x}^3}}{m_3}\right), \tag{42}$$

$$\hat{V} = -\left[g_{12}\frac{\exp\left[-k_{12} \cdot r_{12}\right]}{r_{12}} + g_{13}\frac{\exp\left[-k_{13} \cdot r_{13}\right]}{r_{13}} + g_{23}\frac{\exp\left[-k_{23} \cdot r_{23}\right]}{r_{23}}\right] + V_{123}, \tag{43}$$

m_1, m_2, m_3 are mass of the particles in the triad interior, $\Delta_{\bar{x}^i}$ is Laplace operator for *i*-th particle, where *i* can be equal on the one of the values 1, 2 or 3 and V_{123} is pure three-particle interaction, which require a separate consideration. Then the stationary Schrödinger equation for such a system can be written as:

$$\hat{H}\Psi_{123}\left(\bar{x}^1, \bar{x}^2, \bar{x}^3\right) = E_{123}\Psi_{123}\left(\bar{x}^1, \bar{x}^2, \bar{x}^3\right) \tag{44}$$

where $\Psi_{123}\left(\bar{x}^1, \bar{x}^2, \bar{x}^3\right)$-the wave function of the "swaddled" triad, and E_{123} is the energy of the stationary state of the system. The last equation allows, in this approximation, to determine the quantum mechanical state of the triad of "swaddled" particles:

$$\Psi_{123}\left(\bar{x}^1, \bar{x}^2, \bar{x}^3\right) = \frac{1}{\sqrt{3}}\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\Psi_1\left(\bar{x}^1\right) + \begin{pmatrix}0\\1\\0\end{pmatrix}\Psi_2\left(\bar{x}^2\right) + \begin{pmatrix}0\\0\\1\end{pmatrix}\Psi_3\left(\bar{x}^3\right)\right\} \tag{45}$$

However, we note that the problem of the quantum mechanical description of triads should be formulated within the framework of a general relativistic generalization of the three particle Faddeev equations[11]. There is no such complete

[10]We will note the boundaries of this approximation below…

[11]In this approximation, the radius of the Gaussian curvature is much larger than the Compton wavelength of the "staples".

theory at the moment.

And the last remark about the existence of bound states in the space of the Universe before Big Bang. If we consider in this case the multidimensional space, then there must be a generalization of the Bohr-Sommerfeld quantization rules: $\oint_{n\text{-dimension}} p\mathrm{d}q \sim \hbar N$, where integration is carried out in this multidimensional phase space, N is the number of the bound quantum states of the system. Hence, it follows that in the case of a phase transition associated with a decrease in the dimension of the original space of the embryo of the Universe, phase transitions of the different nature induced in matter are possible. In this case, the bound states of matter structures existing in a higher dimensional space, but forbidden in a lower dimensional space, will be destroyed. Then, the energy released due to the binding energy of these structures of the "higher" world can be colossal and, in principle, it can be a mechanism of BB[12].

8. The Road to the Quantum Gravity

The triumph of Newton's theory of gravity was impressive, but even here we are faced with selectivity of the human mind in relation to facts. Almost all textbooks write about the discovery of the planet Neptune by Adams and Leverrier "at the tip of the pen", which is undoubtedly the greatest achievement of theoretical science. But it is very rarely mentioned that the existence of the planet Vulcan, between the Sun and Mercury, was predicted to explain the secular displacement of the perihelion of Mercury's orbit, but this planet was not discovered. The reason for this, as a rule, remained outside the scope of discussion. However, if gravity is inherent in all matter, then how to describe the gravitational interaction of two rays of light, which are certainly material? And here, too, Newton's theory of gravity could not give the right answer. Einstein's theory of the gravitational field described classical effects that Newton's theory of gravitation could not explain, but it also turned out to be incomplete in terms of "internal perfection" and "external justification" [22]. Its incompleteness necessarily led to the construction of new hypotheses, such as the theory of twistors, noncommutative geometry, the theory of "strings", supergravity, quantum loop gravity and other alternative approaches [23] [24] [25], the main purpose of which was to build a quantum theory of the gravitational field and use it for the explanation of the Universe origin. However, the task of constructing a complete quantum theory of the gravitational field still remains unsolved and a simple analysis gives us some idea of the difficulties that arise in the way of constructing such a theory. Let us dwell on them in more detail, having considered Einstein's theory of the gravitational field and the most conceptually and mathematically developed theories that claim to be a quantum description of the gravitational field—string theory and loop gravity. Indeed, the very recording of the gravitational field equations in the form:

[12]And it can be estimated [1], that is, it is possible to calculate the energy released in the Big Bang (BB).

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{46}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R, and $T_{\mu\nu} = (p + \varepsilon) u_\mu u_\nu - p g_{\mu\nu}$, accordingly are the Ricci tensor, the metric tensor, the scalar curvature, the energy-momentum tensor of matter, p—pressure, ε—energy density, u_μ—the 4-velocity vector, G and c—are the gravitational constant and the speed of light, Einstein considered incorrect, in the sense that it implies "mixing" tensors of different nature when in the left parts (46) are tensors of the pure geometric nature of space-time, and in the right hand side are tensors associated with ordinary, continuously distributed matter[13], which is the source of the gravitational field.

Another reason is that at the consideration of the vacuum equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0, \tag{47}$$

where scalar curvature $R = g^{\alpha\beta} R_{\alpha\beta} = R^\alpha_\alpha$, we come to the equation:

$$R_{\mu\nu} = 0, \tag{48}$$

for which there are no solutions with matter continuously distributed in space, since when integrating Equation (48), we get a solution expressed in terms of the integration constant, which in the limit of a weak gravitational field is the mass of a body with a singular distribution of matter expressed throughout Dirac δ-function:

$$m(\boldsymbol{r}) = m \cdot \delta(\boldsymbol{r}) \tag{49}$$

As a result, for the spherically symmetric case, we come to the solution obtained by K. Schwarzschild [26], which can be represented as:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \mathrm{d}r^2 - r^2 \left(\mathrm{d}\vartheta^2 + \sin^2 \vartheta \mathrm{d}\varphi^2\right), \tag{50}$$

having two singular regions: a point $r = 0$ and the surface of a sphere representing an event horizon, centered at a point $r = 0$ and with a radius $r = R_g = 2\frac{GM}{c^2}$.

The singularity $r = R_g$ is a coordinate one (it can be eliminated by choosing the appropriate coordinate system [27]) and the solution can be continued into the domain $0 < r \le R_g$. But the singularity at the point $r = 0$ is irremediable, which indicates the incompleteness of the theory and the need for its further development. Thus, the vacuum solution of Equation (43) is already performed only for a discrete source of the gravitational field and has an irreducible singularity.

The analysis of the nature of singularities and the concepts of singular space-time arising in the general theory of relativity is sufficiently fully consi-

[13]This is fundamentally different from the usual notation of Maxwell's equations $F^{\mu\nu}{}_{,\nu} = -\frac{4\pi}{c} j^\mu$, where electromagnetic field is described by $A_\mu = (\varphi, -\boldsymbol{A})$ potential, $F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}$, $j^\mu = (c\rho, \boldsymbol{j})$ and $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$. In the right and left hand of side in Maxwell's equations we have the characteristics of material fields and their sources. Is space and time material? If so, how does this materiality manifest itself?

dered in [28]. This consideration is based on the properties of geodesic incompleteness and the characteristics of isometric expansion, taking into account the features of the invariants of the studied space-time and the Riemann curvature tensor along time-like curves.

In a whole series of early papers [29] [30] [31] [32], it was proved that in standard general theory of relativity singularities inevitably arise when solving its field equations. Therefore, in the case of vacuum solutions of the field equations, for a spherically symmetric source [26], we encounter the irreducible point singularity we have noted. Moreover, for the axially symmetric case, we obtain a ring singularity with splitting of the event horizon, in the solution of R.P. Kerr [33]. Further, a whole class of cosmological models has been obtained, starting with the Friedman model, the characteristic feature of which is the presence of a singularity at the time of the birth of the Universe.

However, we note one very important circumstance. The fact is that, being a classical theory, theory of general relativity necessarily leads us to the existence of the minimum possible length in space, which lies in the region of Planck quantities. A simple consideration based on the use of the Heisenberg uncertainty principle and the existence of an event horizon for black holes arising from Einstein's theory requires that when: $\Delta R \sim \frac{\hbar}{M \cdot c} = R_H = 2\frac{\gamma \cdot M}{c^2} \to M \approx 1.54 \times 10^{-7}\ \text{gramm}$. Localization of this matter in $\Delta R = R_H \sim \frac{\hbar}{M \cdot c} = 2.281 \times 10^{-36}\ \text{cm}$, it leads to the fact that it becomes invisible to an external observer [34], being under the event horizon, but manifesting itself gravitationally. In this context, matter "locked" inside the event horizon can claim the role of Dark Matter, provided that the time required for its evaporation is longer than the current lifetime of the Universe. However, we should be careful in our conclusions here, since the behavior of matter in the region of Planck scales is not known to us for certain. If we assume that classical Hawking radiation is valid on Planck scales, then this is unlikely for the simple reason that the very concept of the surface of the event horizon of a black hole becomes blurred, due to the uncertainty principle. And here the question arises—can there in principle be a stable structure from the totality of black holes, taking into account Hawking radiation? How can the gas from the microscopic primary black holes be described? In this formulation, we can talk about the behavior of gas from primary black holes at the early stages of the evolution of the Universe, the formation of a condensate of black holes and the quantum processes of their decay. Solving this problem can clarify a lot in our understanding of the birth and evolution of the Universe and is an open problem. Here we can focus on the study [35], where, in multidimensional space, singular solutions for configurations in the form of black strings, black rings and multiple black holes are considered. The paper uses the inverse scattering problem method to obtain regular solutions without naked singularities, closed time-like curves and Dirac-Misner strings [36].

In addition to the problems associated with singularities, there are technical and conceptual problems. So, in the case of an electromagnetic field, we can unambiguously introduce the concepts of energy density and energy flow:

$$\rho_E = \frac{1}{8\pi} g_{\mu\nu} \left(E^\mu E^\nu + H^\mu H^\nu \right), \quad \boldsymbol{S} = \frac{c}{4\pi} \boldsymbol{E} \times \boldsymbol{H} \to S_i = \frac{c}{4\pi} \cdot e_{ijk} E_j H_k , \tag{51}$$

here $i, j, k = \{1,2,3\}$ and e_{ijk} is antisymmetric Levi-Civita symbol. Pulse density:

$$\boldsymbol{P} = \frac{\boldsymbol{S}}{c^2} = \frac{\boldsymbol{E} \times \boldsymbol{H}}{4\pi \cdot c} \to P_i = \frac{1}{4\pi \cdot c} \cdot e_{ijk} E^j H^k , \tag{52}$$

and momentum flux density expressed in terms of the Maxwell tension tensor:

$$\Phi_i = \oint \sigma_{ik} \mathrm{d}\Pi_k , \quad \sigma_{ik} = \frac{1}{4\pi} \left(\frac{E^2 + H^2}{2} \delta_{ik} - E_i E_k - H_i H_k \right), \tag{53}$$

where $\mathrm{d}\Pi$ is an element of the surface covering the localization area of the electromagnetic field. In this case, Maxwell's vacuum equations are:

$$F^{\mu\nu}{}_{,\nu} = 0 , \tag{54}$$

and they lead to the existence of electromagnetic waves in a vacuum. If we now compare Equation (54) with Equation (48), we will see that the equations of the gravitational field are nonlinear, unlike the equations of the electromagnetic field. This leads to significant conclusions. The fact is that in linear theory, we can independently construct the equations of motion of particles and the equations of change of the field in which they move. This is the case in the classical theory of the electromagnetic field. Assuming the independent existence of the field and particles, we establish an incomplete, dualistic picture of the world. It is valid only in the case of linear theories, where only binary relations are essential. In this case, we need to add the equations of motion of the sources of this field to the field equations. In nonlinear theories, things are much more complicated, but the picture they describe is a more complete approximation to reality. In this case, the division into the sources of the field and the field itself is no longer so obvious and we have the interaction of the sources of the field and the interaction field itself arising between them. At the same time, the carriers of the field are particles of a different nature than the sources of the field. For such theories, the simplified dualistic approach does not work, and the equations of motion are determined by the equations of the field itself. Moreover, in the mathematical aspect, the equations of motion are conditions for the integrability of field equations [37]. In this case, an adequate description is a theory based not on dualism, but on a deeper and universal principle—the Principle of three Beginnings.

The analysis of the right-hand side of the Equation (46) describing matter is also nontrivial. For example, it was concluded that gravitational waves do not exist at all [38], where it was shown that when solving field equations for empty space, the total energy of gravitational waves is zero. This conclusion seems hasty, since standing waves exist, although energy transfer does not occur in them. Moreover, considering an extended body within the framework of Einstein's classical approach poses the problem of adequately determining the energy-momentum tensor of the system, and here we face the problem of synchronization when determining the physical characteristics of various points of this body. This is a conceptual problem, due to the fact that due to the curvature of spacetime, proper time flows differently at each of its points.

The nonlinearity of the gravitational field equations and the lack of universal solution methods greatly complicates their analysis, not to mention the construction of a more general, quantum theory of the gravitational field. We believe that both matter and spacetime should be described in a single way, and only in this case we can claim a complete theory, at the quantum level. The theory that claims to describe everything we observe should be based on a universal approach in which matter, field and spacetime are described in a single way when we stand at the very origins of the birth of the Universe.

The modern approach to building a unified theory of spacetime and matter is to build a theory that reconciles Einstein's general theory of relativity with quantum theory. However, since we do not have a full-fledged theory describing matter on the Planck scale, and not just spacetime, the problem is even deeper than just unification of these two wonderful theories in one unified theory.

This is reminiscent of the problem solved by P.A.M. Dirac, when, from attempts to reconcile quantum mechanics and special relativity, he came to the discovery of an equation that is a relativistic generalization of the Schrodinger equation for fermions with spin $s = 1/2$:

$$\left(i\hbar c \gamma^{\mu} \frac{\partial}{\partial x^{\mu}} - mc^2 \right) \psi_{\sigma}(x) = 0, \tag{55}$$

here γ^{μ} are hypercomplex numbers defined by commutation relations: $\gamma^{\mu}\gamma_{\nu} + \gamma^{\nu}\gamma_{\mu} = 2\delta^{\mu}{}_{\nu}$, where $\mu, \nu = \{0,1,2,3\}$ [39]. In our case, a general relativistic generalization is required, which is certainly more difficult. Let's explain this. Dirac's approach was based on an attempt to write the relativistic equation of the energy of a free particle in a Lorentz-invariant form, since the non-relativistic Schrodinger equation is not Lorentz-invariant. It came down to the representation of a relativistic equation for the energy of a free particle:

$$E^2 = P^2c^2 + m^2c^4, \tag{56}$$

where the value E and momentum $\boldsymbol{P}$ corresponds to the operators:

$$E \to \hat{E} = i\hbar \frac{\partial}{\partial t} \text{ and } P \to \hat{P} = -i\hbar \nabla, \tag{57}$$

in the form corresponding to the linear representation of spatial derivatives and the time derivative. In its final form, this idea led to a matrix equation in the form (55), written in a rational system of units.

It would seem that the idea of continuing the development of Dirac's approach for further general relativistic generalization can be developed in the same way, splitting curved spacetime into finite Euclidean pieces interconnected by coupling equations. Then the partial derivatives, when defining the operators, will be replaced by covariant derivatives, taking into account the Lorentz invariance on each locally Euclidean domain, into which we have divided the entire Riemannian manifold describing curved spacetime. But there are problems here.

Firstly, we are building quantum mechanics in a curved spacetime, and we claim more, because we want to build a unified complete quantum theory of the gravitational field and matter.

Secondly, we note the conceptual difficulty that arises when constructing a quantum theory of the gravitational field, if it is constructed in the spirit of ordinary quantum mechanics using the concepts of hermiticity and self-conjugacy of operators on a Riemannian manifold. For example, the hermitian and conjugacy of operators, in the range of Planck quantities, in the conditions of the early Universe and at its birth, can change dramatically [40]. Let's show this with an example.

Let's construct an operator $\hat{L}^{+}$, conjugate to the operator $\hat{L}=\frac{\partial}{\partial x}$ [41]. Then, for two continuous and differentiable functions f and g, the conjugacy condition is: $\int\left[g^{*}\hat{L}(f)-\left(\hat{L}^{+}(g)f\right)^{*}\right]\mathrm{d}\tau=0$, and substituting $\hat{L}$, we come to the:

$$\int_{a}^{b} g^{*}\frac{\partial f}{\partial x}\mathrm{d}\tau-\int_{a}^{b}\left[\hat{L}^{+}(g)f\right]^{*}\mathrm{d}\tau=0, \tag{58}$$

from where:

$$\int_{a}^{b}\left[\hat{L}^{+}(g)f\right]^{*}\mathrm{d}\tau=\int_{a}^{b} g^{*}\frac{\partial f}{\partial x}\mathrm{d}\tau=g^{*}f\Big|_{a}^{b}-\int_{a}^{b}\frac{\partial g^{*}}{\partial x}f\cdot\mathrm{d}\tau \tag{59}$$

This last one relation can be written as:

$$\int_{a}^{b} g^{*}\frac{\partial f}{\partial x}\mathrm{d}\tau+\int_{a}^{b}\frac{\partial g^{*}}{\partial x}f\cdot\mathrm{d}\tau=g^{*}f\Big|_{a}^{b} \tag{60}$$

In the right-hand side of the obtained relation, if $g^{*}f\Big|_{a}^{b}=0$, then $\hat{L}^{+}=-\frac{\partial}{\partial x}$, however, this conclusion requires that at the boundaries of the field of the determination $[a,b]$, $x=a$ and $x=b$, functions g^{*} and f tends to zero. This condition is satisfied because in the asymptotics[14] real wave functions vanish. However, at Planck scales, at the birth of the Universe, this condition is hardly feasible, since the radius of curvature of the Universe at birth is negligibly small and a mathematical apparatus of operators on a Riemannian manifold is needed here. Since the construction of hermitian operators requires an integration procedure, this operation, in the case of a curved Riemannian spacetime, must be mathematically correctly defined.

And, thirdly, with the discreteness of spacetime, Lorentz invariance is violated and this circumstance requires the imposition of new requirements on the field equations, taking into account the fact that in the limiting case of transition from discreteness to continuity, Lorentz invariance must be restored. Also, Lorentz invariance should be preserved in the case of averaging the field equations on a

[14]At the distances of $r\to\infty$, all real wave functions asymptotically tends to zero.

discrete spatial lattice, but in this case the problem of averaging scale arises. If the scale invariance of the averaged equations is required, then this should impose restrictions on the field equations.

Therefore, quantum mechanics, on the Planck scales that characterize the early Universe, should be built on deeper principles with an adequate mathematical apparatus.

Here, in addition to the above approaches, the path proposed by Heisenberg [42] may also be interesting. Non-relativistic quantum mechanics is built on an infinite-dimensional flat Hilbert space. However, it is interesting to construct quantum mechanics on a curved Hilbert space.

The indefinite metric of such a space is built on the basis of the fundamental vectors of the state of the Hilbert space Φ_i, as $g_{ik} = \langle \Phi_i | \Phi_k \rangle$, where $g_{ik} = g_{ki}{}^*$. Then for the arbitrary vector of state $|\psi\rangle$ we have $|\psi\rangle = \sum_l |\Phi_l\rangle \langle \psi^l|$, where $\langle \psi^l|$ are contravariant components of the vector $|\psi\rangle$. The invariant in such a space will be the full probability of the process under consideration. The possibility of constructing a theory of a gravitational field determined by the geometrical characteristics of a curved Hilbert manifold is a matter of separate research.

Here we would like to mention the following features in the construction of a generally covariant quantum theory of the gravitational field. So, when we define a function, it acts on a set of numbers, translating one numeric set into another numeric set, but both numeric sets belong to the same numeric space, for example:

$$\forall x \in X, \exists y \in Y \,|\, x \xrightarrow{f} y, y = f(x), X, Y \subset R \tag{61}$$

If we talk about the operators defined by us, then they map some functions in the functional space to others defined on the same numerical space, as it is shown in **Figure 5**. For example, the operator $\hat{L} = \frac{\partial}{\partial x}$ transforms a differentiable function on a set of real numbers into another function, as defined on a set of the field of real numbers and being its derivative.

In the general relativistic case, we must well define operators belongs in different spaces tangent to the nearby points on the manifold under consideration. In this case, the differential operator should be defined as: $\hat{P}(x, T_x)$, where T_x is the space tangent to the considered manifold at the point of the manifold *x*. At the same time, the differential operators defined on the manifold must continuously pass into each other: $\hat{P}(x, T_x) \to \hat{P}'(x', T'_{x'})$, $x \to x'$. As an example, let's consider a differentiation operator $\hat{P}(x_\alpha) = \boldsymbol{e}_\mu(x_\alpha) \frac{\partial}{\partial x_\mu}$. Then, at $x_\alpha \to x'_\alpha = x_\alpha + \delta x_\alpha$, will be obtained an operator, defined as:

$$\begin{aligned} &\hat{P}(x_\alpha) = \boldsymbol{e}_\mu(x_\alpha) \frac{\partial}{\partial x_\mu} \\ &\to \hat{P}'(x'_\alpha) = \boldsymbol{e}'_\mu(x_\alpha + \delta x_\alpha) \frac{\partial}{\partial x'_\mu} \approx \left(\boldsymbol{e}_\mu(x_\alpha) + \frac{\partial \boldsymbol{e}_\mu}{\partial x_\alpha} \delta x_\alpha + \cdots \right) \frac{\partial x_\beta}{\partial x'_\mu} \frac{\partial}{\partial x_\beta}, \end{aligned} \tag{62}$$

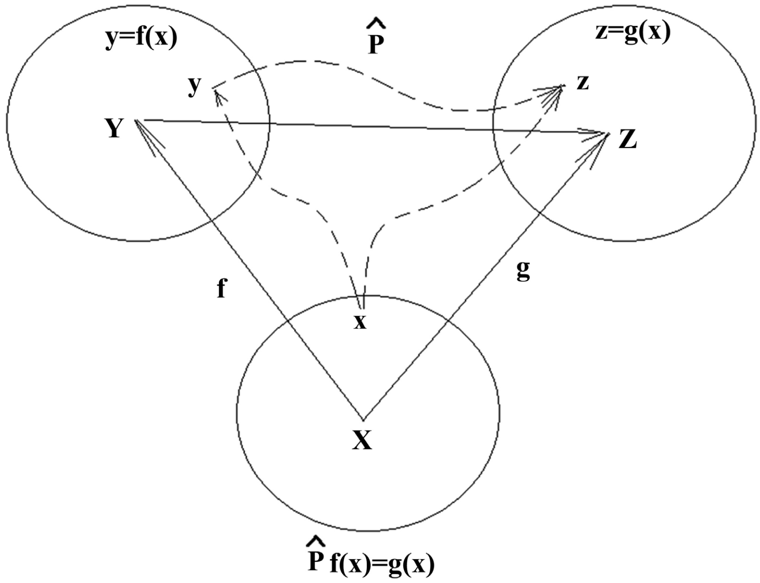

Figure 5. Explanation of operator mapping $f(x) \xrightarrow{\hat{P}} g(x)$.

where vector $\boldsymbol{e}_\mu(x_\alpha + \delta x_\alpha)$ is defined in the tangent to the considering manifold space, defined by the point $x'_\alpha = x_\alpha + \delta x_\alpha$ of the manifold. But in this case, we again come not to a quantum general relativistic description of the gravitational field, but to a general relativistic description of physical fields in a curved spacetime given by this variety. In order for our description to have the necessary completeness, it is necessary to immerse a curved 4-dimensional spacetime into a flat multidimensional space, for which relativistic quantum mechanics in the spirit of Dirac works, for the multidimensional case. The simplest way is to rewrite Dirac Equation (55) in a more general form, taking into the account the gravitation field is following:

$$\left(i\hbar c\gamma^\mu \frac{\partial}{\partial x^\mu} - mc^2\right)\psi_\sigma(x) + i\hbar c\gamma^\mu \Gamma^\alpha_{\mu\sigma}\psi_\alpha(x) = 0, \tag{63}$$

which in the limit of the flat spacetime transform into the usual Dirac equation. But it is done by formal way and we needs in a more deep theory based on the new ideas.

Among the approaches we listed at the beginning of the article, the most mathematically developed are the theory of "strings" and the theory of quantum "loop" gravity. Nevertheless, both of these directions have proved to be far from their final completion. In the theory of "strings", the predictions of supersymmetry, the mass spectrum of elementary particles did not come true; problems arose with the description of the decay of a proton and the calculation of its spin. "Loop" gravity looks preferable, however, it is not without disadvantages of using phenomenological parameters [43]; the conclusion about the violation of Lorentz invariance as a consequence of the theory; mathematical problems related to the correct derivation of the Wheeler-Dewitt equation, which exists in several versions. However, cosmology has encountered not only the problems of the left side of the GRT equations, which are characterized by the absence, at the moment, of a quantum theory of spacetime. We do not have an adequate quantum theory of matter for the right side of the Einstein equation in the field of Planck quantities. If we consider the existing theories for quantization of the gravitational field, then at the level of the fundamental, primary objects of string theory and loop gravity, we have, respectively, strings and "atoms" of space, which consider space and time in a discrete way. In our approach, the fundamental object of the theory is the triad [4] [44], consisting from the three fundamental Beginnings: active-A, passive-P and neutral-N, connected between each other by "braces" S_{NA}, S_{AP}, S_{PN} are the carriers of interaction between these Beginnings, according the their indexes.

9. Main Conclusions and Results

Proceeding from the concept of worlds, we introduce the universal Principle of 3 Beginnings, from where with necessary come to the existence of a Primary Parent Particle having the structure of Borromeo rings, which was a particle of the Primary Matter that existed before the Big Bang. This particle is the carrier of the cosmic genetic code that works the birth and formation of the observable Universe.

It was derived generalized Bohr-Sommerfeld quantization rule, from where follows possibility to the existence of the new structures in higher dimensional spaces in comparison with Lower dimensions.

Geometrical properties of the multidimensional O_{SP}-Space and conditions of the embeddability of the low dimensional space into the high dimensional space were discussed.

For the Primary Parent Particle, it was calculated dipole vibrations energy $E_O \approx 3.75\times 10^{22}$ MeV, what is beyond the capabilities of modern accelerators.

It was obtained the condition of the rings rupture and its connection with the string tension and contribution in to the multidimensional Gauss-Bonnet space generalized gravitational field equations (for dimensions $n \geq 5$).

At different distribution of the profile functions of the DM densities was obtained distribution of the gravitation potentials and velocities for the rotation curves.

The problem for the DM radiation was investigated and expressions for the intensity of the radiation expressed throughout the probabilities of internal transitions in the DM particles were derived.

For the DE, an expression for the constant acceleration in any point of the Universe induced by the oscillation of the external shell was obtained.

In the frame of standard quantum mechanics, an equation for triads structure description was formulated with taking into the account isospin-like formalism.

The duality principle was criticized as a principle which can be applicable only in the case of linear theories as it also was pointed out in [37].

The problem of the quantum gravity was discussed and was made a conclusion: how usual definition of the hermitian operators of quantum mechanics should be reformulated in the curved spacetime.

Conflicts of Interest

The authors declare no conflict interest regarding the publication of this paper.

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