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Kinetic Modeling of an Opinion Model on Social Networks

Hanxiao He

School of Mathematics, East China University of Science and Technology, Shanghai, China
Email: y30201298@mail.ecust.edu.cn

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Abstract

It is commonly accepted that, on social networks, the opinion of the agents with a higher connectivity, *i.e.*, a larger number of followers, results in more convincing than that of the agents with a lower number of followers. By kinetic modeling approach, a kinetic model of opinion formation on social networks is derived, in which the distribution function depends on both the opinion and the connectivity of the agents. The opinion exchange process is governed by a Sznajd type model with three opinions, ±1, 0, and the social network is represented statistically with connectivity denoting the number of contacts of a given individual. The asymptotic mean opinion of a social network is determined in terms of the initial opinion and the connectivity of the agents.

Keywords

Kinetic Model, Social Network, Connectivity, Mean Opinion

1. Opinions on Social Networks

With the widespread of multimedia and internet in the last several decades, a large amount of social media emerge such that people are ubiquitously connected with friends, family or peers, through all kinds of social networks, which increasingly shape or influence their opinions and judgments on many topics such as consumer products, politics, lifestyles or celebrities, thus the problem of modeling how users update opinions on social networks becomes an important research topic in Sociodynamics [1], since understanding how users in social networks update opinions based on their neighbors' opinions, as well as the emergence of aggregate social trends or global opinion structure from the opinion update of each individual, are important in the context of virtual marketing, information dissemination or targeting messages to users in social networks.

To investigate opinions on social networks, one needs to prescribe the network mathematically. There are basically two types of models, *i.e.*, graph based models and statistically structured models. For graph models of social networks, one can see for example [2], in which some collective dynamics of "small-world" networks were studied.

The statistically structured models for social networks are appealing [3], since that in some applications, the number of nodes and links of a social network is so large that a detailed description by means of classical graphs would be largely unfeasible. Later on, various mathematical approaches are developed for theoretical investigations of opinion formation and/or wealth distribution on social networks, see for example [4] [5] [6] [7] [8], to name just a few, ranging from microscopic models, mesoscopic models, to corresponding macroscopic limits. Among these models, the mesoscopic kinetic models are important, especially for dealing with large scale social networks.

Next, we notice that the key factor of a network is the connectivity. The connectivity of an agent represents one's number of contacts/followers. The opinion of the agents with a higher connectivity, *i.e.*, a larger number of followers, results in more convincing than that of the agents with a lower number of followers [9]. Note that agents with high number of contacts/followers are sometimes identified as *influencers*. Influencers are *not* necessarily recognized as leaders, since they obey the same mechanism of opinion sharing as any other agents but do not operate in a coordinated manner. For opinion dynamics with leaders, we refer to [10] and references therein.

In the present study, we consider a kinetic model of an opinion model with three opinions ± 1, 0, allowing indecisive individuals who cannot make a clear choice between ± 1 in the study, which extends the result in [11] where only binary opinions, ± 1, were considered. Note that the kinetic modeling of the Sznajd model with three opinions is studied in [12], but without considering connectivity of the social network. By the kinetic modeling approach, we will derive the asymptotic mean opinion of a social network in terms of the initial opinion and the connectivity of the agents. The plan of the paper is that, in the next section, a discretized model, say, Sznajd model [13], will be introduced, and two revisitations of Sznajd model provide the interaction rules for the further derivation of the Boltzmann-type kinetic equations. In section 3, the kinetic model will be derived. Next, the large time behaviour of the mean opinions of the model is studied in Section 4. Finally, some concluding remarks and research perspectives are given in Section 5.

2. The Sznajd Model

As one important opinion model in computational sociophysics, we mention the important Sznajd model [13], which is considered a "Ising model of opinion dynamics". The dynamics are based on the principle that if two agents share the same opinion, they may succeed in convincing their acquaintances of their opinion ("united we stand, divided we fall"). By using this model, one tries to model the interaction of a system of agents distributed on a 1-dimensional string, with the opinion variables taking only two extreme opinions ±1 denoting positive or negative opinions, mimicking the up/down in the context of Ising model.

Note that the original Sznajd model was introduced as the binary-opinion model, which is very natural for statistical physicists. However, it was later generalized to accommodate three or larger number of discrete states [14] [15] [16] [17], or even a continuous set of opinions [18] [19]. Moreover, the original Sznajd model on a string has also been generalized and studied on square lattice, randomly diluted square lattice, triangular lattice, as well as on various networks. See [20] for more on a review of the Sznajd model. The Sznajd type model for opinion exchange will provide the basic interaction rules for the kinetic modeling of the opinion dynamics on social networks.

Motivated by the study of Loy-Raviola-Tosin [11], in this paper, we consider the kinetic modeling of a Sznajd type opinion model on social networks, in which the distribution function depends on both the *opinion* and the *connectivity* of the agents. Note that unlike classical modeling of opinion formation in a multi-agent society [21] [22] [23] [24] [25] where agents are considered indistinguishable, the *connectivity* of the agents in a social network is intrinsically considered in the kinetic modeling in [11].

Let $W_t, W_t^*, W_t^{**} \in \{-1, 0, 1\}$ be the opinions of three agents in the social network at time *t*. Assume that the connectivity described by the number of followers of a given individual changes more slowly than opinion, so it can be taken as a constant in time. With the notations used in [26], which are some revisitations of the Sznajd model [13], we consider the following two interaction schemes:

1) *The two-against-one model.* This model assumes that the third agent takes the opinion of the first two agents if the latter have same opinions. Otherwise, no interaction occurs. This model can be represented as

$$\begin{cases} W_{t+\Delta t} = W_t, \\ W_{t+\Delta t}^* = W_t^*, \\ W_{t+\Delta t}^{**} = (1-\Theta) W_t^{**} + \Theta W_t, \end{cases} \tag{2.1}$$

where $W_t, W_t^* \in \{-1, 0, 1\}$ are the opinions of the first two agents, $\Delta t > 0$ is the time interval in which the interaction may happen, $\Theta \in \{0, 1\}$ is a Bernoulli random variable which describes whether the interaction happens or not, with the probability

$$P(\Theta = 1) \triangleq \mu \chi\left(W_t = W_t^*\right) C C^* \Delta t,$$

in which $\mu > 0$ is a constant, $\chi(\cdot)$ denotes the characteristic function, and C, C^* are the connectivities of the first two agents. This formula indicates that the interaction takes place only if $W_t = W_t^*$, with the probability proportional to the connectivities of first two individuals, C, C^*, and the time interval Δt. In order to obtain a meaningful probability, the value of probability cannot exceed the extreme value 1, thus there is a constraint on Δt that

$$\Delta t \le \frac{1}{\mu \max\left\{CC^*\right\}},$$

where the maximum is taken over the connectivities of all agents.

2) *The Ochrombel-type simplification.* Instead of assuming that the interactions take place under the condition that two individuals have the same opinions, it is assumed that every agent can affect each of their neighbors' opinion, then the interaction rule becomes

$$\begin{cases} W_{t+\Delta t} = W_t, \\ W^*_{t+\Delta t} = (1-\Theta)W^*_t + \Theta W_t, \end{cases} \tag{2.2}$$

where the probability for occurrence of interaction, $\Theta = 1$, is given by

$$P(\Theta = 1) \triangleq \mu C \Delta t,$$

that is, the probability is proportional to the connectivity of the first individual, *C*, and the time interval Δt. The constraint for Δt becomes

$$\Delta t \le \frac{1}{\mu \max\left\{C\right\}},$$

where the maximum is also taken over the connectivities of all agents.

3. Kinetic Modeling

We consider a social network with large number of agents, each agent is associated with an opinion-connectivity pair of states (w,c), $w \in \{-1,0,1\}$ is a discrete opinion variable and $c \in \mathbb{R}_+$ is a continuous non-negative connectivity variable: Note that the opinion variable w takes values ±1 denoting conventionally two opposite opinions in the same spirit as the original Sznajd model, the third opinion $w = 0$ allowing for indecisive agents who cannot make a clear choice between ±1 and prefer therefore to abstain. The connectivity c is assumed to be a representative measure of the contacts/followers of a given agent, which is considered a continuous variable in accordance with the reference literature on the connectivity distribution of social networks. For simplicity, we take the statistical distribution of the connectivity of the agents constant in time as in [11], which corresponds to the assumption that the connectivity distribution possibly evolves over a timescale much slower than that of the opinion changes such that it can be considered as an almost stationary background.

Let $f(w,c,t)$ be the distribution function of the opinion-connectivity pair (w,c) at time $t \ge 0$. Due to the discreteness of *w*, the distribution function *f* can be represented as

$$f(w,c,t) = p(c,t)\delta(w-1) + q(c,t)\delta(w+1) + r(c,t)\delta(w), \tag{3.1}$$

where $\delta(w-w_0)$ denotes the Dirac delta function centered at $w = w_0$, and the coefficients $p(c,t), q(c,t), r(c,t) > 0$ are the percentage of individuals expressing opinion $w = \pm 1, 0$, respectively. Moreover,

$p(\cdot,t), q(\cdot,t), r(\cdot,t) \in L^1(\mathbb{R}_+)$ for all $t \geq 0$ is assumed, together with the normalisation condition

$$\int_{\mathbb{R}_+}\int_{\{-1,0,1\}} f(w,c,t)\,\mathrm{d}w\mathrm{d}c = 1 \quad \forall\, t \geq 0,$$

meaning that there is a constant-in-time number of agents and connections of the social network.

1) The *two-against-one model* describes multiple interactions, *i.e.* the interactions among three agents. So we consider multiple-interaction Boltzmann-type equation following the derivation process in [11] [27]. Let $\phi = \phi(w,c): \{-1,0,1\} \times \mathbb{R}_+ \to \mathbb{R}$ be an arbitrary observable quantity (test function). Taking the two-against-one model (2.1) as the interaction rule, the Boltzmann-type equation reads

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}_+}\int_{\{-1,0,1\}} \phi(w,c) f(w,c,t)\,\mathrm{d}w\mathrm{d}c \\
&= \frac{1}{3}\int_{\mathbb{R}_+^3}\int_{\{-1,0,1\}^3} B(w,c,w_*,c_*)\big(\phi(w,c_{**}) - \phi(w_{**},c_{**})\big) \\
&\quad \times f(w,c,t) f(w_*,c_*,t) f(w_{**},c_{**},t)\,\mathrm{d}w\mathrm{d}w_*\mathrm{d}w_{**}\mathrm{d}c\mathrm{d}c_*\mathrm{d}c_{**},
\end{aligned} \tag{3.2}$$

with collision kernel

$$B(w,c,w_*,c_*) \triangleq \mu\chi(w = w_*)cc_*.$$

2) The *Ochrombel-type simplification.* Similarly, the Boltzmann-type equation with interaction rule corresponding to (2.2) reads [11]

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}_+}\int_{\{-1,0,1\}} \phi(w,c) f(w,c,t)\,\mathrm{d}w\mathrm{d}c \\
&= \frac{1}{2}\int_{\mathbb{R}_+^2}\int_{\{-1,0,1\}^2} B(c)\big(\phi(w,c_*) - \phi(w_*,c_*)\big) f(w,c,t) f(w_*,c_*,t)\,\mathrm{d}w\mathrm{d}w_*\mathrm{d}c\mathrm{d}c_*,
\end{aligned} \tag{3.3}$$

in which the collision kernel takes

$$B(c) \triangleq \mu c.$$

4. Behaviour of the Mean Opinion

To study the opinion evolution process in the kinetic models (3.2) or (3.3), one defines the mean opinion of the system

$$m_w(t) \triangleq \int_{\mathbb{R}_+}\int_{\{-1,0,1\}} wf(w,c,t)\,\mathrm{d}w\mathrm{d}c = \int_{\mathbb{R}_+}\big(p(c,t) - q(c,t)\big)\mathrm{d}c.$$

Here f is taken with the form (3.1). We will mainly concern the evolution of the mean opinion m_w of the system. For later use, we also denote

$$\begin{aligned}
&m_{wc}(t) \triangleq \int_{\mathbb{R}_+}\int_{\{-1,0,1\}} wcf(w,c,t)\,\mathrm{d}w\mathrm{d}c = \int_{\mathbb{R}_+}\big(cp(c,t) - cq(c,t)\big)\mathrm{d}c, \\
&m_{w^2c}(t) \triangleq \int_{\mathbb{R}_+}\int_{\{-1,0,1\}} w^2cf(w,c,t)\,\mathrm{d}w\mathrm{d}c = \int_{\mathbb{R}_+}\big(cp(c,t) + cq(c,t)\big)\mathrm{d}c, \\
&g(c,t) \triangleq \int_{\{-1,0,1\}} f(w,c,t)\,\mathrm{d}w = p(c,t) + r(c,t) + q(c,t).
\end{aligned} \tag{4.1}$$

Note that we have assumed the number of followers of an agent changes more slowly over time than their opinions, the marginal distribution g is constant in time, that is, $g(c,t)=g(c)$ for all $t\geq 0$, and the mean connectivity m_c is a constant

$$m_c \triangleq \int_{\mathbb{R}_+} cg(c)\mathrm{d}c = \int_{\mathbb{R}_+} c\left(p(c,t)+r(c,t)+q(c,t)\right)\mathrm{d}c.$$

4.1. The Two-Against-One Model

For the two-against-one model, let $\phi(w,c)=w$ in the Boltzmann-type Equation (3.2), we have

$$\dot{m}_w = \frac{\mu}{3}\left[m_{wc}m_{w^2c} + m_w\left(-\frac{1}{2}m_{wc}^2 - \frac{3}{2}m_{w^2c}^2 + 2m_c m_{w^2c} - m_c^2\right)\right]. \tag{4.2}$$

in which

$$\begin{aligned} \int_{\mathbb{R}_+} cp(c,t)\mathrm{d}c &= \frac{m_{wc}+m_{w^2c}}{2}, \\ \int_{\mathbb{R}_+} cr(c,t)\mathrm{d}c &= \frac{m_{w^2c}-m_{wc}}{2}, \\ \int_{\mathbb{R}_+} cq(c,t)\mathrm{d}c &= m_c - m_{w^2c}. \end{aligned} \tag{4.3}$$

Note that m_{wc} and m_{w^2c} are not constants. To study the behavior of m_w, we need to obtain the evolution of m_{wc} and m_{w^2c}. For this purpose, we take $\phi(w,c)=wc$ and $\phi(w,c)=w^2c$ in the Boltzmann-type Equation (3.2) to get

$$\begin{aligned} \dot{m}_{wc} = \frac{\mu}{3}\Bigg[&(m_c - m_{wc})\left(\int_{\mathbb{R}_+} cp(c,t)\mathrm{d}c\right)^2 - (m_c + m_{wc})\left(\int_{\mathbb{R}_+} cq(c,t)\mathrm{d}c\right)^2 \\ &- m_{wc}\left(\int_{\mathbb{R}_+} cr(c,t)\mathrm{d}c\right)^2\Bigg], \\ \dot{m}_{w^2c} = \frac{\mu}{3}\Bigg[&(m_c - m_{w^2c})\left(\int_{\mathbb{R}_+} cp(c,t)\mathrm{d}c\right)^2 + (m_c + m_{w^2c})\left(\int_{\mathbb{R}_+} cq(c,t)\mathrm{d}c\right)^2 \\ &- m_{w^2c}\left(\int_{\mathbb{R}_+} cr(c,t)\mathrm{d}c\right)^2\Bigg]. \end{aligned}$$

The two equations can be simplified with the help of (4.3). Together with (4.2), we get

$$\begin{cases} \dot{m}_w = \frac{\mu}{3}\left[m_{wc}m_{w^2c} + m_w\left(-\frac{1}{2}m_{wc}^2 - \frac{3}{2}m_{w^2c}^2 + 2m_c m_{w^2c} - m_c^2\right)\right], \\ \dot{m}_{wc} = \frac{\mu}{3}\left[-m_{wc}\left(\frac{1}{2}m_{wc}^2 + \frac{3}{2}m_{w^2c}^2 - 3m_c m_{w^2c} + m_c^2\right)\right], \\ \dot{m}_{w^2c} = \frac{\mu}{3}\left[\frac{1}{2}m_c m_{wc}^2 - m_{w^2c}\left(\frac{1}{2}m_{wc}^2 + \frac{3}{2}m_{w^2c}^2 - \frac{5}{2}m_c m_{w^2c} + m_c^2\right)\right]. \end{cases} \tag{4.4}$$

Note that this is a closed system of ordinary differential equations with unknowns (m_w, m_{wc}, m_{w^2c}). To analyze the asymptotic behaviour of the system, we denote

$$\boldsymbol{Y} = (y_1, y_2, y_3)^{\mathrm{T}} \triangleq (m_w, m_{wc}, m_{w^2c})^{\mathrm{T}}, \quad \boldsymbol{F} = (f_1, f_2, f_3)^{\mathrm{T}},$$

with

$$\begin{cases} f_1 = \dfrac{\mu}{3}\left[y_2y_3 + y_1\left(-\dfrac{1}{2}y_2^2 - \dfrac{3}{2}y_3^2 + 2m_cy_3 - m_c^2\right)\right], \\ f_2 = \dfrac{\mu}{3}\left[-y_2\left(\dfrac{1}{2}y_2^2 + \dfrac{3}{2}y_3^2 - 3m_cy_3 + m_c^2\right)\right], \\ f_3 = \dfrac{\mu}{3}\left[\dfrac{1}{2}m_cy_2^2 - y_3\left(\dfrac{1}{2}y_2^2 + \dfrac{3}{2}y_3^2 - \dfrac{5}{2}m_cy_3 + m_c^2\right)\right], \end{cases}$$

then the system (4.4) can be rewritten in the form

$$\frac{\mathrm{d}\boldsymbol{Y}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{Y}). \tag{4.5}$$

This dynamic system has seven rest points:

$$P_1 = (1, m_c, m_c),\ P_2 = (-1, -m_c, m_c),\ P_3 = (0,0,0),$$

and

$$P_4 = \left(\frac{1}{2}, \frac{1}{2}m_c, \frac{1}{2}m_c\right), P_5 = \left(-\frac{1}{2}, -\frac{1}{2}m_c, \frac{1}{2}m_c\right), P_6 = (0,0,m_c), P_7 = \left(0,0,\frac{2}{3}m_c\right).$$

The stability of an equivalent point of a differential system depends on the sign of the eigenvalues of the Jacobian matrix. For this, we compute the Jacobian matrix of $\boldsymbol{F}$:

$$\boldsymbol{J} = \begin{pmatrix} \dfrac{\partial f_1}{\partial y_1} & \dfrac{\partial f_1}{\partial y_2} & \dfrac{\partial f_1}{\partial y_3} \\ \dfrac{\partial f_2}{\partial y_1} & \dfrac{\partial f_2}{\partial y_2} & \dfrac{\partial f_2}{\partial y_3} \\ \dfrac{\partial f_3}{\partial y_1} & \dfrac{\partial f_3}{\partial y_2} & \dfrac{\partial f_3}{\partial y_3} \end{pmatrix} \tag{4.6}$$

where

$$\begin{aligned} &\frac{\partial f_1}{\partial y_1} = \frac{\mu}{3}\left[-\frac{1}{2}y_2^2 - \frac{3}{2}y_3^2 + 2m_cy_3 - m_c^2\right], \\ &\frac{\partial f_1}{\partial y_2} = \frac{\mu}{3}[y_3 - y_1y_2],\ \frac{\partial f_1}{\partial y_3} = \frac{\mu}{3}[y_2 - 3y_1y_3 + 2m_cy_1], \\ &\frac{\partial f_2}{\partial y_1} = 0,\ \frac{\partial f_2}{\partial y_2} = \frac{\mu}{3}\left[-\frac{3}{2}y_2^2 - \frac{3}{2}y_3^2 + 3m_cy_3 - m_c^2\right],\ \frac{\partial f_2}{\partial y_3} = \frac{\mu}{3}[-3y_2y_3 + 3m_cy_2], \\ &\frac{\partial f_3}{\partial y_1} = 0,\ \frac{\partial f_3}{\partial y_2} = \frac{\mu}{3}[m_cy_2 - y_2y_3],\ \frac{\partial f_3}{\partial y_3} = \frac{\mu}{3}\left[-\frac{1}{2}y_2^2 - \frac{9}{2}y_3^2 + 5m_cy_3 - m_c^2\right]. \end{aligned}$$

Now we study the Jacobian matrix at each equivalent point $P_i, i = 1, \cdots, 7$. For $P_1 = (1, m_c, m_c)^{\mathrm{T}}$, one computes

$$\boldsymbol{J} = \begin{pmatrix} -\dfrac{\mu}{3}m_c^2 & 0 & 0 \\ 0 & -\dfrac{\mu}{3}m_c^2 & 0 \\ 0 & 0 & -\dfrac{\mu}{3}m_c^2 \end{pmatrix}.$$

The characteristic equation is

$$|\boldsymbol{J}-\lambda \boldsymbol{I}|=\left(-\frac{\mu}{3}m_c^2-\lambda\right)^3=0.$$

The three eigenvalues

$$\lambda_1=\lambda_2=\lambda_3=-\frac{\mu}{3}m_c^2$$

are all negative, then the equivalent point $(1,m_c,m_c)$ is asymptotically stable. Similarly, the eigenvalues for $P_2=(-1,-m_c,m_c)^{\mathrm{T}}$ are $\lambda_1=\lambda_2=\lambda_3=-\frac{\mu}{3}m_c^2$, the eigenvalues for $P_3=(0,0,0)^{\mathrm{T}}$ are $\lambda_1=\lambda_2=\lambda_3=-\frac{\mu}{3}m_c^2$, they are asymptotically stable.

For $P_4=\left(\frac{1}{2},\frac{1}{2}m_c,\frac{1}{2}m_c\right)^{\mathrm{T}}$, the three eigenvalues are

$$\lambda_1=-\frac{\mu}{6}m_c^2,\quad \lambda_2=-\frac{\mu}{6}m_c^2,\quad \lambda_3=\frac{\mu}{6}m_c^2.$$

Note that the third eigenvalue λ_3 is positive, then the rest point $\left(\frac{1}{2},\frac{1}{2}m_c,\frac{1}{2}m_c\right)$ is not asymptotically stable. The eigenvalues for $P_5=\left(-\frac{1}{2},-\frac{1}{2}m_c,\frac{1}{2}m_c\right)^{\mathrm{T}}$ are

$$\lambda_1=-\frac{\mu}{6}m_c^2,\quad \lambda_2=-\frac{\mu}{6}m_c^2,\quad \lambda_3=\frac{\mu}{6}m_c^2.$$

The eigenvalues for $P_6=(0,0,m_c)^{\mathrm{T}}$ are

$$\lambda_1=-\frac{\mu}{6}m_c^2,\quad \lambda_2=-\frac{\mu}{6}m_c^2,\quad \lambda_3=\frac{\mu}{6}m_c^2.$$

The eigenvalues for $P_7=\left(0,0,\frac{2}{3}m_c\right)^{\mathrm{T}}$ are

$$\lambda_1=-\frac{\mu}{9}m_c^2,\quad \lambda_2=\frac{\mu}{9}m_c^2,\quad \lambda_3=\frac{\mu}{9}m_c^2.$$

In summary, the three stable equivalent points of (4.4) are

$$P_1=(1,m_c,m_c),\ P_2=(-1,-m_c,m_c)\ \text{and}\ P_3=(0,0,0),$$

which correspond to three possible asymptotically limits of the mean opinion, ±1 or 0, respectively. The final asymptotically limit of the mean opinion is determined by the competition of the *connectivity-weighted percentage* of agents expressing the initial opinion 1, −1, 0, that is, which one of the three equivalent points will be the asymptotically limit depends on the initial opinion-connectivity distribution located in the attraction domain of each rest point.

4.2. The Ochrombel-Type Simplification

For the Ochrombel-type simplification, let $\phi(w,c)=w$ in the Boltzmann-type

Equation (3.3), we get the following equation after some calculations:

$$\dot{m}_w = \frac{\mu}{2}\left(m_{wc} - m_c m_w\right). \tag{4.7}$$

This equation is coupled to m_{wc} and m_c. Recall that m_c is assumed to be constant in time. Taking $\phi(w,c) = wc$ in (3.3), one gets the equation for m_{wc} as

$$\dot{m}_{wc} = \frac{1}{2}\int_{\mathbb{R}_+^2}\int_{\{-1,0,1\}^2} \mu c\left(wc_* - w_* c_*\right) f\left(w,c,t\right) f\left(w_*,c_*,t\right) \mathrm{d}w\mathrm{d}w_*\mathrm{d}c\mathrm{d}c_* = 0,$$

which implies that m_{wc} is also constant in time, *i.e.* $m_{wc} = m_{wc}^0$, where m_{wc}^0 is the initial value of m_{wc}, defined with a given initial opinion-connectivity distribution $f^0(w,c)$ of the social network. Then (4.7) becomes

$$\dot{m}_w = \frac{\mu}{2}\left(m_{wc}^0 - m_c m_w\right),$$

which is solved with the initial mean opinion m_w^0 that

$$m_w = \mathrm{e}^{-\frac{\mu}{2}m_c t} m_w^0 + \frac{m_{wc}^0}{m_c}\left(1 - \mathrm{e}^{-\frac{\mu}{2}m_c t}\right), \tag{4.8}$$

this asymptomatically converges to the *connectivity-weighted* mean initial opinion:

$$m_w \to m_w^\infty \triangleq \frac{m_{wc}^0}{m_c} = \frac{\int_{\mathbb{R}_+}\int_{\{-1,0,1\}} wcf^0\left(w,c\right)\mathrm{d}w\mathrm{d}c}{\int_{\mathbb{R}_+}\int_{\{-1,0,1\}} cf^0\left(w,c\right)\mathrm{d}w\mathrm{d}c} \quad \text{as } t \to \infty. \tag{4.9}$$

5. Conclusions and Perspectives

In this paper, a kinetic model of opinion formation on social networks is derived, in which the distribution function depends on both the opinion and the connectivity of the agents. The opinion exchange process is governed by a Sznajd type model considering three opinions, ±1, 0, and the social network is represented statistically with connectivity denoting the number of contacts of a given individual. Two revisitations of the Sznajd model, the two-against-one model and its Ochrombel-type simplification, are considered the microscopic interaction rules. The large time behaviour of the mean opinions of the model is studied by using the macroscopic system of differential equations derived from the Boltzmann-type kinetic equations.

For the case of the two-against-one model, three possible asymptotically limits of the mean opinion, ±1 or 0, are derived. The final asymptotically limit of the mean opinion is determined by the competition of the *connectivity-weighted percentage* of agents expressing the initial opinion 1, −1, 0.

For the case of the Ochrombel-type simplification, the mean opinion converges asymptotically to the *connectivity-weighted* mean initial opinion as in (4.9).

The results demonstrated the fact in social networks, that the opinion of the agents with a higher connectivity, *i.e.*, with a larger number of contacts/followers, the so-called influencers, results more convincing than that of the agents with a lower number of followers.

Further extension of the proposed model may include a kinetic description of opinion distribution of agents on *evolving network*, that is, the case in which the connectivity is not constant in time. This may be resolved by combining an opinion update based on interactions between agents, together with a dynamic creation and removal process of new connections [5]. Since only three opinions are considered in the present study, the model may be extended to the case of more number of discrete states or even a continuous set of opinions.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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