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Pricing European Options Based on a Logarithmic Truncated *t*-Distribution

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Abstract

The *t*-distribution has a "fat tail" feature, which is more suitable than the normal probability density function to describe the distribution characteristics of return on assets. The difficulty of using *t*-distribution to price European options is that a fat tail can lead to a deviation in one integral required for option pricing. We use a distribution called logarithmic truncated *t*-distribution to price European options. A risk neutral valuation method was used to obtain a European option pricing model with logarithmic truncated *t*-distribution.

Keywords

Option Pricing, Logarithmic Truncated *t*-Distribution, Asset Returns, Risk-Neutral Valuation Approach

1. Introduction

An option is a contract that gives its owner the right to purchase (call option) or sell (put option) a fixed number of assets of a specific common stock at a fixed price (exercise price) on or before a specified date (expiration date) (see Vazquez [1]). Options that can only be exercised on the expiration date are called European options, while American options can be exercised at any time before the expiration date. Option pricing is one of the key issues in financial engineering research. It is not only used for investment decision-making, but also for the entire chain of enterprise operations, such as production pricing, marketing, material and accessory supply, after-sales service, and even for macroeconomic research (see Song [2]).

The European option pricing model is the basis for pricing other financial derivatives. Therefore, providing a reasonable pricing method for European options is of great significance. Davis *et al.* [3] considered the pricing of European options when the underlying asset needs to be charged transaction fees during the trading process. They proved that the value function of this problem is a unique viscous solution of a fully nonlinear quasi-variational inequality under different boundary conditions. Wu [4] provided a fuzzy pattern of Black-Scholes formula and the corresponding put-call parity relationship when the related input parameters are fuzzy values. Based on the model proposed by them, financial analysts can be allowed to select any European option price for later usage with an acceptable confidence level of belief. The fuzzy price of European options was also proposed in this study of Wu [5] using the fuzzy set theory concept of "Resolution Identity". Under these assumptions, the European option price will become a fuzzy number. When a continuous-time Markov chain with a limited number of "economic states" modulates the dynamics of the short rate and volatility of the underlying price process, Mamon *et al.* [6] offered closed-form solutions for European option values. Fang *et al.* [7] provided a novel European option pricing method (called COS method) based on the Fourier-cosine series. By utilizing the equivalent martingale measure transformation method, Lin *et al.* [8] considered the pricing issue for European options for two underlying assets with delays Numerical analysis showed that there is a greater chance of a price difference between delayed options and Black Scholes options as the delay lengthens. He *et al.* [9] proposed a new random volatility model. By assuming that the long-term average of volatility in the Heston model is random, a closed pricing formula for European options was derived. Empirical research showed that the current model can generally obtain more accurate option prices than the Heston model. Kil and Kim [10] used the scale version of the double-mean-reverting model and got the closed-form formula for European option pricing. Nzokem [11] studied a novel approach for pricing European option, and he obtained the corresponding pricing model based on the stochastic volatility method. The above literature did not investigate which distribution of underlying securities' returns is most realistic.

In order to price options more accurately, we must construct a reasonable underlying asset price model. Black *et al.* [12] assumed that the asset price follows the standard Geometric Brownian motion model and obtained a partial differential equation, called B-S equation, to describe the option price function. By solving the partial differential equation, they obtained an analytical solution for the option pricing model. Later empirical research showed that return on assets has the characteristics of "peak and fat tail", and the standard Geometric Brownian motion model of asset price return is inconsistent with the actual characteristics of market prices (see Mandelbrot [13], Cassidy *et al.* [14], Fam [15], Zhu and Zhang [16]). To explain the fat tail, Lim *et al.* [17] studied the pricing of currency options using a generalized student distribution. The generalized t-distribution they adopt is a function of a distribution multiplied by a function of the form. Cassidy *et al.* [14] believed that stock returns fit the t-distribution, and then used the t-distribution to price European options.

Although the student distribution has the feature of "fat tail" and is more suitable for asset returns than the normal distribution, the difficulty of pricing options with *t*-distribution is that fat tail will lead to a deviation in an integral required for option pricing. In this paper, we consider a new distribution, called logarithmic truncated *t*-distribution, to describe stock returns, and use risk neutral valuation method (see Hull [18]) to provide a stochastic model for stock prices, and then price European options, and then obtain a European call option pricing model based on logarithmic truncated *t*-distribution. Finally, we use the put-call parity relationship to find the price of European put option.

2. Stock Prices Model

As we all known, there are a close correlation between the value of European option and the underlying stock prices. A reasonable stock prices model is helpful to estimate the option value more precisely. A new probability distribution needs to be defined in order to create the stock price model.

Definition 2.1. *A continuous random variable follows a truncated t-distribution, if it has a probability density function as follows*:

$$f(x,n,\alpha,\beta)=\begin{cases} h(x,n) & \alpha\le x\le\beta, \\ \dfrac{h(\alpha,n)\left[1+(x-\alpha)h(\alpha,n)-\int_\alpha^\beta h(x,n)\mathrm{d}x\right]}{1-\int_\alpha^\beta h(x,n)\mathrm{d}x} & x_1<x<\alpha, \\ \dfrac{h(\beta,n)\left[1+(\beta-x)h(\beta,n)-\int_\alpha^\beta h(x,n)\mathrm{d}x\right]}{1-\int_\alpha^\beta h(x,n)\mathrm{d}x} & \beta<x<x_2, \\ 0 & \text{otherwise,} \end{cases} \tag{2.1}$$

where α, β are constant, $x_1=\alpha-\dfrac{1-\int_\alpha^\beta h(x,n)\mathrm{d}x}{h(\alpha,n)}$, $x_2=\beta+\dfrac{1-\int_\alpha^\beta h(x,n)\mathrm{d}x}{h(\beta,n)}$,

$\Gamma(\cdot)$ is the Gamma function, and $h(x,n)=\dfrac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}}\left(1+\dfrac{x^2}{n}\right)^{-\frac{n+1}{2}}$.

$f(x,n,\alpha,\beta)$ is called the probability density function of X with n degrees of freedom.

For simplification, we can rewrite the above probability density function (2.1) as follows:

$$f(x,n,\alpha,\beta)=\begin{cases} h(x,n) & \alpha\le x\le\beta, \\ \dfrac{h(\alpha,n)}{\alpha-x_1}(x-x_1) & x_1<x<\alpha, \\ \dfrac{h(\beta,n)}{\beta-x_2}(x-x_2) & \beta<x<x_2, \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

Definition 2.2. *If X follows a truncated t-distribution, then* $Y=\mathrm{e}^X$ *is called* *be a logarithmic truncated t-distributed.*

We denote the strike prices by the symbol *K*, and the expiration data by *T*. The stock price at the time *t* is denoted by S_t. Let $v(x,n,\alpha,\beta)=\int_{-\infty}^{x} f(t,n,\alpha,\beta)\mathrm{d}t$ be the distribution function of *X* with *n* degrees of freedom. We suppose the stock price satisfies

$$\frac{\ln\left(\frac{S_T}{S_0}\right)-u(T-t)}{\sqrt{T-t}\sigma}\sim v(x,n,\alpha,\beta),$$

where u is the compound rate of stock return, σ is the volatility of stock return and α,β,n can be obtained by historical data.

W_t is defined as a random variable, with the probability density function $f(x,n,\alpha,\beta)$.

$$\ln\left(\frac{S_T}{S_t}\right)=u(T-t)+\sigma\sqrt{T-t}W_t. \tag{2.3}$$

From Equation (2.3), the stock price at time *T* can be written as

$$S_T=S_t\mathrm{e}^{u(T-t)+\sigma\sqrt{T-t}W_t}. \tag{2.4}$$

3. European Option Pricing Model

Risk neutral valuation approach can be used to price European call option. In a risk neutral world, the returns of all assets equal the interest rate *r*. In the risk neutral world, the stock price can be drove by Equation (3.5), which is similar to Equation (2.4).

$$\tilde{S}_T=S_t\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}W}, \tag{3.5}$$

where $\bar{u}$ is the compound rate of stock return in the risk neutral world, and $\tilde{S}_T$ is the modified stock price at the time *T*. The expected value of $\tilde{S}_T$ in the risk neutral world is given as follows:

$$\begin{aligned}E^*\left(\tilde{S}_T\right)&=E^*\left[S_t\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}W}\right]=S_t\mathrm{e}^{\bar{u}(T-t)}E^*\left(\mathrm{e}^{\sigma\sqrt{T-t}W}\right)\\&=S_t\mathrm{e}^{\bar{u}(T-t)}\int_{-\infty}^{+\infty}f(x,n,\alpha,\beta)\mathrm{e}^{\sigma(T-t)x}\mathrm{d}x\\&=S_t\mathrm{e}^{\bar{u}(T-t)}\left[\int_{\alpha}^{\beta}h(x,n)\mathrm{e}^{\sigma(T-t)x}\mathrm{d}x+\int_{x_1}^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x\right.\\&\quad\left.+\int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\sigma(T-t)x}\mathrm{d}x\right].\end{aligned}$$

The above equation can be simplified to the following form

$$E^*\left(\tilde{S}_T\right)=S_t\mathrm{e}^{\bar{u}(T-t)}\left[A_1+A_2+A_3\right], \tag{3.6}$$

where $A_1=\int_{\alpha}^{\beta}h(x,n)\mathrm{e}^{\sigma(T-t)x}\mathrm{d}x$, $A_2=\int_{x_1}^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x$,

$A_3=\int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\sigma(T-t)x}\mathrm{d}x$.

In the risk neutral world, the following equation is established.

$$E^*\left(\tilde{S}_T\right) = S_t \mathrm{e}^{r(T-t)}. \tag{3.7}$$

Comparing Equation (3.6) with Equation (3.7), we have

$$S_t \mathrm{e}^{r(T-t)} = S_t \mathrm{e}^{\bar{u}(T-t)}\left[A_1 + A_2 + A_3\right]. \tag{3.8}$$

From Equation (3.8), we get

$$\bar{u} = r - \frac{1}{T-t}\ln\left[A_1 + A_2 + A_3\right].$$

Let C_t be the option value at the time *t*. The cost of an European call option, calculated at the time of expiration *T*, is $C_t = E\left[\left(\tilde{S}_T - K\right)^+\right]$. According to the risk-neutral valuation approach, C_t can be written as

$$C_t = \mathrm{e}^{-r(T-t)} E\left[\left(\tilde{S}_T - K\right)^+\right] \tag{3.9}$$

From Equation (3.9), we have

$$\begin{aligned} C_t &= \mathrm{e}^{-r(T-t)} E\left[\left(S_t \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}W} - K\right)^+\right] \\ &= \mathrm{e}^{-r(T-t)} \int\limits_{\tilde{S}_T - K > 0} f\left(x, n, \alpha, \beta\right)\left[S_t \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x} - K\right] \mathrm{d}x \\ &= \mathrm{e}^{-r(T-t)} \int\limits_{\tilde{S}_T - K > 0} f\left(x, n, \alpha, \beta\right) S_t \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x} \mathrm{d}x \\ &\quad - \mathrm{e}^{-r(T-t)} K \int\limits_{\tilde{S}_T - K > 0} f\left(x, n, \alpha, \beta\right) \mathrm{d}x. \end{aligned} \tag{3.10}$$

Considering the inequality $\tilde{S}_T - K > 0$, and using the model given by Equation (3.5), we obtain

$$S_t \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}W} - K > 0 \tag{3.11}$$

Solve the inequality (3.11), so that $W > \dfrac{\ln\left(\dfrac{K}{S_t}\right) - \bar{u}\left(T-t\right)}{\sigma\sqrt{T-t}} = d$. Then Equation (3.10) can be rewritten as

$$C_t = \mathrm{e}^{-r(T-t)}\left[S_t \int_d^{+\infty} f\left(x, n, \alpha, \beta\right) \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x} \mathrm{d}x - K \int_d^{+\infty} f\left(x, n, \alpha, \beta\right) \mathrm{d}x\right]. \tag{3.12}$$

If $d > x_2$, from Equation (3.12), the option value at time *t* is zero, *i.e.*, $C_t = 0$. In fact, when $d > x_2$, we get the inequality $K > S_t \mathrm{e}^{x_2\sigma\sqrt{T-t}+\bar{u}(T-t)}$. That is to say, *K* exceeds $\mathrm{e}^{x_2\sigma\sqrt{T-t}+\bar{u}(T-t)}$ times S_t. Since the value of the European call option is approximated zero as *K* is large enough, $C_t = 0$ accords with the actual financial meaning.

3.1. The Case of $\beta < d < x_2$

By Equation (2) and Equation (12), we get the European call option value at the time *t*, which can be calculated by using the following equation:

$$C_t = \mathrm{e}^{-r(T-t)}\left[S_t \int_d^{+\infty} f\left(x, n, \alpha, \beta\right) \mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x} \mathrm{d}x - K \int_d^{+\infty} f\left(x, n, \alpha, \beta\right) \mathrm{d}x\right]$$

$$= S_t \mathrm{e}^{-r(T-t)} \int_d^{x_2} \frac{h(\beta,n)}{\beta - x_2}(x-x_2)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x$$

$$- K\mathrm{e}^{-r(T-t)} \int_d^{x_2} \frac{h(\beta,n)}{\beta - x_2}(x-x_2)\mathrm{d}x.$$

Simplifying the above formula, we have

$$C_t = \mathrm{e}^{-r(T-t)}\left[S_t B_1 - KB_2\right], \tag{3.13}$$

where $B_1 = \int_d^{x_2} \frac{h(\beta,n)}{\beta - x_2}(x-x_2)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x$, $B_2 = \int_d^{x_2} \frac{h(\beta,n)}{\beta - x_2}(x-x_2)\mathrm{d}x$.

3.2. The Case of $\alpha \le d \le \beta$

If $\alpha \le d \le \beta$, substituting Equation (2.2) into Equation (3.12), we have

$$C_t = S_t\mathrm{e}^{-r(T-t)}\left[\int_d^{\beta} h(x,n)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x + \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x\right]$$

$$- K\mathrm{e}^{-r(T-t)}\left[\int_d^{\beta} h(x,n)\mathrm{d}x + \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{d}x\right]$$

$$= S_t\mathrm{e}^{(\bar{u}-r)(T-t)}\left[\int_d^{\beta} h(x,n)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x + \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x\right]$$

$$- K\mathrm{e}^{-r(T-t)}\left[\int_d^{\beta} h(x,n)\mathrm{d}x + \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{d}x\right].$$

Let $C_1 = \int_d^{\beta} h(x,n)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x$, $C_2 = \int_d^{\beta} h(x,n)\mathrm{d}x$, and $C_3 = \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{d}x$, and we get

$$C_t = S_t\mathrm{e}^{(\bar{u}-r)(T-t)}\left[C_1 + A_3\right] - K\mathrm{e}^{-r(T-t)}\left[C_2 + C_3\right]. \tag{3.14}$$

3.3. The Case of $x_1 < d < \alpha$

If $x_1 < d < \alpha$, the first integral of the Equation (3.12) can be calculated by using the following method

$$\int_d^{+\infty} f(x,n,\alpha,\beta)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x$$

$$= \int_d^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x + \int_{\alpha}^{\beta} h(x,n)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x$$

$$+ \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x$$

$$= \mathrm{e}^{\bar{u}(T-t)}\left[\int_d^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x + \int_{\alpha}^{\beta} h(x,n)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x\right.$$

$$\left.+ \int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)\mathrm{e}^{\sigma\sqrt{T-t}x}\mathrm{d}x\right].$$

Then the first integral of Equation (3.12) becomes

$$\int_d^{+\infty} f(x,n,\alpha,\beta)\mathrm{e}^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}\mathrm{d}x = \mathrm{e}^{\bar{u}(T-t)}\left[D_1 + A_1 + A_3\right], \tag{3.15}$$

where $D_1=\int_d^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)e^{\sigma\sqrt{T-t}x}dx$. The second integral of Equation (3.12) becomes the sum of the three integrals as follows:

$$\int_d^{+\infty}f(x,n,\alpha,\beta)dx=\int_d^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)dx+\int_{\alpha}^{\beta}h(x,n)dx$$
$$+\int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_2}(x-x_2)dx.$$

Then we get

$$\int_d^{+\infty}f(x,n,\alpha,\beta)dx=C_3+D_2+D_3, \tag{3.16}$$

where $D_2=\int_d^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)dx$, $D_3=\int_{\alpha}^{\beta}h(x,n)dx$. Substituting Equation (3.15) and Equation (3.16) into Equation (3.12), we obtain

$$C_t=S_te^{(\bar{u}-r)(T-t)}(D_1+A_1+A_3)-Ke^{-r(T-t)}(C_3+D_2+D_3). \tag{3.17}$$

3.4. The Case of $d<x_1$

When $d<x_1$, the first integral of Equation (3.12) can be simplified as follows:

$$\int_d^{+\infty}f(x,n,\alpha,\beta)e^{\bar{u}(T-t)+\sigma\sqrt{T-t}x}dx$$
$$=e^{\bar{u}(T-t)}\Bigg[\int_{x-1}^{\alpha}\frac{h(\alpha,n)}{\alpha-x_1}(x-x_1)e^{\sigma\sqrt{T-t}x}dx+\int_{\alpha}^{\beta}h(x,n)e^{\sigma\sqrt{T-t}x}dx$$
$$+\int_{\beta}^{x_2}\frac{h(\beta,n)}{\beta-x_1}(x-x_2)e^{\sigma\sqrt{T-t}x}dx\Bigg]$$
$$=e^{\bar{u}(T-t)}[A_1+A_2+A_3].$$

When $d<x_1$, the second integral of Equation (3.12) equals the integral of the probability density function $f(x,n,\alpha,\beta)$ from $-\infty$ to $+\infty$. That is

$$\int_d^{+\infty}f(x,n,\alpha,\beta)dx=\int_{-\infty}^{+\infty}f(x,n,\alpha,\beta)dx=1.$$

Since the two integrals above can be solved, the option value at the time t can be calculated by using the following formula:

$$C_t=S_te^{(\bar{u}-r)(T-t)}(A_1+A_3+A_3)-Ke^{-r(T-t)}. \tag{3.18}$$

3.5. Pricing for European Call and Put Options

When the parameters n,α,β,r,T,t and the volatility of stock prices σ are given, the integrals A_1, A_2, A_3, B_1, B_2, C_1, C_2, C_3, D_1, D_2 and D_3 are easy to be calculated. The expiration date T, the present moment t and the interest rate r always are predictable, and the values of the parameters n, α,β,σ can be obtain by using data mining technology.

According to Equations (3.13) (3.14) (3.17) and (3.18), the value of the European call option at the time t can be given as follows:

$$C_t = \begin{cases} 0 & d \geq x_2, \\ \left(S_t B_1 - K B_2\right) \mathrm{e}^{-r(T-t)} & \beta < d < x_2, \\ \S_t \left(C_1 + A_3\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - K\left(C_2 + C_3\right) \mathrm{e}^{-r(T-t)} & \alpha \leq d \leq \beta, \\ \S_t \left(A_1 + A_3 + D_1\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - K\left(C_3 + D_2 + D_3\right) \mathrm{e}^{-r(T-t)} & X_1 < d < \alpha, \\ S_t \left(A_1 + A_2 + A_3\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - K \mathrm{e}^{-r(T-t)} & d \leq x_1. \end{cases} \quad (3.19)$$

We use put-call parity relation to find the price of the European put option with the same parameters as earlier. The put-call parity relation can be written as

$$P_t = C_t - S_t + \mathrm{e}^{-r(T-t)} K. \quad (3.20)$$

Substitute Equation (3.19) into Equation (3.20), we obtain the value of the European put option at the time t, namely

$$P_t = \begin{cases} \mathrm{e}^{-r(T-t)} K - S_t & d \geq x_2, \\ \left(S_t B_1 - K B_2 + K\right) \mathrm{e}^{-r(T-t)} - S_t & \beta < d < x_2, \\ \S_t \left(C_1 + A_3\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - K\left(C_2 + C_3 - 1\right) \mathrm{e}^{-r(T-t)} - S_t & \alpha \leq d \leq \beta, \\ \S_t \left(A_1 + A_3 + D_1\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - K\left(C_3 + D_2 + D_3 - 1\right) \mathrm{e}^{-r(T-t)} - S_t & X_1 < d < \alpha, \\ S_t \left(A_1 + A_2 + A_3\right) \mathrm{e}^{(\bar{u}-r)(T-t)} - S_t & d \leq x_1. \end{cases}$$

4. Conclusions

Previous empirical tests have shown that returns do not follow a Gaussian distribution. Through data fitting, it can be found that its true distribution has a fat tail feature. In this paper, we assumed that stock returns follow a logarithmic truncated t-distribution and proposed a stock price model. By discussing the parameters, we got the mathematical model of the value of European call option. Finally, we used the put-call parity relationship to find the value of European put option.

However, we only studied the European option when the underlying stock returns follow a logarithmic truncated t-distribution. Many issues have not been resolved yet. Implied volatility parameters may change, interest rates may fluctuate, and option sensitivity may exhibit unexpected behavior. These risks may not be the "cost" of maintaining positions, but they can affect pricing and play an important role in option trading. In addition, it is also worth studying how to price other financial derivatives such as American options and exotic options under the assumption of logarithmic truncated t-distribution.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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