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On the Existence of a Minimum Universal Speed of Physical Transmissions Associated with Matter Wave in Special Relativity

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Abstract

In this work, we show that it is possible to establish coordinate transformations between inertial reference frames in the theory of special relativity with a minimum universal speed of physical transmissions. The established coordinate transformations, referred to as modified Lorentz transformations because they have almost identical form to the Lorentz transformations, also comply with the requirement of invariance of the Minkowski line element. Particularly, the minimum universal speed can be associated with the phase speed of de Broglie matter wave. As application, we also discuss the possibility to formulate relativistic classical and quantum mechanics for the special relativity associated with the modified Lorentz transformations, which describes physical processes that represent an expansion or a collapsing of massive quantum particles.

Keywords

Special Relativity, Minkowski Line Element, Minimum Universal Speed, Modified Lorentz Transformations, Phase Velocity, De Broglie Matter Wave, Relativistic Mechanics

1. Introduction

In physics, the special theory of relativity is formulated from the postulate of the principle of relativity [1] [2] [3] [4], which states that all physical laws are identical in all inertial reference frames, and the postulate of constancy of maximum universal speed of physical transmissions, which is identified with the speed of light in vacuum and can be verified by the Michelson-Morley experiment [5]. Consequently, the Lorentz transformations can be derived and the Minkowski spacetime can be introduced [6]. There are profound features that emerge from the relativistic formulation of classical physics with fundamental changes to the Newtonian concepts from which physical laws of dynamics are based, particularly on the perception of the geometrical structure of space and time, such as time dilation and space contraction, due to the relative motion between inertial reference frames [7] [8] [9] [10] [11].

In this work, we examine mathematically possible conditions that can be imposed on the Minkowski line element and the Lorentz transformations for the derivation of spacetime dilation and contraction in the theory of special relativity. We refer to the universal constant in special relativity simply as Minkowski constant. If the Minkowski constant is assumed to be the maximum speed of physical transmissions, which is normally identified with the speed of light in vacuum, then it is known that the usual Lorentz transformations can be derived. However, we will show that, in fact, the Minkowski constant can be assumed to be the minimum universal speed of physical propagations, and then we are able to establish a system of modified Lorentz transformations for the minimum Minkowski constant that also leave the Minkowski line element invariant. Particularly, we will show that it is possible to associate the minimum universal speed with the phase speed of de Broglie matter wave. Within the framework of special relativity associated with the modified Lorentz transformations, we also discuss the possibility to formulate relativistic classical and quantum mechanics, in the forms of Klein-Gordon and Dirac wave equation [12] [13], which can be used to describe physical processes that involve expanding or collapsing of massive quantum particles.

2. Derivation of Spacetime Dilation and Contraction Using Minkowski Line Element

In the theory of special relativity, the Minkowski line element is formed on the Minkowski spacetime, which is a union of space and time into a four-dimensional manifold. The Minkowski line element can be written in the general form

$$\mathrm{d}s^2 = g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta \tag{1}$$

where the metric tensor g_{ij} is given as

$$g_{\alpha\beta} = \pm\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

In the current formulation of relativistic physics, both signs of the metric tensor are equivalent, which leads to assigning positive, negative, or zero values to the Minkowski line element $\mathrm{d}s^2$. In this work, however, we will assume that the Minkowski line element is only assigned with positive or zero values, that is $\mathrm{d}s^2 \geq 0$, therefore we will need to consider the positive and negative signs of the metric tensor separately and, as shown below, the separation will lead to consideration of the Minkowski constant being assigned to either a maximum or minimum universal value, rather than the only maximum value as currently being identified with the speed of light in vacuum.

Consider an inertial reference frame S in which a Minkowski coordinate system (t, x, y, z) is defined. If we use the metric signature $g_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$ then the Minkowski line element $\mathrm{d}s^2$ is written as

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \tag{3}$$

For real physical analysis, the Minkowski should satisfy the condition $\mathrm{d}s^2 \ge 0$. This requirement implies that the speeds of all physical propagations are less than the maximum universal speed c, which has been identified with the speed of light in vacuum. Therefore, within the time interval $\mathrm{d}t$ the distance travelled by all physical fields are less than the distance travelled by the electromagnetic field.

On the other hand, when the distance travelled by all physical fields greater than the distance travelled by the physical field whose speed is determined by the Minkowski constant c then, also for real physical analysis, that is $\mathrm{d}s^2 \ge 0$, we need to assume a lower universal speed limit instead of an upper universal speed limit. And in this case we need to use the metric signature $g_{\alpha\beta} = \mathrm{diag}(-1, 1, 1, 1)$ and the Minkowski line element $\mathrm{d}s^2$ is written as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{4}$$

2.1. Relative Time Rates between the Temporal Coordinates

Now, we consider another inertial reference frames S' in which a Minkowski coordinate system (ct', x', y', z') is also established. We then assume that the form of the line element $\mathrm{d}s^2$ given in Equation (3) is invariant with respect to the two frames so that we can obtain the following relation

$$c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = c^2\mathrm{d}t'^2 - \mathrm{d}x'^2 - \mathrm{d}y'^2 - \mathrm{d}z'^2 \tag{5}$$

In this case the two inertial reference frames are connected only by the assumption that their Minkowski line elements have identical forms. In the following, we first examine, from the relation given in Equation (5), the kinematics of physical objects, or observers, in the two frames and study how the respective values of spacetime dilation and contraction can be compared in the two frames. Then, we examine the spacetime dilation and contraction in the traditional way by using the coordinate transformations between the two frames when one frame is assumed to be moving with respect to the other. We need to establish transformations of spacetime coordinates that leave the Minkowski line element invariant with respect to the coordinates in the two frames, such as the Lorentz transformations. In fact, as we will discuss in Section 3 that the Minkowski line element $\mathrm{d}s^2$ can also be made invariant under a system of modified Lorentz transformations in which the Minkowski constant c is assigned with a minimum universal speed of physical transmissions rather than a maximum universal speed as being associated with the Lorentz transformations, and in particular we will show that the modified Lorentz transformations are associated with de Broglie matter wave.

In the inertial reference frame S we define the velocity $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$, where $\boldsymbol{r} = (x, y, z)$, and, similarly, in the inertial reference frame S' we also define the velocity $\boldsymbol{v}' = \mathrm{d}\boldsymbol{r}'/\mathrm{d}t'$, where $\boldsymbol{r}' = (x', y', z')$. The velocities $\boldsymbol{v}$ and $\boldsymbol{v}'$ may be assumed to be the velocities of physical objects with regard to their motion, respectively, in the two inertial reference frames. It is observed that if $v = v' = c$, then the equation for the Minkowski line elements given in Equation (5) satisfies automatically. On the other hand, if $v \neq c$ and $v' \neq c$, and $\mathrm{d}t \neq 0$ and $\mathrm{d}t' \neq 0$, then we can rewrite Equation (5) in the form

$$c^2\mathrm{d}t^2\left(1-\frac{\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2}{c^2\mathrm{d}t^2}\right)=c^2\mathrm{d}t'^2\left(1-\frac{\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2}{c^2\mathrm{d}t'^2}\right) \tag{6}$$

From Equation (6) we obtain the following kinematic relation for the time intervals

$$\frac{\mathrm{d}t}{\mathrm{d}t'}=\frac{\sqrt{1-\frac{v'^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

This equation gives a relation between the time rates of the time coordinates in the two frames S and S', which may be regarded as due to the motion of physical objects. If we consider v as being the magnitude of the velocity of a particle in the frame S, and v' being that of a different particle in the frame S' then Equation (7) gives either a time dilation or a time contraction depending on the relative values of the magnitudes of the velocities. If $v > v'$ then $\mathrm{d}t > \mathrm{d}t'$ and in this case we have a time dilation for the time t in S relative to the time t' in S'. On the other hand, if $v < v'$ then $\mathrm{d}t < \mathrm{d}t'$ and we have a time contraction for the time t relative to the time t'. In particular, there is no time dilation or contraction when $v = v'$. As an illustration, we now apply the time relation given in Equation (7) to a clock at rest in the frame S', which is defined by the spatial coordinate condition $|\mathrm{d}\boldsymbol{r}'| = \sqrt{\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2} = 0$ thus $v' = 0$, and the clock is assumed to move with a uniform velocity $\boldsymbol{v}$ with respect to the frame S. Then, we obtain the familiar time dilation given by the relation

$$\mathrm{d}t = \gamma_t \mathrm{d}t' \tag{8}$$

where the time dilation factor γ_t is defined as follows

$$\gamma_t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

This is a measure of the elapse of time in the frame S compared to the proper time recorded by a clock at rest in the frame S', where the frame S' is assumed to move with the uniform velocity $\boldsymbol{v}$ with respect to the frame S. On the other hand, if $v = 0$ but $v' \neq 0$, then we obtain the time contraction given by the relation

$$\mathrm{d}t=\sqrt{1-\frac{v'^2}{c^2}}\mathrm{d}t' \tag{10}$$

2.2. Relative Space Rates between the Spatial Coordinates

Similar to the examination of the relative time rates between the temporal coordinates, we can also examine the space rates between the spatial coordinates of the two reference frames. We first define the spatial line elements in the two frames S and S' as $\mathrm{d}\chi^2=\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2$ and $\mathrm{d}\chi'^2=\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2$, respectively. However, in the present situation, we assume the form of the Minkowski line element $\mathrm{d}s^2$ given in Equation (4), rather than Equation (3), to be invariant with respect to the two inertial reference frames. Then, we obtain the relation

$$-c^2\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2=-c^2\mathrm{d}t'^2+\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2 \tag{11}$$

We now rewrite Equation (11) in the form

$$\begin{aligned}&\left(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)\left(1-\frac{c^2\mathrm{d}t^2}{\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2}\right)\\&=\left(\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2\right)\left(1-\frac{c^2\mathrm{d}t'^2}{\mathrm{d}x'^2+\mathrm{d}y'^2+\mathrm{d}z'^2}\right)\end{aligned} \tag{12}$$

From Equation (12) we obtain the following relation for the spatial coordinate intervals

$$\frac{\mathrm{d}\chi}{\mathrm{d}\chi'}=\frac{\sqrt{1-\frac{c^2}{v'^2}}}{\sqrt{1-\frac{c^2}{v^2}}} \tag{13}$$

Using Equation (13) we can examine the space dilation and contraction in a similar way to that for the time rate relation given in Equation (7). However, for real analysis, the examination also requires the existence of physical transmissions with speeds $v>c$ and $v'>c$. Therefore, in order to apply Equation (13) into real physical events we need to assume that the Minkowski constant c is a minimum universal speed of physical transmissions. According to classical physics, the minimum speed would simply be zero. Yet, this assumption may not be necessary in quantum physics, for example, as in the quantum harmonic motion or the quantum vacuum fluctuation in quantum field theory. In Section 4 we will discuss further these quantum physics topics when we formulate relativistic classical and quantum mechanics from modified Lorentz transformations. From Equation (13), if we impose the condition $v'\gg c$ we then obtain the space dilation given by the relation

$$\mathrm{d}\chi=\gamma_s\mathrm{d}\chi' \tag{14}$$

where the space dilation factor γ_s is defined as

$$\gamma_s = \frac{1}{\sqrt{1 - \frac{c^2}{v^2}}} \tag{15}$$

3. Derivation of Spacetime Dilation and Contraction by a System of Modified Lorentz Transformations

In this section we show that a system of modified Lorentz transformations can be established with the space dilation factor γ_s given in Equation (15) so that the Minkowski line element is invariant under such coordinate transformations. We consider two inertial reference frames S and S' in which two Minkowski coordinate systems (t, x, y, z) and (t', x', y', z') are respectively defined. It is assumed that the x-axis and the x'-axis coincide, and the other two pairs of axes are parallel. It is also assumed that the frame S' moves with respect to the frame S with the velocity $\boldsymbol{v}$ in the positive direction along the x-axis. However, unlike the usual Lorentz transformations, which are briefly presented in Appendix below for comparison and reference, the required modified Lorentz transformations are assumed to be given in the following form

$$x = \gamma_s \left(x' + \frac{c^2}{v} t' \right) \tag{16}$$

$$y = y' \tag{17}$$

$$z = z' \tag{18}$$

$$t = \gamma_s \left(t' + \frac{1}{v} x' \right) \tag{19}$$

It can be shown that the Minkowski line element given in Equation (4) is invariant under the coordinate transformations given by Equations (16)-(19). It is also observed that by comparison to the Lorentz transformations given in the **Appendix**, the two terms $c^2 t'/v$ and x'/v in the modified Lorentz transformations play inverse roles to the two terms vt' and vx'/c^2 in the Lorentz transformations in the sense that the terms x'/v and vt' can be interpreted kinematically but the terms $c^2 t'/v$ and vx'/c^2 cannot, because we do not know exactly what the speed c^2/v and the reciprocal speed v/c^2 would represent in classical physics with regard to Lorentz transformations. However, the speed c^2/v in Equation (16) can be interpreted as the phase speed of matter wave motion in quantum mechanics. It is shown in relativistic mechanics that the momentum and energy of a free particle can be written in the relativistic forms, respectively, as

$$\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{20}$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{21}$$

From the above formulas given for the momentum and energy, we obtain the relation

$$\frac{E}{p}=\frac{c^2}{v} \tag{22}$$

On the other hand, in quantum mechanics the Planck quantum energy and de Broglie matter wave relation are given, respectively, by

$$E=h\nu \tag{23}$$

$$p=\frac{h}{\lambda} \tag{24}$$

From Equations (22)-(24) we then obtain the relation

$$\frac{c^2}{v}=\lambda\nu \tag{25}$$

The quantity c^2/v is the phase velocity associated with de Broglie matter wave, therefore, we may associate the speed $c^2/v=\lambda\nu$ with the speed of de Broglie matter wave [14]. With this association, we can suggest that the physical field that propagates with the minimum universal speed should be a matter wave.

The inverse transformations of the modified Lorentz transformations can also be found as

$$x'=\gamma_s\left(x-\frac{c^2}{v}t\right) \tag{26}$$

$$y'=y \tag{27}$$

$$z'=z \tag{28}$$

$$t'=\gamma_s\left(t-\frac{1}{v}x\right) \tag{29}$$

Additionally, from the modified Lorentz transformations, we can establish the relativistic law of addition for velocities. If we define the components of velocities as

$$u_x=\frac{\mathrm{d}x}{\mathrm{d}t}\quad u_y=\frac{\mathrm{d}y}{\mathrm{d}t}\quad u_z=\frac{\mathrm{d}z}{\mathrm{d}t}\quad u'_x=\frac{\mathrm{d}x'}{\mathrm{d}t'}\quad u'_z=\frac{\mathrm{d}y'}{\mathrm{d}t'}\quad u'_z=\frac{\mathrm{d}z'}{\mathrm{d}t'} \tag{30}$$

then we can obtain the relativistic addition laws for velocities

$$u_x=\frac{u'_x+\frac{c^2}{v}}{1+\frac{1}{v}u'_x}\quad u_y=\frac{\sqrt{1-\frac{c^2}{v^2}}u'_y}{1+\frac{1}{v}u'_x}\quad u_z=\frac{\sqrt{1-\frac{c^2}{v^2}}u'_z}{1+\frac{1}{v}u'_x} \tag{31}$$

It is seen from the above addition laws for the velocities that the Minkowski constant c is in fact a minimum universal speed because if $u'_x=c$ then we also have $u_x=c$.

Now, we consider the two events (t'_1,x'_1,y'_1,z'_1) and (t'_2,x'_2,y'_2,z'_2) in the frame S'. The corresponding events (t_1,x_1,y_1,z_1) and (t_2,x_2,y_2,z_2) in the frame S are respectively obtained from the modified Lorentz transformations as follows

$$x_1 = \gamma_s \left(x_1' + \frac{c^2}{v} t_1' \right) \quad x_2 = \gamma_s \left(x_2' + \frac{c^2}{v} t_2' \right) \tag{32}$$

$$y_1 = y_1' \quad y_2 = y_2' \tag{33}$$

$$z_1 = z_1' \quad z_2 = z_2' \tag{34}$$

$$t_1 = \gamma_s \left(t_1' + \frac{1}{v} x_1' \right) \quad t_2 = \gamma_s \left(t_2' + \frac{1}{v} x_2' \right) \tag{35}$$

Then, we obtain the following results

$$x_2 - x_1 = \gamma_s \left(x_2' - x_1' + \frac{c^2}{v} \left(t_2' - t_1' \right) \right) \tag{36}$$

$$y_2 - y_1 = y_2' - y_1' \tag{37}$$

$$z_2 - z_1 = z_2' - z_1' \tag{38}$$

$$t_2 - t_1 = \gamma_s \left(t_2' - t_1' + \frac{1}{v} \left(x_2' - x_1' \right) \right) \tag{39}$$

In the following, based on Equations (36) and (39), we will provide a mathematical examination on the spacetime dilation and contraction between spacetime coordinates of the two frames S and S'. Physically, the separation of two events in space can be identified with the length of a physical object and the separation in time can be taken as a measure of the time interval of two successive ticks of a clock.

3.1. The Condition on the Spatial Coordinate of the Frame S': $x_2' - x_1' = 0$

This condition is equivalent to examine two events that occur at the same spatial position in the frame S'. By applying the spatial coordinate condition $x_2' - x_1' = 0$ into Equations (36) and (39) we then obtain the following equations

$$x_2 - x_1 = \gamma_s \frac{c^2}{v} \left(t_2' - t_1' \right) \tag{40}$$

$$t_2 - t_1 = \gamma_s \left(t_2' - t_1' \right) \tag{41}$$

Equation (41) represents the time dilation relation between the time coordinates of two inertial reference frames. From Equations (40) and (41) we also obtain the kinematic equation

$$x_2 - x_1 = \frac{c^2}{v} \left(t_2 - t_1 \right) \tag{42}$$

We may interpret this equation by stating that the spatial point $x_2' - x_1' = 0$ in the frame S' moves with the speed c^2/v relative to the frame S in the positive direction along the x-axis.

3.2. The Condition on the Time Coordinate of the Frame S': $t_2' - t_1' = 0$

This condition is equivalent to examine two events that occur at the same temporal position in the frame S'. By applying the temporal coordinate condition $t'_2 - t'_1 = 0$ into the equations given in Equations (36) and (39) we then obtain the following equations

$$x_2 - x_1 = \gamma_s \left(x'_2 - x'_1\right) \tag{43}$$

$$t_2 - t_1 = \gamma_s \frac{1}{v}\left(x'_2 - x'_1\right) \tag{44}$$

It is observed that the relation between the spatial coordinates given in Equation (43) can be interpreted as space dilation. It has the same mathematical status as that of the time dilation given in Equation (41) for the time coordinates. However, from Equations (43) and (44) we obtain the kinematic equation

$$x_2 - x_1 = v\left(t_2 - t_1\right) \tag{45}$$

Unlike the kinematic equation given in Equation (42) for the time dilation, Equation (45) describes the motion of a physical object with the speed v, which is the speed of the frame S' with respect to the frame S in this case. With regard to spacetime symmetry, we may state that the temporal point $t'_2 - t'_1 = 0$ moves in the frame S in the positive direction along the x-axis with the speed v.

3.3. The Condition on the Spatial Coordinate of the Frame S: $x_2 - x_1 = 0$

This condition is equivalent to examine two events that occur at the same spatial position in the reference frame S. By applying the spatial coordinate condition $x_2 - x_1 = 0$ into the equations given in Equations (36) and (39) then we obtain the following equations

$$0 = \gamma_s \left(x'_2 - x'_1 + \frac{c^2}{v}\left(t'_2 - t'_1\right)\right) \tag{46}$$

$$t_2 - t_1 = \gamma_s \left(t'_2 - t'_1 + \frac{1}{v}\left(x'_2 - x'_1\right)\right) \tag{47}$$

From the above relations we obtain the time contraction relation

$$t_2 - t_1 = \frac{1}{\gamma_s}\left(t'_2 - t'_1\right) \tag{48}$$

We can describe the situation as follows. If we locate a clock at a particular spatial position in the frame S and compare the time rate of the clock with the time rate in the frame S' then we see that the time of the clock runs faster than the time of the frame S'.

3.4. The Condition on the Time Coordinate of the Frame S: $t_2 - t_1 = 0$

This condition is equivalent to examine two events occur at the same temporal position in the frame S. By applying the temporal coordinate condition $t_2 - t_1 = 0$ into the equations given in Equations (36) and (39) then we obtain the following equations

$$x_2 - x_1 = \gamma_s\left(x_2' - x_1' + \frac{c^2}{v}\left(t_2' - t_1'\right)\right) \tag{49}$$

$$0 = \gamma_s\left(t_2' - t_1' + \frac{1}{v}\left(x_2' - x_1'\right)\right) \tag{50}$$

From Equations (49) and (50), a contraction relation can be obtained for the spatial intervals in the two reference frames as

$$x_2 - x_1 = \frac{1}{\gamma_s}\left(x_2' - x_1'\right) \tag{51}$$

This is the usual space contraction when the interval $x_2' - x_1'$ in the frame S' being observed moving with the speed v relative to the frame S and the corresponding Lorentz transformed spatial coordinates in the frame S are assumed to be measured simultaneously. Since there are no kinematic equations of motion that can be established from Equations (49) and (50) between the spatial and temporal coordinates in the frame S therefore we cannot determine how the space contraction given in Equation (51) can be interpreted in terms of the kinematics of the motion. However, if we rewrite Equations (50) and (51) in the following form

$$x_2' - x_1' = -v\left(t_2' - t_1'\right) \tag{52}$$

$$x_2' - x_1' = \gamma_s\left(x_2 - x_1\right) \tag{53}$$

then we regain the situation as discussed in Subsection 3.2 in which the roles of the two frames are reversed.

4. Relativistic Classical and Quantum Mechanics

In this section we discuss the possibility to formulate the relativistic classical and quantum mechanics based on the Minkowski line element given in Equation (4). Within the framework of special relativity associated with the modified Lorentz transformations, we can also establish relativistic wave equations, which have similar forms to the relativistic Klein-Gordon and Dirac wave equation, and the established equations can be used to describe physical processes that involve expanding or collapsing of massive quantum particles. The Minkowski line element given in Equation (4) can be rewritten in the form

$$\mathrm{d}s^2 = \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)\left(1 - \frac{c^2}{v^2}\right) \tag{54}$$

where $v^2 = \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)/\mathrm{d}t^2 = v_x^2 + v_y^2 + v_z^2$. Again, for real physical analysis, we also assume the condition $\mathrm{d}s^2 \geq 0$, that is $v > c$. Since we have the condition $\mathrm{d}s^2 \geq 0$, thus from Equation (54) we obtain the line element written as

$$\mathrm{d}s = \left(1/\gamma_s\right)\mathrm{d}\chi \tag{55}$$

where the quantity γ_s is specified in Equation (15), and $\mathrm{d}\chi = \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}$.

The relativistic classical and quantum mechanics under the modified Lorentz transformations can be formulated by also applying the principle of least action.

First, we define the action for a free particle associated with the line element given in Equation (55) as follows

$$S = mc\int \mathrm{d}s = mc\int \sqrt{1-\frac{c^2}{v^2}}\mathrm{d}\chi = mc\int \sqrt{v^2-c^2}\,\mathrm{d}t \tag{56}$$

Thus, the Lagrangian takes the form

$$L = mc\sqrt{v^2-c^2} \tag{57}$$

The momentum $\boldsymbol{p} = \left(p_x, p_y, p_z\right)$ is also assumed to be defined according to the formulas

$$p_x = \frac{\partial L}{\partial v_x} \quad p_y = \frac{\partial L}{\partial v_y} \quad p_z = \frac{\partial L}{\partial v_z} \tag{58}$$

Using the Lagrangian given in Equation (57) we then obtain

$$p_x = \frac{c}{v}\frac{mv_x}{\sqrt{1-\frac{c^2}{v^2}}} \quad p_y = \frac{c}{v}\frac{mv_y}{\sqrt{1-\frac{c^2}{v^2}}} \quad p_z = \frac{c}{v}\frac{mv_z}{\sqrt{1-\frac{c^2}{v^2}}} \tag{59}$$

Equation (59) can be rewritten in a vector form as

$$\boldsymbol{p} = \frac{c}{v}\frac{m\boldsymbol{v}}{\sqrt{1-\frac{c^2}{v^2}}} \tag{60}$$

The energy E of the particle is also defined by the relation

$$E = \boldsymbol{p}\cdot\boldsymbol{v} - L \tag{61}$$

We then obtain

$$E = \frac{c}{v}\frac{mc^2}{\sqrt{1-\frac{c^2}{v^2}}} \tag{62}$$

From Equations (60) and (62), a relationship between the momentum and energy can also be established as

$$\frac{E^2}{c^2} = p^2 - m^2c^2 \tag{63}$$

It is observed that the relation between the momentum and energy in Equation (63) differs from the familiar relation $E^2/c^2 = p^2 + m^2c^2$ under the Lorentz transformations by the negative m^2c^2. In fact, this difference has profound effects on physical processes of quantum particles. If we replace the mass m in Equation (63) by the imaginary mass im, then the relation given in Equation (63) can be rewritten in the form

$$\frac{E^2}{c^2} = p^2 + (im)^2c^2 \tag{64}$$

When the energy and momentum in the energy-momentum relation in Equation (64) are replaced by the differential operators [15] [16] [17], respectively, as

$$E \to i\hbar\frac{\partial}{\partial t} \tag{65}$$

$$\boldsymbol{p} \to -i\hbar\nabla \tag{66}$$

then we obtain a wave equation similar to the Klein-Gordon equation given by

$$\left(\frac{\partial^2}{c^2\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \frac{(im)^2 c^2}{\hbar^2}\right)\psi = 0 \tag{67}$$

Solutions to Equation (67) for free particles can be found as

$$\psi = N\exp\left[-\frac{i}{\hbar}(Et - \boldsymbol{p}\cdot\boldsymbol{r})\right] \tag{68}$$

In particular, for particles at rest in which $\boldsymbol{p} = 0$, then from Equation (64) we obtain the relation $E = \pm imc^2$, and Equation (68) reduces to

$$\psi = N\exp\left(\pm\frac{mc^2}{\hbar}t\right) \tag{69}$$

Since the exponential of Equation (68) is real, therefore we may interpret the obtained solutions as physical processes that involve an expansion or collapsing of quantum particles.

On the other hand, also by using the energy-momentum relation given by Equation (64) with an imaginary mass, an equation similar to the Dirac relativistic equation can be constructed for massive spin-half particles as

$$(i\hbar\gamma^\mu\partial_\mu - imc)\psi = 0 \tag{70}$$

where the wavefunction ψ is the four-component vector $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^{\mathrm{T}}$, and the γ^μ matrices are defined in terms of the Pauli matrices σ_k and the unit matrix as

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \tag{71}$$

From Equation (70), particular solutions for spin-half quantum particles at rest, $\boldsymbol{p} = 0$, can be found as

$$\psi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \mathrm{e}^{\frac{mc^2}{\hbar}t} \quad \psi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \mathrm{e}^{\frac{mc^2}{\hbar}t} \quad \psi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \mathrm{e}^{-\frac{mc^2}{\hbar}t} \quad \psi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mathrm{e}^{-\frac{mc^2}{\hbar}t} \tag{72}$$

Since the exponentials of Equation (72) are all real, therefore, as in the case with the Klein-Gordon equation, we may also attempt to interpret the solutions given in Equation (72) as physical processes that involve an expansion or collapsing of massive spin-half quantum particles.

5. Conclusion

We have analyzed possible mathematical conditions imposed on the Minkowski line element, the modified Lorentz transformations, and the Lorentz transformations for the derivation of spacetime dilation and contraction in the theory of special relativity. We have shown that the Minkowski universal constant in the theory of special relativity can be identified either as the maximum universal speed of physical transmissions, which is normally identified with the speed of light in vacuum, or the minimum universal speed of physical transmissions. For the case of identifying the Minkowski constant as the minimum universal speed, we have shown that it is possible to establish modified Lorentz transformations that also leave the Minkowski line element invariant. We have also shown that it is possible to identify the minimum universal speed with the phase speed of de Broglie matter wave. Within the framework of special relativity associated with the modified Lorentz transformations, we have also established relativistic wave equations, which have similar forms to the relativistic Klein-Gordon and Dirac wave equation, that describe physical processes that involve expanding or collapsing of massive quantum particles.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Poincaré, H. (1905) On the Dynamics of the Electron. Wikisource.

[2] Poincaré, H. (1907) The Fundamental Equations for Electromagnetic Processes in Moving Bodies. Wikisource.

[3] Lorentz, H.A. (1952) The Principle of Relativity: Electromagnetic Phenomena in a System Moving with Any Velocity Less than that of Light. Dover Publications, New York.

[4] Einstein, A. (1952) The Principle of Relativity: On the Electrodynamics of Moving Bodies. Dover Publications, New York.

[5] Michelson, A.A. and Morley, E.W. (1887) On the Relative Motion of the Earth and the Luminiferous Ether. *American Journal of Science*, **34**, 333-345. https://doi.org/10.2475/ajs.s3-34.203.333

[6] Minkowski, H. (1952) The Principle of Relativity: Space and Time. Dover Publications, New York.

[7] Landau, L.D. and Lifshitz, E.M. (1987) The Classical Theory of Fields. Pergamon Press, Oxford.

[8] French, A.P. (1968) Special Relativity. W. W. Norton & Company Inc., New York.

[9] Ryder, R. (2009) Introduction to General Relativity. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511809033

[10] Ho, V.B. (2020) Classification of Relativity. *Journal of Modern Physics*, **11**, 535-564. https://doi.org/10.4236/jmp.2020.114036

[11] Ho, V.B. (2018) Fluid State of Dirac Quantum Particles. *Journal of Modern Physics*,

9, 2402-2419. https://doi.org/10.4236/jmp.2018.914154

[12] Dirac, P.A.M. (1947) The Principles of Quantum Mechanics. Oxford University Press, Oxford.

[13] Thaller, B. (1992) The Dirac Equation. Springer-Verlag, Berlin. https://doi.org/10.1007/978-3-662-02753-0

[14] de Broglie, L.-V. (1927) On the Theory of Quanta. Translated by Kracklauer, A.F., Fondation Louis de Broglie, Paris.

[15] Weinberg, S. (1995) The Quantum Theory of Fields, Volume I: Foundations. Cambridge University Press, Cambridge.

[16] Berestetskii, V.B., Lifshitz, E.M. and Pitaevskii, L.P. (1982) Chapter 14-Electrodynamics of Hadrons. In: Berestetskii, V.B., Lifshitz, E.M. and Pitaevskii, L.P., Eds., *Quantum Electrodynamics*, Elsevier, London, 624-647. https://doi.org/10.1016/B978-0-08-050346-2.50020-9

[17] Greiner, W. (1990) Relativistic Quantum Mechanics. Springer-Verlag, Berlin. https://doi.org/10.1007/978-3-662-02634-2

Appendix. Derivation of Spacetime Dilation and Contraction Using Lorentz Transformations

This section briefly presents the derivation of spacetime dilation and contraction by using the well-known Lorentz transformations in the theory of special relativity [8] [9]. The purpose of the presentation is for comparison and reference to what have been examined in the previous section with the modified Lorentz transformations. Because the resulted equations are almost identical to those obtained from the modified Lorentz transformations therefore we will only list the results without further interpretations, even though there are new features that emerge from the mathematical examination that are worth being examined thoroughly.

For the Lorentz transformations, we also consider two inertial reference frames in which the frame S' moves with the velocity $\boldsymbol{v}$ relative to the frame S in the positive direction along the x-axis. Then, it can be verified that the invariance of the Minkowski line element given in Equation (3) is satisfied under the Lorentz transformations given by

$$x = \gamma_t \left(x' + vt' \right) \tag{73}$$

$$y = y' \tag{74}$$

$$z = z' \tag{75}$$

$$t = \gamma_t \left(t' + \frac{v}{c^2} x' \right) \tag{76}$$

where the time dilation factor γ_t is given in Equation (9). We next consider the two events $\left(t_1', x_1', y_1', z_1'\right)$ and $\left(t_2', x_2', y_2', z_2'\right)$ in the frame S'. The corresponding events $\left(t_1, x_1, y_1, z_1\right)$ and $\left(t_2, x_2, y_2, z_2\right)$ in the frame S are obtained, respectively, as follows

$$x_1 = \gamma_t \left(x_1' + vt_1' \right) \quad x_2 = \gamma_t \left(x_2' + vt_2' \right) \tag{77}$$

$$y_1 = y_1' \quad y_2 = y_2' \tag{78}$$

$$z_1 = z_1' \quad z_2 = z_2' \tag{79}$$

$$t_1 = \gamma_t \left(t_1' + \frac{v}{c^2} x_1' \right) \quad t_2 = \gamma_t \left(t_2' + \frac{v}{c^2} x_2' \right) \tag{80}$$

Consequently, we obtain the following results

$$x_2 - x_1 = \gamma_t \left(x_2' - x_1' + v\left(t_2' - t_1'\right) \right) \tag{81}$$

$$y_2 - y_1 = y_2' - y_1' \tag{82}$$

$$z_2 - z_1 = z_2' - z_1' \tag{83}$$

$$t_2 - t_1 = \gamma_t \left(t_2' - t_1' + \frac{v}{c^2}\left(x_2' - x_1'\right) \right) \tag{84}$$

By imposing different mathematical conditions on the transformation equations given in Equation (81) and (84), we obtain the results listed below.

A1. Imposing the Condition $x_2' - x_1' = 0$

When we impose the spatial condition $x_2' - x_1' = 0$ into Equations (81) and (84) then we obtain the following kinematic equations

$$x_2 - x_1 = \gamma_t v(t_2' - t_1') \tag{85}$$

$$t_2 - t_1 = \gamma_t (t_2' - t_1') \tag{86}$$

$$x_2 - x_1 = v(t_2 - t_1) \tag{87}$$

These equations are similar to Equations (40), (41), and (42) in Subsection 3.1, respectively, for the modified Lorentz transformations.

A2. Imposing the Condition $t_2' - t_1' = 0$

When we impose the temporal condition $t_2' - t_1' = 0$ into Equations (81) and (84) then we obtain the following kinematic equations

$$x_2 - x_1 = \gamma_t (x_2' - x_1') \tag{88}$$

$$t_2 - t_1 = \gamma_t \frac{v}{c^2}(x_2' - x_1') \tag{89}$$

$$x_2 - x_1 = \frac{c^2}{v}(t_2 - t_1) \tag{90}$$

These equations are similar to Equations (43), (44), and (45) in Subsection 3.2, respectively.

A3. Imposing the Condition $x_2 - x_1 = 0$

When we impose the spatial condition $x_2 - x_1 = 0$ into Equations (81) and (84) then we obtain the following kinematic equations

$$0 = \gamma_t \left(x_2' - x_1' + v(t_2' - t_1')\right) \tag{91}$$

$$t_2 - t_1 = \gamma_t \left(t_2' - t_1' + \frac{v}{c^2}(x_2' - x_1')\right) \tag{92}$$

$$t_2 - t_1 = \frac{1}{\gamma_t}(t_2' - t_1') \tag{93}$$

These equations are similar to Equations (46), (47), and (48) in Subsection 3.2, respectively.

A4. Imposing the Condition $t_2 - t_1 = 0$

When we impose the temporal condition $t_2 - t_1 = 0$ into Equations (81) and (84) then we obtain the following kinematic equations

$$x_2 - x_1 = \gamma_t \left(x_2' - x_1' + v(t_2' - t_1')\right) \tag{94}$$

$$0 = \gamma_t \left(t_2' - t_1' + \frac{v}{c^2}(x_2' - x_1')\right) \tag{95}$$

$$x_2 - x_1 = \frac{1}{\gamma_t}(x_2' - x_1') \tag{96}$$

These equations are similar to Equations (49), (50), and (51) in Subsection 3.4, respectively.