

# A Map Coloring Method

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## Abstract

Different vertices are colored in a plan. Adjacent vertices are colored differently from nonadjacent vertices, which are colored the same color. One color is used for a single point, two colors are used for points without a loop, and a maximum of four colors are used for points with a loop. A maximum of four colors are used to color all points.

## Keywords

Map, Four Colors, Four Colors Conjecture, Graph Theory

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## 1. Introduction

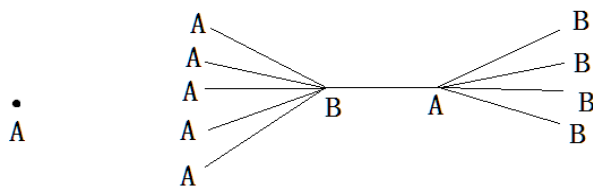
In a plan, different vertices are colored. Adjacent vertices are colored differently, whereas non-adjacent vertices are colored the same. A maximum of four colors are required.

### 1.1. Points without a Loop

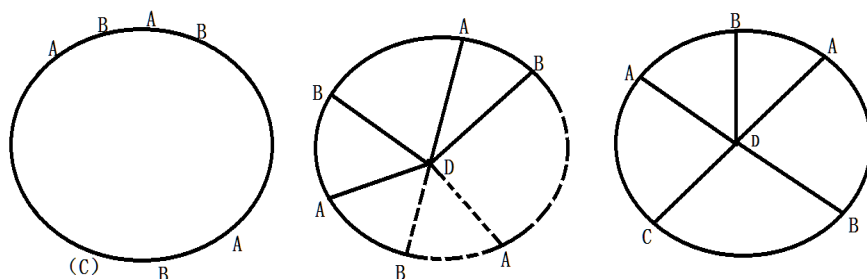
In a plan, only one color is necessary for a single point, while two colors are enough for points without a loop. (**Figure 1**)

### 1.2. Points with a Loop

In a plan, points with a loop create a “circle”. Two colors are used for even numbers of points, and three colors are used for odd numbers of points (left in **Figure 2**). The central point in the “circle” formed by the central point and adjacent points (middle and right in **Figure 2**) is colored with a single color. The points around the center point create a “circle”. When the number of “circle” points is odd, three colors are utilized (right in **Figure 2**). Otherwise, only two colors are utilized (the middle in **Figure 2**). For the central point and all adjacent points, a maximum of four colors are used.



**Figure 1.** 1-color and 2-color chromatic graphs.



**Figure 2.** 3-color and 4-color chromatic graphs.

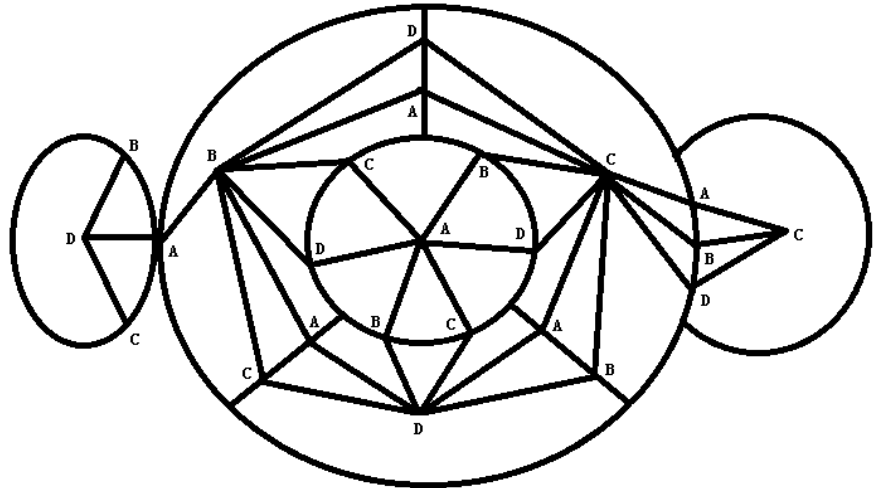
### 1.3. Coloring Relationship between Four Colors

The “circles” created by the central point and neighboring points can be joined by points of one, two, or three colors (**Figure 3**). **Figure 3** represents a schematic diagram. The color of the central point is subject to change, as is the color of the connecting points.

1) 1 color: When a point on the “circle” is connected, this point is colored with one color. If there is no central point, color from the connected point in turn and utilize a maximum of three colors. If there is a central point, select one of the other three colors other than the connected point’s color for the central point. Then color each connected point in turn with the remaining three colors, excluding the color of the central point (including the color of the connected point on the “circle”). Use a maximum of four colors.

2) 2 colors: When two center points have different colors, two “circles” are connected, and the adjacent points (points on the “circles”) between the two central points are colored with two different colors other than the color of the two central points. The “circles” of a number of central points with different colors are connected in pairs. When the “circles” of the central points in **Figure 3**’s colors A, B, C, and D are connected, the points on the “circle” of color A’s central point are linked to the points on color B’s central point. The common points are represented by the colors C and D. The common points that are connected to the central point’s “circle” with color C are represented by the colors B and D. The common points connected to the “circle” of the central point with color D and so on are represented by colors B and C. The color of the central point and the color of the adjacent points on the “circle” can only be a maximum of 4 colors.

3) 3 colors: If two central points share the same color, the other three colors can be used for the points on the “circle” that connect with the two central



**Figure 3.** Coloring relationship between four colors.

points. A maximum of four colors may be used for the color of the central point and the color of neighboring points on the “circle”.

#### 1.4. Color Selection and Order (Figure 4)

In **Figure 4**, point “?” will be colored with Color A if it is adjacent to the points with 4 colors and cannot be colored. Color B will be applied to the point colored in Color A above. The example in **Figure 4** serves as an illustration of how the choice of colors and the order in which they are colored might give the impression that one is unable to color.

#### 1.5. How to Color (Figure 5)

Any local center in **Figure 5** is highlighted in color A (1A), and the five neighboring points are highlighted in colors 1-1B, 1-2C, 1-3B, 1-4C, and 1-5D. These five points can each have a different color.

Choose the second local central point that is adjacent to the first local central point but not the points that are adjacent to it, and then color it with 2A. (The “circle” that is formed by this second local central point and the adjacent point is adjacent to 1-1B and 1-5D on the first local central point’s colored “circle.” The connecting points between the “circles” are these two points. Colors A and C can be selected for the second local central point). Choose three colors for the second local center point, except Color 2A. Color the uncolored points (points on the “circle”) close to the second local central point with colors 2-1B, 2-2C, and 2-3D (\*Select color and coloring order).

Choose 3A, 4C, 5B, and 6D in turn as the local central points to color the adjacent points (points on the “circle”). A maximum of four colors are needed.

#### 1.6. Summary

Different vertices are colored in a plan. Adjacent vertices are colored differently from nonadjacent vertices, which are colored the same color.

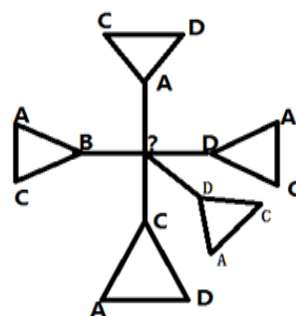


Figure 4. Reasons for 4 colors failing to be colored.

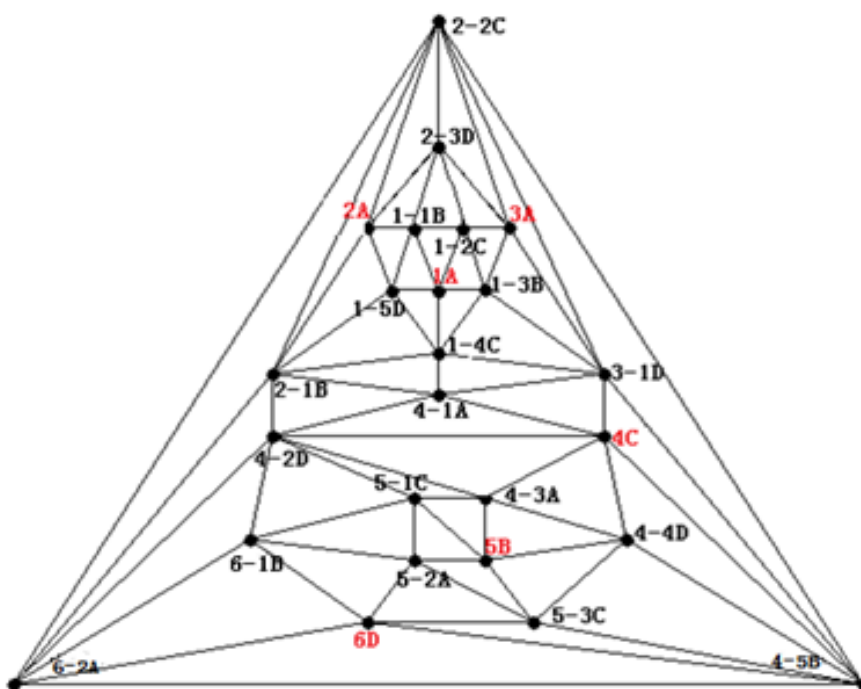


Figure 5. 4-color coloring method.

One color is used for a single point, while two colors are used for connected points that do not create a loop.

For points with a loop without a central point (points on the “circle”), three colors are used.

For points with a loop and a central point, choose one local central point and color it with a single color. Select the remaining three colors in turn to color the adjacent points (points on the “circle”). A maximum of four colors are used.

Continue to search for an adjacent local central point outside the colored “circle” (the local central point and the adjacent points form a “circle”). To color the local central point, choose one of the colors that was not used for the adjacent colored points. Then, paint the nearby non-colored points (points on the “circle”) with the remaining three colors except the color of the local central point.

Continue to color. To color every point, a maximum of four colors is required.

## 2. Using the Triangle Method for Map Coloring

In the plan is not in the same straight line, three points determine a plane, then three points form a triangle is a plane figure in the most basic, most simple, the most stable, the closed graph [1].

In the plan of the numerous points, take an arbitrary three adjacent points of the adjacent  $\triangle ABC$  (**Figure 6**), then 3 color A B C, in the plan to take a point D and A B C three adjacent, while D and A B C three is connected form a triangle. Let take a little E and A, B, C, D four-color. E will be with four-color colored the same, namely E in  $\triangle ABD$  and C the same colour, in  $\triangle ACD$  and B the same colour, in  $\triangle BCD$  and A the same colour, in  $\triangle ABC$  and D the same color, E and another three-color connected to form a new triangle [2].

In the triangle with three points, let take a point only in the triangle interior and exterior two kinds of circumstances, the two cases of points are not adjacent, this point and triangle with three points are connected and form a new triangle [3].

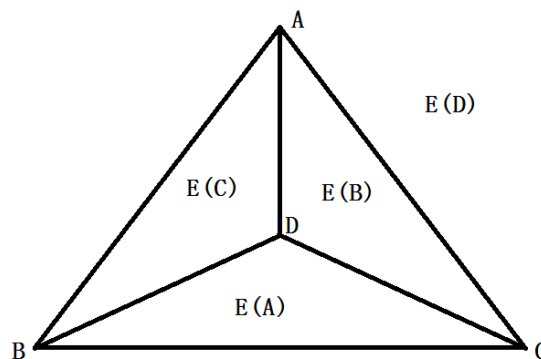
To choose a point for coloring, the points are the same and a triangle with three points are connected, and forming a new triangle, the point of at least one color of four colors. Point by point coloring to all point by one color of A, B, C, D four color [4].

## 3. History of the Four Color Conjecture

Four color conjecture is one of the three mathematical conjectures in the world, namely Fermat conjecture, four color conjecture and Goldbach conjecture [5].

This is a topological problem, that is, to find the minimum number of different colors required for coloring a spherical (or planar) map. When shading, it is necessary to make the area without two adjacent (*i.e.*, with a common boundary line segment) have the same color [6].

In 1852, Francis Guthrie, who graduated from London University, came to a scientific research unit to do map coloring work, and found an interesting phenomenon: each map can be colored with four colors, making countries with common borders different colors. Can this phenomenon be strictly proved mathematically? He and his younger brother, who was studying in college, decided



**Figure 6.** 4-color relationship.

to try, but there was a large stack of manuscript paper, but there was no progress in the research work. On October 23, 1852, his brother consulted his teacher, the famous mathematician De Morgan, about the proof of this problem. Morgan could not find a way to solve this problem, so he wrote to his friend, Sir Hamilton, the famous mathematician, for advice. But until Hamilton died in 1865, the problem could not be solved.

In 1872, Kelly, the most famous mathematician in Britain at that time, formally raised this question to the London Mathematical Society, so the four-color conjecture became a concern of the world's mathematical community. Many first-class mathematicians in the world participated in the four-color conjecture war.

Between 1878 and 1880, Alfred Kempe and Peter Guthrie Tait, famous lawyers and mathematicians, respectively submitted papers to prove the four-color conjecture and announced that they had proved the four-color theorem. Everyone thinks that the four-color conjecture has been solved since then, but actually Kemp has not proved the four-color problem. Eleven years later, in 1890, He-wood, 29, who was studying at Oxford University, pointed out the loopholes and counterexamples in Kemp's proof with his own accurate calculation. Soon Taylor's proof was also denied [7].

It is found that they actually proved a weak proposition, the Five Color Theorem. That is to say, five colors are enough for coloring the map. The bottom line is the nature of mathematicians. On the one hand, five colors are enough; on the other hand, there are indeed examples that show that three colors are not enough. So are four colors enough? With slow progress, people found that the four-color problem was unexpectedly difficult. Many people once published proofs or counterexamples of the four-color problem, but they were proved to be wrong. Later, more and more mathematicians struggled with this, but they got nothing. Therefore, people begin to realize that this seemingly easy topic is actually a difficult problem. Since the 20th century, scientists have been basically following Kemp's ideas in proving the four-color conjecture. The essence of the four-color conjecture is that it is impossible to construct five or more regions connected in pairs on a plane or sphere [8]. If there are more than five regions connected in pairs, the fifth region is at least the same color as one region. In 1913, the famous American mathematician, Berkov of Harvard University, used Kemp's idea to prove that some large configurations were reducible in combination with his new ideas. Later, American mathematician Franklin proved in 1939 that maps under 22 countries can be colored in four colors. In 1950, Winn advanced from 22 countries to 35 countries. In 1960, it was proved that maps under 39 countries can be colored with only four colors; then it was pushed to 50 countries. It seems that this progress is still very slow.

After the advent of the electronic computer, the process of proving the four-color conjecture has been greatly accelerated due to the rapid improvement of the calculation speed and the emergence of human-computer dialogue. In June 1976, American mathematicians Apel and Hacken spent 1200 hours on two

different electronic computers at the University of Illinois to make 10 billion judgments. As a result, none of the maps needed five colors. Finally, they verified the four-color conjecture and stirred the world. This is a great event that has attracted many mathematicians and mathematical enthusiasts for more than a hundred years. When the two mathematicians published their research results, the local post office stamped all mails sent on the same day with a special post-mark of “four colors enough” to celebrate the solution of this problem. The four color problem is the first theory mainly verified by computers. This verification is not accepted by all mathematicians, because it is not a mathematical proof of countless maps. People must fully trust all kinds of maps, the correctness of computer compilation and the hardware devices running this program? People always hope to prove the four color problem by mathematical method. But the proof does not stop. Computer proof cannot give a convincing thinking process.

For more than a century, mathematicians have racked their brains to prove the four-color conjecture, and the concepts and methods introduced have stimulated the development of topology and graph theory. In the research process of the “four color problem”, many new mathematical theories came into being and many mathematical calculation skills were developed. For example, turning the coloring problem of maps into a problem of graph theory enriches the content of graph theory [9]. Moreover, the “four color problem” has promoted engineering technology, computer technology, electronic circuit design and other aspects.

“I don’t know the true face of Lushan Mountain, but only because I am in this mountain”. From the theory of topology and graph theory, if I change my thinking—look for a method from plane geometry, I will be “a mountain heavy and a river full of doubts, and a village full of dark flowers”.

#### **4. New Application of the Four Color Theorem**

Sometimes, the influence of a theory goes far beyond its original intention. This situation is now applicable to the four-color theorem in the field of mathematics. This theory, which was used by the original generation of cartographers to draw maps hundreds of years ago, can now be used to understand the crystal structure and the magnetic properties of complex materials.

“Can you dye all maps with only four colors?” This proposition was first proposed by a British cartographer in 1852. His question was whether every map without enclaves (*i.e.*, two disconnected regions belong to the same country) could be colored with no more than four colors, and no two adjacent regions would have the same color? Surprisingly, the four-color problem is extremely difficult to verify. It was not until the 1970s that mathematicians were fully proved for the first time with the help of computers, and the four color problem eventually became the four color theorem, which was described as: if some adjacent finite regions were drawn on the plane, then no more than four colors could be used to color these regions, making every two adjacent regions different colors.

The four-color theorem has been considered to be quite limited in practical applications. But now it has new uses. According to the report of Physicists' Organization Network on June 10, a team of international researchers found that this mathematical theory can be used to understand the crystal structure and the magnetic properties of complex materials [10]. This group of members, including mathematicians, physicists and chemists from the United States, South Korea and Japan, proved for the first time that the configuration of domain structures can be understood from the theory of mathematics.

Researchers analyzed the new ferromagnet  $\text{Fe}_x\text{TaS}_2$ , which belongs to a class of materials called layered transition metal disulfide compounds (TMDs). Compared with graphene sheets, they are chemically versatile. In this material, the thin layer of layered transition metal tantalum disulfide is sandwiched with iron ions, which generates a new crystal structure in the upper part, changing the physical properties of the material.

In the experiment, the researchers inserted iron ions into the upper part of the structure in different order and form. It can be observed by transmission electron microscope (TEM) that different upper part of the structure produced very different crystal domain structure patterns. Crystal domains are local categories separated by interfaces in crystal structures. If these different types of domains are regarded as different countries on the map, and they can be colored according to the four-color theorem, then mathematically speaking, they can be "four-color". Researchers gave examples to illustrate its feasibility, and one of the complex special cases even needs to be applied to the special version of "three colors, two steps" derived from the four-color theorem. For example, among the three colors of red, blue and green, after coloring, the dark red area will never be adjacent to the light red area, nor to any dark area of blue and green.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References

- [1] Han, S.J. (2003) Four Color Conjecture. *Journal of Educational Research and Practice*, **11**, 140.
- [2] Han, S.J. (2020) Four Color Conjecture. Happy World, 2020-A-00021917, National Copyright Administration of the People's Republic of China.
- [3] Han, S.J. (2022) Using the Triangle Method for Map Coloring. The All Committee Meeting of the Professional Committee of Combinatorial Mathematics and Graph Theory of the Chinese Mathematical Society and the Tenth National Congress of Combinatorial Mathematics and Graph Theory, Harbin.
- [4] Han, S.J. (2009) Four Color Conjecture. <http://happy200810011.blog.sohu.com/117118828.html>
- [5] Zuo, X.L. (1987) Euler Diagram and Hamilton Diagram, Discrete Mathematics.



Shanghai Science and Technology Literature Press, Shanghai.

- [6] Zuo, X.L. (1987) Dual Graph and Coloring, Discrete Mathematics. Shanghai Science and Technology Literature Press, Shanghai.
- [7] Zuo, X.L. (1987) Method of Proving Hewood's Five Color Theorem, Discrete Mathematics. Shanghai Science and Technology Literature Press, Shanghai.
- [8] Zuo, X.L. (1987) Welch Powell Coloring Method, Discrete Mathematics. Shanghai Science and Technology Literature Press, Shanghai.
- [9] Zuo, X.L. (1987) Judgment Method of Plan and Non Plan, Discrete Mathematics. Shanghai Science and Technology Literature Press, Shanghai.
- [10] Zhang, M.R. (2014) New Application of the Four Color Theorem: It Can Analyze the Magnetic Properties of Crystal Structures and Complex Materials.  
<http://scitech.people.com.cn/n/2014/0611/c1007-25131879.html>