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Simplified Method of Stability Analysis of Nonlinear Systems without Using of Lyapunov Concept

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Abstract

In this paper a new simplified method of stability study of dynamical nonlinear systems is proposed as an alternative to using Lyapunov's method. Like the Lyapunov theorem, the new concept describes a sufficient condition for the systems to be globally stable. The proposed method is based on the assumption that, not only the state matrix contains information on the stability of the systems, but also the eigenvectors. So, first we will write the model of nonlinear systems in the state-space representation, then we use the eigenvectors of the state matrix as system stability indicators.

Keywords

Stability Criterion of Nonlinear Systems, Eigenvectors, State-Space Representation, Lyapunov Method

1. Introduction

For nonlinear systems, Lyapunov methods play a vital role in both stability analysis and control synthesis [1]. The Lyapunov method is made on two parts. The first part consists on the knowledge and exploitation of the state model of the nonlinear system under study. Then, we have to propose a candidate energy function [2] (Lyapunov function) which allows to prove the stability of the nonlinear system [3] [4]. The disadvantage of this method is that it does not provide the tools to build the Lyapunov's function. Despite this, the Lyapunov method is the most used for the stability analysis of nonlinear system [5] [6] [7].

In this paper, we propose a method of study of stability of nonlinear systems simpler than the Lyapunov method. The simplicity of this proposed method consists in the fact that it uses only the state model of the nonlinear systems, without the need to propose a candidate energy function to prove the stability of systems. Thus, by using this proposed method, the study of stability of nonlinear systems will become simpler and easier, and this, because we managed to circumvent the principal difficulty of using Lyapunov method.

The new method of stability study of nonlinear systems is based first:

1) On writing the model of nonlinear systems in state-space representation, and the deduction of eigenvectors of these systems. We must note here that, since the systems considered, are nonlinear, so, the eigenvectors will not contain constant values, but they contain terms that are functions of state variables.

2) Then, to prove the stability of systems, we make an assumption that, not only the state matrix contains information on the stability of systems, but the eigenvectors contain also information on the stability of these systems. Other research works say that, the eigenvector defines the meaning of evolution of the states of system [8].

Also, in this paper, we needed to introduce a new notion that we called a stable matrix point (SMP), and this, in order to describe the points that compose eigenvectors. The new proposed method should simplify the stability study of nonlinear systems, because it carries out the system stability study without the need to find a candidate energy function, as for the case of the Lyapunov method, which is the most inconvenient of the Lyapunov's method.

This paper is organized as follow. In Section 2, we present the new proposed method of the stability study of nonlinear systems. In Section 3, we prove the effectiveness of the new proposed method of checking of the stability of nonlinear systems, by the application of this method on several nonlinear systems given by their state representation. In Section 5 we give a conclusion of this work.

2. Simplified Method of the Stability Analysis

2.1. The State Model of Nonlinear Systems

The nonlinear systems under consideration in this paper have the following mathematical form [6]:

$$\dot{x} = f(x) + h(x)u \tag{1}$$

where: $x \in R^n$ is the state, $u \in R^p$ is the control input, $f : R^n \to R^n$.

And: $h : R^n \to R^p$ are Lipshiz functions such that: $f(0) = 0$ [9].

To simplify the calculus, we consider that:

$$h(x) \cdot u = u(x) \tag{2}$$

So we write the following model of nonlinear systems:

$$\dot{x} = f(x) + u(x) \tag{3}$$

Writing the physical formula of a system in the form of a state matrix is specific to linear systems. In this work, we will likewise write nonlinear systems in the state matrix representation [10], because, this will allow us to propose our new criterion for the stability study of nonlinear systems. However, writing a nonlinear system as a state-space representation is generally not used, we can write nonlinear system in state-space representation just according to the nonlinear system formula [10].

To explain the new proposed concept of stability study of nonlinear systems, we will take following example of second order nonlinear system:

To explain the new proposed concept of stability study of nonlinear systems, we will take following example of second order nonlinear system:

$$\textbf{Sys1:}\quad \begin{aligned} \dot{x}_1 &= -x_1^3 - x_2^2 \\ \dot{x}_2 &= x_1 \cdot x_2 - x_2^3 \end{aligned} \tag{4}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A(x)\cdot x = \begin{bmatrix} -x_1^2 & -x_2 \\ x_2 & -x_2^2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5}$$

with $A(x)$ is the state matrix of the nonlinear system, which is not made up of constant values (as for linear systems), but is made up of the state variables x_1 and x_2 Now, in the aim to define the stability of nonlinear systems, we will use the notion of eigenvectors [11] [12] [13].

We know that in automation field, the calculation of eigenvectors is done only for linear systems, not for nonlinear systems. Despite this, in this work, aim to define the stability of this system, we will propose to calculate the eigenvectors of nonlinear systems.

2.2. The Simplified Method of the Stability Study of Nonlinear Systems

In the aim to give the best explanation of the new proposed method of stability study of nonlinear systems, we will give the following Assumptions and Lemmas.

ASSUMPTION 1: This consists in considering that the matrix representation of nonlinear systems (like for the systems given by Equation (5)) contains the eigenvectors of the system.

Obviously, what we have just assumed in *Assumption 1* is neither always mathematically corrects nor verified. However we will give, an intuitive explanation on the concept by which we posed this supposition:

- First we talk about the eigen-values of linear and nonlinear [14] [15] [16] systems. The calculation of eigen-values makes it possible to study the stability of linear systems. The study of stability of nonlinear systems is more complex, it requires a linearization [17] [18] of the state matrix of the system, because, for nonlinear systems, the state matrix is composed of functions of state variables (not constant values as for linear systems).

- Now, we will talk about the eigenvectors (EV) of systems. The calculation of the eigenvectors is done by the calculation of the eigen-values. For example, for a second-order system, computing the eigen-values involves transforming the state matrix into a diagonal matrix whose second term and third term are zero.

So, Assumption 1 is only an approximation of the calculation of the eigenvectors of nonlinear systems, because the non-diagonal elements of the state matrices are not zero: Assumption 1 implies that we will not consider the true eigenvectors of the nonlinear system, and the eigen-vectors obtained by this assumption can be considered as offset from to the true eigen-vectors of the nonlinear system. So, we can call them: approximate eigenvectors (AEV).

- Now, we will also assume that in spite of the offset of the approximate eigenvectors compared to the real eigenvectors, the information on the behavior of the nonlinear system which exists in the real eigen-vectors, will be the same information which is in the eigenvectors.

- Although this first supposition lacks accuracy, but, thereafter, and through the application of Lemma1 to check the stability of different nonlinear systems, we can confirm that the approximate eigenvectors are a faithful reproduction of the true eigenvectors (concerning the description of the stability of nonlinear systems) since these approximate eigenvectors allow each time to verify the stability of nonlinear systems.

To simplify the drawing of this paper, we will simply use the term eigenvector (EV) to designate the approximate eigen-vector (AEV).

Assuming that the state matrix of the nonlinear system of Equation (5) is in eigenvector form, this implies that we have the following two eigenvectors $\boldsymbol{S}_1$ and; $\boldsymbol{S}_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{6}$$

Eigenvectors are: $\boldsymbol{S}_1 = \begin{bmatrix} -x_1^2 \\ x_2 \end{bmatrix}$, and $\boldsymbol{S}_2 = \begin{bmatrix} -x_2 \\ -x_2^2 \end{bmatrix}$

The state matrix $A(x)$ is composed of the two vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. This matrix defines the evolution of the system. We can deduce like shown in **Figure 1**, the resultant of these two vectors: $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$. We can say that this resultant vector represent totally the state matrix.

In general case, the state matrix of systems $A(x)$ contains information on the properties of the systems, in particular the stability of systems. For linear systems, eigenvectors are used to diagionize the state matrix of systems, while the eigenvalues provide information on the stability of linear systems [8]. On the other hand, we know that a eigenvector represents the direction of the state matrix, so the eigenvector also defines the meaning of evolution of the states of system. Thus, we are going to propose a second assumption concerning the relation between eigenvector of state matrix, and the stability of nonlinear systems.

ASSUMPTION 2: For any nonlinear system, if we manage to define the values of eigenvectors, this will allow us to have information on the stability of systems. Mathematically, we can express this assumption as:

For any second order nonlinear system (like for the system of Equation (4)), we have that, each of the two eigenvectors of the system is composed of two points, and, for a eigenvector to be a finite (stable), the two points that define

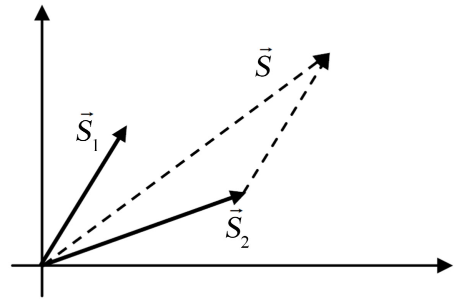

Figure 1. Eigenvector for state matrix.

this vector, must be finite points. Still considering that the matrix $A(x)$ mentioned above is composed of eigenvectors, now, we will write the elements of the state matrix as a state function. So, for nonlinear system of second order, we have:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1(x) & A_3(x) \\ A_2(x) & A_4(x) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (7)$$

Later, we will see that the points which define the eigenvectors will not be only finite (constant) values, but these are a state functions (depending on the state variable *x*). The new concept of stability study of nonlinear systems is finally given by following two lemmas.

LEMMA 1: (First case) Where the four elements $A_1(x)$, $A_2(x)$, $A_3(x)$, $A_4(x)$ are the elements that define the state matrix $A(x)$. To deduce the stability of the nonlinear system at origin ($x=(0,0)$), we must check that each of the four elements $A_1(x)$, $A_2(x)$, $A_3(x)$, $A_4(x)$ is either negative definite function, or is constant value. So, the principle of this lemma can be represented by the following **Figure 2**.

According *Lemma1*, we can propose the following arbitrary nonlinear system, with a state matrix composed of elements depending on state variables, and other elements which are constant values.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{V}_1(x) & cte \\ \dot{V}_2(x) & \dot{V}_3(x) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (8)$$

In Equation (8), the functions that make up the state matrix are mentioned with the derivative, to allude to Lyapunov's negative definite functions [14]. So, we can write:

$$\dot{V}_j(x) = \frac{\mathrm{d}V_j(x)}{\mathrm{d}t} \prec 0 \quad \text{with:} \quad j = 1,2,3,4 \quad (9)$$

we have $j = 1,2,3,4$ in Equation (9), because for a second order nonlinear system, the maximum number of negative definite functions we can have is 4. Obviously for a nonlinear system, the four elements of the state matrix cannot all be constant values, otherwise, the system would be a linear system.

Thus, for this nonlinear system to be stable, we have to prove that elements 1, 2 and 4 for the state matrix are negative definite functions, and that the third

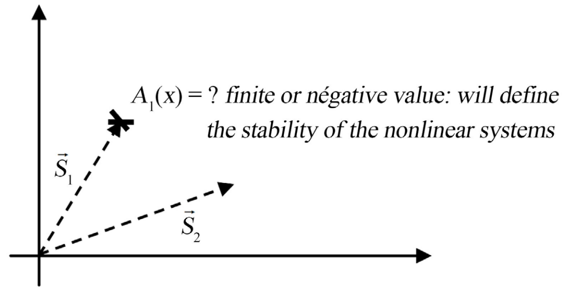

Figure 2. Eigenvectors define the stability of systems.

element of the state matrix is a constant negative or positive value. On the other hand, if we want to define the eigenvectors of Equation (6): $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are composed by points, which are negative definite functions, or constant values.

In this paper, we will purpose a new term to express the two possibilities for the elements of the state matrix $A(x)$ that give the stability of nonlinear systems (a constant value or a negative definite function). The new term is called a ***stable matrix points*** (SMP), and in the inverse case, ***unstable matrix points*** abbreviated by UMP. So, for example, and as for Lyapunov functions, a point $\dot{V}_2(x) = -x_1^2$ is a SMP, and $\dot{V}_1(x) = x_1^3$ is UMP.

As we will see later, when we will make the application of the new method of the stability study to verify the stability of several nonlinear systems, the proposed *Lemma 1* is not sufficient to solve the stability of all nonlinear systems.

In other way, as already mentioned, for certain nonlinear systems, it is necessary to check not the sign of the elements of eigenvector, but we must check the sign of the elements of the resultant of the eigenvectors. We give the following second Lemma.

LEMMA 2: (second case) In some case, to resolve the stability of some nonlinear systems, it is necessary to verify that the resultant vector $\boldsymbol{S}$ of vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, is composed of two stable matrix points SMP (which are either constant values or negative definite functions). Notice that in this second case, the summation of the two vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ must always begins with the second vector $\boldsymbol{S}_2$.

3. Application of the Proposed Method to Nonlinear Systems

Thus, for the first example of nonlinear system given by Equation (4), we can imagine that we have two eigenvectors: $\boldsymbol{S}_1$ which starts from a stable matrix point (SMP): $-x_1^2$ and goes towards the unstable matrix point (UMP): x_2 (x_2 cannot be a SMP because it is neither constant value or negative definite function).

Then the vector $\boldsymbol{S}_1$ will be added to the second vector $\boldsymbol{S}_2$ which starts from the UMP: $-x_2$, towards the SMP (negative definite function $-x_2^2$). Thus: $\dot{V}_1(x) = -x_1^2$, and $\dot{V}_2(x) = -x_2^2$ are two points that define two negative definite functions, like defined in Lyapunov's theory. For this first nonlinear system, we must make the two following remarks: * From *Lemma* 2, we conclude that the two vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are necessarily added together to obtain the resultant $\boldsymbol{S}$, and this, because there is a possibility (and need) to link these two vectors in order to prove the stability of the nonlinear system at origin: We have two stable matrix points, for each eigenvector, and one point in common, which is the UMP point x_2.

* We must note that we have reversed the vector $\boldsymbol{S}_2$, to have a common point x_2, thus, to obtain a resultant vector $\boldsymbol{S}$, which is a stable vector, because it starts from a SMP to another SMP. Thus, we arrive at our final goal, that of proving the stability of the nonlinear system given by Equation (4) at origin.

Remark: The most important remark to make, is that we proved the stability of the origin of the system of Equation (4), according to the method proposed in this paper, in a simple way and above all, without the need to propose an energy function, as in the case of the Lyapunov method. The stability study of the nonlinear system of Equation (4) at origin using the Lyapunov method, gives the same result with the following Lyapunov function candidate:

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right) \tag{10}$$

Example 2: Consider the following nonlinear system:

$$\textbf{Sys 2:} \quad \begin{aligned} \dot{x}_1 &= -x_1^2 + x_2^3 \\ \dot{x}_2 &= -2x_2^2 \cdot x_1 \end{aligned} \tag{11}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 & x_2^2 \\ -2x_2^2 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{12}$$

We obtain two eigenvector for this system:

$$\boldsymbol{S}_1 = \begin{bmatrix} -x_1 \\ -2x_2 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} x_2^2 \\ 0 \end{bmatrix}$$

From *Lemma* 2, we have that the eigenvector $\boldsymbol{S}_2$ starts from a SMP 0, and goes to the UMP: x_2^2. Then the eigenvector $\boldsymbol{S}_2$ will be added to $\boldsymbol{S}_1$ which starts from the point: $-2x_2$ (same that x_2^2 of $\boldsymbol{S}_2$) towards the UMP: $-x_1$. The evolution of the resultant $\boldsymbol{S}$ goes from vector $\boldsymbol{S}_2$, to vector $\boldsymbol{S}_1$. So, the resultant $\boldsymbol{S}$ goes from a SMP: 0, to an unstable point: $-x_1$ (or semi unstable point because: $-x_2$ is not included here). This implies that we have a local stability of the nonlinear system of Equation (11) at origin, because $-x_1$ which the first point of the vector $\boldsymbol{S}_1$ (and final point of resultant $\boldsymbol{S}$) is not an SMP. It can take positive or negative values. The stability study of the nonlinear system given by Equation (11) at origin, using Lyapunov method gives the same result: The local stability of this nonlinear system may be verified using following candidate Lyapunov function:

$$V(x) = x_1^2 + \frac{1}{2}x_2^2 \tag{13}$$

Example 3: Consider the third nonlinear system:

$$\textbf{Sys 3:} \quad \begin{aligned} \dot{x}_1 &= -x_1^3 + u \\ \dot{x}_2 &= x_1 \end{aligned} \tag{14}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1^2 & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{15}$$

We obtain the two eigenvector for this nonlinear system:

$$\boldsymbol{S}_1 = \begin{bmatrix} -x_1^2 \\ 1 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

From *Lemma* 2, the second eigenvector of this system is the vector $\boldsymbol{S}_2$ that starts from a SMP: 0, and goes to the SMP 0. Then, $\boldsymbol{S}_2$ will be added to $\boldsymbol{S}_1$ which starts from the point 1, towards the SMP $-x_1^2$. The stability study of this nonlinear system at origin using Lyapunov method gives the same result. The global stability may be obtained using following candidate function:

$$V(x) = x_1^2 + x_2^2 \tag{16}$$

Example 4: Consider the fourth nonlinear system:

$$\textbf{Sys 4:} \quad \begin{aligned} \dot{x}_1 &= -x_1 + x_2^2 \\ \dot{x}_2 &= -x_2 \end{aligned} \tag{17}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & x_2 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{18}$$

We obtain the two eigenvector for this system:

$$\boldsymbol{S}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} x_2 \\ -1 \end{bmatrix}$$

From *Lemma* 2, this system, $\boldsymbol{S}_2$ starts from a UMP: x_2 and goes to the stable point: −1. Then $\boldsymbol{S}_2$ will be added to the eigenvector $\boldsymbol{S}_1$ which starts from the point: −1 towards the SMP: 0.

For this example, the vector $\boldsymbol{S}_2$ starts from a UMP, and this directs by summation to the vector $\boldsymbol{S}_1$ towards a stable point: 0. This example can be solved by first and by second *Lemma*. We will assume that the resultant which is the vector $\boldsymbol{S}$ behaves exactly the same way as the nonlinear system. We conclude that this nonlinear system et globally stable at origin. The stability study of the nonlinear system at origin given by Equation (17) using Lyapunov method gives the same result: the global stability at origin may be obtained using following Lyapunov function:

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right) \tag{19}$$

Example 5: Consider the following example:

$$\textbf{Sys 5:} \quad \begin{aligned} \dot{x}_1 &= 2x_1 \cdot x_2^2 - 2x_1 \\ \dot{x}_2 &= -x_1 \cdot x_2^2 - x_2 \end{aligned} \tag{20}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 2x_1 \cdot x_2 \\ -x_1 \cdot x_2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{21}$$

Thus the eigenvectors which form the matrix $A(x)$ are:

$$\boldsymbol{S}_1 = \begin{bmatrix} -2 \\ -x_1 \cdot x_2 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} 2x_1 \cdot x_2 \\ -1 \end{bmatrix}$$

From *Lemma* 2, we have that the vector $\boldsymbol{S}_2$ starts from a SMP −1, and goes to the UMP $2x_1 \cdot x_2$. Then the vector $\boldsymbol{S}_2$ will be added to $\boldsymbol{S}_1$. The eigenvector $\boldsymbol{S}_1$ starts from the point $x_1 \cdot x_2$ towards the SMP: −2. The stability study of the nonlinear system of Equation (20) at origin using Lyapunov method gives the same result: The global stability may be obtained using following Lyapunov function.

$$V(x) = \frac{1}{2}\left(x_1^2 + 2x_2^2\right) \tag{22}$$

Example 6: Consider the following nonlinear system:

$$\textbf{Sys 6:} \quad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 - 2x_2 - 4x_1^3 \end{aligned} \tag{23}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 - 4x_1^2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{24}$$

We obtain two vectors for this nonlinear system:

$$\boldsymbol{S}_1 = \begin{bmatrix} 0 \\ -2 - 4x_1^2 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

From *Lemma* 1, So, we conclude that the vector $\boldsymbol{S}_2$ starts from a SMP: −2 and goes to the SMP: 1. Then $\boldsymbol{S}_1$ starts from the SMP: $-2 - 4x_1^2$ towards the SMP: 0.

For both eigenvectors we have only stables matrix points. Here, it is not necessary to add vector $\boldsymbol{S}_2$ to vector $\boldsymbol{S}_1$. The stability study of this nonlinear system at origin, using Lyapunov method gives the same result.

The global stability is verified using following Lyapunov function:

$$V(x) = 4x_1^2 + 2x_2^2 + 4x_1^4 \tag{25}$$

Example 7: Consider the following nonlinear system:

$$\textbf{Sys 7:} \quad \dot{x} = u(x) \cdot x \tag{26}$$

This nonlinear system is of first order. So, if we consider a state-space model, we have a state matrix with only one element.

$$u(x) = A_1(x) = \dot{V}_1(x) \tag{27}$$

Obviously, for this example there is no eigenvectors. In the aim to prove that the origin of this nonlinear system is a global stable point, the unique element of the state matrix must be negative definite function. So, we can choose:

$$u(x) = \dot{V}_1(x) = -x^2 \tag{28}$$

So, $\dot{V}_1(x) = -x^2$ is an SMP, and the global stability of this system at origin may be obtained using following Lyapunov function:

$$\dot{V}(x) = \frac{1}{2}x^2 \tag{29}$$

Similarly, the local stability of the origin can be obtained by a command:

$$u(x) = \dot{V}_1(x) = -x \tag{30}$$

Example 8: Consider now the following unstable nonlinear system:

$$\textbf{Sys 8:} \quad \begin{aligned} \dot{x}_1 &= 3x_2 \\ \dot{x}_2 &= -5x_1 + x_1^3 - 2x_2 \end{aligned} \tag{31}$$

Writing this system in state matrix form gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -5 + x_1^2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{32}$$

We obtain two vectors for this nonlinear system:

$$\boldsymbol{S}_1 = \begin{bmatrix} 0 \\ -5 + x_1^2 \end{bmatrix} \text{ and } \boldsymbol{S}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

From *Lemma* 1, So, we conclude that the vector $\boldsymbol{S}_2$ starts from a SMP: −2 and goes to the SMP: 1. Then $\boldsymbol{S}_1$ starts from the matrix point: $-5 + x_1^2$ towards the SMP: 0. So, we conclude that we have the stability of the nonlinear system with the condition that:

$$-5 + x_1^2 \le 0 \Rightarrow x_1^2 \le 5 \tag{33}$$

Here, it is not necessary to add vector $\boldsymbol{S}_2$ to vector $\boldsymbol{S}_1$. The stability study of this nonlinear system at origin, using Lyapunov method gives the same result. The global stability is verified using following Lyapunov function:

$$V(x) = 12x_1^2 - x_2^4 + 6x_1x_2 + 6x_2^2 \tag{34}$$

4. Conclusions

In this paper we proposed a new method of studies of stability of nonlinear systems. The new method is called simplified method of stability study of nonlinear system. This method is based on first writing the formula of nonlinear systems in the form of state matrix, then, we deduce the convergence of nonlinear systems by an evaluation of the eigenvectors of the state matrix of these nonlinear systems.

Also, we proposed a lemma to define the stability of systems directly from the obtained eigenvectors. To prove the effectiveness of this method, we applied this proposed method for the study of stability of six nonlinear systems. Results show that this method makes it possible to quickly deduce the stability of these systems.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Abbreviations and Acronyms

SMP: stable matrix point.

UMP: unstable matrix point.

EV: eigenvector.

AEV: approximate eigenvector.