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Quantum Gravity Based on Generalized Thermodynamics

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Abstract

This paper proposes a novel approach and simplified model of Quantum Gravity based on the unification framework of Generalized Thermodynamics which suggests cross-related terms and modified equations of General Relativity and Quantum Mechanics. To address the "background problem", a metric tensor is introduced into stationary Schrödinger equations via curved coordinates yielding quantum spacetime variation term. Then quantum Lagrangian is added to Einstein-Hilbert functional yielding quantum stress-energy tensor. Obtained from one variational principle, two theories are linked by a common quantum spacetime field. The theory offers some interpretations of the quantum vacuum spacetime fluctuations, zero-point-fields, quantum fields shifting towards high spacetime densities, the quantum nature of spacetime, and black hole singularity.

Keywords

Quantum Gravity, General Relativity, Quantum Spacetime, Generalized Thermodynamics, Unified Theory

1. Introduction

Quantum Gravity (QG), the theory pretending to combine General Relativity (GR) [1] [2] and Quantum Mechanics [3] [4] [5] [6] (QM), is included in the list of the most important unresolved problems in physics [7] with the following formulation: "*How can the theory of quantum mechanics be merged with the theory of general relativity/gravitational force and remain correct at microscopic length scales? What verifiable predictions does any theory of quantum gravity make?*" Another formulation includes the following [8]: "*Gravity is negligible in many areas of particle physics, so that unification between general relativity and* *quantum mechanics is not an urgent issue in those particular applications. However, the lack of a correct theory of quantum gravity is an important issue in physical cosmology and the search by physicists for an elegant 'Theory of Everything' (TOE). Consequently, resolving the inconsistencies between both theories has been a major goal of 20th- and 21st-century physics. This TOE would combine not only the models of subatomic physics but also derive the four fundamental forces of nature from a single force or phenomenon.*"

After a brief historical review, the goal of this work is to describe a novel approach to build equations of Quantum Gravity (QG) based on Generalized Thermodynamics (GT) which provides the framework for uniting fields in interconnected physical theories like thermo-elasticity, thermo-electricity, electromagnetism, quantum field theory, and others. For the last century, GT evolved into some kind of Theory-of-Everything based on the most common laws of nature capable to formulate unification principles also applicable to Quantum Gravity. After defining GT's extensive and intensive factors, energy, and state laws, GT suggests cross-related terms and modified GR and QM equations leading to a quite simple QG model, especially in stationary conditions. Modified GR equations include additional *quantum stress-energy tensor* reflecting quantum effects on spacetime while modified QM Schrödinger equations in curved spacetime include *GR metric tensor* reflecting the effect of curved spacetime onto quantum fields. Obtained from one variational principle, QM and GR become connected by a common *quantum spacetime field*. As discussed, the effect of a curved spacetime on quantum mechanical fields becomes significant only at high spacetime densities near black holes while quantum effects onto spacetime are significant for close-to-vacuum states.

The novelty of the described QG theory is that it is based on Generalized Thermodynamics (GT) which evolved from Classical Thermodynamics dealing with heat and entropy into a generic framework for uniting interconnected fields from different theories based on common GT laws. GT simply says that if some two theories—QM and GR in our case—are connected, there should be some cross-reference terms in state, energy, and transfer equations. So, curved GR spacetime should be introduced into QM Schrödinger equations while QM wave function should be introduced into Einstein's GR equations making the two theories connected. However, this GT unification idea requires a challenging procedure described here leading to the GTQG model which correlates with some known Lagrangian QG formulations observed hereafter.

1.1. Brief Historical Review of Quantum Gravity

This brief review of Quantum Gravity development [1]-[75] is based on more comprehensive reviews of Rovelli [6], Stachel [33], and Hamber [34] [35]. Quantum Gravity theory to combine GR with QM started in the 1920s soon after the onset of GR in 1915 and QM in 1926. Einstein himself in 1916 pointed out that quantum effects should lead to modifications of GR [6] [26]. The pioneering work on building a new theory has started by Klein [27], Rosenfeld [28], Fierz and Pauli [29] [30], Blokhintsev and Galperin [31], Bohr and Heisenberg [32], and by the 1940s the notions of "gravitons" and "quantum theory of gravitational fields" have become familiar concepts [6]. Early QG theories have been summarized in some comprehensive reviews by Stachel [33], Rovelli [6], and Hamber [34] [35]. There are multiple major streams of QG research [1]-[16], [27]-[62]—covariant QG formulation [27] [28] [29] [30] [31] [35], canonical QG [11] [24] [25] [34] [37], loop quantum gravity [6] [38] [53] [55] [62], Euclidean quantum gravity [34] [36] [37], supergravity [60], string [45] [46] [49] [56] [57] [58] [59], and quantum field theories [15] [16] [47] [50] [55] [64] [65].

The covariant line of research [6], [27]-[35], started by Rosenfeld, Fierz, and Pauli [27] [28] [29] [30] in the 1930s, assumes building QG using fluctuations of the metric over a flat Einstein-Minkowski space, or another metric. The Feynman rules of GR were laboriously found by DeWitt and Feynman in the 1960s [6] [11] [21] [24]. In the 1970s, firm evidence of a non-renormalizability problem [6] [34] [35] [61] has been found which triggered a search for an extension of GR for renormalizable or finite perturbation expansions [6] [34] [35] [61]. Through high derivative theory and supergravity [60], the search converged successfully [6] to string theory [45] [46] [49] [56] [57] [58] [59] in the 1980s.

The canonical QG stream [6] [11] [24] [25] [26] [34] [35] [36] [37] involves constructing QG based on the Hamiltonian formulation of GR without a background metric to be fixed. The program set by Bergmann, Dirac, Peres, Deser, and Misner in the 1950s and 1960s [6] [9] [11], has led to central Wheeler-DeWitt equations [1] [6] [11] [33] [34] [35] originally called by DeWitt as "Schrödinger-Einstein" equations in 1967. The Wheeler-DeWitt equation [11] [33] [34] [35], as a functional-differential equation on the space of three-dimensional spatial metrics, has the form of a Hamiltonian constraint operator acting on a wave functional. While important in theoretical physics, this equation was found as "too ill-defined" [6] [34] and complicated to do real calculations. Canonical Hamiltonian formulations are complicated not only for GR but also for QM equations which need to be rewritten in curved spacetime to address "the background problem" of QM [24] [25] where Schrödinger equations were originally formulated in flat spacetime. For example, the authors of a recent paper [25] provide a very complex Hamiltonian for Schrödinger equation in curved spacetime which includes many terms like wave function and metric tensors with their derivatives in complex-number coordinates. Also, canonical QG theories employ Feynman path integrals [21] [24] [35] [37] typically used to derive the QM Schrödinger equation from a variational principle. The need to address QM's "background problem" leads to even more complicated expressions for path integrals over curved spacetime.

There were multiple attempts to build simplified versions of QG like Euclidean Quantum Gravity [6] [34] [36] [37] based on Wick's rotation and related to ADM formalism finally leading to a generalization of Einstein-Hilbert functional by including a path integral of QM. This approach allowed S. Hawking to derive theoretically significant black hole radiation [6] [20] [36] [37] called "Hawking Radiation" nowadays. Hawking's result is not directly connected to quantum gravity—it is a skillful application [6] of quantum field theory in curved spacetime—but had a very strong impact on the field [6] [18] [34] [35] [43] [67] [68] leading to a new research stream in "black hole thermodynamics" reviewed in [6] [18] [43]. The research points out the existence of a general relationship between quantum theory, gravity, and thermodynamics [18] [43] [67] [68] also explored in this work. Later, Hartle and Hawking [69] introduce the notion of the "wave function of the universe" opening up a new intuition on quantum gravity and quantum cosmology. But the Euclidean integral [36] [37] did not provide a better way of computing field theoretical quantities than the Wheeler-DeWitt equation [6] [11]. Later, Hartle [70] and Isham [71] develop the idea of a sum over histories formulation of GR into an extension of quantum mechanics to the general covariant setting. But still, the Euclidean functional integral for field theoretical quantities was proved as "a weak calculation tool" [6] [34] as the Wheeler-DeWitt equation [11] [34].

Further development of canonical QG in the late 1980s has led to Loop Quantum Gravity (LQG) which claims to provide better calculation tools for quantum spacetime quantities [5] [6] [38] [39] [53] [54]. LQG, pioneered by Rovelli, Smolin, Ashtekar, and Jacobson [5] [6] [38] [39] [53] [54], is formally background-independent, leads to solutions in the form of intersecting loops and discrete spectrum of volumes correlating well with the quantum nature of spacetime in QG theories. LQG is based on a formulation where quantum GR equations are the Hamiltonian constraints [38] [53] [54] which are different from string and quantum field theories based on Lagrangians and involve many fields rather than only quantized gravity primarily focused in LQG. Other difficulties of LQG are related to problems of deriving standard GR equations as LQG semiclassical limit [38], as well as a reconciliation of the discrete combinatorial nature of quantum states with the continuous nature of spacetime in classical theories [38] [39].

An interesting interpretation of Quantum Gravity has been proposed in 1986 by R. Penrose [63] who suggested that the wave function collapse in quantum mechanics might be of quantum gravitational origin. This radical [6] idea implies rethinking the basis of QM and GR, but with the prospects of an experimental test. Other contributions of R. Penrose include twistor theory and spin networks [64] found some confirmations in loop quantum gravity later.

Other major physical theories—quantum field theory (QFT) [15] [16] [47] [50] [55] [64] [65] and string theory [45] [46] [49] [56] [57] [58] [59]—are also addressing the problem of quantum gravity from the generic theory of many fields. After Green, Schwarz, and Witten in the book on superstring theory [48] proclaim that strings might describe "our universe", the long search [6] for computable theory delivered perturbative string theory [48] and heterotic strings [46] in the 1980s. The central string theory assumption of ten dimensions that need to be reduced to four-dimensional QG was studied in terms of compactification on Calabi-Yau manifolds in [49] and has led to conformal field theory by Belavin, Polyakov, and Zamolodchikov [50]. Later, Witten introduced topological quantum field theory [51] [52] with Jones polynomials relevant to knot theory invariant and "loop transform" from loop quantum gravity [53] [54]. Enhanced by Atiyah [55], topological QFT strongly affected later QG development [6] [34]. General topological theories in any dimension have been introduced by Horowitz [56]. In the 1990s, the QFT has been expanded by Turaev, Viro, and Ooguri [57] [58] with the model of quantization of 2D gravity extended later to 4D. Then, n string theory, more complex branes, matrix models, and M-theory reviewed in [59] have emerged giving hopes of building a unique fundamental theory based on strings. In recent years, the new theory variants like supergravity and conformal theories [34] [35] [60] by Maldacena have merged into string theory leading to an explosion of interest [6] in the "holographic principle" promoted by Susskind [44] and t'Hooft [61]. Reconciliation of loop quantum gravity and string theory has been attempted by Smolin [62], but due to different fundamentals, the theories have kept their separation [6]. In general, the complexity of ten-dimensional string models and still undiscovered supersymmetric particles put some skepticism [6] on the completeness of background-independent theory based on strings. Nevertheless, QFT and string theory, along with loop quantum gravity, have emerged as the two main contenders [6] for truly fundamental Quantum Gravity theory as a part of a sought-after unified theory combining all fundamental fields of nature.

Apart from precise continuous QG formulation, there are also approximate semi-classical theories [6] [10] [12] [13] [14] [34] [35] [52] [53] [54] [62] [72] assuming constructing computable algorithms and some discretization of spacetime. A comprehensive review of discrete and continuous theories was provided by Hamber [34].

The author's contribution to Quantum Spacetime research includes the development since 2017 of Atomic Spacetime theory [10] [12] [13] [14] based on finite Atomic AString Functions and Atomic Solitons where using Atomization Theorems [14] spacetime and other fields are composed of flexible overlapping "solitonic atoms" resembling quanta also leading to the fields unification idea based on Atomic Solitons.

In summary, the outcome of the century-long QG journey can be summarized as a "work-in-progress", with a few supporting citations: "*Even though the predictions of both quantum theory and general relativity have been supported by rigorous and repeated empirical evidence, their abstract formalisms contradict each other and they have proven extremely difficult to incorporate into one consistent, cohesive model.*" [8] "*So, where we are after* 70 *years of research? There are well-developed tentative theories, in particular, strings and loops, and several other intriguing ideas. There is no consensus, no established theory…*" [6].

1.2. Brief Review of Generalized Thermodynamics as a Theory-of-Everything

This work offers a novel approach to build the QG model based on the unification framework of Generalized Thermodynamics (GT) [17] [18] [19] [40] [41] [42] [43] [44] [65]. Evolved from Classical Thermodynamics in some kind of Theory-Of-Everything, GT formulates the universal laws of nature and provides the framework for uniting fields in many physical theories dealing with inter-connected phenomena/fields like thermo-elasticity, thermo-electricity, electromagnetism, and quantum field theory. Classical Thermodynamics originated from the science of heat and thermal processes in XIX century by pioneering works of R. Clausius, L. Boltzmann, and W. Gibbs who formulated the first and second laws of thermodynamics introduced the core notion of entropy, reversible and irreversible processes, and corresponding variational principles [17] [18] [19] [40] [41] [42] [43] [44] [65]. Comprehensive reviews of thermodynamics are provided by Tolman [18], Lavenda [19], de Groot and Mazur [65], Veinik [17], and Eu [40]. Thermodynamical models have also been used for special and general relativity, with comprehensive reviews and theories of relativistic thermodynamics to cosmological models provided by Tolman [18]. For quantum gravity and black holes, thermodynamics has been used by Bekenstein [43], and Hawking [6] [36] [37] which resulted in the discovery of the black hole radiation effect using the concepts of black hole temperature and entropy. It is strongly impacted [6] not only quantum gravity studies but also leads to novel concepts of black hole information paradox and the "holographic principle".

In the late XX century, by the efforts of Onsager [66], Glansdorff, Prigogine [41], de Groot, Mazur [65], Lavenda [19], and Veinik [17] amongst others, classical Thermodynamics has evolved into *Generalized Thermodynamics* (GT) beyond the scope of thermal problems trying to formulate the universal laws applicable for all physical theories. GT assumes the separation of phenomena/fields into *extensors*, *intensials*, and energy [4] [17] [18] [19] [39] [66]. Along with energy, extensors (eq an electric charge, spin, momentum, mass) follow the laws of conservation, unlike intensials which define the intensity of fields (eq velocity, temperature, voltage, stress). Intensials depend on extensors via state laws [17] [18] [19] where the intensity of one field may depend on extensors of other fields, with the postulate that the state of a system is fully defined by the set of conservable extensors and energy. The evolution of a system in time leads to thermodynamic flows which obey Onsager's reciprocity principle discovered in the 1950s and express symmetric cross-influence between flows and thermodynamic forces [17] [18] [19] [40] [65] [66]. The principle of minimum entropy production [17] [18] [19], generalized in GT into the principle of least dissipation of energy [17] [18] [19] [65], leads to variational principles including Lagrangian formulations widely used in quantum field theories [2] [3] [4] [5] [6] [15] [16] [24] [50] [55] [64] [65].

A significant contribution to GT has been done by A. Veinik in the 1960s-

1990s summarizing many of his books in the final book [17] frequently cited in this paper. Apart from systematic descriptions of GT theories with seven generic laws, he pioneered some ideas of applicability of GT not only to conventional fields but also to spacetime itself proposing the candidates for intensials, extensors, and energy of space and time following conservation laws [17].

1.3. Quantum Gravity Unification Idea Based on Generalized Thermodynamics

The efforts of many scientists have expanded Classical Thermodynamics from the theory of heat to Generalized Thermodynamics as a "Theory-of-Everything" [15] [16] [17] [48] which provides the generic framework for the unification of multiple fields. Namely in this way, the GT is used in this work for uniting GR with QM into the GT variant of Quantum Gravity (GTQG). GT offers a relatively simple and intuitive framework that treats a quantum field (a field described by a QM wave function) in the typical GT way of conservable extensors and related intensials as any other GT field. Despite being probabilistic and uncertain, the quantum field possesses energy altering the fabric of spacetime hence it can be injected into Einstein's GR equation via stress-energy tensor. On the other side, strongly curved spacetimes should alter the quantum energy distributions, probabilistic wave-particle positions, trajectories, and results of quantum experiments. So, on the fundamental level, spacetime and quantum phenomena are deeply connected, hence should be suitable for GT unification principles.

The theory described here involves the identification of GT intensials and extensors, addressing the QM background problem by rewriting QM equations in curved space, and formulating the GT state, energy laws, and variational principles leading to modified Einstein's and Schrödinger equations. According to GT state and Onsager's reciprocity relations [17] [18] [19] [66], if two phenomena/fields are connected, there must be some cross-reference terms in energy and state equations as well as a common principle to derive linked equations. In application to QG where quantum and spacetime fields affect each other, it means that independent schematic equations $QM = 0$ and $GR = 0$ can be replaced by unified QG equations

$$QM + QMGR = 0, GRQM + GR = 0. \tag{1}$$

The cross-influence terms $QMGR$ and $GRQM$ reflect the connections between the theories. As described later, after addressing the QM background problem, GT suggests the following structure of the QG equations:

$$-\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi + V\psi + SQE\left(\psi, g_{\mu\nu}\right) = E\psi, \tag{2}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right). \tag{3}$$

The first equation is a modification of the time-independent Schrödinger equation of QM while the second one is a modification of Einstein's GR equation. They describe a common *quantum spacetime field* where GR affects QM via additional *curved spacetime quantum energy* $SQE(\psi, g_{\mu\nu})$ while QM affects GR via additional *quantum stress-energy tensor* $QSE_{\mu\nu}(g_{\mu\nu}, \psi)$. These equations are obtained from the one Lagrangian variational principle making QM and GR connected:

$$L(\psi, g_{\mu\nu}) = \int \sqrt{-g} \mathrm{d}^4 x \left(\mathcal{L}_{GR}(g_{\mu\nu}) + \mathcal{L}_{QM}(\psi, g_{\mu\nu}) \right); \ \delta L = 0. \tag{4}$$

This theory as a symbiosis of GT, GR, and QM may supplement QG research streams described above with GT field unification ideas. Based on Lagrangian formulation, it seems close to covariant QG [6] [27]-[35] and quantum field theories [15] [16] [47] [50] [55] [64] [65]. But regarding final equations, it seems related to canonical QG leading to some simplified variants of Wheeler-DeWitt equations [11] [34] [35] and Euclidean Quantum Gravity equations [34] [36] [37]. But instead of dealing with "canonical" Hamiltonians and path integrals, which become very complicated in curved spacetime, we use simpler Lagrangians leading to easy-to-comprehend combinations of familiar QM and GR functions especially simple in a stationary case.

The main goal of this work is to describe the challenging procedure of deriving simplified time-independent QG equations from the GT unification framework and provide an overview of potential applications of the theory to interpret the quantum nature of spacetime, quantum vacuum spacetime fluctuations [22] [23], quantum fields shifting towards high spacetime densities, atoms ionization near black holes, and black hole singularity [20] [25] [36] [37].

2. Generalized Thermodynamics

Generalized Thermodynamics (GT) [17] [18] [19] [40] [41] [42] [43] [44] [65] developed by L. Onsager, I. Prigogine, A. Veinik, and others as an extension of Classical Thermodynamics formulates the most common laws of nature and provides the framework for uniting fields in many physical theories like thermoelasticity, thermo-electricity, electromagnetism, and, as some kind of Theory-of-Everything, can be used for Quantum Mechanics (QM) and General Relativity (GR) too. In GT, the state of a body/system is completely defined by a set of *extensors* $E_i, i = 1, \cdots, n$ —conservable quantities, charges, or degrees of freedom (mass, momentum, spin, electric charge, etc.) as well as energy $U(E_i)$ as a universal measure of interactions between different fields. Due to fundamental laws of *energy and extensors conservation*, during an exchange, the energy and extensors of a system composed of m parts are conserved and follow the rule of addition:

$$E_i = E_{i1} + \cdots + E_{im}, \tag{5}$$

$$U = U_1 + \cdots + U_m. \tag{6}$$

Examples are conservation laws for momentum, mass, spin, electric charge, and energy. Supplying a portion of extensor $\mathrm{d}E_i$ to the body changes its energy (summation on indices assumed):

$$\mathrm{d}U = \frac{\partial U}{\partial E_i}\mathrm{d}E_i = P_i \mathrm{d}E_i;\ P_i = \frac{\partial U}{\partial E_i};\ [P_i] = \frac{Joules}{[E_i]}. \tag{7}$$

P_i are *intensials* [17], or intensities (velocity, pressure, temperature, voltage...), the units of which are determined by dividing the units of energy (Joules) and corresponding extensor (7). Body intensials $P_i(E_j)$ also depend on extensors E_j via *the state law* [17]

$$\mathrm{d}P_i = \frac{\partial P_i}{\partial E_j}\mathrm{d}E_j = a_{ij}(E_k)\mathrm{d}E_j, \tag{8}$$

with examples being the GR $T_{\mu\nu} = k\varepsilon_{\mu\nu}$ and elasticity theory $\sigma_{ij} = d_{ijkl}\varepsilon_{kl}$. *State coefficients* $a_{ij}(E_k)$ are symmetric expressing the principle that fields symmetrically affect each other:

$$a_{ij}(E_k) = a_{ji}(E_k) = \frac{\partial^2 U}{\partial E_i \partial E_j}. \tag{9}$$

For *ideal* systems [17] [18] [19] with constant a_{ij}, the Equations (6)-(8) become linear and lead to a familiar quadratic form for energy, which can be expressed either via extensors or intensials:

$$P_i = a_{ij}E_j;\ E_i = c_{ij}P_j;\ U = \frac{1}{2}\left(a_{ii}E_i^2 + 2a_{ij}E_iE_j\right);$$

$$U = \frac{1}{2}\left(c_{ii}P_i^2 + 2c_{ij}P_iP_j\right). \tag{10}$$

The examples are kinetic $U = mv_i^2$ and elastic energies $U = d_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$.

According to the *law of transfer* [17] [18] [19], the gradients δP_j of intensials between a body and environment create a *flow*/*flux* of matter/extensor J_i typically measured in relation to time:

$$\mathrm{d}E_i = \frac{\partial E_i}{\partial t}\mathrm{d}t = J_i\mathrm{d}t;\ J_i = b_{ij}(P_k)\delta P_j. \tag{11}$$

Balancing the influx and outflow of an extensor with changes in intensials produces different [17] dynamic equations between flows J_i and thermodynamical forces $X_j = \Delta P_j$ often expressed via Laplacians:

$$J_i = b_{ij}X_j;\ J_i = \rho\frac{\partial P_i}{\partial t};\ X_j = \Delta P_j;\ \rho\frac{\partial E_i}{\partial t} = b_{ij}\Delta P_j. \tag{12}$$

Examples are heat conductivity, electro-magnetism, diffusion, and quantum mechanics. As per Onsager's *reciprocity relations* [17] [18] [19] [66], the resistance, capacity, or conductivity coefficients b_{ij}, constant for linear systems, are also symmetric which reflects the mutual balance of forces:

$$b_{ij}(P_k) = b_{ji}(P_k). \tag{13}$$

This law important for QG tells that changes of one extensor, for example, deformations of spacetime, would engage other extensors, like momenta of a quantum particle, and the opposite. Together with state law (8), this reciprocity law (13) makes the fields and theories interconnected.

According to *the law of dissipation-screening* [17] [19], during the transfer of extensor $\mathrm{d}E_i$ caused by some gradient of intensials $\mathrm{d}P_i$, it is released (usually as heat) the energy of dissipation:

$$\mathrm{d}Q_d = -\mathrm{d}E_i \mathrm{d}P_i = -J_i \mathrm{d}t. \tag{14}$$

The tendency of nature's processes to minimize the wastage of energy $Q_d \to \min$ and to move in the direction of the least resistance leads to the well-known *least-action principle* [17] [19]. It minimizes some *action A* typically expressed via Lagrangian energy $L = P - K$ as a difference between potential P and kinetic K energies of a system with n fields:

$$A = L(P_i) = \int \mathrm{d}V\mathrm{d}t\left(\mathcal{L}_1(P_i) + \cdots + \mathcal{L}_n(P_i)\right);\ \delta L(P_i) = 0. \tag{15}$$

$\mathcal{L}_i$ are *energy densities* over some spacetime volume $\mathrm{d}V\mathrm{d}t$. The variational equation

$$\delta L(P_i) = \frac{\delta L(P_i)}{\delta P_k}\delta P_k = \int\left(\frac{\delta \mathcal{L}_1(P_i)}{\delta P_k} + \cdots + \frac{\delta \mathcal{L}_n(P_i)}{\delta P_k}\right)\delta P_k \mathrm{d}V\mathrm{d}t = 0 \tag{16}$$

produces the system of differential equations of transfer (12) and connects multiple fields:

$$\frac{\delta \mathcal{L}_1(P_i)}{\delta P_k} + \cdots + \frac{\delta \mathcal{L}_n(P_i)}{\delta P_k} = 0;\ \rho\frac{\partial P_i}{\partial t} - b_{ij}\Delta P_j = 0. \tag{17}$$

Examples are GR derived from the Einstein-Hilbert principle [1] [2] [21] [75], classical mechanics, and quantum field theories [15] [16] [47] [50] [55] [64] [65]. These GT laws offer a unified framework for linking field theories, including GR and QM.

3. Generalized Thermodynamics for Quantum Mechanics and General Relativity

QM is governed by the Schrödinger equation [3] [4] [9] for a complex wave function $\psi(x, y, z, t)$ of a moving particle in *flat* static spacetime under the influence of some energy potential $V(x, y, z)$:

$$i\hbar\frac{\partial \psi(x, y, z, t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(x, y, z, t) + V(x, y, z)\psi(x, y, z, t). \tag{18}$$

Time-independent $\psi(x, y, z, t) = \psi(x, y, z) f(y)$ eigenvalue equation for quantized energy E is [3] [4] [9]:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x, y, z) + V(x, y, z)\psi(x, y, z) = E\psi(x, y, z). \tag{19}$$

GR describes the shape/metric $g_{\mu\nu}$ of spacetime curved by stress-energy $T_{\mu\nu}$ and cosmological expansion with Einstein's curvature tensor $\varepsilon_{\mu\nu}$ expressible via Ricci tensor $R_{\mu\nu}$ and its invariant R:

$$\varepsilon_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu};\ \varepsilon_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{20}$$

GT, as a universal framework for connected physical fields, can be used to build a unified QG theory. But it possesses some challenges. QM and GR equations describe two different things: the probabilistic position of a quantum particle in *static* flat spacetime and the shape of the *dynamic* GR spacetime. This is the essence of the background problem [24] [25]—QM does not know what curved space is, while GR does not know what wave function is. Also, Schrödinger Equation (19), with potential energy *PE* and kinetic energy *KE*, is the manifestation of the law of energy conservation:

$$E\psi = PE(\psi) + KE(\psi);\ PE(\psi) = V\psi;\ KE(\psi) = -\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi. \quad (21)$$

But Einstein's GR Equation (20) is not related to energies and expresses different GT state law (8) connecting stress-energy and curvature tensors. To apply GT to the QM and GR unification problem, a deeper analysis is required.

First of all, we need to realize that QM and GR describe connected fields affecting each other. The peculiarity of a quantum phenomenon is that it introduces a probabilistic degree of freedom. Due to the uncertainty principle [3] [4] [9], we do not know the exact location of a particle that becomes a wave-particle represented by a superposition of quantum states. Nevertheless, any quantum system (hydrogen atom, particle-in-the-box, quantum vacuum) has energy and hence can alter spacetime via Einstein's stress-energy tensor. Also, the minimal energy of a quantum system is nonzero and defined by ground-state energy [3] [4] [9] which also can exert some pressure onto spacetime, especially with a vast amount of energy in the quantum vacuum of the universe [1] [23] [24]. Despite QM being the probabilistic theory, the quantum energy field is real and can affect spacetime while curved spacetime can also affect the probabilistic trajectories of quantum particles moving along the curved geodesics. So, QM and GR describe connected fields for which GT is well applicable.

Let us denote E_{QM}, P_{QM} to be extensors and intensials for QM, and E_{GR}, P_{GR} for GR. Extensors should obey the conservation law (11) by being able to add to each other:

$$E_{QM} = E_{QM1} + \cdots + E_{QM2},\ E_{GR} = E_{GR1} + \cdots + E_{GR2}. \quad (22)$$

Total energy (4) of quantum spacetime field $U(E_{QM}, E_{GR})$ should be fully defined by the set of extensors and variation of extensors $\mathrm{d}E_{QM}, \mathrm{d}E_{GR}$ should alter the energy:

$$U = U(E_{QM}, E_{GR});\ \mathrm{d}U = P_{QM}\mathrm{d}E_{QM} + P_{GR}\mathrm{d}E_{GR};\ P_{QM} = \frac{\partial U}{\partial E_{QM}},\ P_{GR} = \frac{\partial U}{\partial E_{GR}}. \quad (23)$$

Here, $P_{QM}(E_{QM}, E_{GR})$ and $P_{GR}(E_{QM}, E_{GR})$ are intensials which also should be fully defined by the set of extensors, or in the differential form (8):

$$\mathrm{d}P_{QM} = A_{QMQM}\mathrm{d}E_{QM} + A_{QMGR}\mathrm{d}E_{GR},\ \mathrm{d}P_{GR} = A_{GRQM}\mathrm{d}E_{QM} + A_{GRGR}\mathrm{d}E_{GR}. \quad (24)$$

In inverse form (10), this state equation can be rewritten as

$$\mathrm{d}E_{QM} = B_{QM}\mathrm{d}P_{QM} + B_{QMGR}\mathrm{d}P_{GR},\ \mathrm{d}E_{GR} = B_{GRQM}\mathrm{d}P_{QM} + B_{GRGR}\mathrm{d}P_{GR};$$

$$B = A^{-1}. \tag{25}$$

In this case, the total quantum spacetime energy (23) can be expressed in terms of intensials:

$$U = U\left(P_{QM}, P_{GR}\right); \mathrm{d}U = \frac{\partial U}{\partial P_{QM}} \mathrm{d}P_{QM} + \frac{\partial U}{\partial P_{GR}} \mathrm{d}P_{GR}. \tag{26}$$

For QM, there are two related candidates for conservable extensors E_{QM} — momentum p of a quantum wave-particle and energy E itself. Momentum p obeys the law of conservation (22) but does not explicitly reflect the specificity of a probabilistic quantum phenomenon of energy quantization. Instead, we can use some conservable quantum energy $E_{QM} = E$ and wave function–related factor ψ noting that they appear as cofactors in energy Equation (19). Because of units $[\psi] = \mathrm{m}^{-3/2}$, we can use $P_{QM} = \alpha[\psi]$ where α is some scaling factor $[a] = \mathrm{m}^{3/2}$ which does not affect QM Equations (18), (19), so we can often omit α.

For GR, the candidates for extensors that obey the law of summation (22) $\tilde{L} = \tilde{L}_1 + \tilde{L}_2 + \tilde{L}_3 + \cdots$, would be some lengths $\tilde{L}$ or displacements along curved geodesics. But they are not convenient to represent curved/deformed spacetime expressed in tensors (20). Instead, by analogy with close elasticity theory, it is possible to use some relative measures like metric $g_{\mu\nu}\left(x_n\right)$ widely used in GR, or more complex tensors related to deformations, like curvature tensor $\varepsilon_{\mu\nu}$ used by Einstein in GR equations in conjunction with stress-energy tensor $T_{\mu\nu}$:

$$\varepsilon_{\mu\nu} = kT_{\mu\nu}, \; k = \frac{8\pi G}{c^4}; \; \varepsilon_{\mu\nu} = R_{\mu\nu}\left(g_{ij}\right) - \frac{1}{2} R\left(g_{ij}\right) g_{\mu\nu}. \tag{27}$$

This makes GR equations similar to the elasticity theory where deformations ε_{ij} are also some complex derivatives of displacements u_i linearly related to stresses σ_{ij}:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\mathrm{d}u_i}{\mathrm{d}x_j} + \frac{\mathrm{d}u_j}{\mathrm{d}x_i} + \frac{\mathrm{d}u_k}{\mathrm{d}x_i}\frac{\mathrm{d}u_k}{\mathrm{d}x_j}\right), \; \sigma_{ij} = D_{ijkl}\varepsilon_{kl}. \tag{28}$$

Deformations $\varepsilon_{\mu\nu}$ are always expressible via metric tensor $g_{\mu\nu}$ (27), so energy variation equations can be expressed via $g_{\mu\nu}$ widely used in GR.

In summary, GR has the extensors $E_{GR} = \varepsilon_{\mu\nu}$ and intensials $P_{GR} = T_{\mu\nu}$, while the QM extensor is some energy $E_{QM} = E$ and the intensial is the wave function $P_{QM} = \psi$:

$$E_{GR} = \varepsilon_{\mu\nu}, \; P_{GR} = T_{\mu\nu}; \; E_{QM} = E, \; P_{QM} = \psi. \tag{29}$$

Identifying the extensors and intensials allows for applying the GT unification framework (5)-(17).

Matching GR and QM Equations (18)-(20) to GT Equations (5)-(17) yields the conclusion that QM expresses some energy conservation law (5) while GR expresses a different GT state Equation (25) $\mathrm{d}E_{GR} = B_{GRGR}\mathrm{d}P_{GR}$, $B_{GRGR} = k$. This is why linking QM and GR is so challenging. To unite two theories presumably describing connected fields, we have to express the equations in common terms of energies. Following (24)-(26) and separating full energy (23) into QM-only, GR-only, and mixed part

$$U = U\left(P_{QM}, P_{GR}\right) = U_{QM}\left(P_{QM}\right) + U_{GR}\left(P_{GR}\right) + U_{QMGR}\left(P_{QM}, P_{GR}\right), \tag{30}$$

suggests that GR fields can be injected into the QM Schrödinger equation via some additional *spacetime quantum energy* $SQE = U_{QMGR}\left(\psi, \varepsilon_{\mu\nu}\right)$ which can be expressed either via deformations $\varepsilon_{\mu\nu}$, related stress-energy $T_{\mu\nu} = k\varepsilon_{\mu\nu}$ or metric tensor ($\varepsilon_{\mu\nu} = R_{\mu\nu}\left(g_{ij}\right) - \frac{1}{2}R\left(g_{ij}\right)g_{\mu\nu}$):

$$E\psi = KE\left(\psi\right) + PE\left(\psi\right) + SQE\left(\psi, \varepsilon_{\mu\nu}\left(g_{ij}\right)\right),$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi + V\psi + SQE\left(\psi, g_{\mu\nu}\right). \tag{31}$$

Concrete expressions will be obtained later after reformulating QM in curved spacetime.

From the other side, the QM wave function can be injected into GR via the GT state Equation (25)

$$d\varepsilon_{\mu\nu} = B_{GRQM}\left(T_{\mu\nu}, \psi\right)d\psi + B_{GRGR}dT_{\mu\nu}, B_{GRGR} = k \tag{32}$$

yielding the following expressions:

$$\varepsilon_{\mu\nu} = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right); \ \varepsilon_{\mu\nu} + \gamma_{\mu\nu}\left(g_{\mu\nu}, \psi\right) = kT_{\mu\nu};$$

$$\gamma_{\mu\nu} = -QSE_{\mu\nu}/k. \tag{33}$$

QM effect onto spacetime can be expressed either via additional *quantum stress-energy tensor* $QSE_{\mu\nu}$ or via additional *quantum spacetime deformation* $\gamma_{\mu\nu}$. Physically it means that the quantum field can assert some additional pressure $QSE_{\mu\nu}$ onto spacetime causing it to curve more $\gamma_{\mu\nu}\left(g_{\mu\nu}, \psi\right)$ similar to the cosmological expansion $g_{\mu\nu}\Lambda$ pressure often included in GR:

$$\varepsilon_{\mu\nu} + g_{\mu\nu}\Lambda + \gamma_{\mu\nu}\left(g_{\mu\nu}, \psi\right) = kT_{\mu\nu}. \tag{34}$$

Concrete expressions for these factors will be obtained in the next sections.

Let us note that incorporating external fields via GT state equations is often used in other theories [17] [18] [19] [40], for example, in thermo-elasticity studying deformations of materials under temperature gradient:

$$\sigma_{ij} = D_{ijkl}\left(\varepsilon_{kl} - \alpha_{kl}\Delta T\right). \tag{35}$$

Exactly in the same way, Einstein introduced the cosmological expansion factor [1] [2] [26]:

$$\varepsilon_{\mu\nu} + g_{\mu\nu}\Lambda = kT_{\mu\nu}. \tag{36}$$

In summary, GT suggests the following form of united QG equations:

$$-\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi + V\psi + SQE\left(\psi, g_{\mu\nu}\right) = E\psi,$$

$$\varepsilon_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right). \tag{37}$$

It describes a united *quantum spacetime* field where GR affects QM via additional curved spacetime energy $SQE(\psi, g_{\mu\nu})$ while QM affects GR via additional pressure $QSE_{\mu\nu}(g_{\mu\nu}, \psi)$. Assuming that QM and GR describe the related phenomena, GT suggests the cross-influence factors between QM and GR. Injecting scalar QM functions into GR can be done by modifying state equations while injecting GR spacetime field into QM can be achieved by reformulating QM equations in curved coordinates. Let us note that GT mainly provides a generic framework to understand how the theories can be united. Concrete expressions can be obtained after a detailed analysis of field equations and variational principles described hereafter.

4. Lagrangian Quantum Gravity Formulation

GT provides some guidance on how QM and GR can be united by incorporating curved spacetime-related energy terms into the QM Schrödinger equation and wave-function-related deformation terms into GR state equations. To obtain concrete expressions, we have to apply more detailed variational equations derived from GT Lagrangian least-action principles (15)-(17).

If we introduce QM and GR symbolic operators as

$$QM(\psi) = -\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi + V\psi - E\psi;\ GR(g_{\mu\nu}) = \varepsilon_{\mu\nu}(g_{\mu\nu}) - kT_{\mu\nu}(g_{\mu\nu}), \quad (38)$$

the pair of standard classical QM and GR Equations (19), (20) can be rewritten in schematic form:

$$QM(\psi) = 0;\ GR(g_{\mu\nu}) = 0\,. \quad (39)$$

These are independent equations. But this is not an adequate description of physical quantum spacetime reality where quantum and spacetime fields affect each other. Instead, one can propose some connected equations with cross-reference terms:

$$QM(\psi) + QMGR(g_{\mu\nu}, \psi) = 0,\ GRQM(g_{\mu\nu}, \psi) + GR(g_{\mu\nu}) = 0. \quad (40)$$

Following GT framework (22)-(37), to incorporate a scalar QM wave function into GR tensor fields, it is necessary to bring both equations to the same energetic form expressing some generic guiding principle. The Lagrangian least-action principle formulated in scalar energies (15) is specifically designed for this. If we denote Lagrangians for QM and GR as L_{QM}, L_{GR}, assume they depend on both intensities $P_{QM} = \psi$, $P_{GR} = \varepsilon_{\mu\nu}(g_{\mu\nu})$ and can be split into specific and cross-reference terms, the total quantum spacetime Lagrangian can be written as

$$L = L_{QM}(\psi) + L_{GR}(g_{\mu\nu}) + L_{QMGR}(\psi, g_{\mu\nu}). \quad (41)$$

As per the least-action principle (16), its variation should be zero yielding the pair of equations:

$$\delta L = \left(\frac{\delta L_{QM}}{\delta\psi} + \frac{\delta L_{QMGR}}{\delta\psi}\right)\delta\psi + \left(\frac{\delta L_{GR}}{\delta g_{\mu\nu}} + \frac{\delta L_{QMGR}}{\delta g_{\mu\nu}}\right)\delta g_{\mu\nu} = 0;$$

$$\frac{\delta L_{QM}}{\delta \psi}+\frac{\delta L_{QMGR}}{\delta \psi}=0,\ \frac{\delta L_{GR}}{\delta g_{\mu\nu}}+\frac{\delta L_{QMGR}}{\delta g_{\mu\nu}}=0. \tag{42}$$

Comparing them to (40) leads to the following schematic equations:

$$QM+QMGR=0,\ GRQM+GR=0;$$

$$QM=\frac{\delta L_{QM}}{\delta \psi},\ QMGR=\frac{\delta L_{QMGR}}{\delta \psi},\ GR=\frac{\delta L_{GR}}{\delta g_{\mu\nu}},\ GRQM=\frac{\delta L_{QMGR}}{\delta g_{\mu\nu}}. \tag{43}$$

This unification idea stays true even if the clear split (41) is not achievable, eq when total energy is:

$$L=L_{QM}\left(\psi,g_{\mu\nu}\right)+L_{GR}\left(g_{\mu\nu}\right). \tag{44}$$

Here we have got:

$$\delta L=\left(\frac{\delta L_{QM}}{\delta \psi}\right)\delta\psi+\left(\frac{\delta L_{GR}}{\delta g_{\mu\nu}}+\frac{\delta L_{QM}}{\delta g_{\mu\nu}}\right)\delta g_{\mu\nu}=0;\ \frac{\delta L_{QM}}{\delta \psi}=0,\ \frac{\delta L_{GR}}{\delta g_{\mu\nu}}+\frac{\delta L_{QM}}{\delta g_{\mu\nu}}=0, \tag{45}$$

and a final pair of connected equations become:

$$QM\left(g_{\mu\nu},\psi\right)=0,\ GRQM\left(g_{\mu\nu},\psi\right)+GR\left(g_{\mu\nu}\right)=0. \tag{46}$$

Let us note that Lagrangians L_{GR} and L_{QM} are well-known or can be derived. For GR, it is known as Einstein-Hilbert action [2] [21] [75] with Ricci scalar R and metric tensor determinant g:

$$L_{GR}\left(g_{\mu\nu}\right)=\frac{1}{2k}\int R\sqrt{-g}\,\mathrm{d}^4x;\ \delta L_{GR}=0. \tag{47}$$

For QM, the generic Lagrangian is known as Dirac-Feynman path-integral [3] [4] [25], but for the time-independent QM Equation (19), it can be simplified to the form

$$L_{QM}\left(\psi\right)=\frac{\hbar^2}{2m}\left(\nabla\psi\right)^2+V\frac{\psi^2}{2}-E\frac{\psi^2}{2}. \tag{48}$$

It is extendable to the desired mixed form $L_{QM}\left(\psi,g_{\mu\nu}\right)$ after rewriting QM equations in curved space. Let us note that Lagrangian (48) is easier to rewrite in curved spacetime than Dirac-Feynman's path integral defined in complex space with a time dimension. Later it will be shown how to derive standard GR and QM equations from Lagrangians (47), (48). But to build a unified QG theory, it is possible to extend them with cross-reference terms which can be found after solving the QM "background problem" discussed next.

5. Addressing the Background Problem of Quantum Mechanics

QM is governed by the Schrödinger equation for a wave function $\psi\left(x,y,z,t\right)$ in *flat* static spacetime under the influence of energy potential $V\left(x,y,z\right)$:

$$i\hbar\frac{\partial\psi\left(x,y,z,t\right)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi\left(x,y,z,t\right)+V\left(x,y,z\right)\psi\left(x,y,z,t\right). \tag{49}$$

But static spacetime is not an adequate description of physical reality. In strong gravitational fields, the spacetime is curved, and all QM wave particles should be moving by the shortest path along geodesics in curved space $\tilde{x},\tilde{y},\tilde{z}$ rather than along the straight lines in flat spacetime x,y,z prescribed by the Schrödinger Equation (49). Actually, in QM a path is uncertain [3] [4] [11] [36], and we can only think about the probability of a movement along a path; nevertheless, QM contains derivatives over unphysical flat coordinates and needs to be improved.

5.1. One-Dimensional QM Equations in Curved Space

For a more adequate description, the QM equations have to be reformulated in curved coordinates:

$$i\hbar\frac{\partial\psi\left(\tilde{x},\tilde{y},\tilde{z},\tilde{t}\right)}{\partial\tilde{t}}=-\frac{\hbar^2}{2m}\tilde{\nabla}^{\mathrm{T}}\tilde{\nabla}\psi\left(\tilde{x},\tilde{y},\tilde{z},\tilde{t}\right)+V\left(\tilde{x},\tilde{y},\tilde{z}\right)\psi\left(\tilde{x},\tilde{y},\tilde{z},\tilde{t}\right);$$
$$\tilde{\nabla}^{\mathrm{T}}=\left(\frac{\partial}{\partial\tilde{x}},\frac{\partial}{\partial\tilde{y}},\frac{\partial}{\partial\tilde{z}}\right), \tag{50}$$

or in the 1D case

$$i\hbar\frac{\partial\psi\left(\tilde{x},\tilde{t}\right)}{\partial\tilde{t}}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi\left(\tilde{x},\tilde{t}\right)}{\partial\tilde{x}^2}+V\left(\tilde{x}\right)\psi\left(\tilde{x},\tilde{t}\right). \tag{51}$$

For an internal observer/experimenter in curved space $\tilde{x}$, the QM equations would describe the wavy distributions $\psi\left(\tilde{x}\right)$ of particle locations $\tilde{x}=\tilde{x}_0$ in a double-slit QM experiment, quantized energy levels in a hydrogen atom with function $\psi_n\left(\tilde{\rho}\right)=A_n\exp\left(-k_n\tilde{\rho}\right)\tilde{\rho}^m$, or eigenvalue modes for quantum particles-in-the-box solution [3] [4] [9]:

$$\psi_n\left(\tilde{x},\tilde{t}\right)=A\sin\left(k_n\left(\tilde{x}-\tilde{L}/2\right)\right)\mathrm{e}^{-i\omega_n\tilde{t}},\ k_n=n\pi/\tilde{L}. \tag{52}$$

In all of these equations, the background coordinates $\tilde{x},\tilde{\rho}$ are curved, a length $\tilde{L}$ is flexible—because according to GR spacetime and a ruler are flexible too.

Let us examine the 1D model of strongly curved space with known density $\rho\left(x\right)$ and metric of space $g\left(x\right)=\rho\left(x\right)^2$, for example, from Schwarzschild metric near a black hole $g_r\left(r\right)=\left(1-\frac{r_s}{r}\right)^{-1}$, $r=x$,

$$\rho\left(x\right)=\frac{\mathrm{d}\tilde{x}}{\mathrm{d}x};\ \mathrm{d}\tilde{x}^2=\rho\left(x\right)^2\mathrm{d}x^2=g\left(x\right)\mathrm{d}x^2;\ \tilde{L}=\int_A^B\mathrm{d}\tilde{x}\left(x\right)=\int_A^B\rho\left(x\right)\mathrm{d}x. \tag{53}$$

It allows transforming the curved QM equations and solutions (50)-(52) into flat spacetime *x*, which is required to match QM with other theories including GR. What were the straight lines or sinusoidal waves (52) in curved space would become squashed and stretched in flat spacetime, as shown in **Figure 1**. This is how GR affects QM—it defines background coordinates $\tilde{x}\left(x\right)$ where quantum mechanical and other physical fields are evolving. If spacetime is strongly uneven, it will affect the positions and trajectories of particles and the results of quantum physics experiments. This is a more adequate description of a dynamic spacetime reality rather than a flat QM Equation (49).

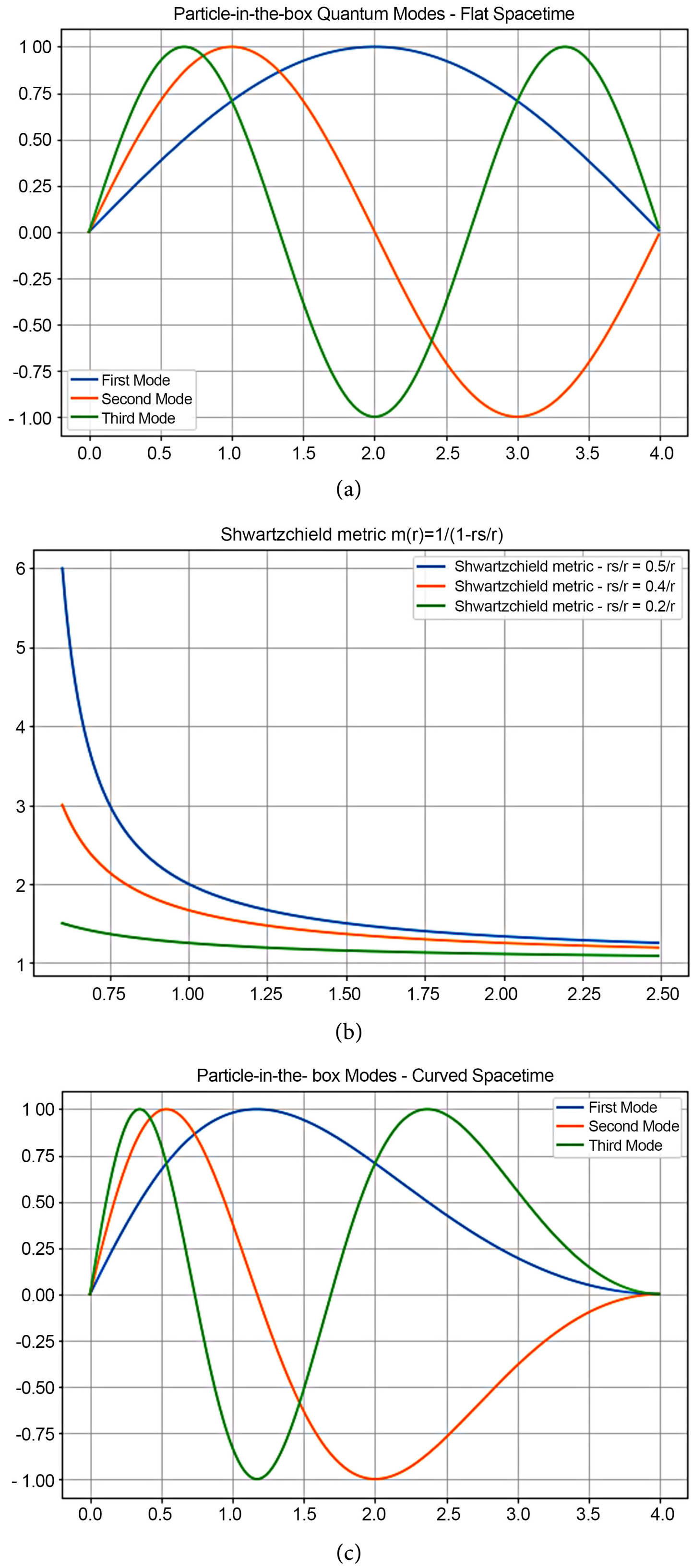

Figure 1. Three wavefunctions modes for particle-in-the-box (a) Flat spacetime; (b) Schwarzschild metric function; (c) Curved spacetime shifts pulses closer to denser spacetime zones.

Assuming that the spacetime density and metric (53) are known, let us rewrite the 1D curved QM Equation (51) for wave function $\psi(\tilde{x})=\psi(\tilde{x}(x))$ measured in static flat spacetime:

$$\frac{\mathrm{d}\psi}{\mathrm{d}x}=\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{x}}\frac{\mathrm{d}\tilde{x}}{\mathrm{d}x}=\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{x}}\rho(x);\ \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=\frac{\mathrm{d}^2\psi}{\mathrm{d}\tilde{x}^2}\rho^2(x)+\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{x}}\rho'(x)\rho(x). \tag{54}$$

Assuming for now that within the boundaries of a quantum system the metric change $\rho'(x)\sim 0$ and time dilation effects can be ignored $\tilde{t}=t$ yields the following equation:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\tilde{x}^2}\approx\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}\frac{1}{g(x)},\ g(x)=\rho^2(x). \tag{55}$$

Injected into (51), it leads to

$$i\hbar\frac{\partial\psi(\tilde{x}(x))}{\partial\tilde{t}}=-\frac{\hbar^2}{2m}\frac{1}{g(x)}\frac{\partial^2\psi(\tilde{x}(x),t)}{\partial x^2}+V(\tilde{x}(x))\psi(\tilde{x}(x)). \tag{56}$$

Denoting scalars as $V(\tilde{x}(x))=V(x),\psi(\tilde{x}(x))=\psi(x)$ yields 1D QMGR equation notably including GR metric function $g(x)$:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{1}{g(x)}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x)\psi(x,t). \tag{57}$$

Time-independent equation [3] [4] [9] can be obtained using typical substitution $\psi(x,t)=\psi(x)f(t)$ leading to the eigenvalue problem for total energy E:

$$-\frac{\hbar^2}{2m}\frac{1}{g(x)}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x). \tag{58}$$

This equation generalizes Schrödinger Equation (49) which becomes classical at $g(x)=1$. It describes the effect of gravity and curved spacetime on the QM field, for example, for particle-in-the-box solutions (52).

5.2. Metric Mass and Metric Wave Function

Interestingly, metric $g(x)$ becomes a cofactor of mass m allowing us to introduce a *metric mass function* $\tilde{m}(x)=mg(x)$ and relevant QMGR equation:

$$\tilde{m}(x)=mg(x);\ i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2\tilde{m}(x)}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x)\psi(x,t). \tag{59}$$

This mass becomes similar to variable relativistic mass from special relativity [1] [2]. It implies that at very dense spacetimes near black holes, the quantum wave-particles become "heavy" and hard-to-move, quantum wave-particle distribution effects become less pronounced, and particles are mostly localized rather than distributed, as shown in **Figure 1**. At $mg(x)\to\infty$ wave function $\psi(x)$ disappears from the QM Equation (58). But for low-density vacuum-like states $mg(x)\ll m$, quantum effects play a major role and particles behave like distributed wave clouds. This leads to another effect of gravity on QM. If for a hypothetical particle-in-very-long-box quantum experiment (52) the metric

$g(x)$ changes from low $g(x=0)=g_0$ to high $g(x=L)=ng_0$ level, the wave function would be wobblier at low g and more concentrated at high g implying a higher probability of finding a quantum particle at those locations (**Figure 1**). It seems like the wave function "gravitates" to higher-density spaces. As described later, it means the classic quantum double-slit experiment [3] [4] [5] [9] [25] between strong gravitational fields should show a tendency of probabilistic wave patterns to concentrate closer to heavier mass, which hopefully can be confirmed experimentally.

Another interesting expression can be obtained by multiplying the QMGR Equation (57) by $g(x)$:

$$i\hbar\frac{\partial g(x)\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x)g(x)\psi(x,t). \tag{60}$$

It leads to the appearance of a new function $g\psi$ which unites QM wave function $\psi(x,t)$ with unitless GR metric function $g(x)$ in one conglomerate:

$$g\psi(x,t)=g(x)\psi(x,t). \tag{61}$$

This *metric wave function* becomes "the bridge" between GR and QM linking the two theories together. For uniform spacetime $g=const$, it becomes the ordinary quantum wave function $\psi(x,t)$ while for uniform quantum field $\psi=1$ it becomes the ordinary metric function. There is another useful implication of metric wave function $g\psi(x,t)$—it gives a hint about how the QM wave function may be incorporated into Einstein's GR Equation (34) which explicitly contains metric tensor $g_{\mu\nu}$.

5.3. Quantum Mechanical Spacetime Variation Energy

QMGR Equation (60) can be also rewritten in the supplementary form assuming, like in covariant QG models [6] [27]-[35], $g(x)=1+\delta g(x)$:

$$\begin{aligned}&i\hbar\frac{\partial\psi(x,t)}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}-V(x)\psi(x,t)+i\hbar\frac{\partial\delta g(x)\psi(x,t)}{\partial t}\\&-V(x)\delta g(x)\psi(x,t)=0.\end{aligned} \tag{62}$$

The first term expresses the standard QM Schrödinger operator while the second one expresses the effect of "spacetime background" change δg to QM, or schematically:

$$QM(\psi)+QMGR(g,\psi)=0. \tag{63}$$

This exactly matches the representations (40) and (43) from GT and Lagrangian principles. Because the QM Schrödinger equation, especially in the form (58), manifests the law of energy conservation [3] [4] [9], it can be expressed in the form matching the GT representations (31), (37):

QM Kinetic Energy + QM Potential Energy + QM Space Variation Energy = QM Full Energy;

$$KE(\psi)+PE(\psi)+SQE(\psi,g_{ij})=E\psi, \tag{64}$$

The third term shows how curving of space adds extra *space quantization energy* $SQE(\psi, g_{ij})$ to QM supporting GT ideas (30), (40). This term can be easily understood intuitively—QM energy equations for a wave function in x, y, z, t have to be modified because x, y, z, t is not static anymore. Uneven spacetime should alter both quantum energy level distributions and concentrations of energies at certain spots (**Figure 1**), and additional energy is required to account for those effects. As described later, it also means that space quantization energy $SQE(\psi, g_{ij})$ near a black hole may be so dominant that it may significantly shift the quantized energy levels at atoms forcing some external electrons to escape and leaving atoms ionized hypothetically contributing to known Hawking's radiation effect [20] [36] [37]. In summary, in super-strong gravitational fields, the distributions of quantum energy states may be significantly altered [25].

5.4. Einstein's Relativity and Equivalence Principles for Quantum Mechanics

Concerning the additional energy seemingly appearing after switching from flat to curved spacetime coordinates, it seems important to reiterate Einstein's relativity principle. Additional energy term (64) is not just magically appearing in QM—it always exists because spacetime is curved and holds energy which needs to be accounted for in all equations dealing with spacetime. The curved QM equation is the reality while the classical QM equation with missing spacetime energy only approximates this reality. The energy statement (64) should be considered more fundamental while ignoring of spacetime energy variation term should be treated as a simplification of the real picture. This is similar to Newton's and Einstein's gravity laws. Einstein's law does not add energy, it always exists in curved spacetime. It is just when we ignore this energy, we come up with simplified Newtonian solutions.

In every curved coordinate system, the QM Schrödinger equation has the same form (50), and this is another manifestation of Einstein's relativity and the equivalence principle [1] [2]. For an internal observer, even near a black hole, the particle-in-the-box or double-slit quantum experiments would look the same and produce the same wave patterns in local coordinates. But external observers may notice the heavy concentration of quantum fields at denser spacetimes, slowness of time, and stretching of space because space itself is a dynamic fabric. This is reflected in curved QM equations.

It is also tempting to recall the famous John Wheeler's saying "*Spacetime tells matter how to move*; *matter tells spacetime how to curve*" [1]. This wisdom should be relevant not only to macro-matter but also to probabilistic QM wave-particle trajectories along the least-resistance geodesics, as well as to the results of quantum experiments in curved spacetime. So, spacetime should tell quantum wave-particles where to concentrate, especially near black holes. Quantum fluctuations also make spacetime wobbly.

6. Quantum Mechanical Background Problem in Three-Dimensional Space

The background problem of QM, when Schrödinger Equation (49) is formulated in flat static spacetime and does not account for the curvature of real space, can also be resolved in the 3D case (curvature of time is excluded for now) with the following procedure.

6.1. Quantum Mechanical Equations in Curved 3D Space

When particles are moved and distances are measured along the shortest paths of curved geodesics $\tilde{x}, \tilde{y}, \tilde{z}$, QM Equation (50) becomes

$$i\hbar \frac{\partial \psi(\tilde{x},\tilde{y},\tilde{z},\tilde{t})}{\partial \tilde{t}} = -\frac{\hbar^2}{2m} \tilde{\nabla}^{\mathrm{T}} \tilde{\nabla} \psi(\tilde{x},\tilde{y},\tilde{z},\tilde{t}) + V(\tilde{x},\tilde{y},\tilde{z}) \psi(\tilde{x},\tilde{y},\tilde{z},\tilde{t});$$

$$\tilde{\nabla}^{\mathrm{T}} = \left\{ \frac{\partial}{\partial \tilde{x}}, \frac{\partial}{\partial \tilde{y}}, \frac{\partial}{\partial \tilde{z}} \right\}. \tag{65}$$

In differential geometry [1] [2], curved coordinates $\tilde{x}(x,y,z), \tilde{y}(x,y,z), \tilde{z}(x,y,z)$ are related to flat coordinates x, y, z via Jacobian matrix $[J]$, and back via transpose matrix $[J^{\mathrm{T}}]$:

$$\begin{Bmatrix} \mathrm{d}\tilde{x} \\ \mathrm{d}\tilde{y} \\ \mathrm{d}\tilde{z} \end{Bmatrix} = \begin{bmatrix} \partial\tilde{x}/\partial x & \partial\tilde{x}/\partial y & \partial\tilde{x}/\partial z \\ \partial\tilde{y}/\partial x & \partial\tilde{y}/\partial y & \partial\tilde{y}/\partial z \\ \partial\tilde{z}/\partial x & \partial\tilde{z}/\partial y & \partial\tilde{z}/\partial z \end{bmatrix} \begin{Bmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{Bmatrix} = [J] \begin{Bmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{Bmatrix}, \quad \begin{Bmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{Bmatrix} = [J^{\mathrm{T}}] \begin{Bmatrix} \mathrm{d}\tilde{x} \\ \mathrm{d}\tilde{y} \\ \mathrm{d}\tilde{z} \end{Bmatrix}. \tag{66}$$

Derivation of the scalar function $\psi(\tilde{x},\tilde{y},\tilde{z})$ from (65) (ignoring presumably small second-order spacetime derivatives $\partial J_{ij}/\partial x_k$ within a quantum system) leads to

$$\tilde{\nabla}\psi = [J^{\mathrm{T}}] \nabla \psi; \tilde{\nabla}^{\mathrm{T}} \tilde{\nabla} \psi \approx \nabla^{\mathrm{T}} [J][J^{\mathrm{T}}] \nabla \psi = \nabla^{\mathrm{T}} [G]^{-1} \nabla \psi; [G]^{-1} = [J][J^{\mathrm{T}}]. \tag{67}$$

Here, $[G]^{-1}$ is the inverse metric tensor matrix (3 × 3) well-known as the multiplication of Jacobians [2] transforming flat to curved coordinates. This tensor is the inverse of the curvature tensor $[G]$ from 3D GR equations which do the opposite transformation. Noting (67) brings QM Equation (65) to the form:

$$i\hbar \frac{\partial \psi(x,y,z,t)}{\partial \tilde{t}} = -\frac{\hbar^2}{2m} \nabla^{\mathrm{T}} [G]^{-1} \nabla \psi(x,y,z,t) + V(x,y,z) \psi(x,y,z,t);$$

$$\nabla^{\mathrm{T}} = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}. \tag{68}$$

Because $[G]^{-1}$ is a matrix/tensor, $\nabla^{\mathrm{T}} [G]^{-1} \nabla$ cannot generally be brought to the form $\Delta\psi$ with Laplacian $\Delta = \nabla^{\mathrm{T}} \nabla$ appearing in classical 3D QM equations. But, similar to GR tensor equations, this seems correctly reflect the curved spacetime reality when spacetime derivatives may alter the perfection of Laplacian and Hamiltonian operators unknown in GR.

6.2. Metric Mass Tensor and Quantum Spacetime Variation Energy

Similar to the 1D case (59), it is possible to introduce *metric mass tensor* and relevant equation:

$$\tilde{m} = m[G];$$

$$i\hbar \frac{\partial \psi(x,y,z,t)}{\partial \tilde{t}} = -\frac{\hbar^2}{2} \nabla^{\mathrm{T}} [\tilde{m}]^{-1} \nabla \psi(x,y,z,t) + V(x,y,z)\psi(x,y,z,t). \quad (69)$$

It implies that at dense spacetime areas, the mass becomes "heavier" and harder to move, quantum wave-particle distribution effects become less pronounced, and wave-particles are mostly localized rather than distributed. At $\tilde{m} \to \infty$, $[\tilde{m}]^{-1} \sim 0$ wave function $\psi(x)$ disappears from Equation (69) implying that quantum effects in strong gravitational fields become negligible.

Assuming $[G]^{-1} = I + [\delta G]^{-1}$, with I being the identity matrix, brings QMGR Equation (68) to the schematic forms (43), (63) with additional terms reflecting the effect of curved space onto QM fields:

$$\left[i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^{\mathrm{T}} \nabla \psi - V\psi \right] + \left[\frac{\hbar^2}{2m} \nabla^{\mathrm{T}} [\delta G]^{-1} \nabla \psi \right] = 0;$$

$$QM(\psi) + QMGR(\psi, g_{ij}) = 0. \quad (70)$$

As we can see, 3D Equations (65)-(70) mostly correlate with simpler 1D QMGR interpretations (57)-(63) making them quite representative.

6.3. Deriving Quantum Mechanical Equations from QM Lagrangian

Let us note that classical time-independent Schrödinger equations can be formally obtained from Lagrangian variational principle with the following functional including energy density $\mathcal{L}_{QM}(\psi)$:

$$L_{QM}(\psi) = \int \mathcal{L}_{QM} \mathrm{d}^3 x; \ \delta L_{QM}(\psi) = 0;$$

$$\mathcal{L}_{QM}(\psi) = \frac{\hbar^2}{2m} \left(\nabla^{\mathrm{T}} \psi \nabla \psi \right) + V\psi^2 - E\psi^2. \quad (71)$$

Indeed, the QM equation follows from $\delta L_{QM}(\psi) = 0$ with transformations:

$$\begin{aligned} \delta L_{QM} &= 2\int \left[\frac{\hbar^2}{2m} (\nabla \psi) \delta \nabla \psi + V\psi \delta \psi - E\psi \delta \psi \right] \mathrm{d}^3 x \\ &= 2\int \left[-\frac{\hbar^2}{2m} \nabla^{\mathrm{T}} \nabla \psi + V\psi - E\psi \right] \delta \psi \mathrm{d}^3 x. \end{aligned} \quad (72)$$

Lagrangian for curved space yielding (65) has the same form expressed via curved coordinates:

$$L_{QM}(\psi) = \int \left[\frac{\hbar^2}{2m} \tilde{\nabla}^{\mathrm{T}} \psi \tilde{\nabla} \psi + V\psi^2 - E\psi^2 \right] \mathrm{d}^3 \tilde{x}; \ \delta L_{QM}(\psi) = 0. \quad (73)$$

After noting (67) and ignoring $\partial J_{ij} / \partial x_k$, it can be transferred to flat space:

$$L_{QM}(\psi,G)=\int \mathcal{L}_{QM}(\psi,G)\det(G)\mathrm{d}^3x;$$
$$\mathcal{L}_{QM}(\psi,G)\approx\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi[G]^{-1}\nabla\psi\right)+V\psi^2-E\psi^2. \tag{74}$$

Important to notice the appearance of the determinant $\det(G)$ of metric tensor making the integral invariant against coordinates' transformations as well as complex scalar $\nabla^{\mathrm{T}}\psi[G]^{-1}\nabla\psi$ linking together GR metric tensor and QM wave function ψ. Lagrangian density should have units of energy over some volume $[\mathcal{L}_{QM}]=\mathrm{J/m^3}$ which is correctly reflected in (74) noting that due to QM normalization of the wave function $\int\psi^2\mathrm{d}^3\tilde{x}=1$, $[\psi^2]=\mathrm{m}^{-3}$ and $[V\psi^2]=\mathrm{J/m^3}$.

Typically, Lagrangian should be considered in the spacetime of certain dimensionality, eq 3D for (74). For typical GR 4D spacetime, it needs to be integrated over 4D volume with the inclusion of typical GR term $\sqrt{-g}$ providing invariance [2] [4] over 4D coordinate transformations:

$$L_{QM}(\psi,G)=\int \mathcal{L}_{QM}(\psi,G)\sqrt{-g}\,\mathrm{d}^4x;$$
$$\mathcal{L}_{QM}(\psi,G)=\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi[G]^{-1}\nabla\psi\right)+V\psi^2-E\psi^2. \tag{75}$$

6.4. Spatially Isotropic Equations and Metric Wave Function

In a special case of spatially isotropic space with metric $[G]^{-1}=I/g$, where I is the identity matrix, is the 3D equations become simpler:

$$\mathcal{L}_{QM}(\psi,G)=\frac{\hbar^2}{2mg}\left(\nabla^{\mathrm{T}}\psi\nabla\psi\right)+V\psi^2-E\psi^2;$$
$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2mg}\nabla^{\mathrm{T}}\nabla\psi+V\psi,\ i\hbar\frac{\partial(g\psi)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi+Vg\psi. \tag{76}$$

Similar to the 1D case (61), the metric wave function $g\psi$ expresses the connection between GR and QM.

In summary, QM "background problem" can be solved for both 1D and 3D models by introducing curved spacetime which links QM to GR.

7. Introducing Quantum Mechanical Wave Function into General Relativity Equations

Addressing the QM background problem by introducing curved coordinates is only the first step toward unifying QM and GR. It was done by many authors before [11] [16] [25] but it does not complete the united QG theory. It only explains how the curved GR spacetime affects QM equations. But assuming that *quantum spacetime* is one inseparable field, we need to explore the opposite—how QM affects GR. Here, the universal GT concepts (4)-(37) become useful. GT, via state law, Onsager reciprocity principle, and energy Equations (8)-(26) states that if two connected fields are defined by two sets of extensors (23), there must be cross-influence terms and additional energies (30) reflecting those relations. It means if the GR field affects QM, the QM field should affect the GR field too. GR field has been introduced into GM (67) via inverse metric tensor $[G]^{-1}$ while the QM field should be introduced into GR via a scalar term (33) related to the scalar wave function. From GT (33), (37), we know the structure of the desired equation:

$$\varepsilon_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right). \tag{77}$$

Now the goal is to obtain concrete expressions for quantum stress-energy tensor $QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)$ reflecting the effect of QM fields on spacetime. Lagrangian formulations (41)-(45) can help with that.

7.1. Deriving General Relativity Equations from Einstein-Hilbert Lagrangian

First, let us evaluate how classic GR equations can be obtained from the Lagrangian variational principle, and the theory of this has been well-known [2] [21] [75] since 1915. This principle minimizes the action *S*, or Lagrangian $L_{GR} = T - V$ traditionally defined as a difference between kinetic T and potential energy V. For GR it has been known as the Einstein-Hilbert action [2] [21] [75]:

$$L_{GR}\left(g_{\mu\nu}\right) = \frac{1}{2k} \int R\sqrt{-g}\,\mathrm{d}^4 x;\ \delta L_{GR} = 0. \tag{78}$$

Here, $k = \frac{8\pi G}{c^4}$, R—Ricci scalar, $g = \det\left(g_{\mu\nu}\right)$—the determinant of matrix components of the metric tensor $g_{\mu\nu}$ from GR Equation (20). Extending this action with some external Lagrangian density $\mathcal{L}_T$

$$L_{GR}\left(g_{\mu\nu}\right) = \int\left(\frac{1}{2k} R + \mathcal{L}_T\right)\sqrt{-g}\,\mathrm{d}^4 x;\ \delta L_{GR} = 0 \tag{79}$$

allows obtaining all GR equations by following a well-known procedure [2] [21] [75] for finding L_{GR} variations for the metric tensor $g_{\mu\nu}$:

$$\delta L_{GR} = \int\left[\frac{1}{2k}\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{2k}\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} + \frac{\delta L_T}{\delta g^{\mu\nu}} + \frac{L_T}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}\right]\delta g^{\mu\nu}\sqrt{-g}\,\mathrm{d}^4 x \tag{80}$$

Considering Ricci and metric tensor properties [2] [21] [75]

$$\frac{\delta R}{\delta g^{\mu\nu}} = R_{\mu\nu};\ \frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2} g_{\mu\nu} \tag{81}$$

and introducing stress-energy tensor

$$T_{\mu\nu} := \mathcal{L}_T g_{\mu\nu} - 2\frac{\delta \mathcal{L}_T}{\delta g^{\mu\nu}}, \tag{82}$$

from variation $\delta L_{GR} = 0$ it is possible to obtain the desired Einstein's GR equation [75]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu};\ k = \frac{8\pi G}{c^4}. \tag{83}$$

This known procedure suggests the way how an external field (electromagnetic, quantum, cosmological, etc.), with some Lagrangian density $\mathcal{L}_T$ can be injected into GR equations. Namely in this way by adding a constant $-\Lambda/k$ into (79), Einstein introduced the cosmological expansion factor:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{84}$$

7.2. Adding Quantum Lagrangian to Einstein-Hilbert Functional

Einstein-Hilbert least-action principle, also discussed in §4, suggests a way how to inject the QM field into GR equations. We need to get QM Lagrangian density $\mathcal{L}_{QM}\left(\psi, g_{\mu\nu}\right)$ (75) and add it to Einstein-Hilbert functional (79):

$$L = \int \left(\frac{1}{2k}(R - 2\Lambda) + \mathcal{L}_T + \mathcal{L}_{QM} \right) \sqrt{-g}\,\mathrm{d}^4 x;\ \delta L = 0. \tag{85}$$

Let us note that in Quantum Field Theory [15] [16] [47] [50] [55] [64] [65], the external to GR Lagrangians $\mathcal{L}_{em}$ are typically inserted with a minus sign, eq $R/2k - \mathcal{L}_{em}$, but they are negative as well $\mathcal{L}_{em} < 0$, so the overall sign would be positive. So, in (85) we would safely assume that $\mathcal{L}_{QM}$ is positively defined in (75).

Following GT procedure §4, we can represent the total Lagrangian of a quantum spacetime field as the sum of GR and QM parts:

$$L\left(\psi, g_{\mu\nu}\right) = L_{GR}\left(g_{\mu\nu}\right) + L_{QM}\left(\psi, g_{\mu\nu}\right). \tag{86}$$

This Lagrangian depends on two field variables $\psi, g_{\mu\nu}$. Variation regarding ψ should yield the curved QM Schrodinger equitation while variation for $g_{\mu\nu}$ the desired GR equation with some addition reflecting Quantum Gravity:

$$\delta L = \left(\frac{\delta L_{QM}}{\delta \psi} \right) \delta\psi + \left(\frac{\delta L_{GR}}{\delta g^{\mu\nu}} + \frac{\delta L_{QM}}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0,\ \frac{\delta L_{QM}}{\delta \psi} = 0,\ \frac{\delta L_{GR}}{\delta g^{\mu\nu}} + \frac{\delta L_{QM}}{\delta g^{\mu\nu}} = 0; \tag{87}$$

$$QM\left(g_{\mu\nu}, \psi\right) = 0,\ GR\left(g_{\mu\nu}\right) + GRQM\left(g_{\mu\nu}, \psi\right) = 0; \tag{88}$$

$$QM = \frac{\delta L_{QM}}{\delta \psi},\ GR = \frac{\delta L_{GR}}{\delta g^{\mu\nu}},\ GRQM = \frac{\delta L_{QM}}{\delta g^{\mu\nu}}. \tag{89}$$

$L_{GR}\left(g_{\mu\nu}\right)$ is Einstein-Hilbert action (79) while $L_{QM}\left(\psi, g_{\mu\nu}\right)$ has been obtained in QM (75):

$$L_{QM}\left(\psi, g_{\mu\nu}\right) = \int \mathcal{L}_{QM}\left(\psi, g_{\mu\nu}\right) \sqrt{-g}\,\mathrm{d}^4 x;$$

$$\mathcal{L}_{QM} := (V - E)\psi^2 + \Upsilon\left(g_{\mu\nu}, \psi\right). \tag{90}$$

$\mathcal{L}_{QM}$ is the invariant Lagrangian energy density properly measured in Joules over volume (due to wave function normalization rule $\int \psi^2 \mathrm{d}V = 1$, ψ^2 has inverse units of volume).

7.3. Deriving Quantum Gravity Equations and the Quantum Stress-Energy Tensor

So, to accommodate QM, Einstein-Hilbert functional can be extended in the following way:

$$L\left(\psi, g_{\mu\nu}\right)=\int\left(\left[\frac{1}{2k}R\left(g_{\mu\nu}\right)+\mathcal{L}_T\left(g_{\mu\nu}\right)\right]+\left[V\psi^2-E\psi^2\right]\right.$$
$$\left.+\left[\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi\left[G\right]^{-1}\nabla\psi\right)\right]\right)\sqrt{-g}\mathrm{d}^4x. \tag{91}$$

The last scalar term explicitly depends on both spacetime $g_{\mu\nu}$ and wave function ψ:

$$\Upsilon\left(g_{\mu\nu},\psi\right)=\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi\left[G\right]^{-1}\nabla\psi\right). \tag{92}$$

Variation of it regarding ψ according to QM formulation (72) yields the curved derivative:

$$\delta\Upsilon\left(\psi, g_{\mu\nu}=const\right)=-\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\left[G\right]^{-1}\nabla\psi\right)\delta\psi. \tag{93}$$

Let us now vary the full $L\left(\psi, g_{\mu\nu}\right)$ (91) in terms of ψ at $g_{\mu\nu}=const$:

$$\frac{\delta L\left(\psi, g_{\mu\nu}\right)}{\delta\psi}\delta\psi=\frac{\delta L_{QM}}{\delta\psi}\delta\psi$$
$$=2\int\left(\left[V\psi-E\psi\right]-\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\left[G\right]^{-1}\nabla\psi\right)\delta\psi\sqrt{-g}\mathrm{d}^4x. \tag{94}$$

Demanding $\frac{\delta L_{QM}}{\delta\psi}=0$ as per (87) yields the desired curved QM equation:

$$\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\left[G\right]^{-1}\nabla\psi+V\psi=E\psi\,. \tag{95}$$

Next, let us vary $L\left(\psi, g_{\mu\nu}\right)$ in terms of $g_{\mu\nu}$ at $\psi=const$. As per (80)-(84), it yields the desired GR equation accounting for QM:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=k\left(T_{\mu\nu}+QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right)\right), \tag{96}$$

$$QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right):=\mathcal{L}_{QM}\,g_{\mu\nu}-2\frac{\delta\mathcal{L}_{QM}}{\delta g^{\mu\nu}}, \tag{97}$$

$$\mathcal{L}_{QM}:=\left(V-E\right)\psi^2+\Upsilon\left(g_{\mu\nu},\psi\right), \tag{98}$$

$$\Upsilon\left(g_{\mu\nu},\psi\right):=\frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi\left[G\right]^{-1}\nabla\psi\right). \tag{99}$$

Quantum stress-energy tensor $QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right)$ is introduced in the same way as Einstein did for the stress-energy tensor $T_{\mu\nu}:=\mathcal{L}_T g_{\mu\nu}-2\frac{\delta\mathcal{L}_T}{\delta g^{\mu\nu}}$ (82) defined by some external factors with known Lagrangian density $\mathcal{L}_T$. Interestingly, the new GR Equation (96) can be rewritten in terms of additional spacetime curvatures added to Einstein's cosmological constant Λ:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+g_{\mu\nu}\left(\Lambda+\frac{\mathcal{L}_{QM}}{k}\right)=k\left(T_{\mu\nu}-2\frac{\delta\mathcal{L}_{QM}}{\delta g^{\mu\nu}}\right);\ k=\frac{8\pi G}{c^4}. \tag{100}$$

It means quantum effects may act as some expansions and contractions of spacetime. Also, interesting to observe the appearance of ψ^2 which in QM reflects the probability density of finding a quantum particle at a certain location [3] [4] [9]. QG Equations (96)-(99) also include two external factors—one is Einstein's stress-energy tensor $T_{\mu\nu}$ defining the "background" metric of spacetime (for example near a black hole) and another one is QM potential $V(x,y,z)$ affecting quantum wave-particles distribution (for example, inside a hydrogen atom). The QG Equation (95) also includes quantized/discrete energy E typically calculated in classical QM which now needs to be altered to account for curved spacetime. It means that on the fundamental level, spacetime may have some discrete component, as discussed later. Scalar quantum spacetime factor $\Upsilon(g_{\mu\nu},\psi)$ uniting GR metric tensor and QM wave function shows that dense spacetimes at large $g_{\mu\nu}$ act as an additional metric mass $mg_{\mu\nu}$ and suppress quantum effects.

8. Main Quantum Gravity Model Equations

In summary, derived from the unification principles of Generalized Thermodynamics and common variational principle, the following pair of QG equations unite Schrödinger's and Einstein's equations:

$$\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}[G]^{-1}\nabla\psi+V\psi=E\psi\,, \tag{101}$$

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+g_{\mu\nu}\Lambda=k\left(T_{\mu\nu}+QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right)\right). \tag{102}$$

These equations obtained using the unification framework of General Thermodynamics, and logically to be called GTQG equations, are the main result of this work. GR curved spacetime is included in QM via space components of the inverse metric tensor $[G]^{-1}$ while QM included in GR via additional quantum stress-energy tensor $QSE_{\mu\nu}(g_{\mu\nu},\psi)$ making two theories connected. The extensions of these equations for time dependency and other factors are discussed in §10. The next chapters provide interpretations of these GTQG equations.

9. One-Dimensional Quantum Gravity Equations

1D GTQG equations, for example, describing simple quantum particle-in-the-box QM model [3] [4] [9] in 1D curved space $\tilde{x}(x)$ with metric $\mathrm{d}\tilde{x}^2=g(x)\mathrm{d}x^2$, allows evaluating QG equations in the simplest forms. Curved QM Equation (101), also discussed in §5, becomes:

$$-\frac{\hbar^2}{2m}\frac{1}{g(x)}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x). \tag{103}$$

For GR Equations (102), (96)-(99), we have:

$$\Upsilon(g,\psi)=\frac{\hbar^2}{2mg}\left(\frac{\partial\psi(x)}{\partial x}\right)^2;\ \ \mathcal{L}_{QM}(g,\psi)=\Upsilon(g,\psi)+\left(V(x)-E\right)\psi^2. \tag{104}$$

So, the additional stress that a quantum wave particle with mass m exerts on spacetime would be:

$$QSE_{\mu\nu}(g,\psi) := \mathcal{L}_{QM} g_{\mu\nu} - 2\frac{\delta\mathcal{L}_{QM}}{\delta g^{\mu\nu}} = \Upsilon(g,\psi)g + (V(x)-E)\psi^2 - 2\frac{\delta\mathcal{L}_{QM}(g,\psi)}{\delta g}. \tag{105}$$

Let us note that in curved space, we need to be mindful of what x, V, and ψ mean in these equations. As described in §5, for an internal observer in curved space $\tilde{x}(x)$, the curved QM Equation (51) is formulated for the "local" wave function $\psi(\tilde{x})$ influenced by some local external factor $V(\tilde{x})$. But when we translate QM functions like (52) into flat spacetime x, where we would like to unite QM with GR or other theories, we need to treat these functions as complex ones $V(\tilde{x}(x)) = V(x)$, $\psi(\tilde{x}(x)) = \psi(x)$ along with derivatives $\frac{\partial^2\psi(\tilde{x})}{\partial\tilde{x}^2}$. This leads to (104), but we should understand that $V(\tilde{x}(x))$ and $\psi(\tilde{x}(x))$ may also depend on metric function $g(x)$ which is hidden in $\tilde{x}(x)$. Hence, we should not assume that the derivative $\frac{\delta\mathcal{L}_{QM}}{\delta g^{\mu\nu}}$ in (105) would only depend on $\Upsilon(g,\psi)$.

Quantum stress-energy tensor $QSE_{\mu\nu}(g,\psi)$ (105) depends on probability density ψ^2 of finding a particle at a certain location, the squared speed of charge of the wave function $\Upsilon(\psi)$ like some kinetic energy, external QM potential $V(x)$, discrete quantum energy levels E, and other terms. The higher mass and density of space $g(x)$, the less pronounced the quantum effects because $g(x)$ acts as an additional metric mass $mg(x)$ making a particle "heavier". The lower metric mass $mg(x)$, the more distinct quantum fluctuations which are especially significant for quantum vacuum states discussed later.

10. More Generic Quantum Gravity Model Equations

Main GTQG Equations (96)-(102) have been obtained for time-independent QM equations with the assumption that metric change can be ignored within the boundaries of a quantum system. While it is sufficient to build a common basic QG theory, let us evaluate how they can be generalized.

The first enhancement would be to account for the variability of metric $g_{\mu\nu}$ within the size of a quantum system when second derivatives in curved space $\partial^2\psi/\partial\tilde{x}_i^2$ need to be translated into flat derivatives with Jacobian $\mathrm{d}\tilde{x}_i = J_{ij}\mathrm{d}x_j$ and metric $g_{\mu\nu} = \frac{\partial\tilde{x}_i}{\partial x_\mu}\frac{\partial\tilde{x}_i}{\partial x_\nu}$ transformations. This will bring desired curved QM equation to the typical form:

$$H(\psi) + V\psi = E\psi \tag{106}$$

with Hamiltonian H operator provided, for example, in paper [25] with a complex mix of second and first derivatives multiplied by metric tensor and its derivatives:

$$H = A_{kl}\left(g_{\mu\nu}\right)\partial^2/\partial x_k \partial x_l + B_{lkmn}\left(g_{\mu\nu}\right)\partial g_{mn}/\partial x_l\, \partial/\partial x_k + \cdots. \tag{107}$$

For example, for the 1D case with metric function $\mathrm{d}\tilde{x}^2 = g(x)\mathrm{d}x^2$, the curved QM equation looks like

$$-\frac{\hbar^2}{2m}\left(\frac{\psi''}{g} - \frac{\psi' g'}{2g^2}\right) + V(x)\psi(x) = E\psi(x). \tag{108}$$

These generalizations would only be important for hypothetical metric fields quickly changing within the boundaries of a quantum system. They would alter the simple expression for scalar $\Upsilon = \frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi[G]^{-1}\nabla\psi\right)$. But generic Lagrangian presentation (98) would have the same form.

For time-dependent QM equations with the accounting of gravitational time charge $\mathrm{d}\tilde{t} = g_t \mathrm{d}t$ and complex wave function (18), the Lagrangian would include temporal components:

$$\mathcal{L}_{QM} := \psi^* V\psi + \frac{\hbar}{2ig_t}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right) + \Upsilon^*\left(g_{\mu\nu},\psi\right);$$

$$\Upsilon^*\left(g_{\mu\nu},\psi\right) = \frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\psi^*[G]^{-1}\nabla\psi. \tag{109}$$

Another improvement for future research would be to replace QM Lagrangian (98) with Feynman's path integral [3] [11] [21] [25] [36] [37] in complex space which needs to be generalized for curved spacetime. Logically, it should lead to known covariant [27] [28] [29] [30] [31] [35] and canonical [11] [24] [25] [34] [35] [36] [37] QG formulations which should be evaluated in future research regarding GT concepts important in this theory.

11. Curved Spacetime Effect on Quantum Fields

Quantum Gravity GTQG Equations (101), (102) link together quantum and spacetime fields into connected unity of quantum spacetime pioneered in the 1930s. Let us evaluate how within this model the strong gravitational fields affect some known QM models and what known effects and predictions this theory may support.

11.1. Particle-in-the-Box

This simple well-known 1D QM model also analyzed in §5 describes an uncertain position of a quantum particle inside an impenetrable box of size *L*. In curved coordinates $\tilde{x},\tilde{t}$, it is described by QM Equation (51):

$$i\hbar\frac{\partial\psi(\tilde{x},\tilde{t})}{\partial\tilde{t}} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(\tilde{x},\tilde{t})}{\partial\tilde{x}^2}. \tag{110}$$

The solution (52) is the superposition of eigenvalue modes with quantized energy E_n [3] [25]:

$$\psi\left(\tilde{x},\tilde{t}\right)\equiv\sum_{n}d_n\psi_n\left(\tilde{x},\tilde{t}\right);\ \psi_n\left(\tilde{x},\tilde{t}\right)=A\sin\left(k_n\left(\tilde{x}-\frac{\tilde{L}}{2}\right)\right)\mathrm{e}^{-i\omega_n\tilde{t}},$$

$$k_n=n\pi/\tilde{L};\quad E_n=\frac{n^2\pi^2\hbar^2}{2m\tilde{L}^2}. \tag{111}$$

In these equations, the local background coordinates $\tilde{x}(x)$ and length $\tilde{L}$ are flexible—because spacetime and a ruler are flexible too. If the box is located near a black hole with known metric $g_r(r)=\left(1-\frac{r_s}{r}\right)^{-1}, r=x$ from Schwarzschild GR solution [1] [2] [9], the background coordinates $\tilde{x}(x)$ can be directly expressed via flat coordinates via $\mathrm{d}\tilde{x}=\rho(x)\mathrm{d}x$, $g(x)=\rho(x)^2$. So, what were the straight lines or sinusoidal waves in curved space, would become squashed and stretched (**Figure 1**) from the viewpoint of an external observer in flat spacetime. Injecting curved metric into $g(x)$ and omitting second-order terms $\rho'(x)\sim 0$ would lead to the equation where mass and metric become cofactors $\tilde{m}(x)=mg(x)$:

$$\tilde{m}(x)=mg(x);\ i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2\tilde{m}(x)}\frac{\partial^2\psi(x,t)}{\partial x^2}. \tag{112}$$

Existence of this *metric mass* $mg(x)$ (§5) implies that at very dense spacetimes near black holes, the quantum wave-particles become "heavy" and hard-to-move, quantum wave-particle distribution effects become less pronounced, and wave-particles are mostly localized rather than distributed, as shown in **Figure 1**. At $mg(x)\to\infty$ wave function $\psi(x)$ disappears from equations. But for low-density vacuum-like states $mg(x)\ll m$, quantum effects play a major role, and particles behave like distributed QM wave clouds. This leads to another effect of gravity on QM. Considering a hypothetical very long box where space metric $g(x)$ changes from low $g(x=0)=g_0$ to high $g(x=L)=ng_0$ level, the wave function would be wobblier at low g and more concentrated at high g (**Figure 1**). It implies a higher probability of finding a quantum particle at those locations. It seems like the wave function "gravitates" toward higher-density spaces/heavier masses. It means not only matter "attracts" to heavy masses but also probabilistic QM wave-particle distributions evolved in curved spacetimes; this is the result of spacetime derivatives in QM equations and Einstein's concept that spacetime is curved. Intuitively, it is more probable and energetically economical for a quantum particle to move along the shortest part of curved geodesics rather than along a straight line in curved space.

Let us note that in this particles-in-the-box model, we ignore the presumably little effect of the QM field on GR spacetime assuming strong gravitational fields. If this is not the case, we should be finding Lagrangian (104) from QM particle-in-the-box equations, inject it into GR via quantum stress-energy tensor $QSE_{\mu\nu}(g,\psi)$ (97) and obtain a pair of GTQG equations to be solved simultaneously.

In summary, even the simplest model confirms known effects and yields interesting insights into the effect of gravity on the probabilistic quantum fields.

11.2. Quantum Hydrogen Atom in Curved Spacetime

A hydrogen atom is another classic QM model [3] [9] yielding curved radial Schrödinger equation (65):

$$-\frac{\hbar^2}{2m}\tilde{\nabla}^2\psi(\tilde{r})+V(\tilde{r})\psi(\tilde{r})=E\psi(\tilde{r}),\ V(\tilde{r})=-\frac{e^2}{4\pi\varepsilon_0\tilde{r}};$$

$$\tilde{\nabla}^2=\frac{1}{\tilde{r}^2}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}^2\frac{\partial\psi}{\partial\tilde{r}}\right). \tag{113}$$

In curved Schwarzschild spacetime metric $\mathrm{d}\tilde{r}^2=g(r)\mathrm{d}r^2$, $g_r(r)=\left(1-\frac{r_s}{r}\right)^{-1}$, the second space derivative would inevitably lead to the appearance of mg factor instead of the electron's reduced mass m:

$$-\frac{\hbar^2}{2mg(r)}\nabla^2\psi(r)+V(r)\psi(r)=E\psi(r). \tag{114}$$

Due to the similarity of models, all the insights of the particle-in-the-box QM model would apply to the hydrogen atom too. So, in a vicinity of a black hole, an electron mass mg in the atom would appear heavier and harder to move in comparison with the same atom far away. Also, the atom's orbitals would be shifted/gravitated towards a black hole, in agreement with the paper [25]. For atoms with many electrons, it is possible to hypothesize that some external electron layers may be torn apart leaving some atoms ionized, and probably this effect is known to specialists. Also, the presence of a black hole would shift wave function modes $\psi_n(\tilde{\rho})=A_n\exp(-k_n\tilde{\rho})\tilde{\rho}^m$ and break their spherical symmetry confirming the findings in [25].

11.3. Quantum Double-Slit-Experiment Interpretation

Discussed above sensitivity of a wave function to high-density spaces and the existence of metric mass mg and metric wave function $g\psi$ cofactors (§5.2) should also affect the results of the quantum double-slit experiment [3] [4] [9], from which QM ideas had originated in the 1920s. This experiment performed in a strong gradient of gravitational fields, eq in a presence of a very heavy mass from one side/slit, should show the tendency of probabilistic wave patterns to concentrate closer to heavier mass, and most likely this effect is known to physicists.

11.4. Quantum Ground State and Gaussians

Quantum Ground State function [3] [4] [9] expresses the minimal energy state of a quantum system, for example in the 1D form of the symmetric Gaussian function $\psi_0(x)=\exp(-ax^2)$. In highly curved spacetime $\tilde{x}(x)$, it may not be symmetrical but squashed towards the higher spacetime density $\psi_0(\tilde{x})=\exp(-a\tilde{x}^2)$. This is also related to other QM models where Gaussians are often used [3] [4] [9] for both probability distributions and localized quantum states.

11.5. Quantum Wave Function Collapse/Localization

In 1986, R. Penrose [63] [64] suggested the wave function collapse [3] [4] [9] in quantum mechanics might be of quantum gravitational origin. According to [6], this idea implies re-thinking the basis of QM and GR but offers the prospects of an experimental test. Interestingly, it can be interpreted in the proposed GTQG model (96)-(102) in the following way. A particle wave function "collapse" into a certain position $\boldsymbol{x} = \boldsymbol{x}_0$ implies probability density $\int \psi^2 \left(\boldsymbol{x} - \boldsymbol{x}_0\right) \mathrm{d}^4 x = 1$ at some moment of measurement. ψ^2 included in Lagrangian density (98) $\mathcal{L}_{QM} = \left(V - E\right)\psi^2 + \Upsilon\left(g_{\mu\nu}, \psi\right)$ yields quantum stress-energy tensor $QSE_{\mu\nu}\left(g, \psi\right)$ which creates an additional point-like (localized/collapsed) warp of spacetime. So, based on GR concepts, one can say that a collapsed quantum particle just "falls" into this spacetime warp. It means a wave function collapse can be associated with spacetime localization [12] [13] [14] [63] correlating with R. Penrose's arguments [63].

12. Quantum Effect on Spacetime

Quantum Gravity GTQG Equations (96)-(102) are based on the Generalized Thermodynamics idea of mutual influence between QM and GR fields. It means not only spacetime affects QM equations and experiments but also QM affects spacetime. This is reflected in the appearance of an extra term (96) in the Einstein-Hilbert least-action principle in the form of the Lagrangian energy density component $\mathcal{L}_{QM}$:

$$L = \int \left(\frac{1}{2k}\left(R - 2\Lambda\right) + \mathcal{L}_T + \mathcal{L}_{QM} \right) \sqrt{-g}\,\mathrm{d}^4 x;\ \delta L = 0. \tag{115}$$

It is related to the quantum field energy (98) in curved space

$$\mathcal{L}_{QM} := \left(V - E\right)\psi^2 + \Upsilon\left(g_{\mu\nu}, \psi\right);\ \Upsilon\left(g_{\mu\nu}, \psi\right) := \frac{\hbar^2}{2m}\left(\nabla^{\mathrm{T}}\psi \left[G\right]^{-1} \nabla \psi\right) \tag{116}$$

and leads to quantum stress-energy $QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)$ added to Einstein's stress-energy tensor $T_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu}\Lambda = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right),\ k = \frac{8\pi G}{c^4}. \tag{117}$$

The following paragraphs describe some particular cases of Quantum effects on spacetime interpreted within the proposed GTQG model.

12.1. Energy Density Estimations

Some estimations of how strongly QM affects GR can be done by comparing energy-based addition $\mathcal{L}_{QM}$ against Lagrangian $\mathcal{L}_T$ density defining primary spacetime field in (115). To affect the fabric of spacetime for massive objects like the Sun, the mass of a quantum particle should be at least comparable $\mathcal{L}_T \sim \mathcal{L}_{QM}$, and it is hardly reachable for microscopic quantum systems. Also, as was discussed in §5, 11, due to metric mass $mg(x)$, strong gravitational fields make quantum particles harder to move hence altering the fabric of spacetime. The higher the density of space $g(x)$ and the closer a quantum particle is to a big mass, the less pronounced quantum effects would be, and the less wobbly would be the quantum spacetime.

12.2. Spacetime Quantum Fluctuation Interpretations

To make some quantum influence on spacetime, we should have $\mathcal{L}_T \sim \mathcal{L}_{QM}$ in (115), and this seems achievable for sub-atomic particles and especially for vacuum states $\mathcal{L}_T \sim 0$. Let us first evaluate cases where $\mathcal{L}_{QM}$ is comparable with $\mathcal{L}_T$. As per QG Equations (85), (96)-(99) with ignoring $\partial L/\partial g_{\mu\nu}$, we would have

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu}\Lambda = k\left(T_{\mu\nu} + QSE_{\mu\nu}\left(g_{\mu\nu}, \psi\right)\right) \approx k\left(g_{\mu\nu}\mathcal{L}_T + g_{\mu\nu}\mathcal{L}_{QM}\right). \quad (118)$$

Transferring the terms of $g_{\mu\nu}$ into the left part would yield additional terms to the cosmological constant Λ:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu}\left(\Lambda - \frac{\mathcal{L}_T}{k} - \frac{\mathcal{L}_{QM}}{k}\right) \approx 0. \quad (119)$$

The cosmological term $g_{\mu\nu}\Lambda$ was introduced by Einstein (with the famous "biggest blunder" story [1] [2]) to compensate for and later to explain the expansion of the universe, which evolved to modern interpretations of anti-gravity and "dark energy" [1] [20]. From (119) it looks like the quantum energy density $\mathcal{L}_{QM}$ acts as an additional expansion/contraction factor making spacetime "breathing". Vibrations in quantum energy $\mathcal{L}_{QM}$ would make spacetime wobble as well, and the higher $\mathcal{L}_{QM}$ in comparison with some static background $\mathcal{L}_T$, the more pronounced would be the "quantum breathing". It is well-known that quantum spacetime fluctuations are related to quantum ground states and virtual particles seemingly permeating space and popping in and out of existence [22] [23] [74]. Interestingly, Lagrangian (98) includes both positive and negative terms related to quantum potential energy $\pm V(x)$, full energy $(-E)\psi^2 < 0$ and Υ-energy related to wave function speed-of-change $\frac{\hbar^2}{2m}\left[\psi'(x)\right]^2 > 0$. Depending on the relationships between them, we can get either negative $\mathcal{L}_{QM}$ leading to spacetime expansion or positive $\mathcal{L}_{QM}$ for spacetime contraction:

$$\mathcal{L}_{QM} := (V - E)\psi^2 + \Upsilon\left(g_{\mu\nu}, \psi\right); \ \Upsilon\left(g_{\mu\nu}, \psi\right) \sim \frac{\hbar^2}{2mg}\left[\psi'(x)\right]^2. \quad (120)$$

Also, the wave function appears in square ψ^2 which in QM expresses the probability of finding a particle at a certain location [3] [9]. So, quantum spacetime fluctuations depend on the relationships between energies, wave function amplitudes, and gradients. In QM, energy *E*, as the solution of the QM eigenvalue problem, is discrete meaning the effect on spacetime would be discrete too. It does not necessarily mean the spacetime itself is discrete; it means in this GTQG model it can be represented as some superposition of functions with a discrete spectrum.

12.3. Quantum Vacuum States Interpretation

Including the quantum degree of freedom into GTQG Equations (96)-(102) allows interpreting low-density spacetimes close to vacuum states [22] [23] [74]. Spacetime can be affected by quantum stress-energy tensor $QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right)$ even without the presence of matter-related stress-energy tensor $T_{\mu\nu}=0$:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=kQSE_{\mu\nu}\left(g_{\mu\nu},\psi\right),\ k=\frac{8\pi G}{c^4}. \tag{121}$$

Quantum oscillations in quantum energy density $\mathcal{L}_{QM}$, which as discussed can be positive or negative, and can cause pressure $QSE_{\mu\nu}$ onto the spacetime making it "breathing" with expansions and contractions (§12.2). In QM, the minimal energy of a quantum system is nonzero but defined by "ground state" energy [3] [4] [9] which would create "background states" of spacetime making it the dynamic continuum on the fundamental level. It correlates well with well-known "quantum vacuum states" and "zero-point field" physical concepts [22] [23] [74] which assume the vacuum is not truly empty but a dynamic continuum of virtual particles popping in and out of existence and holding some energy hence creating some pressure onto spacetime.

After further research, this GTQG model may also provide some interpretation of the following problem from "Unsolved problem in physics" [7] [73]: "*Why does the zero-point energy of the vacuum not cause a large cosmological constant? What cancels it out?*". The answer to why the zero-point-energy field does not significantly expand the spacetime may be related to the abovementioned fact that Lagrangian (98) includes both positive and negative terms canceling each other on average, as discussed later as well.

12.4. Notes on Discrete or Continuous Spacetime

From the positions of this GTQG model, it is also interesting to interpret the fundamental question of whether spacetime is truly "quantized/discrete", like a chain of "balls" connected by "springs" in the popular Harmonic Oscillator model [3], or it is continuous like a liquid. From one side, Lagrangian $\mathcal{L}_{QM}=\left(V-E\right)\psi^2+\Upsilon\left(g_{\mu\nu},\psi\right)$ explicitly contains quantized energy E_n with discrete values (eq $E_n=\frac{n^2\pi^2\hbar^2}{2m\tilde{L}^2}$ for a quantum particle-in-the-box). But from the other side, those discrete values are multiplied by smooth continuous wave functions ψ^2 and $\Upsilon\left(g_{\mu\nu},\psi\right)$ making the overall factor $\mathcal{L}_{QM}$ continuous, but as a superposition of modes with a *discrete* spectrum. As a result, total spacetime deformations would not be truly discrete but rather continuous like a packet of ocean waves with definite "dense" areas. It means the spacetime can be considered *both* continuous and discrete: continuous in terms of the absence of "angles" and "rigid balls" but discrete in terms of a spectrum of superpositions—

more like a superposition of "vibrating strings" with certain frequencies, as string theory suggests [15] [16] [47] [50] [55] [64] [65]. A similar discrete-continuous quantum spacetime model is discussed in the author's Atomic Spacetime research [10] [12] [13] [14].

12.5. Quantization for Confinements

Let us notice that the Schrödinger Equation (49) by itself does not lead to the quantization of full energy E. As well-known from eigenvalue problems of mathematical physics [3] [4] [9], the eigenvalue frequencies ω appear only for certain boundary conditions, but for free-moving body $\omega = 0$. It means a QM system must be confined, like a particle-the-box or hydrogen atom, to produce some spectrum of discrete energies. For an unconfined spacetime continuum, energy quantization seems not applicable. Only when spacetime/matter gets "locked", does the quantum energy gets quantized producing spacetime deformations described by QG Equations (96)-(102) which include smooth matter-related $T_{\mu\nu}$ and quantum-related terms $QSE_{\mu\nu}$.

12.6. Is Spacetime Quantum in Nature?

From the positions of GTQG theory, this frequently debated question [20] [21] [63] can be interpreted in the following way. The shape of spacetime is defined by many factors (117)—matter-related stress-energy $T_{\mu\nu}$, cosmological dark-energy expansion $g_{\mu\nu}\Lambda$ and quantum stress-energy $QSE_{\mu\nu}$. A quantum component is only one of many factors which may temporarily prevail. When we consider quantum vacuum states with no physical matter $T_{\mu\nu} \sim 0$, quantum component $QSE_{\mu\nu}$ may become dominant, and we can consider space as having some quantum properties like probabilistic uncertainty, wave-particle behavior, superposition of states, quantization of energies, etc. So, one can say that vacuum space could be quantum-in-nature. But when matter appears, the factor $T_{\mu\nu}$ starts competing with $QSE_{\mu\nu}$ and may quickly suppress quantum effects. Due to the core factor $mg_{\mu\nu}$ (§5.2) the higher density of space, the less pronounced quantum effects, and the more quantum wave-particles become just particles. Thus, dense spacetime ceases to have quantum properties. All these influences happen on the background of dark-matter cosmological factor $g_{\mu\nu}\Lambda$ which looks like a constant pressure not having much to do with quantum uncertainties. So, it is quite an exaggeration that spacetime is always quantum-in-nature—all depends on the balance of three factors—$T_{\mu\nu}$, $QSE_{\mu\nu}$ and Λ in the GTQM model.

12.7. Black Hole Singularity Problem Interpretation

Another outcome of the proposed GTQM theory may confirm the well-known idea that quantum effects may prevent singularity for Ricci scalar $R \to \infty$ at the center of a black hole which in the standard GR model leads to equation [1] [2] $R = -\frac{8\pi G}{c^4}T$. In GTQG model (96)-(99), the Ricci scalar is surrounded by other terms with the inclusion of a quantum part $\mathcal{L}_{QM}$ which, unlike $\mathcal{L}_T$, can be negative:

$$\left(-\frac{1}{2}R+\Lambda-\frac{\mathcal{L}_T}{k}-\frac{\mathcal{L}_{QM}}{k}\right)g_{\mu\nu}+\cdots \tag{123}$$

It implies that the growing energy of a compression $\mathcal{L}_T$ at the center of a black hole may not be spent entirely on the growth of R but also the increase of quantum energy $\mathcal{L}_{QM}$ which would act as a "damping factor". Let us recall that $\mathcal{L}_{QM}$ (98) exclusively contains potential energy V used in QM to represent external energy potential. Rather than producing unphysical infinity $R\to\infty$, it may lead to very high $\mathcal{L}_{QM}$ and V which may cause some quantum fluctuation releasing/distributing the concentrated energy of a black hole center because in QM the particles become distributed wave-particle clouds. Also, in Quantum Gravity where GR affects QM and QM affects GR, the infinite energies/extensors would be prohibited by the GT law of dissipation (14) which would release the energy of dissipation into the heat $\mathrm{d}Q_d=T\mathrm{d}S$. It seems to correlate well with Bekenstein [43] and Hawking [6] [36] [37] black hole radiation models, with further research required.

In summary of this chapter, we can see that the proposed GTQM model upholds the known effects and provides some insights into the influence of quantum effects on the fabric of spacetime with easy-to-interpret additions to Einstein's GR equations.

13. Conclusions and Future Research Directions

This paper proposes a novel approach and model to build equations of Quantum Gravity (QG) based on Generalized Thermodynamics (GT) which evolve from Classical Thermodynamics into some kind of Theory-of-Everything providing a common framework for uniting fields from different interrelated physical theories including Quantum Mechanics (QM) and General Relativity (GR). After defining GT extensors, intensials, energy, and state equations, GT suggests cross-reference terms and modified equations of QM and GR making two theories linked together. GR metric tensor introduced into stationary QM Schrödinger equations via curved coordinates which addresses the "background problem" and yields an additional quantum spacetime variation term. Then quantum Lagrangian is added to Einstein-Hilbert functional yielding additional quantum stress-energy tensor into Einstein's GR equation. Obtained from one variational principle, two theories are linked by a common quantum spacetime field.

Obtained GTQM model yields easy-to-interpret factors like metric mass and metric wave function pointing that at dense spacetimes near black holes, the quantum wave-particles become "heavy" and hard-to-move, quantum wave particle distribution effects become less pronounced, and wave-particles are mostly localized rather than distributed. But for low-density vacuum-like states quantum effects play a major role and particles behave like distributed wave clouds. The theory and model uphold and provide interpretations for some known

Quantum Gravity effects of shifting of QM wave-particles towards high spacetime densities, diminishing quantum effects for high metric mass densities, researched for QM models including particle-in-the-box, hydrogen atom, the quantum double-slit experiment, and wave function collapse. Analyzing quantum effects of spacetime, the concepts of spacetime quantum fluctuations, quantum vacuum states, and discrete-continuous spacetime are also discussed. The GTQR model offers the interpretation for some important problems like the quantum nature of spacetime, black hole singularity, and zero-point fields.

Further research directions should include whether the Generalized Thermodynamics approach can be expanded for more complex canonical, covariant, quantum field, string, and loop quantum gravity models. Including temporal components in the GTQM model would make the model more generic. It seems important to clarify which Quantum Gravity stream this research belongs to. Reviewed interpretations of mutual influence quantum and spacetime fields would be interesting to research in more detail.

This theory may contribute not only to Quantum Gravity research but also extend Generalized Thermodynamics as a kind of "Theory-of-Everything" toward important quantum spacetime concepts and beyond.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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