Journal of Applied Mathematics and Physics, 2023, 11, 919-932
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Group of Weakly Continuous Operators Associated to a Generalized Schrödinger Type Homogeneous Model

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How to cite this paper: Ayala, Y.S.S. (2023) Group of Weakly Continuous Operators Associated to a Generalized Schrödinger Type Homogeneous Model. *Journal of Applied Mathematics and Physics*, **11**, 919-932.
https://doi.org/10.4236/jamp.2023.114062

Received: March 8, 2023
Accepted: April 18, 2023
Published: April 21, 2023

Abstract

In this work, we prove the existence and uniqueness of the solution of the generalized Schrödinger type homogeneous model in the periodic distributional space P'. Furthermore, we prove that the solution depends continuously respect to the initial data in P'. Introducing a family of weakly continuous operators, we prove that this family is a group of operators in P'. Then, with this family of operators, we get a fine version of the existence and dependency continuous theorem obtained. Finally, we give some remarks derived from this study.

Keywords

Groups Theory, Weakly Continuous Operators, Existence of Solution, Generalized Schrödinger Type Equation, Homogeneous Equation, Periodic Distributional Space

1. Introduction

First, we begin by commenting that [1] has proven the existence of a solution of the Schrodinger type equation in the Hilbert space H^s_{per}. Also in [1] a family of bounded operators is introduced in the Hilbert space H^s_{per} and it is proved that forms a unitary group. For the justification of the model, we suggest reviewing the references cited in [1]. Thus motivated by these ideas we will solve the problem (Q_{m+1}) in the topological dual of P: P', which is not a Banach space.

In this article, we will prove the existence and uniqueness of the solution of (Q_{m+1}) in P'. Furthermore, we will demonstrate that the solution depends continuously with respect to the initial data in P', considering the weak convergence in P'. And we will prove that the introduced family of operators forms a group of weakly continuous linear operators. Thus, with this family we will rewrite our result in a fine version.

We also want to highlight the wealth of information from Terence [2], Kato [3], Linares and Ponce [4]. We can also cite works of existence solution using Semigroup theory by Liu-Zheng [5], Muñoz [6], Pazy [7], Santiago [8] [9] and Raposo [10].

Our article is organized as follows. In Section 2, we indicate the methodology used and cite the references used. In Section 3, we put the results obtained from our study. This section is divided into three subsections. Thus, in Subsection 3.1 we prove that the problem (Q_{m+1}) has a unique solution and also demonstrate that the solution depends continuously with respect to the initial data. In Subsection 3.2, we introduce families of weakly continuous linear operators in P' that manage to form a group. In Subsection 3.3 we improve Theorem 3.1.

Finally, in Section 4 we give the conclusions of this study.

2. Methodology

As theoretical framework in this article we use the references [1] [11] [12] [13] and [14] for Fourier Theory in periodic distributional space, periodic Sobolev spaces, topological vector spaces, weakly continuous operators, group of operators and existence of solution of a distributional differential equation.

We will use this theory in the analysis of the existence and continuous dependence of the solution of (Q_{m+1}), carrying out a series of calculations and approximations in the process.

Thus, below we will briefly give some definitions necessary for the development (understanding) of this work. It is suggested, for an in-depth study, to refer to the cited references.

Let be

$$P := C^{\infty}_{per}\left(\left[-\pi,\pi\right]\right),$$

that is, the space of the functions $f : \mathbb{R} \to \mathbb{C}$ infinitely differentiable and periodic with period 2π. It's known that this space is a complete metric space.

Also,

$$\begin{aligned} P' := \Big\{ & T : P \to \mathbb{C} \text{ linear such that } \exists\, \psi_n \in P \text{ and} \\ & \langle T,\varphi\rangle = \lim_{n\to\infty}\int_{-\pi}^{\pi}\psi_n(x)\varphi(x)\,\mathrm{d}x, \forall \varphi \in P \Big\} \\ = {} & (P)'. \end{aligned}$$

That is, P' is the topological dual of P. P' is known as the space of periodic distributions.

We want to summarize the properties of P' with the following diagram:

$$\begin{array}{ccccc} P & \to & L^2\left(\left[-\pi,\pi\right]\right) & \to & P' \\ \wedge\downarrow\uparrow\vee & & \wedge\downarrow\uparrow\vee & & \wedge\downarrow\uparrow\vee \\ S(Z) & \to & l^2(Z) & \to & S'(Z) \end{array}$$

where the inclusions are continuous with dense image, $S(Z)$ is the space of Rapidly Decreasing sequences (R.D.), defined by

$$S(Z) := \left\{ \alpha = (\alpha_k)_{k\in Z}, \alpha_k \in \mathbb{C} / \sum_{k=-\infty}^{+\infty} |\alpha_k| < \infty \text{ and } \sum_{k=-\infty}^{+\infty} |\alpha_k| |k|^n < \infty, \forall n \geq 1 \right\}$$

and $S'(Z)$ is the space of Slowly Growing sequences (S.G.), defined by

$$S'(Z) := \left\{ \alpha = (\alpha_k)_{k\in Z}, \alpha_k \in \mathbb{C} / \exists C > 0, \exists N \in \mathbb{N} \text{ with } |\alpha_k| \leq C|k|^N, \forall k \neq 0 \right\}.$$

3. Main Results

The presentation of the results obtained has been organized in subsections and is as follows.

3.1. Solution of the Schrödinger Equation (Q_{m+1})

In this subsection we will study the existence of a solution to the problem (Q_{m+1}) and the continuous dependence of the solution with respect to the initial data in P'.

Theorem 3.1 *Let* $\mu > 0$, $\alpha > 0$, *m even not multiple of four and the distributional problem*

$$(Q_{m+1}) \quad \begin{vmatrix} u \in C(\mathbb{R}, P') \\ \partial_t u - i\mu\partial_x^m u + i\alpha u = 0 \in P' \\ u(0) = f \in P'. \end{vmatrix}$$

then (Q_{m+1}) has a unique solution $u \in C^1(\mathbb{R}, P')$. Furthermore, the solution depends continuously on the initial data. That is, given $f_n, f \in P'$ such that $f_n \xrightarrow{P'} f$ implies $u_n(t) \xrightarrow{P'} u(t)$, $\forall t \in \mathbb{R}$, where u_n is solution of (Q_{m+1}) with initial data f_n and *u* is solution of (Q_{m+1}) with initial data *f*.

Proof.- We have organized the proof as follows.

1) Suppose there exists $u \in C(\mathbb{R}, P')$ satisfying (Q_{m+1}); this will allow us to obtain the explicit form of *u*. Then taking the Fourier transform to the equation

$$\partial_t u - i\mu\partial_x^m u + i\alpha u = 0,$$

we get

$$0 = \partial_t \hat{u} - i\mu(ik)^m \hat{u} + i\alpha\hat{u} = \partial_t \hat{u} + i\mu k^m \hat{u} + i\alpha\hat{u},$$

which for each $k \in \mathbb{Z}$ is an ODE with initial data $\hat{u}(k,0) = \hat{f}(k)$.

Thus, we propose an uncoupled system of homogeneous first-order ordinary differential equations

$$(\Omega_k) \quad \begin{vmatrix} \hat{u} \in C(\mathbb{R}, S'(\mathbb{Z})) \\ \partial_t \hat{u}(k,t) + i\mu k^m \hat{u}(k,t) + i\alpha\hat{u}(k,t) = 0 \\ \hat{u}(k,0) = \hat{f}(k) \text{ with } \hat{f} \in S'(\mathbb{Z}), \end{vmatrix}$$

$\forall k \in \mathbb{Z}$ and we get

$$\hat{u}(k,t) = \mathrm{e}^{-i\mu k^m t} \mathrm{e}^{-i\alpha t} \hat{f}(k),$$

from where we obtain the explicit expression of u, candidate for solution:

$$u(t)=\sum_{k=-\infty}^{+\infty}\hat{u}(k,t)\phi_k=\sum_{k=-\infty}^{+\infty}\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\hat{f}(k)\phi_k, \tag{1}$$

$$=\left[\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\right]^{\vee}. \tag{2}$$

Since $f\in P'$ then $\hat{f}\in S'(Z)$. Thus, we affirm that

$$\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\in S'(Z),\quad \forall t\in I\!R. \tag{3}$$

Indeed, let $t\in I\!R$, since $\hat{f}\in S'(Z)$ then satisfies: $\exists C>0$, $\exists N\in I\!N$ such that $\left|\hat{f}(k)\right|\le C|k|^N$, $\forall k\in Z-\{0\}$, using this we get

$$\left|\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right|=\left|\hat{f}(k)\right|\underbrace{\left|\mathrm{e}^{-i\mu k^m t}\right|}_{=1}\underbrace{\left|\mathrm{e}^{-i\alpha t}\right|}_{=1}=\left|\hat{f}(k)\right|\le C|k|^N.$$

Then,

$$\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\in S'(Z).$$

If we define

$$u(t):=\left[\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\right]^{\vee},\quad \text{for all } t\in I\!R, \tag{4}$$

we have that $u(t)\in P'$, $\forall t\in I\!R$, since we apply the inverse Fourier transform to $\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\in S'(Z)$.

2) We will prove that u defined in (4) is solution of (Q_{m+1}) and $u\in C^1(I\!R,P')$.

Evaluating (2) at $t=0$, we obtain

$$u(0)=\left[\left(\hat{f}(k)\right)_{k\in Z}\right]^{\vee}=\left[\hat{f}\right]^{\vee}=f.$$

Also, the following statements are verified.

a) $\partial_t u(t)=i\mu\partial_x^m u(t)-i\alpha u(t)$ in P', $\forall t\in I\!R$. That is, we will prove that the following equality

$$\underbrace{\lim_{h\to 0}\left\langle\frac{u(t+h)-u(t)}{h},\varphi\right\rangle}_{\langle\partial_t u(t),\varphi\rangle:=}=i\mu\left\langle\partial_x^m u(t),\varphi\right\rangle-i\alpha\left\langle u(t),\varphi\right\rangle,\forall\varphi\in P$$

is satisfied, for all $t\in I\!R$.

Indeed, let $t\in I\!R$, $\varphi\in P$ and $h\in I\!R-\{0\}$, we denote

$$I_{h,t}:=\left\langle\frac{u(t+h)-u(t)}{h},\varphi\right\rangle.$$

Thus, we get

$$\begin{aligned}I_{h,t}&=\frac{1}{h}\left\{\left\langle u(t+h),\varphi\right\rangle-\left\langle u(t),\varphi\right\rangle\right\}\\&=\frac{1}{h}\left\{\lim_{n\to+\infty}\left\langle\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m(t+h)}\mathrm{e}^{-i\alpha(t+h)}\phi_k,\varphi\right\rangle\right.\\&\quad\left.-\lim_{n\to+\infty}\left\langle\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\phi_k,\varphi\right\rangle\right\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{h}\left\{ \lim_{n\to+\infty} \left\langle \sum_{k=-n}^{n} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right) \phi_k, \varphi \right\rangle \right\} \\
&= \lim_{n\to+\infty} \left\langle \sum_{k=-n}^{n} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(\frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right) \phi_k, \varphi \right\rangle \\
&= \lim_{n\to+\infty} \left\{ \sum_{k=-n}^{n} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(\frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right) \underbrace{\langle \phi_k, \varphi \rangle}_{=2\pi\hat{\varphi}(-k)} \right\} \\
&= \lim_{n\to+\infty} 2\pi \left\{ \sum_{k=-n}^{n} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(\frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right) \hat{\varphi}(-k) \right\} \\
&= 2\pi \sum_{k=-\infty}^{+\infty} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(\frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right) \hat{\varphi}(-k).
\end{aligned} \tag{5}
$$

Let $h > 0$, we have

$$
\begin{aligned}
e^{-i\mu k^m h} e^{-i\alpha h} - 1 &= \int_0^h \left[e^{-i\mu k^m s} e^{-i\alpha s} \right]' \mathrm{d}s \\
&= \int_0^h \left(-i\mu k^m - i\alpha \right) e^{-i\mu k^m s} e^{-i\alpha s} \mathrm{d}s.
\end{aligned} \tag{6}
$$

Taking norm to equality (6) we obtain

$$
\begin{aligned}
\left| e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right| &\le \int_0^h \left\{ \mu |k|^m + |\alpha| \right\} \underbrace{\left| e^{-i\mu k^m s} \right|}_{=1} \underbrace{\left| e^{-i\alpha s} \right|}_{=1} \mathrm{d}s \\
&= \left\{ \mu |k|^m + |\alpha| \right\} \underbrace{\int_0^h \mathrm{d}s}_{=h} \\
&= \left\{ \mu |k|^m + |\alpha| \right\} h.
\end{aligned} \tag{7}
$$

That is, from (7) we get

$$
\left| \frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right| \le \mu |k|^m + |\alpha|. \tag{8}
$$

Note that (8) is valid for $h \in \mathbb{R} - \{0\}$.

Using the inequality (8) and that $\hat{f} \in S'(Z)$ we obtain

$$
\begin{aligned}
&\sum_{k=-\infty}^{+\infty} \left| \hat{f}(k) \right| \underbrace{\left| e^{-i\mu k^m t} \right|}_{=1} \underbrace{\left| e^{-i\alpha t} \right|}_{=1} \left| \hat{\varphi}(-k) \right| \left| \frac{e^{-i\mu k^m h} e^{-i\alpha h} - 1}{h} \right| \\
&\le \sum_{k=-\infty}^{+\infty} \left| \hat{f}(k) \right| \left| \hat{\varphi}(-k) \right| \left\{ \mu |k|^m + |\alpha| \right\} \\
&= \mu \sum_{k=-\infty}^{+\infty} \left| \hat{f}(k) \right| \left| \hat{\varphi}(-k) \right| |k|^m + |\alpha| \sum_{k=-\infty}^{+\infty} \left| \hat{f}(k) \right| \left| \hat{\varphi}(-k) \right| \\
&\le C \left\{ \mu \sum_{k=-\infty}^{+\infty} |k|^{N+m} \left| \hat{\varphi}(\underbrace{-k}_{=J}) \right| + |\alpha| \sum_{k=-\infty}^{+\infty} |k|^N \left| \hat{\varphi}(\underbrace{-k}_{=J}) \right| \right\} \\
&= C \left\{ \mu \sum_{J=-\infty}^{+\infty} |J|^{N+m} \left| \hat{\varphi}(J) \right| + |\alpha| \sum_{J=-\infty}^{+\infty} |J|^N \left| \hat{\varphi}(J) \right| \right\} < \infty
\end{aligned}
$$

since $\hat{\varphi} \in S(Z)$.

Using the Weierstrass M-Test, the series $I_{h,t}$ is absolute and uniformly convergent. Then we can take limit and get

$$\begin{aligned}\lim_{h\to 0} I_{h,t} &= 2\pi\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\hat{\varphi}(-k)\underbrace{\lim_{h\to 0}\left\{\frac{\mathrm{e}^{-i\mu k^m h}\mathrm{e}^{-i\alpha h}-1}{h}\right\}}_{=-i\mu k^m-i\alpha}\\ &=(-i\mu)2\pi\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\hat{\varphi}(-k)k^m \\ &\quad -i\alpha 2\pi\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\hat{\varphi}(-k).\end{aligned} \tag{9}$$

Using (9) and that $\left\langle T^{(m)},\varphi\right\rangle=(-1)^m\left\langle T,\varphi^{(m)}\right\rangle=\left\langle T,\varphi^{(m)}\right\rangle$ for $\varphi\in P$, $T\in P'$, we have

$$\begin{aligned}&\lim_{h\to 0} I_{h,t}\\ &=(-i\mu)2\pi\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\underbrace{\hat{\varphi}(-k)}_{=\frac{1}{2\pi}\langle\varphi,\phi_k\rangle}k^m-i\alpha 2\pi\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\underbrace{\hat{\varphi}(-k)}_{=\frac{1}{2\pi}\langle\varphi,\phi_k\rangle}\\ &=i\mu\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\varphi,\underbrace{-k^m\phi_k}_{=(ik)^m\phi_k}\right\rangle-i\alpha\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\varphi,\phi_k\right\rangle\\ &=i\mu\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\underbrace{\left\langle\varphi,\phi_k^{(m)}\right\rangle}_{=\left\langle\varphi^{(m)},\phi_k\right\rangle}-i\alpha\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\varphi,\phi_k\right\rangle\\ &=i\mu\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\phi_k,\varphi^{(m)}\right\rangle-i\alpha\sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\phi_k,\varphi\right\rangle\\ &=i\mu\lim_{n\to+\infty}\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\phi_k,\varphi^{(m)}\right\rangle-i\alpha\lim_{n\to+\infty}\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\left\langle\phi_k,\varphi\right\rangle\\ &=i\mu\lim_{n\to+\infty}\left\langle\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\phi_k,\varphi^{(m)}\right\rangle-i\alpha\lim_{n\to+\infty}\left\langle\sum_{k=-n}^{n}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\phi_k,\varphi\right\rangle\\ &=i\mu\left\langle u(t),\varphi^{(m)}\right\rangle-i\alpha\left\langle u(t),\varphi\right\rangle\\ &=i\mu\left\langle \partial_x^m u(t),\varphi\right\rangle-i\alpha\left\langle u(t),\varphi\right\rangle.\end{aligned} \tag{10}$$

Therefore,

$$\left\langle\partial_t u(t),\varphi\right\rangle=i\mu\left\langle\partial_x^m u(t),\varphi\right\rangle-i\alpha\left\langle u(t),\varphi\right\rangle,\quad\forall\varphi\in P,\quad\forall t\in I\!R.$$

That is,

$$\partial_t u(t)=i\mu\partial_x^m u(t)-i\alpha u(t)\ \text{ in } P',\quad\forall t\in I\!R.$$

b) $u\in C(I\!R,P')$. That is, we will prove that

$$u(t+h)\xrightarrow{P'}u(t)\ \text{ when } h\to 0,\forall t\in I\!R.$$

In effect, let $t\in I\!R$ and $\varphi\in P$, we will prove that

$$H_{t,h}:=\left\langle u(t+h)-u(t),\varphi\right\rangle\to 0,\ \text{ when } h\to 0.$$

We know that if $\varphi\in P$ then $\hat{\varphi}\in S(Z)$. Using (5) we have

$$H_{t,h} = 2\pi \sum_{k=-\infty}^{+\infty} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \left(e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right) \hat{\varphi}(-k).$$

Let $0 < |h| < 1$, from (8) we get

$$\left| e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right| \le \mu |k|^m |h| + |\alpha||h| < \mu |k|^m + |\alpha|. \quad (11)$$

Using (11) and that $\hat{f} \in S'(Z)$ we obtain

$$\sum_{k=-\infty}^{+\infty} \left| \hat{f}(k) \right| \underbrace{\left| e^{-i\mu k^m t} \right|}_{=1} \underbrace{\left| e^{-i\alpha t} \right|}_{=1} \left| e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right| \left| \hat{\varphi}(-k) \right|$$

$$\le C\mu \sum_{k=-\infty}^{+\infty} |k|^{N+m} \left| \hat{\varphi}(\underbrace{-k}_{=J}) \right| + C|\alpha| \sum_{k=-\infty}^{+\infty} |k|^N \left| \hat{\varphi}(\underbrace{-k}_{=J}) \right|$$

$$= C\mu \sum_{J=-\infty}^{+\infty} |J|^{N+m} \left| \hat{\varphi}(J) \right| + C|\alpha| \sum_{J=-\infty}^{+\infty} |J|^N \left| \hat{\varphi}(J) \right| < \infty$$

since $\hat{\varphi} \in S(Z)$.

Using the Weierstrass M-Test we conclude that the series $H_{t,h}$ converges absolute and uniformly. Then it is possible to take limit and obtain

$$\lim_{h \to 0} H_{t,h} = 2\pi \sum_{k=-\infty}^{+\infty} \hat{f}(k) e^{-i\mu k^m t} e^{-i\alpha t} \hat{\varphi}(-k) \underbrace{\lim_{h \to 0} \left\{ e^{-i\mu k^m h} e^{-i\alpha h} - 1 \right\}}_{=0} = 0.$$

Since $t \in \mathbb{R}$ was taken arbitrarily, then we can conclude that

$$u \in C(\mathbb{R}, P').$$

c) $\partial_t u \in C(\mathbb{R}, P')$. That is, we will prove that

$$\partial_t u(t+h) \xrightarrow{P'} \partial_t u(t) \text{ when } h \to 0, \forall t \in \mathbb{R}.$$

In effect, let $t \in \mathbb{R}$ and $\varphi \in P$, using item a) we have

$$\begin{aligned} &\left\langle \partial_t u(t+h), \varphi \right\rangle - \left\langle \partial_t u(t), \varphi \right\rangle \\ &= i\mu \left\{ \left\langle \partial_x^m u(t+h), \varphi \right\rangle - \left\langle \partial_x^m u(t), \varphi \right\rangle \right\} - i\alpha \left\{ \left\langle u(t+h), \varphi \right\rangle - \left\langle u(t), \varphi \right\rangle \right\} \quad (12) \\ &= i\mu \underbrace{\left\langle u(t+h), \varphi^{(m)} \right\rangle - \left\langle u(t), \varphi^{(m)} \right\rangle}_{\to 0} - i\alpha \underbrace{\left\{ \left\langle u(t+h), \varphi \right\rangle - \left\langle u(t), \varphi \right\rangle \right\}}_{\to 0} \to 0 \end{aligned}$$

when $h \to 0$, since item b) is valid with $\varphi^{(r)} \in P$ for $r = 0, m$.

From b) and c) we have that $u \in C^1(\mathbb{R}, P')$.

3) Now, we will prove that the solution depends continuously respect to initial data. That is, if $f_n \xrightarrow{P'} f$ we will prove that:

$$u_n(t) \xrightarrow{P'} u(t), \quad \forall t \in \mathbb{R}.$$

We know that if $f_n \xrightarrow{P'} f$ then $\hat{f}_n \xrightarrow{S'(Z)} \hat{f}$, that is

$$\left\langle \hat{f}_n - \hat{f}, \xi \right\rangle \to 0 \text{ when } n \to +\infty, \quad \forall \xi \in S(Z). \quad (13)$$

For $t \in \mathbb{R}$ fixed and arbitrary, we want to prove that

$$\left\langle u_n(t), \psi \right\rangle \to \left\langle u(t), \psi \right\rangle \text{ when } n \to +\infty, \quad \forall \psi \in P.$$

Thus, let $t \in \mathbb{R}$ be fixed and $\psi \in P$, using the generalized Parseval identity, we obtain the following equalities:

$$\langle u_n(t),\psi\rangle = 2\pi\left\langle \left(\hat{f}_n(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}, \tilde{\hat{\psi}}\right\rangle \tag{14}$$

$$\langle u(t),\psi\rangle = 2\pi\left\langle \left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}, \tilde{\hat{\psi}}\right\rangle. \tag{15}$$

From (14) and (15) we obtain:

$$\langle u_n(t),\psi\rangle - \langle u(t),\psi\rangle = 2\pi\sum_{k=-\infty}^{+\infty}\left\{\hat{f}_n(k)-\hat{f}(k)\right\}\underbrace{\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\tilde{\hat{\psi}}(k)}_{\xi_k :=} \to 0$$

when $n\to+\infty$, since $\xi := (\xi_k)_{k\in Z} \in S(Z)$ and (13) holds. □

Corollary 3.1 *The unique solution of* (Q_{m+1}) *is*

$$u(t) = \sum_{k=-\infty}^{+\infty}\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\phi_k = \left[\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\right]^{\vee},$$

where $\phi_k(x) = \mathrm{e}^{ikx}$, $x\in \mathbb{R}$.

3.2. Group of Operators in P'

In this subsection, we will introduce families of operators $\{T_{\mu,\alpha}(t)\}_{t\in\mathbb{R}}$ in P', with $\mu>0$, $\alpha>0$ and m even not multiple of four; and we will prove that these operators are continuous in the weak sense. That is, $T_{\mu,\alpha}(t)$ is continuous from P' to P' with the weak topology of P', which we will call the weakly continuous operator.

Furthermore, we will prove that $T_{\mu,\alpha}(t)$ satisfies the group properties.

For simplicity, we will denote this family of operators by $\{T(t)\}_{t\in\mathbb{R}}$.

Theorem 3.2 *Let* $t\in\mathbb{R}$, *we define*:

$$T(t): P' \to P'$$

$$f \to T(t)f := \left[\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\right]^{\vee} \in P',$$

then the following statements are satisfied:

1) $T(0) = I$.

2) $T(t)$ is $\mathbb{C}$—linear and weakly continuous $\forall t\in\mathbb{R}$. That is, for every $t\in\mathbb{R}$, if $f_n \xrightarrow{P'} f$ then $T(t)f_n \xrightarrow{P'} T(t)f$.

3) $T(t+r) = T(t)\circ T(r)$, $\forall t,r\in\mathbb{R}$.

4) $T(t)f \xrightarrow{P'} f$ when $t\to 0$, $\forall f\in P'$.

That is, for each $f\in P'$ fixed, the following is satisfied

$$\langle T(t)f,\psi\rangle \to \langle f,\psi\rangle, \text{ when } t\to 0, \forall\psi\in P.$$

Proof.- Let $f\in P'$ then $\hat{f}\in S'(Z)$. Then, from (3) we have

$$\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z} \in S'(Z);$$

taking the inverse Fourier transform, we obtain

$$\underbrace{\left[\left(\hat{f}(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\right)_{k\in Z}\right]^{\vee}}_{=T(t)f} \in P', \quad \forall t\in\mathbb{R}.$$

That is, $T(t)$ is well defined for all $t\in\mathbb{R}$.

1) We easily obtain:

$$T(0)f=\left[\left(\hat{f}(k)e^{-i\mu k^{m}0}e^{-i\alpha 0}\right)_{k\in Z}\right]^{\vee}=\left[\left(\hat{f}(k)\right)_{k\in Z}\right]^{\vee}=\left[\hat{f}\right]^{\vee}=f,\quad \forall f\in P'.$$

2) Let $t\in\mathbb{R}$, we will prove that $T(t):P'\to P'$ is $\mathbb{C}$-linear. In effect, let $a\in\mathbb{C}$ and $(\phi,\psi)\in P'\times P'$, we have

$$\begin{aligned}T(t)(a\phi+\psi)&=\left[\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\left[a\phi+\psi\right]^{\wedge}(k)\right)_{k\in Z}\right]^{\vee}\\&=\left[\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\left[a\hat{\phi}(k)+\hat{\psi}(k)\right]\right)_{k\in Z}\right]^{\vee}\\&=\left[a\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\hat{\phi}(k)\right)_{k\in Z}+\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\hat{\psi}(k)\right)_{k\in Z}\right]^{\vee}\\&=a\left[\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\hat{\phi}(k)\right)_{k\in Z}\right]^{\vee}+\left[\left(e^{-i\mu k^{m}t}e^{-i\alpha t}\hat{\psi}(k)\right)_{k\in Z}\right]^{\vee}\\&=aT(t)\phi+T(t)\psi.\end{aligned}$$

Now, for $t\in\mathbb{R}$ we will prove that $T(t):P'\to P'$ is weakly continuous. That is, if $f_n\xrightarrow{P'}f$ then we will prove that $T(t)f_n\xrightarrow{P'}T(t)f$. Note that the case $t=0$ is obvious.

We know that if $f_n\xrightarrow{P'}f$ then $\hat{f}_n\xrightarrow{S'}\hat{f}$, that is,

$$\left\langle\hat{f}_n,\xi\right\rangle\to\left\langle\hat{f},\xi\right\rangle,\quad\text{when } n\to+\infty,\quad\forall\xi\in S(Z).$$

That is,

$$\left\langle\hat{f}_n-\hat{f},\xi\right\rangle\to 0,\quad\text{when } n\to+\infty,\quad\forall\xi\in S(Z).\tag{16}$$

We want to prove that:

$$\left\langle T(t)f_n,\psi\right\rangle\to\left\langle T(t)f,\psi\right\rangle\quad\text{when } n\to+\infty,\quad\forall\psi\in P.$$

Thus, let $t\in\mathbb{R}$ fixed and $\psi\in P$, using the generalized Parseval identity, we obtain the following equalities

$$\begin{aligned}\left\langle T(t)f_n,\psi\right\rangle&=\left\langle\left[\left(\hat{f}_n(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z}\right]^{\vee},\psi\right\rangle\\&=2\pi\left\langle\left(\hat{f}_n(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z},\tilde{\hat{\psi}}\right\rangle,\end{aligned}\tag{17}$$

$$\begin{aligned}\left\langle T(t)f,\psi\right\rangle&=\left\langle\left[\left(\hat{f}(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z}\right]^{\vee},\psi\right\rangle\\&=2\pi\left\langle\left(\hat{f}(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z},\tilde{\hat{\psi}}\right\rangle.\end{aligned}\tag{18}$$

From (17) and (18) we get

$$\begin{aligned}&\left\langle T(t)f_n,\psi\right\rangle-\left\langle T(t)f,\psi\right\rangle\\&=2\pi\left\{\left\langle\left(\hat{f}_n(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z},\tilde{\hat{\psi}}\right\rangle-\left\langle\left(\hat{f}(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\right)_{k\in Z},\tilde{\hat{\psi}}\right\rangle\right\}\\&=2\pi\left\{\sum_{k=-\infty}^{+\infty}\hat{f}_n(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\tilde{\hat{\psi}}(k)-\sum_{k=-\infty}^{+\infty}\hat{f}(k)e^{-i\mu k^{m}t}e^{-i\alpha t}\tilde{\hat{\psi}}(k)\right\}\\&=2\pi\sum_{k=-\infty}^{+\infty}\left\{\hat{f}_n(k)-\hat{f}(k)\right\}\underbrace{e^{-i\mu k^{m}t}e^{-i\alpha t}\tilde{\hat{\psi}}(k)}_{\xi_k:=}\to 0\end{aligned}$$

when $n \to +\infty$, since $\xi := (\xi_k)_{k\in Z} \in S(Z)$ and (16) holds, that is $\langle \hat{f}_n - \hat{f}, \xi \rangle \to 0$ when $n \to +\infty$.

3) Let $t, r \in I\!R - \{0\}$, we will prove that $T(t) \circ T(r) = T(t+r)$. In effect, let $\phi \in P'$,

$$\begin{aligned} T(t+r)\phi &= \left[\left(\hat{\phi}(k) \mathrm{e}^{-i\mu k^m (t+r)} \mathrm{e}^{-i\alpha(t+r)} \right)_{k\in Z} \right]^{\vee} \\ &= \left[\left(\underbrace{\hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r}} \cdot \mathrm{e}^{-i\mu k^m t} \mathrm{e}^{-i\alpha t} \right)_{k\in Z} \right]^{\vee}. \end{aligned} \tag{19}$$

Since $\phi \in P'$, using (3) we have that

$$\left(\hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r} \right)_{k\in Z} \in S'(Z), \quad \forall r \in I\!R. \tag{20}$$

Then, taking the inverse Fourier transform, we get:

$$\left[\left(\hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r} \right)_{k\in Z} \right]^{\vee} \in P', \quad \forall r \in I\!R.$$

Thus, we define:

$$g_r := \left[\left(\hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r} \right)_{k\in Z} \right]^{\vee} \in P'.$$

That is,

$$g_r := T(r)\phi. \tag{21}$$

Taking the Fourier transform to g_r we get:

$$\hat{g}_r = \left(\hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r} \right)_{k\in Z},$$

that is,

$$\hat{g}_r(k) = \hat{\phi}(k) \mathrm{e}^{-i\mu k^m r} \mathrm{e}^{-i\alpha r}, \quad \forall k \in Z. \tag{22}$$

Using (22) in (19) and from (21) we have:

$$\begin{aligned} T(t+r)\phi &= \left[\left(\hat{g}_r(k) \mathrm{e}^{-i\mu k^m t} \mathrm{e}^{-i\alpha t} \right)_{k\in Z} \right]^{\vee} \in P' \\ &= T(t) g_r \\ &= T(t)(T(r)\phi) \\ &= [T(t) \circ T(r)](\phi), \quad \forall t, r \in I\!R - \{0\}. \end{aligned}$$

So we have proven,

$$T(t+r) = T(t) \circ T(r), \forall t, r \in I\!R - \{0\}. \tag{23}$$

If $t = 0$ or $r = 0$ then equality (23) is also true, with this we conclude the proof of

$$T(t+r) = T(t) \circ T(r), \quad \forall t, r \in I\!R. \tag{24}$$

4) Let $f \in P'$, we will prove that:

$$T(t) f \xrightarrow{P'} f \text{ when } t \to 0.$$

That is, we will prove that

$$\langle T(t) f,\varphi\rangle \to \langle f,\varphi\rangle \text{ when } t\to 0,\quad \forall\varphi\in P.$$

In effect, for $t\in I\!R-\{0\}$ and $\varphi\in P$, we have

$$\begin{aligned} H_t &:= \langle T(t) f,\varphi\rangle - \langle f,\varphi\rangle \\ &= \lim_{n\to+\infty}\left\{\left\langle \sum_{k=-n}^{n}\hat f(k)\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}\phi_k,\varphi\right\rangle - \left\langle \sum_{k=-n}^{n}\hat f(k)\phi_k,\varphi\right\rangle\right\} \\ &= \lim_{n\to+\infty}\left\langle \sum_{k=-n}^{n}\hat f(k)\left(\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right)\phi_k,\varphi\right\rangle \\ &= \lim_{n\to+\infty}\sum_{k=-n}^{n}\hat f(k)\left(\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right)\langle\phi_k,\varphi\rangle \\ &= \lim_{n\to+\infty}2\pi\sum_{k=-n}^{n}\hat f(k)\left(\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right)\hat\varphi(-k) \\ &= 2\pi\sum_{k=-\infty}^{+\infty}\hat f(k)\left(\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right)\hat\varphi(-k). \end{aligned} \tag{25}$$

Since $t\in I\!R-\{0\}$, from (8) we get

$$\left|\frac{\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1}{t}\right| \le \mu|k|^m+|\alpha|. \tag{26}$$

From (26) we obtain

$$\left|\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right| \le \left\{\mu|k|^m+|\alpha|\right\}|t|,\quad \forall t\in I\!R. \tag{27}$$

From (27) with $0<|t|<1$, we have

$$\left|\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right| \le \mu|k|^m+|\alpha|. \tag{28}$$

Then using (28) and that $f\in P'$, we obtain

$$\begin{aligned} &\sum_{k=-\infty}^{+\infty}\left|\hat f(k)\right|\left|\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right|\left|\hat\varphi(-k)\right| \\ &\le C\left\{\mu\sum_{k=-\infty}^{+\infty}|k|^{N+m}\left|\hat\varphi(\underbrace{-k}_{=J})\right|+|\alpha|\sum_{k=-\infty}^{+\infty}|k|^{N}\left|\hat\varphi(\underbrace{-k}_{=J})\right|\right\} \\ &= C\left\{\mu\sum_{J=-\infty}^{+\infty}|J|^{N+m}\left|\hat\varphi(J)\right|+|\alpha|\sum_{J=-\infty}^{+\infty}|J|^{N}\left|\hat\varphi(J)\right|\right\}<\infty \end{aligned}$$

since $\hat\varphi\in S(Z)$.

Using the Weierstrass M-Test we conclude that the H_t series converges absolute and uniformly. So,

$$\lim_{t\to0}H_t = 2\pi\sum_{k=-\infty}^{+\infty}\hat f(k)\hat\varphi(-k)\underbrace{\lim_{t\to0}\left\{\mathrm{e}^{-i\mu k^m t}\mathrm{e}^{-i\alpha t}-1\right\}}_{=0}=0.$$

Thus, we have proved

$$\lim_{t\to0}\langle T(t) f,\varphi\rangle = \langle f,\varphi\rangle.$$

□

Theorem 3.3 *For each* $f\in P'$ *fixed and the family of operators* $\{T(t)\}_{t\in I\!R}$

from Theorem 3.2, then the application

$$M : I\!R \to P'$$
$$t \to T(t) f$$

is continuous in $I\!R$. That is,

$$T(t+h) f \xrightarrow{P'} T(t) f \text{ when } h \to 0, \forall t \in I\!R. \tag{29}$$

(is the continuity at t).

That is, (29) tell us that for each $t \in I\!R$ fixed, the following is satisfied

$$\langle T(t+h) f, \psi \rangle \to \langle T(t) f, \psi \rangle, \text{ when } h \to 0, \forall \psi \in P.$$

And if $t = 0$, we have the continuity of M at 0, which is item 4) of Theorem 3.2.

Proof.- Let $t \in I\!R - \{0\}$, arbitrary fixed and $f \in P'$ then $g := T(t) f \in P'$, using item 4) of Theorem 3.2, we have that $T(h) g \xrightarrow{P'} g$ when $h \to 0$. That is,

$$\underbrace{T(h)(T(t) f)}_{= \underbrace{[T(h) \circ T(t)] f}_{= T(h+t) f}} \xrightarrow{P'} T(t) f \text{ when } h \to 0,$$

where we use item 3) of Theorem 3.2.

□

Remark 3.1 *The results obtain in Theorems 3.2 and 3.3 are also valid for the family of operators* $\{S(t)\}_{t \in I\!R}$, *defined as*

$$S(t) : P' \to P'$$
$$f \to S(t) f := \left[\left(e^{i\mu k^m t} e^{-i\alpha t} \hat{f}(k) \right)_{k \in Z} \right]^{\vee},$$

for $t \in I\!R$. Its proof is similar.

3.3. Version of Theorem 3.1 Using the Family $\{T(t)\}_{t \in I\!R}$

We improve the statement of theorem 3.1, using a family of weakly continuous Operators $\{T(t)\}_{t \in I\!R}$.

Theorem 3.4 *Let* $f \in P'$ *and the family of operators* $\{T(t)\}_{t \in I\!R}$ *from Theorem* 3.2, *defining* $u(t) := T(t) f \in P'$, $\forall t \in I\!R$, *then* $u \in C(I\!R, P')$ *is the unique solution of* (Q_{m+1}). *Furthermore, u continuously depends on f. That is, given* $f_n, f \in P'$ *with* $f_n \xrightarrow{P'} f$ *implies* $u_n(t) \xrightarrow{P'} u(t)$, $\forall t \in I\!R$, *where* $u_n(t) := T(t) f_n$, $\forall t \in I\!R$ (*that is,* u_n *is a solution of* (Q_{m+1}) *with initial data* f_n).

Proof.- It is analogous to the proof of Theorem 3.1.

□

Corollary 3.2 *Let* $f \in P'$ *be fixed and the family of operators* $\{T(t)\}_{t \in I\!R}$ *from Theorem* 3.4, *then* $\exists \partial_t T(t) f$, $\forall t \in I\!R$ *and the mapping*

$$\tilde{\eta} : I\!R \to P'$$
$$t \to \partial_t T(t) f = i\mu \partial_x^m T(t) f - i\alpha T(t) f$$

is continuous at $I\!R$. That is,

$$\partial_t T(t+h) f \xrightarrow{P'} \partial_t T(t) f \quad \text{when } h \to 0, \quad \forall t \in I\!R. \tag{30}$$

(30) tells us that for each $t \in I\!R$ fixed, it holds:

$$\langle \partial_t T(t+h) f, \varphi \rangle \to \langle \partial_t T(t) f, \varphi \rangle \quad \text{when } h \to 0, \quad \forall \varphi \in P.$$

Proof.- Indeed,

$$\begin{aligned}
&\langle \partial_t T(t+h) f, \varphi \rangle - \langle \partial_t T(t) f, \varphi \rangle \\
&= i\mu \left\{ \langle \partial_x^m T(t+h) f, \varphi \rangle - \langle \partial_x^m T(t) f, \varphi \rangle \right\} - i\alpha \left\{ \langle T(t+h) f, \varphi \rangle - \langle T(t) f, \varphi \rangle \right\} \\
&= i\mu \underbrace{\left\{ \langle T(t+h) f, \varphi^{(m)} \rangle - \langle T(t) f, \varphi^{(m)} \rangle \right\}}_{\to 0} - i\alpha \underbrace{\left\{ \langle T(t+h) f, \varphi \rangle - \langle T(t) f, \varphi \rangle \right\}}_{\to 0} \\
&\to 0
\end{aligned}$$

when $h \to 0$, due to Theorem 3.3 with $\psi := \varphi^{(J)} \in P$, for $J = 0, m$.

□

Corollary 3.3 *Let* $f \in P'$ *be fixed and the family of operators* $\{T(t)\}_{t\in I\!R}$ *from Theorem* 3.4, *then the solution of* (Q_{m+1}): $u(t) := T(t) f$, $\forall t \in I\!R$, *satisfies* $u \in C^1(I\!R, P')$.

Proof.- It comes out as a consequence of Corollary 3.2.

□

4. Conclusions

In our study of the generalized Schrödinger type homogeneous model in the periodic distributional space P', we have obtained the following results:

1) We prove the existence, uniqueness of the solution of the problem (Q_{m+1}). Thus we also prove the continuous dependence of the solution respect to the initial data.

2) We introduce families of operators in P': $\{T(t)\}_{t\in I\!R}$ and we prove that they are linear and weakly continuous in P'. Furthermore, we proved that they form a group of weakly continuous operators in P'.

3) With the family of operators $\{T(t)\}_{t\in I\!R}$ we improve Theorem 3.1.

4) In contrast to what was obtained in P' with what has already been studied in H^s_{per}, we see that the weakly continuous operators are not unitary due to the topology of P'.

5) It is mathematically enriched, since we generate families of operators.

6) We must indicate that this technique can be applied to other evolution equations in P'.

7) Finally, for future work we want to emphasize that the results obtained will allow us to apply computational methods to determine the solution with a degree of approximation that is required and with a lower error rate.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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