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Coherent Perfect Absorption in Higher Order Systems

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Abstract

We investigate the phenomenon of coherent perfect absorption in a high-order system with three passive resonators coupled to a super-surface to form this three-state coherent perfect absorber. The effective parity time (PT) symmetry in the passive system has received much attention, and in this open three-state PT symmetric system, the incident wave is used as the effective gain instead of balancing the material gain and loss. We analyze the variation of coherent perfect absorption of this system with the coupling coefficient of the system by simulation.

Keywords

Exceptional Point, PT Symmetry, Non-Hermitian System, Coherent Perfect Absorption

1. Introduction

In recent years, the time-symmetric optical system has attracted a lot of attention in all aspects. Since PT symmetric Hamiltonians have the characteristic of having a fully real-valued energy spectrum below the phase transition point [1]. PT symmetry can extend quantum mechanics to some new areas of research. Ruschhaupt *et al.* have carried out pioneering work in recent years to achieve PT symmetry using gain and loss in optics to achieve PT symmetry Hamiltonian [2], and PT symmetry has emerged in various fields, including negative refraction [3], coherent perfect absorption [4], topological phase [5] [6] [7] [8] [9], electromagnetic impurity immunity [10], gain-loss induced topological edge states [11] [12] [13], laser absorbers [14], power oscillations [15] [16] [17] [18], irreversible Bloch oscillations [19], unidirectional invisibility [20] and various nonlinear effects [21].

According to quantum mechanics, if a Hermitian quantum system is interconnected with the external environment and introduces loss and gain, the system becomes an open non-Hermitian systems. At the same time, the concept of non-Hermitian can be applied to the field of optics, which has become a hot topic in the field of optics in recent years. Non-Hermitian systems can exhibit spontaneous symmetry breaking relative to PT symmetry, accompanied by spectral phase transitions from real to complex numbers [22]. The most obvious feature that distinguishes non-Hermitian systems from Hermitian systems is the singularity. In a non-Hermitian system, due to the exchange of energy between the system and the environment, the eigenstates of the non-Hermitian systems will merge, and the point at which the eigenstates merge together is the singularity. At the singularity, a phase transition occurs in the non-Hermitian systems—multiple eigenstates merge into one. Ideally, such a quantum system, which is closed and has constant energy that does not change with time, is the Hermitian systems. In general, quantum systems are basically open and have a large number of theoretical and experimental studies on non-Hermitian systems with energy varying with time [23] [24] [25]. EPs are used in many different fields, including acoustics [26] [27] [28], PT-symmetric systems [29] [30], coherent perfect absorption [31], photonic lattices [32], photonic crystals [33] [34], Bose-Einstein condensates [35], lasers [36], hydrogen atoms MS [37], microwaves [38] [39]. One direct application that takes advantage of the unique properties of EPs is sensors [40] [41]. More precisely, the feature topology surrounding the eigenvalues of the EP can be used to enhance the sensitivity of such a device.

Coherently, perfect absorption is a peculiar phenomenon that occurs when a non-Hermitian system satisfies the condition of PT symmetry. The loss and gain of the control system are balanced so that the energy is absorbed, thus achieving coherent perfect absorption of non-Hermitian systems. The principle of coherent perfect absorber is the reverse process of generating laser, that is, effectively using the interference effect of coherent light in the absorbing body, so that the absorber has an almost perfect (close to 100%) absorption efficiency, that is, the incident radiation is completely absorbed. We call this optical system a coherent perfect absorber. CPA has attracted great interest due to its wide range of potential applications in optical communications and photonic devices, such as sensors, modulators, optical switches and transistors. The CPA mechanism was first proposed by Chong *et al.* in 2010 [31], and since then, a large number of theories and experiments on CPA have been proposed. Now, CPA phenomenon has been demonstrated in different materials, including graphene [42], photonic crystals [43], waveguides [44], etc.

The study of high-order EPs is a hot research direction in recent years, because the cube root response near EP3 is extremely sensitive to external perturbations compared to the square root response near EP2 [45]. Therefore, we propose a three-state coherent perfect absorber with a PT phase transition consisting of three passive resonators coupled to the metasurface. In our three-state system, it can be found that this three-state PT symmetric system has CPA appearing before and after the PT phase transition, and the effect of system coupling on the absorption peak frequency is observed by calculation and simulation.

2. Theory of Higher-Order Systems

The theoretical model of our three-state system is shown in **Figure 1**, which consists of a "U" shaped comb line and two open resonant rings, and the two open resonant rings are located on the same side of the comb line. Since it needs to meet the 50 ohm matching, the width of the microstrip line is 2.4 mm, the geometric parameters in **Figure 1(a)** are $l_1 = 25\text{ mm}$; $l_2 = 15\text{ mm}$; $l_3 = 22\text{ mm}$, the total size of the two open resonant rings is 8 mm × 8 mm, the line width of both the comb line and the open resonant ring is 0.2 mm, the seam width at the opening of the comb line and the resonant ring is 1 mm, the distance of the open resonant ring from the microstrip line The distance between the open resonant ring and the microstrip line is 10 mm, and the simulation is performed using the microwave simulation software of CST Microwave Studio.

In our whole system, the "U" shaped comb line acts as the bright resonator A ($\tilde{a} = A\text{e}^{-iwt}$), the resonant ring close to the comb line acts as the dark resonator B ($\tilde{b} = B\text{e}^{-iwt}$), and the rightmost resonant ring acts as the dark resonator C ($\tilde{c} = C\text{e}^{-iwt}$), the bright resonator A can be directly excited by the incident wave, while the dark resonance can only be excited by the near-field coupling from the bright resonant cavity, by changing the distance between the comb line and the resonant ring B to adjust the coupling κ_1 between them, and by changing the distance s_1 between the two resonators to adjust the coupling κ_2 between

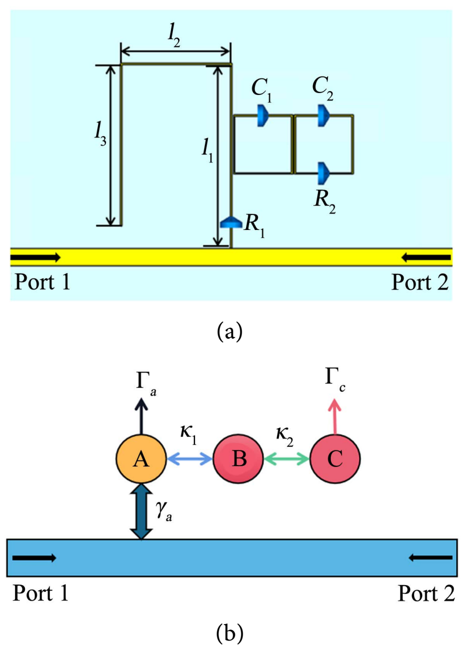

Figure 1. (a) Structural model of the system, (b) Theoretical model of the system where γ_a is the scattering loss of resonator A; Γ_a is the dissipation loss of resonator A; Γ_c the dissipation loss of resonator C; κ_1 is the coupling between resonator A and resonator B; κ_2 is the coupling between resonator B and resonator C.

them. Resistor R_1 is loaded at the opening of resonator A, capacitor C_1 is loaded at the opening of resonator B, and resistor R_2 and capacitor C_2 are loaded at the opening of resonant ring C. As can be seen from the CST simulation, bright resonator A is excited at $w_0 = 0.91\,\text{GHz}$. By adjusting the system parameters capacitor C_1 and C_2, so that the three resonators can be excited at the same frequency. Then $C_1 = C_2 = 2.74\,\text{pf}$.

The coupled mode equation of the three-state Non-Hermitian system is:

$$\begin{aligned}\frac{\mathrm{d}\tilde{a}}{\mathrm{d}t} &= \left(-iw_0 - \gamma_a - \Gamma_a\right)\tilde{a} - i\kappa_1\tilde{b} + i\sqrt{\gamma_a}\tilde{S}_{in} \\ \frac{\mathrm{d}\tilde{b}}{\mathrm{d}t} &= -iw_0\tilde{b} - i\kappa_1\tilde{a} - i\kappa_{12}\tilde{c} \\ \frac{\mathrm{d}\tilde{c}}{\mathrm{d}t} &= \left(-iw_0 - \Gamma_c\right)\tilde{c} - i\kappa_2\tilde{b}\end{aligned} \tag{1}$$

This in turn leads to the derivation of the Hamiltonian quantity H for the three-state non-Ermian system as:

$$H = \begin{pmatrix} w_0 + i\left(\gamma_a - \Gamma_a\right) & \kappa_1 & 0 \\ \kappa_1 & w_0 & \kappa_2 \\ 0 & \kappa_2 & w_0 - i\Gamma_c \end{pmatrix} \tag{2}$$

In the ideal PT symmetric system to satisfy $\kappa_1 = \kappa_2 = \kappa$ and $\gamma_a - \Gamma_a = \Gamma_c$. The eigen solutions of the Hamiltonian quantities are derived by $\det\left(H - wI\right) = 0$:

$$w_1 = w_0;\ w_2 = w_0 - \sqrt{2\kappa^2 - \Gamma_c^2};\ w_3 = w_0 + \sqrt{2\kappa^2 - \Gamma_c^2} \tag{3}$$

3. Simulation Results for Higher-Order Systems

The following parameters were obtained by fitting the system model using CST simulation software: $\gamma_a = 0.13\,\text{GHz}$; $\Gamma_a = 0.006$; $\Gamma_c = 0.009$; the data simulation of the real and imaginary parts of the eigenvalue analytic solution with κ evolution is shown in **Figure 2**.

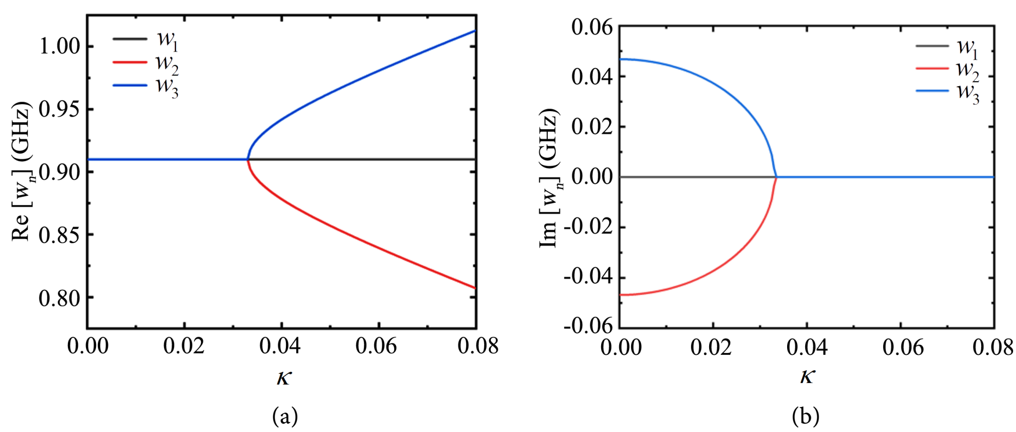

Figure 2. (a) Simulation of the real part of the eigenvalue analytic solution with κ evolution, the real part is equal before $\kappa = 0.033$ GHz. (b) Data simulation of the evolution of the imaginary part of the eigenvalue analytic solution with κ, the imaginary parts are equal after $\kappa = 0.033$ GHz, then the EP point appears at $\kappa = 0.033$ GHz.

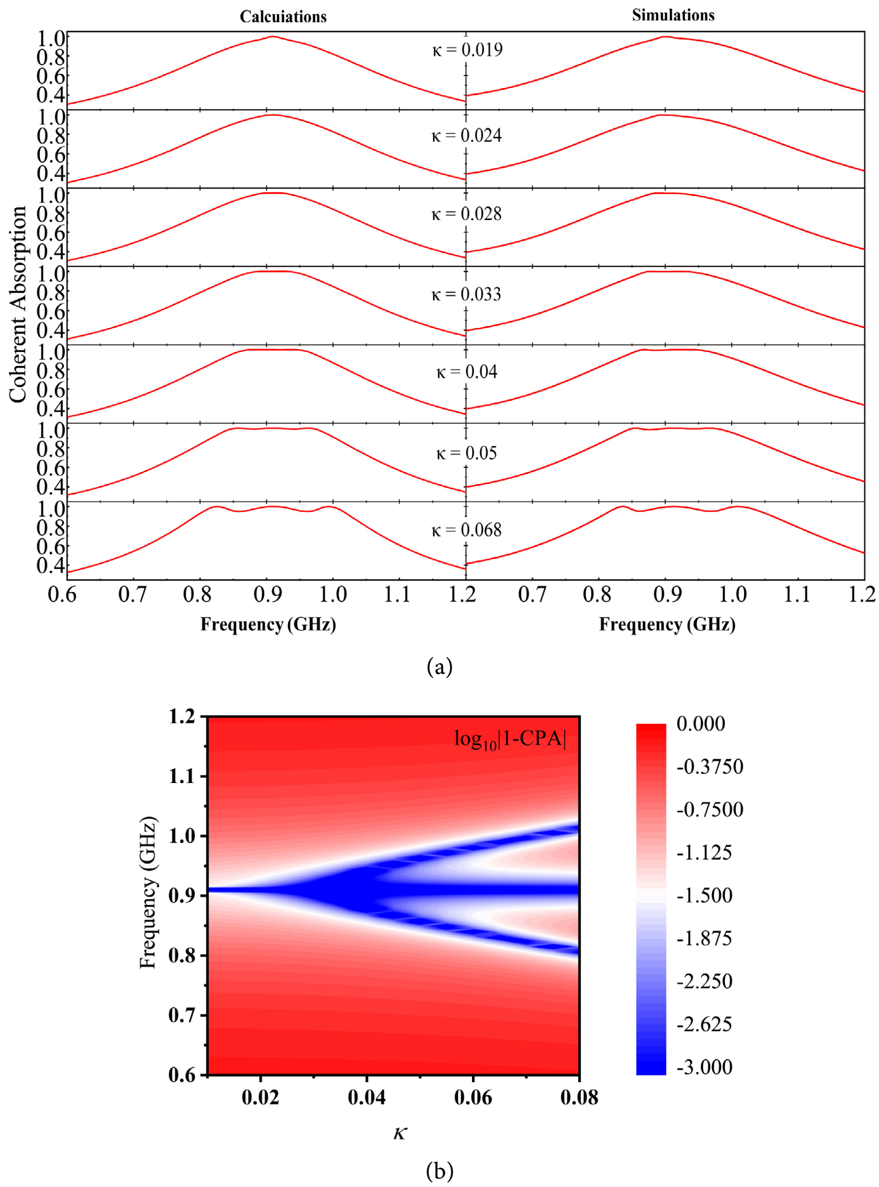

Figure 3. (a) Variation of CPA with κ for the system at ideal PT symmetry, where the left column shows the calculated results of the coupled mode equation and the right column shows the results of the CST simulation. (b) Analytic spectrum of the system at ideal PT symmetry with respect to $\log_{10}|1-\text{CPA}|$.

From **Figure 3**, it can be seen that when κ is less than 0.033 GHz, the peak of coherent perfect absorption is one; when κ is greater than 0.033 GHz, the peak of coherent perfect absorption changes from one to three; this change can be observed more obviously from **Figure 3(b)**.

4. Conclusion

In conclusion, instead of balancing the gain and loss of the material, we use the incident wave as the effective gain to build a three-state system with ideal PT symmetry. The coherent perfect absorption phenomenon in the higher-order system is analyzed by simulation results, and unlike the two-state system, the three-state system has a coherent perfect absorption peak before and after the EP point. This study of coherent perfect absorption in higher-order systems can be applied to optical communication and photonic devices such as sensors, modulators, optical switches and transistors which also have some wide potential applications.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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