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Study on Band Gap Characteristics of Boat-Shaped Phononic Crystal

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Abstract

In this paper, we analyze the two-dimensional Boat-shaped structure based on the finite element method. We calculated its energy band structure and vibration transmission properties and found that the structure has band gaps at both high and low frequencies. Compared with common traditional two-dimensional phononic crystals, the boat-shaped phononic crystal has the advantage of larger bandgap design and modulation parameter space due to their structural complexity. In order to obtain better bandgap characteristics, we studied the influence of four key parameters, such as the rod length and the angle between the rods, on the bandgap. The results show that: for low frequency band gaps, the width of the band gap can be effectively changed by changing the size of the angle between the rods while rod length greatly affects the bandgap position; for high band gaps, the length of rods has a large effect on the band gap position. These laws have guiding significance for the bandgap regulation of boat-shaped phononic crystal.

Keywords

Phononic Crystal, Band Gap, Boat-Shaped, Finite Element Simulation

1. Introduction

Phononic crystals (PCs) are periodic structure that can prohibit the propagation of acoustic or elastic waves with frequency in the range of phononic bandgap [1]. In recent years, theoretical and applied exploration of PCs has attracted extensive research interest due to the unique function of the band gap and has proposed a wealth of applications. Especially, it has a good application prospect in damping and noise reduction and elastic wave control, including but not limited to sound absorbers [2] [3], vibration isolators [4], acoustic cloaks [5] [6], filters [7], and waveguides [8]. It is generally believed that there are two mechanisms for generating band gaps in phononic crystals, namely Bragg scattering [9] and local resonance [10]. For the former, the interaction between unit cells or the periodicity of the structure leads to the appearance of the band gap; for the latter, the resonance properties of a single scatterer play a decisive role in the appearance of the band gap. Based on these two mechanisms, in order to obtain better bandgap properties, a host of academic research has been done on different periodic structures. Based on the Bragg scattering theory, Wang [11] studied the effect of a plate with cross cavities on the band gap. Wang [12] studied the effect of different inclusions in the same matrix on the band gap by the finite element method. Seongmin *et al.* [13] proposed a tapered phononic beam with a broad low-frequency band gap for flexural waves. Gao *et al.* [14] proposed and discussed the elastic wave attenuation of hollow metamaterial beam embedded acoustic black hole. M. Thota *et al.* [15] proposed and studied an origami phononic structure with tunable waveguiding. The boat-shaped phononic crystal we studied in this paper was proposed in a previous work of our group [16]. Compared with conventional Bragger scattering PCs and local resonance PCs, the advantage of Boat-shaped phononic crystal is the larger parameter spaces for design and modulation of bandgaps.

The structure of this paper is as follows: Firstly, we introduce the structural characteristics of Boat-shaped phononic crystal and investigate the band gap properties of this phononic crystal using the finite element method. Secondly, the parametric study was performed and the results of how parameters affect the bandgaps were discussed. Lastly, the conclusions are summarized.

2. Structure and Theoretical Basis

The structure of the Boat-shaped phononic crystal studied in this paper is shown in **Figure 1(a)**. Due to the mirror symmetry of the structure along both the x-axis and the y-axis, the unit cell can be described by the following geometric parameters: The lengths of rods C_1C_2 and C_3C_4 are C_1C_2 = 15.4 mm and C_3C_4 = 15.7 mm respectively. θ_3 is the space angle between rod $L_{C_1C_2}$ and rod $L_{C_3C_4}$, θ_1 is the angle between the plane where notes C_1, C_2 and C_3 are located and the plane where notes C_2, C_3 and C_4 are located. The values of θ_1 and θ_3 are θ_1 = 48.55˚ and θ_3 = 110.7˚ respectively; all rods have a radius R of 1 mm. The material parameters set during the numerical simulation are as follows: ρ = 1800 kg/m^3, E = 0.86 GPa and ν = 0.45 represent the density, Young's modulus and Poisson ratio, respectively.

In a linear elastic, isotropic, homogeneous medium with infinite volume, ignoring the effect of damping, the vector form of the elastic wave equation can be written as:

$$-\rho\omega^2 u(r) = (\lambda+\mu)\nabla(\nabla\cdot u(r)) + \mu\nabla^2 u(r) \tag{1}$$

In Equation (1), r is the spatial position vector, u is the mass displacement vector, ρ is the density of the medium, μ and λ are the Lame constants of the material, ω denotes the characteristic circle frequency. Due to the spatial periodicity and symmetry of the phononic crystal, the eigenfrequency and eigenmode of the wave field have a certain periodicity. According to the Bloch theorem, the eigenwave field in the periodic structure can be expressed as:

$$u(r) = u_k(r)e^{i(k \cdot r)} \tag{2}$$

In Equation (2), k denotes Bloch wave vector. The translation of the amplitude function $u_k(r)$ to the lattice vector R is also periodic, that is:

$$u_k(r+R) = u_k(r) \tag{3}$$

Due to the symmetry of the phononic crystal point group, the wave vector k can only take a value in the first irreducible Brillouin zone. Figure 1(b) shows the first Brillouin zone of Boat-shaped phononic crystal, and the shaded area denotes the irreducible Brillouin zone. Due to the function of setting periodic boundaries, all results in this paper are calculated by commercial code COMSOL. Figure 2 shows the band structure of Boat-shaped phononic crystal when all

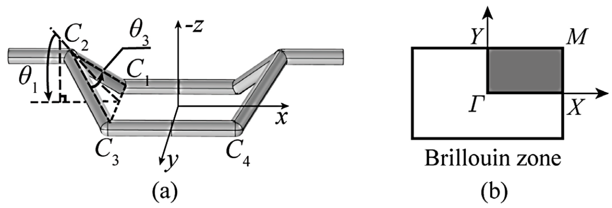

Figure 1. (a) The unit cell of Boat-shaped phononic crystal. C_1, C_2, C_3 and C_4 are the labels at the node, θ_3 is the angle between rod $L_{C_1C_2}$ and rod $L_{C_3C_4}$, θ_1 is the angle between the plane where notes C_1, C_2 and C_3 are located and the plane where notes C_2, C_3 and C_4 are located. (b) Brillouin zone of a rectangular lattice. Shading indicates irreducible Brillouin zone.

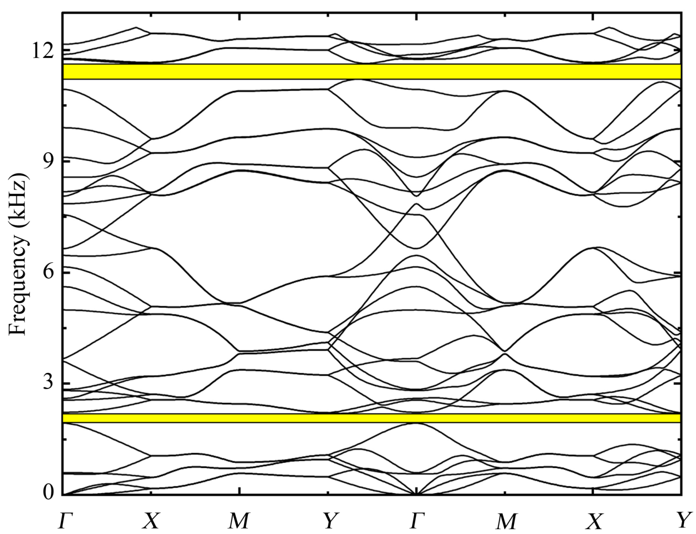

Figure 2. Band structure of the model under original size data. The yellow shaded area are the band gaps.

parameters are initialized. We find that the first band gap appears in the interval (1942 Hz, 2204 Hz) and the second band gap appears in the interval (11,216 Hz, 11,640 Hz). The band gap widths for low and high frequencies are 262 Hz and 424 Hz, respectively.

3. Numerical Simulations and Results

In this section, we discuss the effect of geometric parameters θ_1, θ_3, $L_{C_1C_2}$ and $L_{C_3C_4}$ on the low frequency first band gap and high frequency second band gap of boat-shaped phononic crystal. Noting that other geometric dimensions and material properties remain unchanged when the single parameter changes. As parameter θ_1 increase from 30˚ to 90˚, the lower bound of band gap has a significant decline while the band gap expands, as shown in **Figure 3(a)**. **Figure 3(b)** shows the effect of θ_3 on band gap, the width of the band gap is basically unchanged when θ_3 is between 70˚ - 140˚, but when θ_3 increases from 140˚ to 150˚, the band gap shrinks and closes in 150˚. As geometric parameter $L_{C_1C_2}$ increase from 11 mm to 21 mm, the lower bound and upper bound of first band gap all show a downward trend and the bandwidth widen first and then narrow, and the result is shown in **Figure 3(c)**. **Figure 3(d)** shows the effect of $L_{C_3C_4}$ on band gap, when $L_{C_3C_4}$ changes from 12 mm to 20 mm, the position of the band gap decreases, and the bandwidth shows the same pattern as in **Figure 3(c)**. By analyzing **Figure 3**, we can see that the bandwidth is more sensitive to θ_1, and $L_{C_1C_2}$ has the greatest effect on the bandgap position.

After analyzing the effect of parameters on the low frequency bandgap, now we study the relationship between these four parameters (θ_1, θ_3, $L_{C_1C_2}$, $L_{C_3C_4}$) and the high frequency band gap, the results are shown in **Figure 4**. We can find

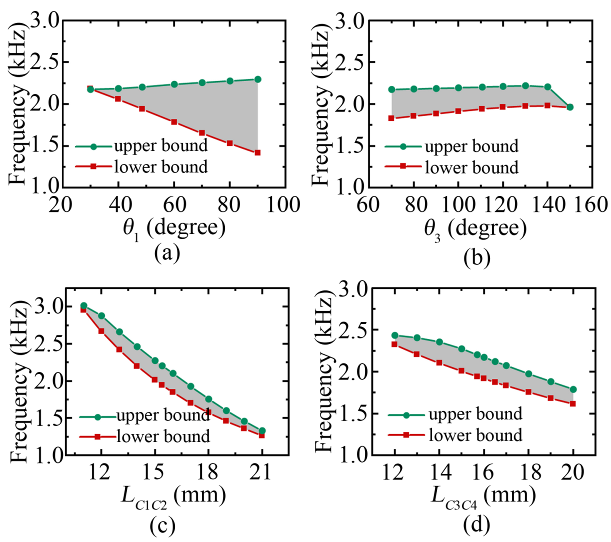

Figure 3. Effect of geometric parameter θ_1, θ_3, $L_{C_1C_2}$ and $L_{C_3C_4}$ on the low frequency first band gap of Boat-shaped phononic crystal. (a), (b), (c) and (d) are change of first band gap as the respective change of θ_1, θ_3, $L_{C_1C_2}$ and $L_{C_3C_4}$. The circle and square lines denote the upper and lower bounds of the low frequency bandgap, respectively.

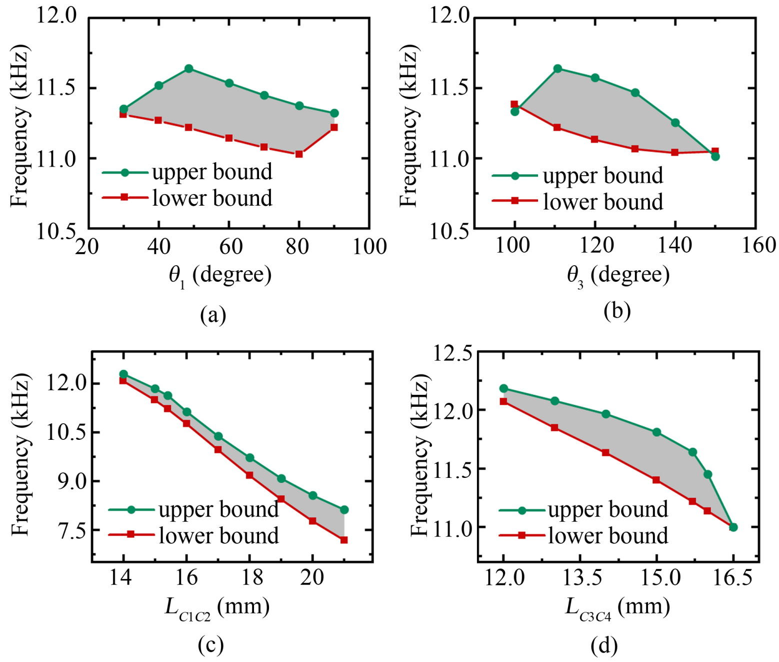

Figure 4. Effect of geometric parameter θ_1, θ_3, $L_{C_1C_2}$ and $L_{C_3C_4}$ on the high frequency second band gap of Boat-shaped phononic crystal. (a), (b), (c) and (d) are change of second band gap as the respective change of θ_1, θ_3, $L_{C_1C_2}$ and $L_{C_3C_4}$. The circle and square lines denote the upper and lower bounds of the high frequency bandgap, respectively.

that as θ_1, θ_3, and $L_{C_3C_4}$ increase within their respective ranges, the corresponding band gaps first open and then close, and bandwidths first widen and then narrow, as shown in Figure 4(a), Figure 4(b) and Figure 4(d). In contrast, as shown in Figure 4(c), as parameter $L_{C_1C_2}$ increase from 14 mm to 21 mm, the position of high frequency bandgap has a significant decrease while the bandwidth keeps increasing. The maximum of bandwidth of High frequency second bandgap occurs at $L_{C_1C_2}$ = 21 mm. Within the variation interval of $L_{C_1C_2}$, the variation of the band gap position is also the largest. These show that both the bandgap position and the bandwidth are most sensitive to the parameter $L_{C_1C_2}$.

4. Conclusion

In summary, we used the finite element method to study the band structure of the Boat-shaped phononic crystal and found that it has elastic wave band gaps at both high and low frequencies. In the parametric study of four key design parameters, the law of the influence of parameters on the band gap was found. For low frequency band gaps, the width of the band gap can be effectively changed by changing the size of θ_1 while $L_{C_1C_2}$ and $L_{C_3C_4}$ greatly affect the bandgap position. For high band gaps, $L_{C_1C_2}$ and $L_{C_3C_4}$ have a large effect on the band gap position. These laws will inspire us to design boat-shaped phononic crystal with more excellent or specific band gaps that can be applied in specific fields.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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