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New Solitary Wave Solutions of the Fisher Equation

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Abstract

In this paper, we use Riccati equation to find new solitary wave solutions of Fisher equation, which describes the process of interaction between diffusion and reaction. It is of great importance to comprehend the equation to solve the problems in chemical kinetics and population dynamics. We resolve the Ricatti equation through diverse function transformation and many types of exact solutions are obtained. Then it is used as an auxiliary equation to solve Fisher equation. In the process, we select different coefficients in the Racatti equation, as a result, abundant solitary wave solutions are obtained, some of which haven't been found in other documents yet. Moreover, these solutions we got in this paper will be favorable for understanding the Fisher equation.

Keywords

Nonlinear Evolution Equations, Solitary Wave, Soliton, Fisher Equation, Riccati Equation

1. Introduction

At present, the study of solitary waves and solitons is a hot topic in nonlinear physics, from chemical kinetics, population dynamics, plasma, optics, and biology, etc [1]-[6]. Through physical mechanisms and facts from experiments, we learned that solitons exist. Under some reasonable conditions, we can establish mathematics models of most physics' laws, and many of these nonlinear identification studies can give the credit to the nonlinear evolution equations eventually. Hence, finding the exact solutions, such as solitary-wave solutions, is very important for exploring the relevant nonlinear problems and understanding their characteristics, which can help scientists apply them into practical researches and studies to solve the problems in chemical kinetics and population dynamics. A large number of scientists have devoted themselves to the solutions in the latest centuries and as a result, many effective and forceful methods have been put forward in other documents to get the exact solutions of Fisher equation, for example, tanh-sech method and the extended tanh-coth method [7] [8], F-expansion method [9] [10], Jacobi elliptic function expansion method [11] [12], auxiliary equation method [13] [14] [15] [16], and so on. However, even some of them are powerful and effective, not all of them can be suitable for exploring the exact solutions of Fisher equation [16], which describes the interaction process between diffusion and reaction. This equation is encountered in chemical kinetics and population dynamics, including problems such as the nonlinear evolution of the population in the one-dimensional conventional neutron population in nuclear reactions. In Ref. [17], Fisher equation is treated with extended tanh method and many new solitary wave solutions were obtained. In this paper, we will use Riccati equation as an auxiliary equation to solve Fisher equation, so that many new results are obtained. The method used in this paper, on the one hand, can simplify the solving process of the nonlinear equation, on the other hand, obtains a new solitary wave solution of the Fisher equation.

The framework of the paper is as follows: Section 2 introduces the construction of abundant exact solutions of Riccati equation; Section 3 establishes how to operate this method for producing new solitary wave solutions of Fisher equation; Section 4 is the conclusion.

2. Abundant Exact Solutions of Riccati Equation

The Riccati equation method is very simple but very effective. Hence, it is an ideal method to solve constant coefficient, variable coefficient and high-dimensional nonlinear evolution equations. In the paper, it first comes to our mind that we can use Racatti equation to solve the problems in the following form:

$$f'(\xi) = p_1 f^2(\xi) + q_1 \tag{1}$$

where p_1 and q_1 are constants and can be determined later. To find out new exact solutions of Equation (1), a new auxiliary function $g(\xi)$ is introduced, which satisfies the following form

$$\left[g'(\xi)\right]^2 = p_2 g^2(\xi) + q_2 \tag{2}$$

where p_2 and q_2 are constants. Equation (2) has the following hyperbolic function solution

$$g_1(\xi) = \sinh(\xi),\ (p_2 = 1, q_2 = 1) \tag{3}$$

$$g_2(\xi) = \cosh(\xi),\ (p_2 = 1, q_2 = -1) \tag{4}$$

$$g_3(\xi) = \cosh^2(\xi) - \frac{1}{2} = \sinh^2(\xi) + \frac{1}{2},\ (p_2 = 4, q_2 = -1) \tag{5}$$

Then we assume $f(\xi)$ and $g(\xi)$ have the following formal solution

$$f(\xi) = \frac{g'(\xi)}{g(\xi) + r} \tag{6}$$

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where r is a constant. Substituting Equation (6) into Equation (1) and using Equation (2), we can obtain

$$\begin{cases} p_1 p_2 + q_1 = 0, \\ p_2 r = 2q_1 r, \\ -q_2 = p_1 q_2 + q_1 r^2. \end{cases} \tag{7}$$

Solving this system, we can obtain

$$\begin{cases} r = 0, \\ p_1 = -1, \\ q_1 = p_2, \end{cases} \quad \text{or} \quad \begin{cases} r = \pm\sqrt{-\dfrac{q_2}{p_2}}, \\ p_1 = -\dfrac{1}{2}, \\ q_1 = \dfrac{p_2}{2}. \end{cases} \tag{8}$$

So, we have the following exact solutions of Equation (1)

$$f_1(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}, \ (p_1 = -1, q_1 = 1) \tag{9}$$

$$f_2(\xi) = \frac{\sinh(\xi)}{\cosh(\xi)}, \ (p_1 = -1, q_1 = 1) \tag{10}$$

$$f_3(\xi) = \frac{2\sinh(\xi)\cosh(\xi)}{\cosh^2(\xi) - \frac{1}{2}}, \ (p_1 = -1, q_1 = 4) \tag{11}$$

$$f_4(\xi) = \frac{\sinh(\xi)}{\cosh(\xi) + \varepsilon}, \ (p_1 = -\frac{1}{2}, q_1 = \frac{1}{2}, \varepsilon^2 = 1) \tag{12}$$

$$f_5(\xi) = \frac{\cosh(\xi)}{\sinh(\xi) + \varepsilon}, \ (p_1 = -\frac{1}{2}, q_1 = \frac{1}{2}, \varepsilon^2 = -1) \tag{13}$$

Next, we use the following another formal solution to solve Equation (1)

$$f(\xi) = \frac{g(\xi) g'(\xi)}{g^2(\xi) + r} \tag{14}$$

where $r \neq 0$. Substituting Equation (14) into Equation (1) and using Equation (2), we can obtain

$$\begin{cases} p_1 p_2 + q_1 = 0, \\ -q_2 + 2p_2 r = p_1 q_2 + 2q_1 r, \\ r q_2 = q_1 r^2. \end{cases} \tag{15}$$

Solving this system, we can obtain

$$\begin{cases} r = \dfrac{q_2}{2p_2}, \\ p_1 = -2, \\ q_1 = 2p_2. \end{cases} \tag{16}$$

So, we can have the following exact solutions

$$f_6(\xi)=\frac{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}},\ (p_1=-2,q_1=8) \tag{17}$$

It is easy to know that $h(\xi)=1/f(\xi)$ can also satisfy Equation (1) in the condition of $p_1'=-q_1,q_1'=-p_1$. Equations (9) and (10) are a pair of solutions on this condition. Therefore, the following equations are also the solutions of Equation (1)

$$f_7(\xi)=\frac{\cosh^2(\xi)-\frac{1}{2}}{2\sinh(\xi)\cosh(\xi)},\ (p_1=-4,q_1=1) \tag{18}$$

$$f_8(\xi)=\frac{\cosh(\xi)+\varepsilon}{\sinh(\xi)},\ \left(p_1=-\frac{1}{2},q_1=\frac{1}{2},\varepsilon^2=1\right) \tag{19}$$

$$f_9(\xi)=\frac{\sinh(\xi)+\varepsilon}{\cosh(\xi)},\ \left(p_1=-\frac{1}{2},q_1=\frac{1}{2},\varepsilon^2=-1\right) \tag{20}$$

$$f_{10}(\xi)=\frac{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)},\ (p_1=-8,q_1=2) \tag{21}$$

Equations (6) and (10) are the new types of exact solutions of Equation (1), which are rarely found in the other documents. Then, we use the Equation (1) and its solutions (9)-(13), (17) and (18)-(21) to solve the Fisher equation, and the solving process can be greatly simplified.

3. Application of the Method

The following Fisher equation [16] [17] is considered

$$u_t-u_{xx}-u(1-u)=0 \tag{22}$$

Then suppose Equation (22) has the traveling wave solution

$$u(x,t)=u(\xi),\xi=\mu x+ct \tag{23}$$

where μ and c are travelling wave parameters. Substituting the traveling wave equations into Equation (22), the following equation can be obtained

$$cu'-\mu^2u''-u(1-u)=0 \tag{24}$$

We assume that Equation (24) has the following formal solution

$$u(\xi)=\sum_{i=1}^{0}a_if^i(\xi)+\sum_{i=1}^{n}b_if^{-i}(\xi) \tag{25}$$

where a_i and b_i are constants to be determined and $f^i(\xi)$ is the solutions of Equation (1) and n can be determined by the homogeneous balance method. In the Equation (24), it is easy to know n = 2, so that the solution can be expressed as

$$u(\xi)=a_0+a_1f(\xi)+a_2f^2(\xi)+b_1f^{-1}(\xi)+b_2f^{-2}(\xi) \tag{26}$$

We bring above equation into Equation (24) and use Equation (1), resulting in a series of equations a set of algebraic equations about a_0, a_1, a_2, b_1, b_2, μ and c.

Then we collect all the terms with the same power of $f(\xi)$, and set each coefficient to zero. Finally, we can obtain

$$\textbf{Case 1}\quad \begin{aligned} &a_0=\frac{1}{4}, a_1=\pm\frac{1}{2}\sqrt{-\frac{p_1}{q_1}}, a_2=-\frac{p_1}{4q_1}, b_1=b_2=0,\\ &k^2=-\frac{1}{24p_1q_1}, c=\mp\frac{5}{12\sqrt{-p_1q_1}} \end{aligned} \tag{27}$$

$$\textbf{Case 2}\quad \begin{aligned} &a_0=\frac{1}{4}, a_1=a_2=0, b_1=\pm\frac{1}{2}\sqrt{-\frac{q_1}{p_1}}, b_2=-\frac{q_1}{4p_1},\\ &k^2=-\frac{1}{24p_1q_1}, c=\pm\frac{5}{12\sqrt{-p_1q_1}} \end{aligned} \tag{28}$$

$$\textbf{Case 3}\quad \begin{aligned} &a_0=\frac{3}{8}, a_1=\pm\frac{1}{4}\sqrt{-\frac{p_1}{q_1}}, a_2=-\frac{p_1}{16q_1}, b_1=\mp\frac{1}{4}\sqrt{-\frac{q_1}{p_1}},\\ &b_2=-\frac{q_1}{16p_1}, k^2=-\frac{1}{96p_1q_1}, c=\mp\frac{5}{24\sqrt{-p_1q_1}} \end{aligned} \tag{29}$$

According to Case 1, we have the following solitary wave solutions of the Fisher equation

$$u_1(\xi)=\frac{1}{4}\pm\frac{1}{2}\coth(\xi)+\frac{1}{4}\coth^2(\xi) \tag{30}$$

where $\xi=\mu x+ct, \mu=\frac{1}{2\sqrt{6}}\varepsilon, c=\mp\frac{5}{12}, \varepsilon^2=1$. Figure 1(a) shows the three-dimensional diagrams of Equation (30), which represents the bright solitary wave solution. Figure 1(b) shows that the amplitude and velocity of this solitary wave remain unchanged during propagation.

$$u_2(\xi)=\frac{1}{4}\pm\frac{1}{2}\tanh(\xi)+\frac{1}{4}\tanh^2(\xi) \tag{31}$$

where $\xi=\mu x+ct, \mu=\frac{1}{2\sqrt{6}}\varepsilon, c=\mp\frac{5}{12}, \varepsilon^2=1$. Figure 2 shows the three-dimensional and two- dimensional plots diagrams of Equation (31), which

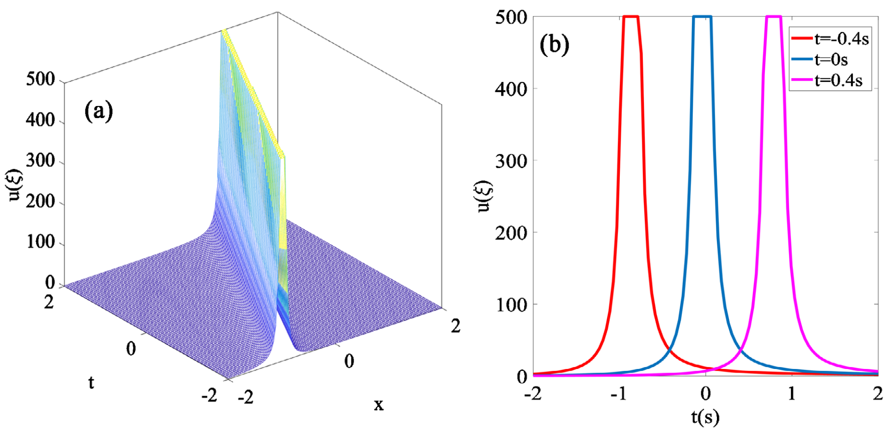

Figure 1. (a) Three dimensional and (b) two dimensional plots represent the bright solitary wave solution of Equation (30), when ± sign takes +, ∓ sign takes – and $\varepsilon=1$.

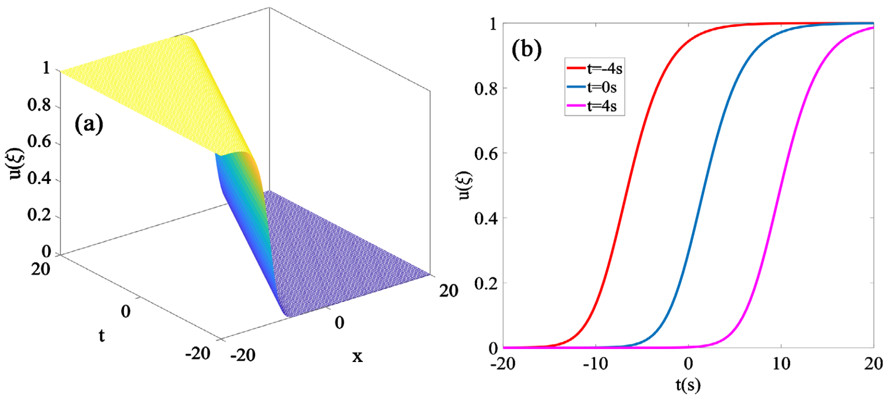

Figure 2. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (31), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

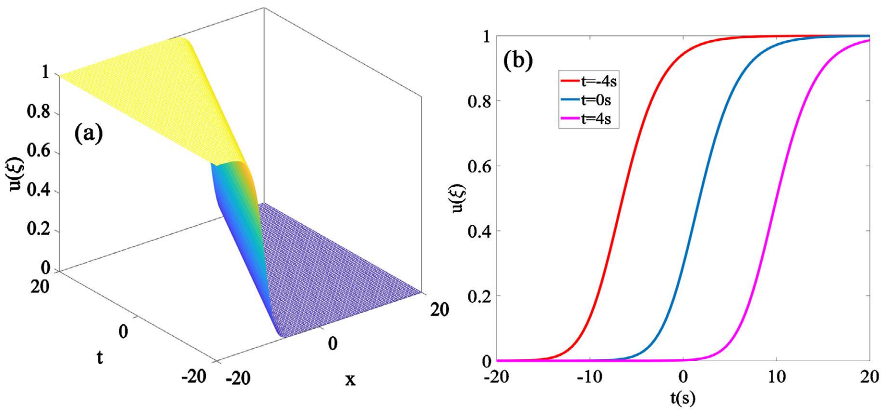

Figure 3. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (32), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

represents the kink solitary wave solution.

$$u_3(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\sinh(\xi)\cosh(\xi)}{\cosh^2(\xi) - \frac{1}{2}} + \frac{1}{4}\left[\frac{\sinh(\xi)\cosh(\xi)}{\cosh^2(\xi) - \frac{1}{2}}\right]^2 \quad (32)$$

where $\xi = \mu x + ct, \mu = \frac{1}{4\sqrt{6}}\varepsilon, c = \mp\frac{5}{24}, \varepsilon^2 = 1$. **Figure 3** shows the three-dimensional and two- dimensional plots diagrams of Equation (32), which also represents the kink solitary wave solution.

$$u_4(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\sinh(\xi)}{\cosh(\xi) + \varepsilon} + \frac{1}{4}\left[\frac{\sinh(\xi)}{\cosh(\xi) + \varepsilon}\right]^2 \quad (33)$$

where $\xi = \mu x + ct, \mu = \frac{1}{\sqrt{6}}\lambda, c = \mp\frac{5}{6}, \varepsilon^2 = 1, \lambda^2 = 1$. **Figure 4** shows the three-dimensional and two- dimensional plots diagrams of Equation (33), which also represents the kink solitary wave solution.

$$u_5(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\cosh(\xi)}{\sinh(\xi) + \varepsilon} + \frac{1}{4}\left[\frac{\cosh(\xi)}{\sinh(\xi) + \varepsilon}\right]^2 \quad (34)$$

where $\xi = \mu x + ct, \mu = \frac{1}{\sqrt{6}}\lambda, c = \mp\frac{5}{6}, \varepsilon^2 = -1, \lambda^2 = 1$. The solution of Equation (34) represents the traveling wave solutions of Equation (24) in complex space.

$$u_6(\xi) = \frac{1}{4} \pm \frac{1}{4}\frac{2\sinh(\xi)\cosh^3(\xi) - \sinh(\xi)\cosh(\xi)}{\left[\cosh^2(\xi) - \frac{1}{2}\right]^2 - \frac{1}{8}} + \frac{1}{16}\left\{\frac{2\sinh(\xi)\cosh^3(\xi) - \sinh(\xi)\cosh(\xi)}{\left[\cosh^2(\xi) - \frac{1}{2}\right]^2 - \frac{1}{8}}\right\}^2 \tag{35}$$

where $\xi = \mu x + ct, \mu = \frac{1}{8\sqrt{6}}\varepsilon, c = \mp\frac{5}{48}, \varepsilon^2 = 1$. Figure 5 shows the three-dimensional and two- dimensional plots diagrams of Equation (35), which represents the kink solitary wave solution.

$$u_7(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\cosh^2(\xi) - \frac{1}{2}}{\sinh(\xi)\cosh(\xi)} + \frac{1}{4}\left[\frac{\cosh^2(\xi) - \frac{1}{2}}{\sinh(\xi)\cosh(\xi)}\right]^2 \tag{36}$$

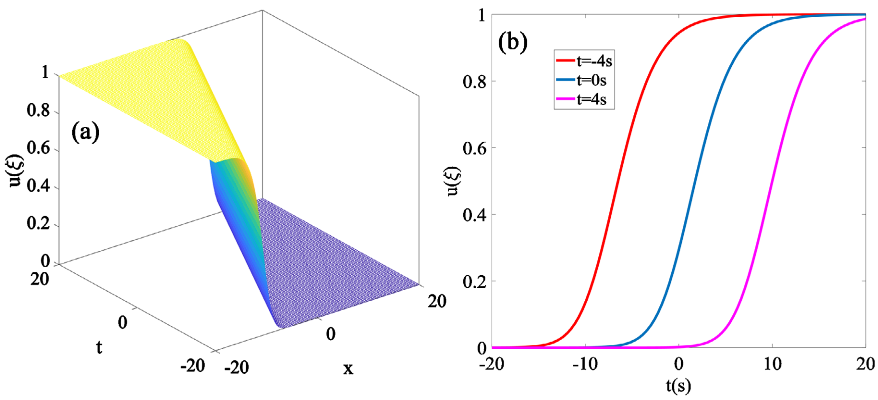

Figure 4. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (33), when ± sign takes +, ∓ sign takes -, $\varepsilon = 1$ and $\lambda = 1$.

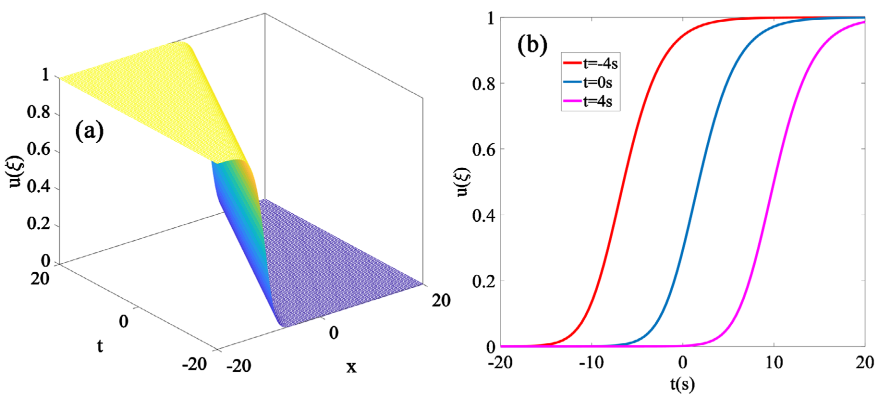

Figure 5. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (35), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

where $\xi = \mu x + ct, \mu = \frac{1}{4\sqrt{6}}\varepsilon, c = \mp\frac{5}{24}, \varepsilon^2 = 1$. Figure 6 shows the three-dimensional and two- dimensional plots diagrams of Equation (36), which represents the bright solitary wave solution.

$$u_8(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\cosh(\xi)+\varepsilon}{\sinh(\xi)} + \frac{1}{4}\left[\frac{\cosh(\xi)+\varepsilon}{\sinh(\xi)}\right]^2 \tag{37}$$

where $\xi = \mu x + ct, \mu = \frac{1}{\sqrt{6}}\lambda, c = \mp\frac{5}{6}, \varepsilon^2 = 1, \lambda^2 = 1$. Figure 7 shows the three-dimensional and two- dimensional plots diagrams of Equation (37), which represents the bright solitary wave solution.

$$u_9(\xi) = \frac{1}{4} \pm \frac{1}{2}\frac{\sinh(\xi)+\varepsilon}{\cosh(\xi)} + \frac{1}{4}\left[\frac{\sinh(\xi)+\varepsilon}{\cosh(\xi)}\right]^2 \tag{38}$$

where $\xi = \mu x + ct, \mu = \frac{1}{\sqrt{6}}\lambda, c = \mp\frac{5}{6}, \varepsilon^2 = -1, \lambda^2 = 1$. The solution of Equation (38) represents the traveling wave solutions of Equation (24) in complex space.

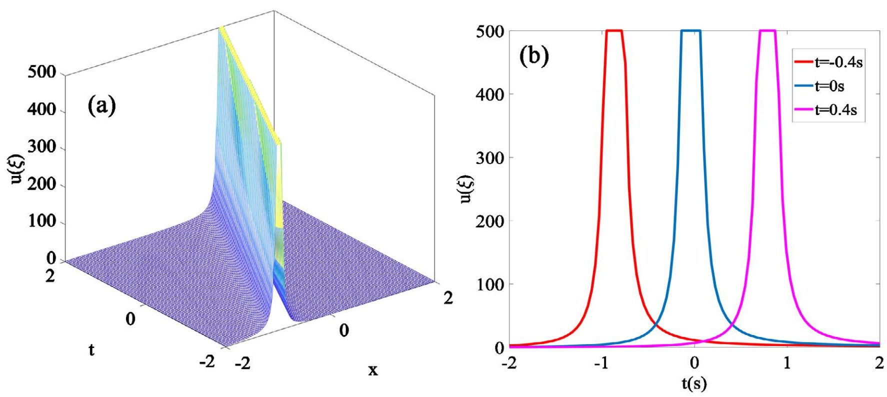

Figure 6. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (36), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

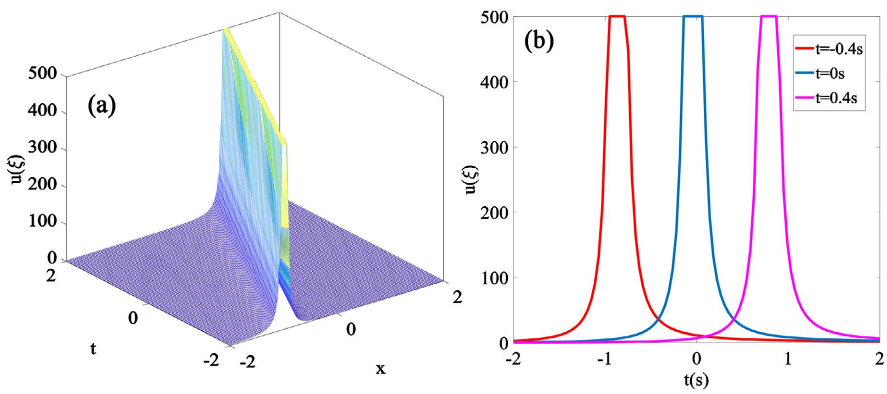

Figure 7. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (37), when ± sign takes +, ∓ sign takes -, $\varepsilon = 1$ and $\lambda = 1$.

$$u_{10}(\xi)=\frac{1}{4}\pm\frac{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}+\left\{\frac{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}\right\}^2 \quad (39)$$

where $\xi=\mu x+ct, \mu=\frac{1}{8\sqrt{6}}\varepsilon, c=\mp\frac{5}{48}, \varepsilon^2=1$. **Figure 8** shows the three-dimensional and two- dimensional plots diagrams of Equation (39), which represents the bright solitary wave solution.

Because the solutions $f_1(\xi)-f_{10}(\xi)$ of Equation (1) contain that corresponding to $h(\xi)=1/f(\xi)$, the solitary wave solutions of the Fisher equation in Case 2 are the same as in Case1. Corresponding to case 3, we express the solitary wave solution of Fisher equation as

$$u_{11}(\xi)=\frac{3}{8}\pm\frac{1}{4}\coth(\xi)+\frac{1}{16}\coth^2(\xi)\mp\frac{1}{4}\tanh(\xi)+\frac{1}{16}\tanh^2(\xi) \quad (40)$$

where $\xi=\mu x+ct, \mu=\frac{1}{4\sqrt{6}}\varepsilon, c=\mp\frac{5}{24}, \varepsilon^2=1$. **Figure 9** shows the three-dimensional and two- dimensional plots diagrams of Equation (40), which represents the bright solitary wave solution.

$$u_{12}(\xi)=\frac{3}{8}\pm\frac{1}{4}\frac{\sinh(\xi)\cosh(\xi)}{\cosh^2(\xi)-\frac{1}{2}}+\frac{1}{16}\left[\frac{\sinh(\xi)\cosh(\xi)}{\cosh^2(\xi)-\frac{1}{2}}\right]^2\mp\frac{1}{4}\frac{\cosh^2(\xi)-\frac{1}{2}}{\sinh(\xi)\cosh(\xi)}+\frac{1}{16}\left[\frac{\cosh^2(\xi)-\frac{1}{2}}{\sinh(\xi)\cosh(\xi)}\right]^2 \quad (41)$$

where $\xi=\mu x+ct, \mu=\frac{1}{8\sqrt{6}}\varepsilon, c=\mp\frac{5}{48}, \varepsilon^2=1$. **Figure 10** shows the

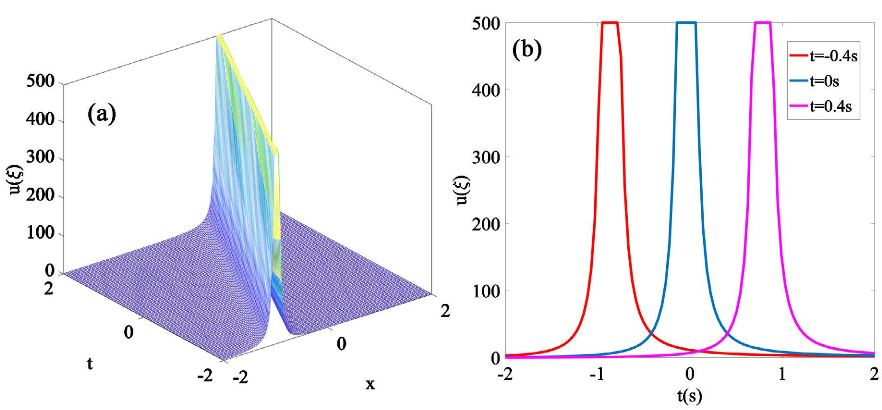

Figure 8. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (39), when ± sign takes +, ∓ sign takes - and $\varepsilon=1$.

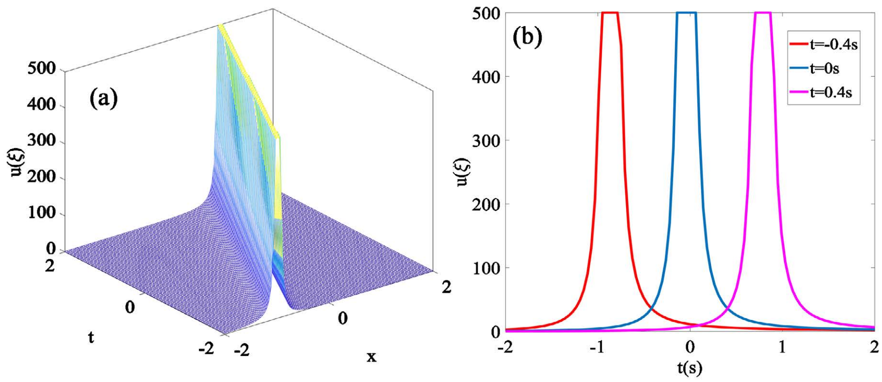

Figure 9. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (40), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

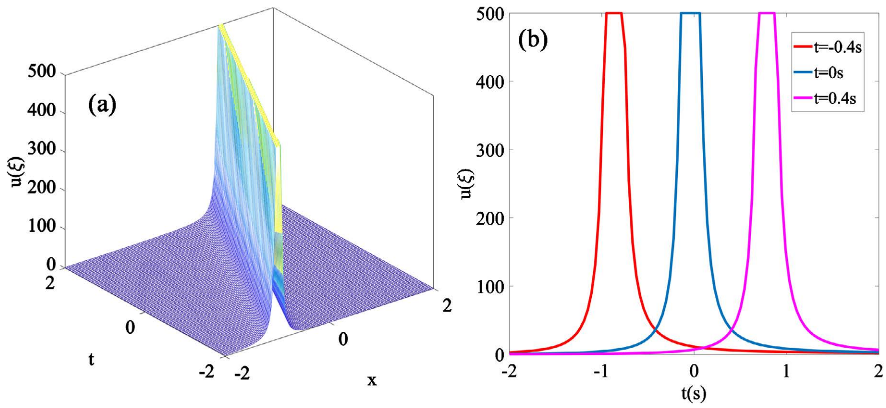

Figure 10. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (41), when ± sign takes +, ∓ sign takes - and $\varepsilon = 1$.

three-dimensional and two- dimensional plots diagrams of Equation (41), which represents the bright solitary wave solution.

$$u_{13}(\xi) = \frac{3}{8} \pm \frac{1}{4}\frac{\cosh(\xi)+\varepsilon}{\sinh(\xi)} + \frac{1}{16}\left[\frac{\cosh(\xi)+\varepsilon}{\sinh(\xi)}\right]^2 \mp \frac{1}{4}\frac{\sinh(\xi)}{\cosh(\xi)+\varepsilon} + \frac{1}{16}\left[\frac{\sinh(\xi)}{\cosh(\xi)+\varepsilon}\right]^2 \qquad (42)$$

where $\xi = \mu x + ct, \mu = \frac{1}{2\sqrt{6}}\lambda, c = \mp\frac{5}{12}, \varepsilon^2 = 1, \lambda^2 = 1$. **Figure 11** shows the three-dimensional and_two- dimensional plots diagrams of Equation (42), which represents the bright solitary wave solution.

$$u_{14}(\xi) = \frac{3}{8} \pm \frac{1}{4}\frac{\cosh(\xi)}{\sinh(\xi)+\varepsilon} + \frac{1}{16}\left[\frac{\cosh(\xi)}{\sinh(\xi)+\varepsilon}\right]^2 \mp \frac{1}{4}\frac{\sinh(\xi)+\varepsilon}{\cosh(\xi)} + \frac{1}{16}\left[\frac{\sinh(\xi)+\varepsilon}{\cosh(\xi)}\right]^2 \qquad (43)$$

where $\xi = \mu x + ct, \mu = \frac{1}{2\sqrt{6}}\varepsilon, c = \mp\frac{5}{12}, \varepsilon^2 = -1$. The solution of Equation (43)

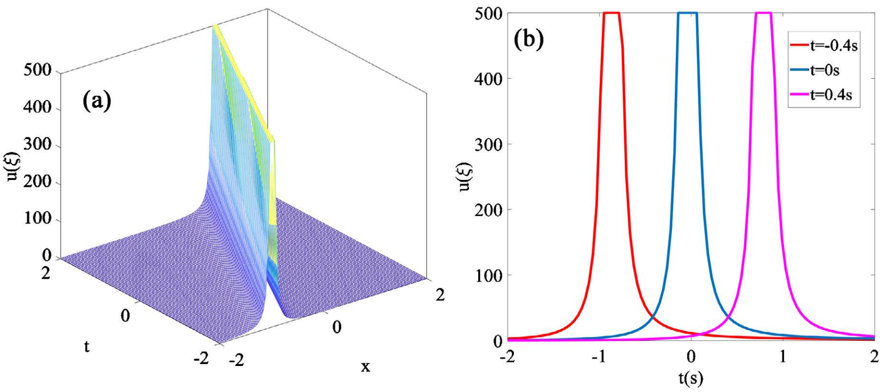

Figure 11. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (42), when ± sign takes +, ∓ sign takes -, $\varepsilon=1$ and $\lambda=1$.

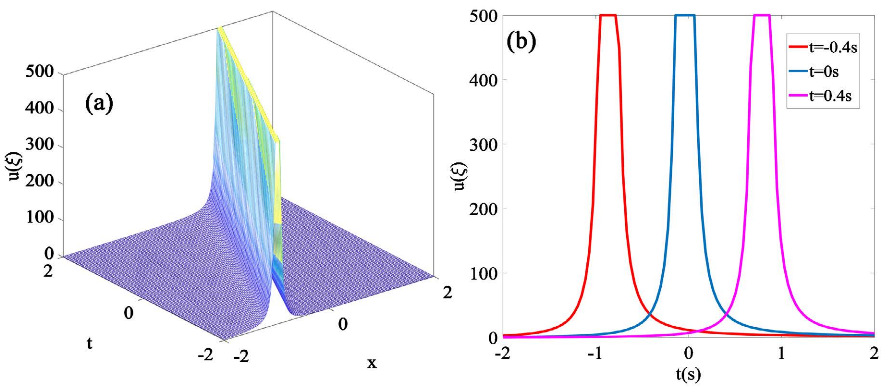

Figure 12. (a) Three dimensional and (b) two dimensional plots represent the kink solitary wave solution of Equation (41), when ± sign takes +, ∓ sign takes - and $\varepsilon=1$.

represents the traveling wave solutions of Equation (24) in complex space.

$$
\begin{aligned}
u_{15}(\xi)=&\frac{3}{8}\pm\frac{1}{2}\frac{[\cosh^2(\xi)-\frac{1}{2}]^2-\frac{1}{8}}{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}\\
&+\frac{1}{4}\left[\frac{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}\right]^2\\
&\mp\frac{1}{8}\frac{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}\\
&+\frac{1}{64}\left[\frac{2\sinh(\xi)\cosh^3(\xi)-\sinh(\xi)\cosh(\xi)}{\left[\cosh^2(\xi)-\frac{1}{2}\right]^2-\frac{1}{8}}\right]^2
\end{aligned}
\tag{44}
$$

where $\xi=\mu x+ct, \mu=\frac{1}{16\sqrt{6}}\varepsilon, c=\mp\frac{5}{96}, \varepsilon^2=-1$. Figure 12 shows the three-dimensional and two- dimensional plots diagrams of Equation (44), which represents the bright solitary wave solution.

4. Conclusion

In this paper, we use Riccati equation to find new solitary wave solutions of Fisher equation. Riccati equation is solved by using of two types of function transformation Equation (6) and Equation (14), so that many new exact solutions are obtained. With the formal solution of Equation (25), we have constructed abundant and new solitary wave solutions for the Fisher equation. The solitary wave solutions expressed by Equations (39), (41)-(44) are rarely found in other documents. The numerical images show that although the new expressions of many solutions are different, the solitary waves represented by them, including amplitude, wave velocity and space-time width, are the same. This method can greatly simplify the calculation process, especially suitable for solving more complex nonlinear systems.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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