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Natural Convection and Thermal Radiation Influence on Nanofluid Flow over a Nonlinearly Stretching Sheet

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Abstract

This paper studies the effects of Natural convection and Thermal radiation on nanofluid over a non-linearly stretching sheet. The governing equation of nonlinear partial differential equations of the flow is transformed to nonlinear ordinary differential equations by using similarity transformation, earlier than being solved numerically through a Rung-Kutta-Fehlberg method with shooting technique. The numerical results have been obtained for The influence of Brownian motion number (Nb), thermophoresis number (Nt), Grashof number (Gr), Lewis number (Le), stretching parameter (n), and thermal radiation parameter (Ra) on the velocity, temperature, and nanoparticles concentration profiles are shown graphically.

Keywords

Nanofluid, Stretching Sheet, Thermal Radiation, Free Convection

1. Introduction

The study of flow over a stretching sheet has generated a lot of interest in recent years in view of its several industrial programs along with the aerodynamic extrusion of plastic sheets, boundary layer behavior over stretching surface is crucial because it consists of several engineering processes, like, glass-fiber and paper production. Natural convection is a kind of flow motion of a liquid which includes water or a gas such as air, wherein the fluid motion isn't always generated with the aid of any external supply but with the aid of some parts of the fluid being heavier than other parts. This force of a natural Convection is gravity. However, the simultaneous impact of thermal radiation and magnetic effects on unsteady laminar viscous nanofluid flow over a shrinking sheet is given by Nandy *et al.* [1]. He found that the dual solutions exist for the flow over a shrinking sheet. Kuznetsov and Nield [2] have finished an analytical study on natural convective flow of a nanofluid past a vertical plate below the influence of Brownian motion and thermophoresis results. Turkyilmazoglu and Pop [3] studied the unsteady and thermal transport characteristics of different nanofluids flowing a protracted vertical plate with the consideration of a radiative effect. The study of temperature and velocity field over warm vertical plate due to natural Convection is investigated by Schmidt and Beckmann. Ostrach [5] applied method of iterative integration to research free convection over a semi-infinite isothermal flat plate. Sparrow and Gregg [6] offered a similar study on numerical solutions for laminar-free convection from a vertical plate with uniform surface heat flux. Since the pioneering work of Sakiadis, various elements of the problem were investigated through many authors, which includes Xu and Liao [8], Cortell [9], Hayat and Sajid [10] and Hayat *et al.* [11]. The effect of magnetic field in viscous flow over a stretching cylinder embedded in a porous medium has been studied by Butt. Ishak *et al.* [13] defined suction/injection influence on regular flow of an incompressible fluid over a permeable stretching tube, they discovered that Reynolds number ascends as mounting in the numerical values of skin friction coefficient. Alok *et al.* [14] study that the collective has an effect on thermal radiation and convection flow of cu-water nanofluid because of a stretching cylinder in porous medium a protracted with viscous dissipation and slip boundary conditions. The role of radiation heat transfer is superficial in lots of engineering approaches which occurs at high temperature. A large number of experimental and theoretical research had been done with the aid of numerous researchers on radiation impact [15] [16] [17] [18]. In most of the mentioned literature, radiation effect was inspected by imposing the linearized Rosseland approximation. This approximation involves the dimensionless parameter as radiation parameter and prandtl number due to the temperature difference between the plate and ambient fluid is small. However, for the large temperature difference, non linearized Rossland approximation is legitimate. M. Archana *et al.* [19] examined the effect of nonlinear thermal radiation on rotating flow of a casson nanofluid. Biliana *et al.* [20] examined the numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. A. Brusly Solomon *et al.* *[21]* examined the natural convection enhancement in a porous cavity with Al_2O_3-Ethylene glycol/water nanofluids. And S M Sohel Murshed *et al.* [22] present experimental research and development on the Natural Convection of Suspensions of Nanoparticles-A Comprehensive Review. A. I. Alsabery *et al.* [23] research the Natural Convection Flow of a nanofluid in an inclined square enclosure partially filled with a Porous Medium. And M. Ghalambaz *et al.* [24] tested Natural convection of nanofluids over a convectively heated vertical plate embedded in a porous medium. In this paper, we investigated numerically and extend the work by P. Rana *et al.* [25]. This research is based on a study on the effects of Natural Convection and Thermal radiation on nanofluid over a stretching sheet. The governing partial differential equations were transformed into ordinary differential equations by using similarity solution transformation. We have solved the ordinary differential equations numerically by using shooting method (Rung-Kutta-Fehlberg) and we get the result for the effects of Brownian motion number (*Nb*), thermophoresis number (*Nt*), Grashof number (*Gr*), Lewis number (*Le*), stretching parameter (*n*), and thermal radiation parameter (*Ra*) on the velocity, temperature, and nanoparticles concentration.

2. Mathematical Formulation

Considering the nano fluid is steady, incompressible laminar two dimensional flow past a flat sheet coinciding with plane y=0 and the flow are confined to y>0. The flow is generated, because of non-linear stretching of the sheet caused by the simultaneous application of equal and opposite force alongside *x*-axis. Keeping the original fixed, the sheet is then stretch with a velocity $u_w = ax^n$ wherein a is a constant, n is a nonlinear stretching parameter and x is the coordinate measured along the stretching surface, varying nonlinearly with the distance from the slit. The problem is within the presence of thermal radiation and free convection. The flow configuration of this problem is illustrated in Figure 1.

The pressure gradient and external force are neglected. The stretching surface is maintained at constant temperature and concentration, T_w and C_w, respectively, and these values are assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞, respectively. The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian coordinates x and y as, see Buongiorno [26], Kuznetsov and Nield [27], Niled and Kuznetsov [28], Bachok *et al.* [29] and Khan and Pop [30],

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\left(1-\varphi_\infty\right)g\beta\left(T-T_\infty\right) \tag{2}$$

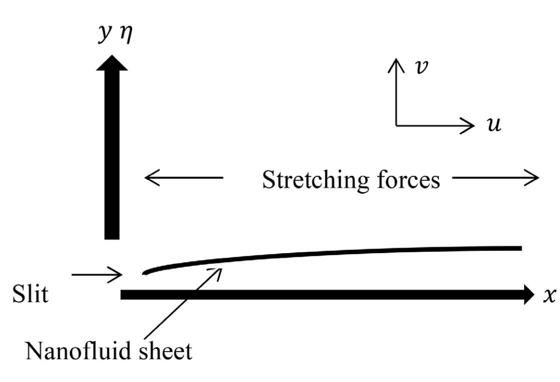

Figure 1. Physical model and coordinate system.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\nabla^2 T+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where

$$\alpha_m=\frac{k_m}{(\rho c)_p},\quad \tau=\frac{(\rho c)_p}{(\rho c)_f} \tag{5}$$

Here u and v are the velocity components along the axes x and y, respectively. α_m is the thermal diffusivity, ρ_f is the density of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, τ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, β is the volumetric coefficient of the thermal expansion, g is the gravitational acceleration, φ_∞ is nanoparticle volume fraction, q_r is the radiative heat flux, C_p is the heat capacity at constant pressure p, C is the concentration of nanoparticles volume fraction, ρ_p is the density of the particle, $(\rho c)_p$ effective heat capacity of the nanoparticle material, $(\rho c)_f$ heat capacity of the fluid and T_∞ is the temperature of the ambient fluid.

The appropriate boundary conditions for the problem are:

$$\begin{aligned}&v=0,\, u_w=ax^n,\, T=T_w,\, C=C_w \quad \text{at } y=0\\&u=v=0,\, T=T_\infty,\, C=C_\infty \quad \text{as } y\to\infty\end{aligned} \tag{6}$$

Using Roseland's approximation, the radiative heat flux q_r is modeled as

$$q_r=-\left(\frac{4\sigma}{3k_1}\right)\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ is the Stefan-Boltzmann constant and k_1 is the absorption coefficient. Assuming that the difference in temperature within the flow is such that T^4 can be expressed as a linear combination of temperature, we expand T^4 in Taylor's series about T_∞ as follows:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots \tag{8}$$

And neglecting higher order terms beyond the first degree in $(T-T_\infty)$, we have

$$T^4\approx 3T_\infty^4+4T_\infty^3 T. \tag{9}$$

Differentiating (7) with respect to y and using (9) we get

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma}{3k_1}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

Using (10) in (3) we obtain

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\nabla^2 T+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{16\sigma T_\infty^3}{3k_1\rho C_P}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

We look for a similarity solution of Equations ((1), (2), (4), (11)) with the boundary conditions (6) of the following form:

$$\eta = y\sqrt{\frac{a(n+1)}{2\nu}}x^{\frac{n-1}{2}},\ u = ax^n f'(\eta),\ v = -\sqrt{\frac{a\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left(f + \left(\frac{n-1}{n+1}\right)\eta f'\right)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{12}$$

The governing Equations ((1), (2), (4), (11)) then reduce to

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 + \left(\frac{2}{n+1}\right)Gr\theta = 0 \tag{13}$$

$$\left(\frac{1}{pr} + Ra\right)\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{14}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{15}$$

The transformed boundary conditions are

$$\begin{aligned} &f = 0,\ f' = 1,\ \theta = 1,\ \phi = 1 \ \text{ at } \eta = 0,\\ &f' = 0,\ \theta = 0,\ \phi = 0 \ \text{ as } \eta \to \infty \end{aligned} \tag{16}$$

where

$pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Le = \frac{\nu}{D_B}$ is the Lewis number,

$Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f \nu}$ is the Brownian motion parameter,

$Nt = \frac{(\rho c)_p D_T (T - T_\infty)}{(\rho c)_f \nu T_\infty}$ is the thermophoresis parameter,

$Gr = \frac{(1-\varphi_\infty) g \beta_T (T_w - T_\infty)}{a^2 x^{2n-1}}$ is the Grashof number, $Ra = \frac{16\sigma T_\infty^3}{3k_1 \rho c_p \nu}$ is the radiation parameter.

3. Result and Discussion

Numerical computation is carried out for several set of values of the parameters that describe the flow characteristics and the results are illustrated graphically. The partial differential Equations ((1), (2), (4), (11)) subject to the boundary condition (6) are transformed into ordinary equation by using similarity transformation method, before solved numerically by a Rung-Kutta method with shooting technique. We studied the impact of several prevailing physical parameters such as stretching parameter *(n)*, Prandtl number *(Pr)*, Brownian motion parameter *(Nb)*, thermophoresis parameter *(Nt)*, Lewis number *(Le)*, radiation parameter (*Ra*), and Grashof number (*Gr*), on flow and heat transfer are elucidated through figures, the region of integration η is considered as 0 to $\eta_\infty = 7$, where η_∞ corresponds to $\eta \to \infty$ which lies very well outside the momentum and thermal boundary layers. We studied the influence of the above parameters on the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration distribution $\phi(\eta)$.

Figures 2-4 we observe that $f'(\eta), \theta(\eta)$ and $\phi(\eta)$ with different value of *Ra*, which *Ra* = (0.2, 0.5, 1, 1.5) with *Pr* = 2, *Nt* = *Nb* = *Gr* = 0.5, *Le* = 2 and *n* = 2. It is observed reduction in concentration $\phi(\eta)$ with increasing in thermal radiation but temperature fluid $\theta(\eta)$ increasing with increases radiation parameter *Ra* and there is no obvious effect on velocity $f'(\eta)$.

Figure 5 & Figure 6 the effect of Brownian motion parameter *Nb* on temperature $\theta(\eta)$ and concentration $\phi(\eta)$, which *Nb* = (0.5, 1, 1.5, 2.5) and *Pr* = *n* = *Le* = 2, *Nt* = *Ra* = *Gr* = 0.5. the temperature $\theta(\eta)$ in the boundary layer

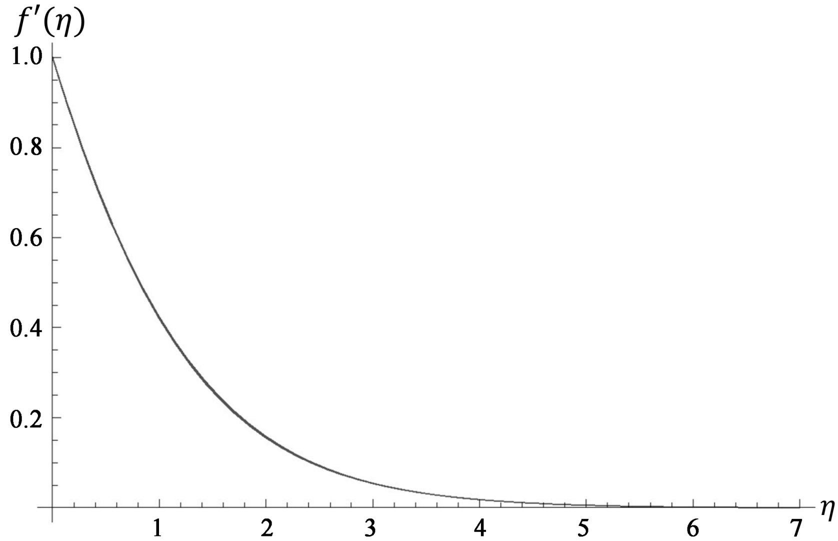

Figure 2. The effect of radiation parameter (*Ra*) on velocity distribution for *Pr* = *Le* = *n* = 2, *Nt* = *Nb* = *Gr* = 0.5.

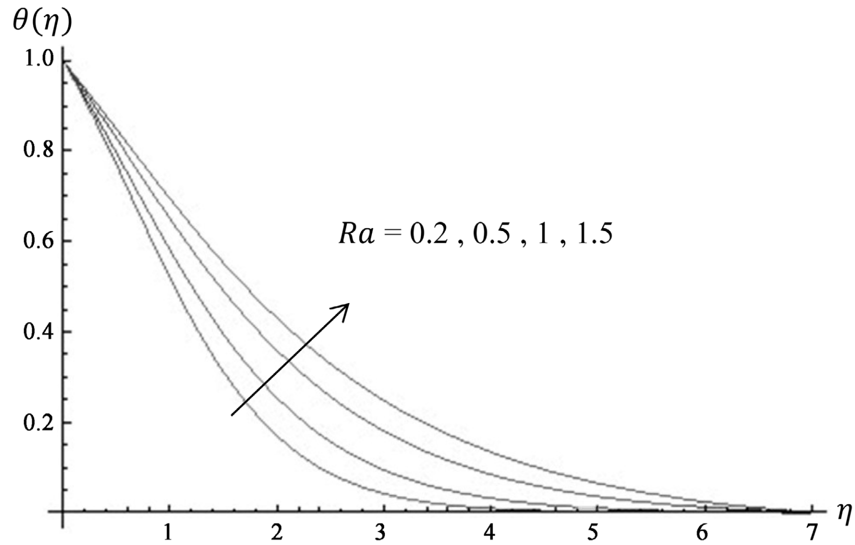

Figure 3. The effect of radiation parameter (*Ra*) on temperature distribution for *Pr* = *n* = *Le* = 2, *Nt* = *Nb* = *Gr* = 0.5.

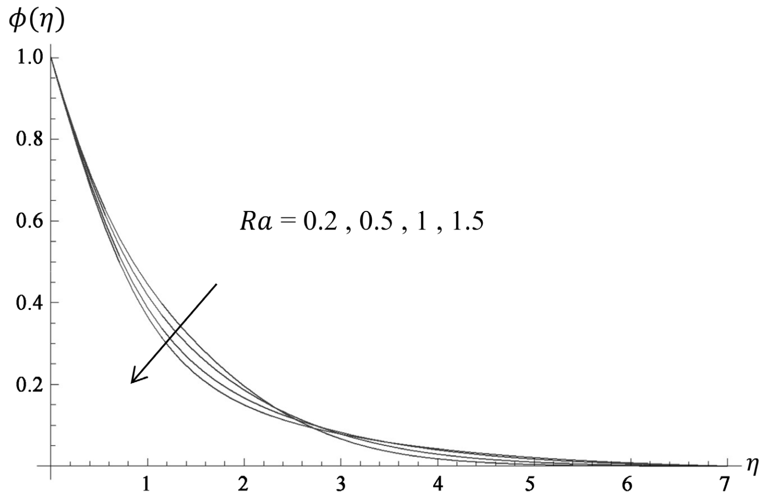

Figure 4. The effect of radiation parameter (*Ra*) on concentration distribution for $Pr = n = Le = 2$, $Nt = Nb = Gr = 0.5$.

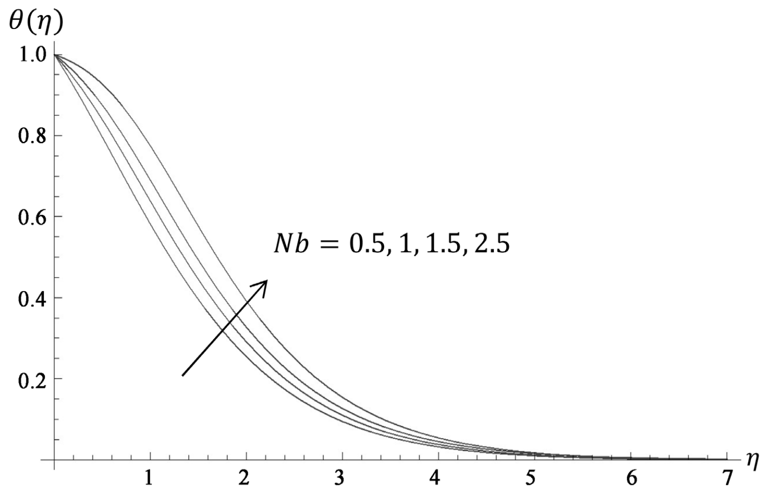

Figure 5. The effect of Brownian motion parameter *Nb* on temperature distribution for $Pr = n = Le = 2$, $Nt = Ra = Gr = 0.5$.

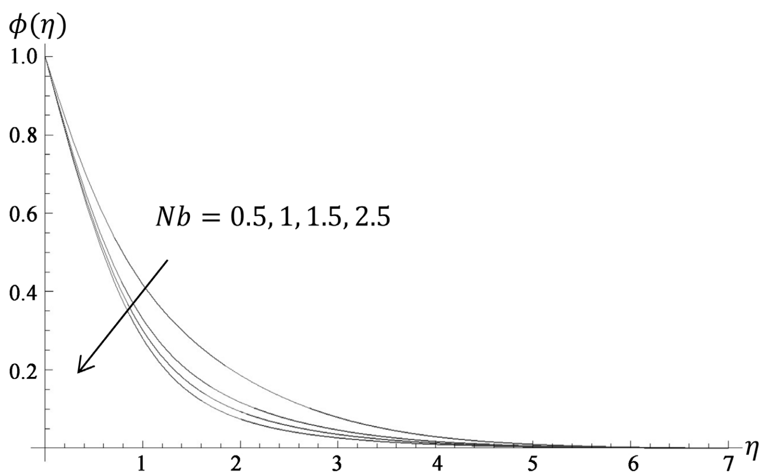

Figure 6. The effect of Brownian motion parameter *Nb* on concentration distribution for $Pr = n = Le = 2$, $Nt = Ra = Gr = 0.5$.

increasing with increasing in the Brownian parameter, but the nano-particle volume fraction profile $\phi(\eta)$ decreases with the increasing in the Brownian motion parameter and there no obvious effect on velocity profiles $f'(\eta)$ The Brownian motion of nanoparticles can enhance thermal conduction via one of two mechanism—either a direct effect owing to nanoparticles the transport heat or alternatively via an indirect contribution due to micro-convection of fluid surrounding individual nanoparticles. For small particles, Brownian motion is strong and the parameter *Nb* will have high values; the converse is the case for large particles and clearly Brownian motion does exert a significant enhancing influence on both temperature and concentration profiles.

Figure 7 & Figure 8 with the different values of $Nt = (0.5, 1, 1.5, 2)$ the effect of thermophretic parameter (Nt) with value of $Pr = 2$, $Nb = Gr = Ra = 0.5$, $Le = 2$

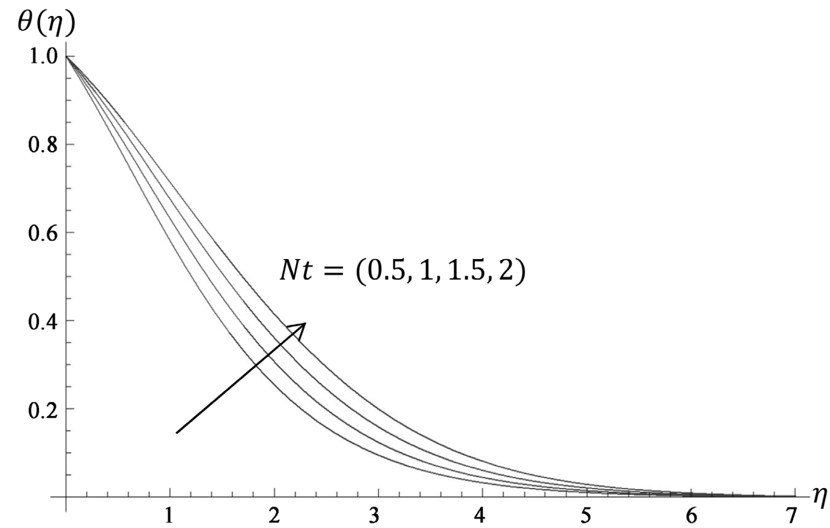

Figure 7. The effect of thermophoresis parameter (Nt) on temperature distribution for $Pr = n = Le = 2$, $Nb = Ra = Gr = 0.5$.

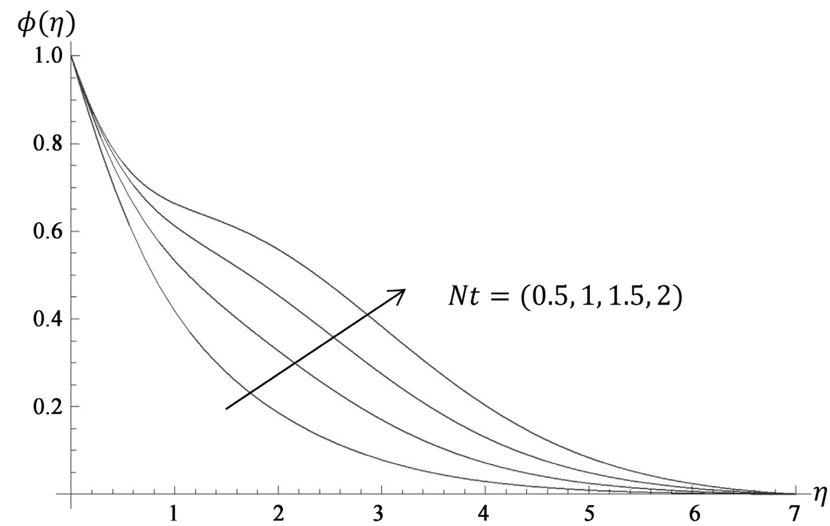

Figure 8. The effect of thermophoresis parameter Nb on concentration distribution for $Pr = n = Le = 2$, $Nb = Ra = Gr = 0.5$.

and $n = 2$, it is observed that an increasing in the thermophoretic parameter (Nt) leads to increase in both fluid temperature $\theta(\eta)$ and concentration $\phi(\eta)$ and there is no obvious effect on velocity profiles $f'(\eta)$.

Figures 9-11 illustrate the effect of stretching parameter (n) on velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ with different values of n = (0, 0.75, 3, 4) and $Pr = Le = 2$, $Nb = Nt = Ra = Gr = 0.5$., it is observed that an increasing in the stretching parameter (n), leads to increasing in temperature and concentration but decreases in velocity i.e. rate of heat transfer and mass transfer decreases with increasing in stretching sheet (n).

Figure 12 & Figure 13 illustrate the effect of Lewis number (Le) which Le = (2, 4, 6, 10) and $Pr = n = 2$, $Nb = Ra = Gr = Nt = 0.5$. It defines as the ratio of thermal diffusivity to mass diffusivity. The increases of Lewis number it's leads to decreasing in both temperature $\theta(\eta)$ and concentration $\phi(\eta)$ and there is no obvious effect on velocity profiles $f'(\eta)$.

Figures 14-16 depict the variation of velocity $f'(\eta)$, temperature $\theta(\eta)$

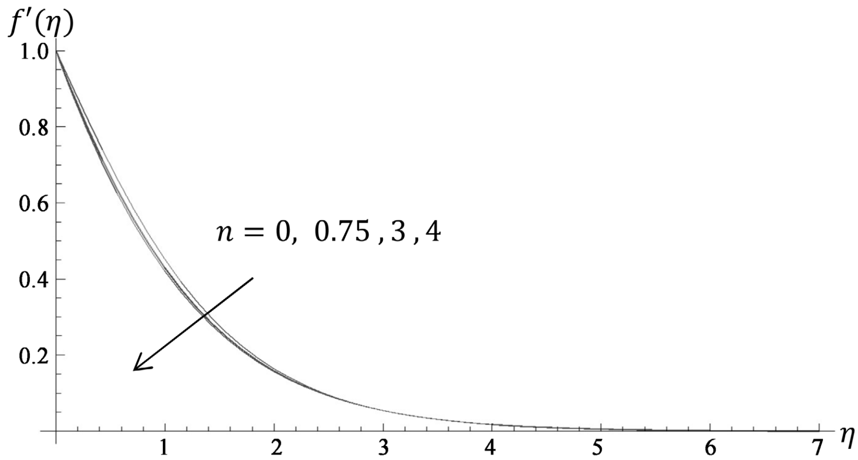

Figure 9. The effect of stretching parameter (n)on velocity distribution for $Pr = Le = 2$, $Nb = Ra = Gr = Nt = 0.5$.

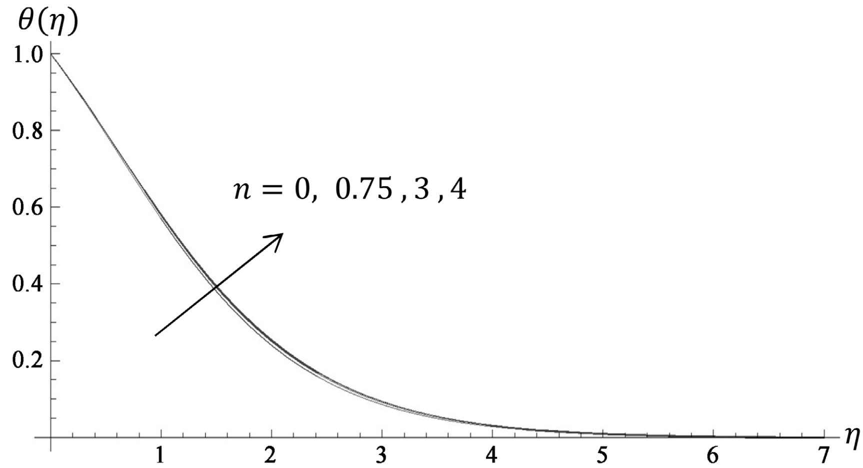

Figure 10. The effect of stretching parameter (n)on temperature distribution for $Pr = Le = 2$, $Nb = Ra = Gr = Nt = 0.5$.

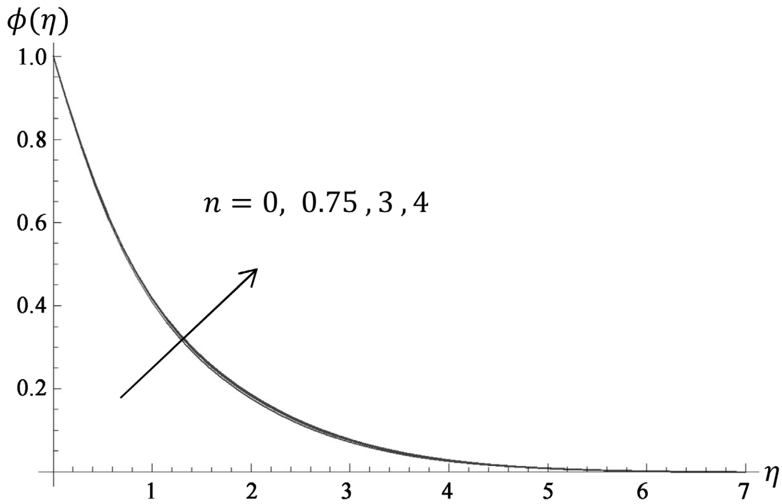

Figure 11. The effect of stretching parameter (n) on concentration distribution for $Pr = Le = 2$, $Nb = Ra = Gr = Nt = 0.5$.

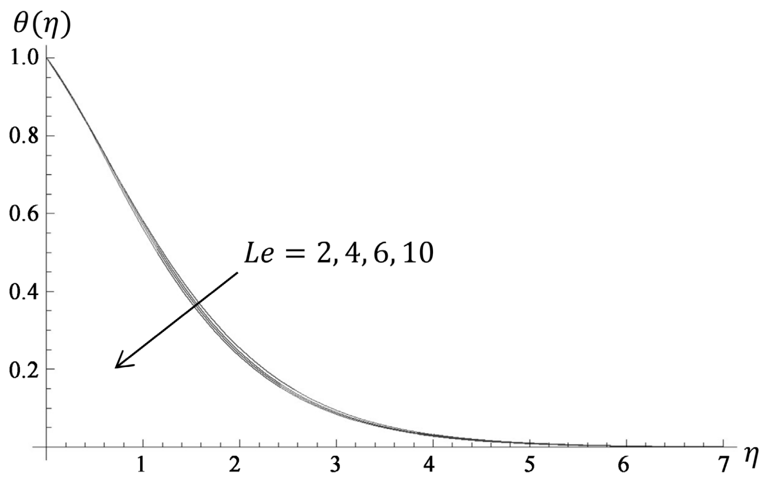

Figure 12. The effect of Lewis number on the temperature distribution for $Pr = n = 2$, $Nb = Ra = Gr = Nt = 0.5$.

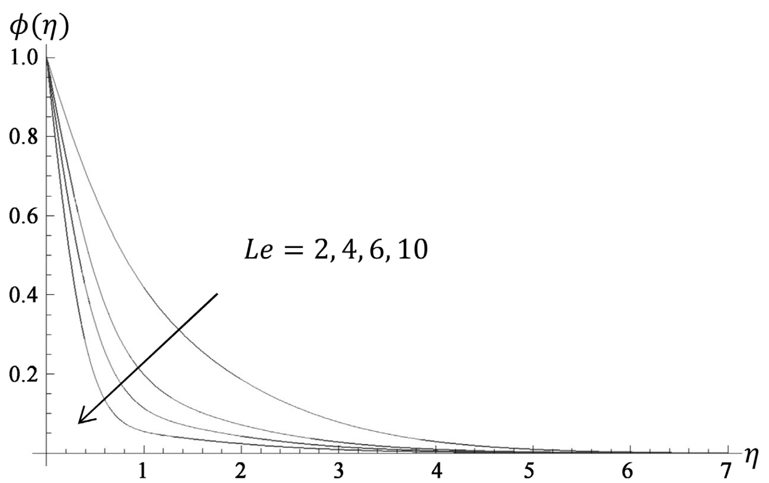

Figure 13. The effect of Lewis number on concentration distribution for $Pr = n = 2$, $Nb = Ra = Gr = Nt = 0.5$.

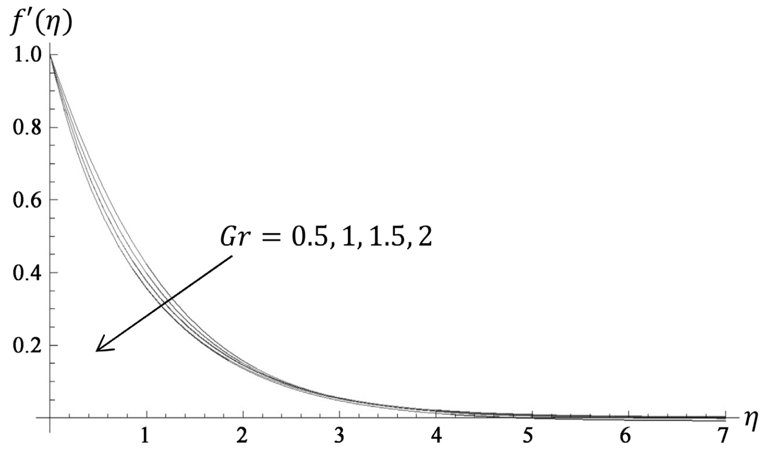

Figure 14. The effect of Grashof number Gr on velocity distribution for $Pr = n = Le = 2$, $Nb = Ra = Nt = 0.5$.

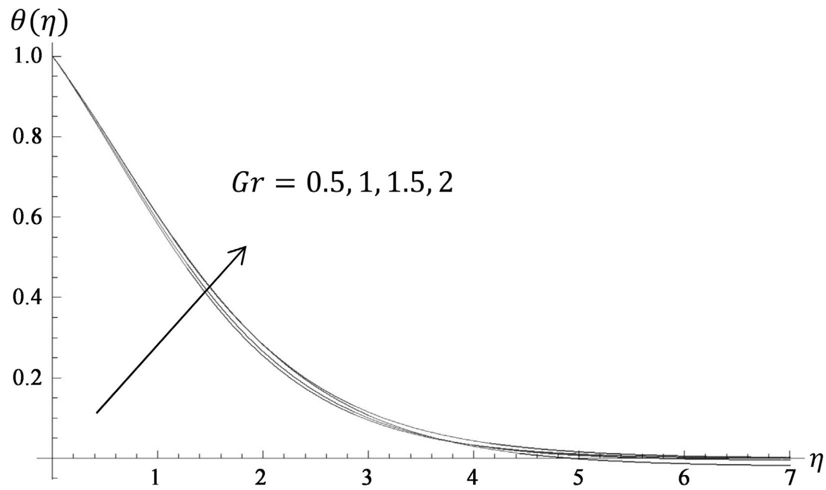

Figure 15. The effect of Grashof number Gr on temperature distribution for $Pr = n = Le = 2$, $Nb = Ra = Nt = 0.5$.

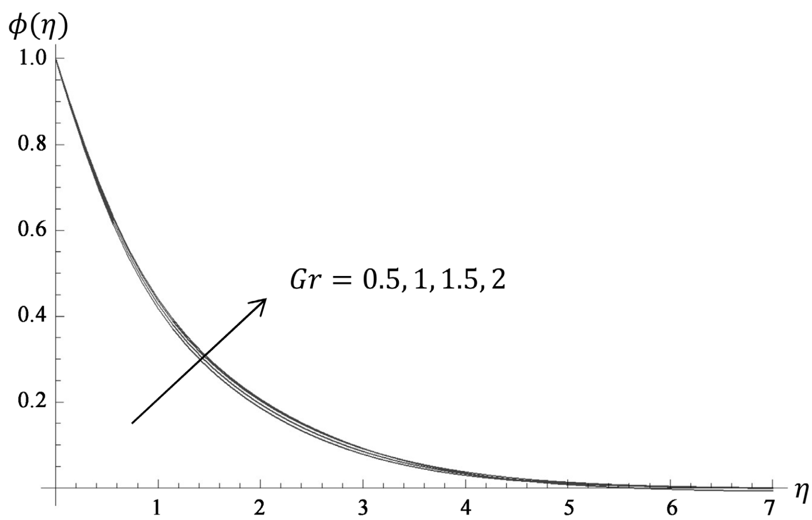

Figure 16. The effect of Grashof number Gr on concentration distribution for $Pr = n = Le = 2$, $Nb = Ra = Nt = 0.5$.

and concentration $\phi(\eta)$ with Grashof number (*Gr*), with various values [0.5, 1, 1.5, 2] and *Pr* = *n* = *Le* = 2, *Nb* = *Ra* = *Nt* = 0.5., Grashof number (*Gr*) defined as the ratio of the buoyant to viscous force acting on a fluid in the velocity boundary layer. It is used in the correlation of heat and mass transfer due to thermally induced natural convection at a solid surface immersed in a fluid. The increasing in Grashof number (*Gr*) leads to decreases in velocity $f'(\eta)$ but increases in both fluid temperature $\theta(\eta)$ and nanoparticles concentration $\phi(\eta)$.

Figure 17 & Figure 18 depict the variation of temperature $\theta(\eta)$ and concentration $\phi(\eta)$ with Prandtl number (*Pr*) = 2, 3, 6.2, 10 and *n* = *Le* = 2, *Nb* = *Ra* = *Gr* = *Nt* = 0.5, which Prandtl number (*Pr*) defined as the ratio of momentum diffusivity to thermal diffusivity. The effect of increasing in Prandtl number on temperature $\theta(\eta)$ and concentration (η), leads to decreses in temperature $\theta(\eta)$ but increases in nanoparticle concentration $\phi(\eta)$ and there is no obvious effect on velocity profiles $f'(\eta)$.

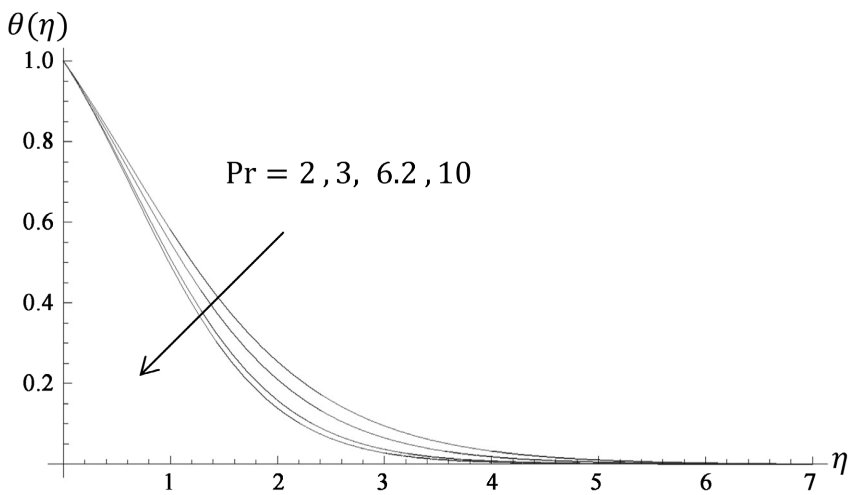

Figure 17. The effect of Prandtl number *Pr* on temperature distribution for *Le* = *n* = 2, *Nt* = *Nb* = *Ra* = *Gr* = 0.5.

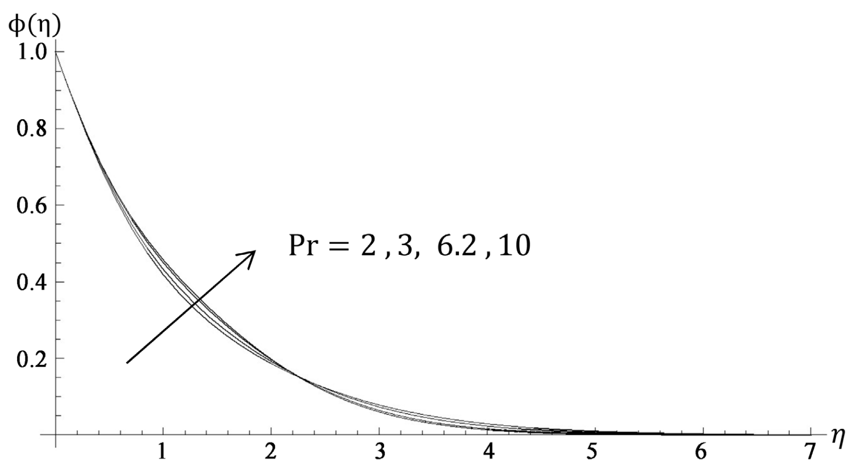

Figure 18. The effect of Prandtl number *Pr* on concentration distribution for *n* = *Le* = 2, *Nb* = *Ra* = *Gr* = *Nt* = 0.5.

4. Conclusions

In the present paper, we have examined the boundary layer flow which is the non-linear stretching of a flat surface in a nanofluid with thermal radiation and free convection with Brownian motion and thermophoresis effects. The governing partial differential equations for mass, momentum, energy and nanoparticles conservation are transformed into ordinary differential equations by using similarity transformation. Then these equations are solved numerically by using shooting method. Effects of various parameters on velocity, temperature and concentration profiles are shown graphically. The result, in summary, has shown that:

1) An analytical solution for boundary layer flow with non-linear stretching of flat surface in nanofluid with thermal radiation and free convection was obtained.

2) The velocity within the boundary layer decreases when a stretching parameter (n) and Grashof number (Gr) are increasing, and there is no effect with increasing of radiation parameter (Ra), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Lewis number (Le), and Prandtl number (Pr).

3) The temperature $\theta(\eta)$ within the boundary decreases when Lewis number (Le), and Prandtl number (Pr) are increasing, and increases with radiation parameter (Ra), Brownian motion parameter (Nb), thermophoresis parameter (Nt) and stretching parameter (n).

4) The concentration $\phi(\eta)$ within the boundary decreases when radiation parameter (Ra), Brownian motion parameter (Nb) and Lewis number (Le) are increasing, and increasing with thermophoresis parameter (Nt), stretching parameter (n), Prandtl number (Pr), and radiation parameter (Ra).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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