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Stability of Generalized Minimax Regret Equilibria with Scalar Set Payoff

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Abstract

In this paper, we first introduce the notion and model of generalized minimax regret equilibria with scalar set payoffs. After that, we study its general stability theorem under the conditions that the existence theorem of generalized minimax regret equilibrium point with scalar set payoffs holds. In other words, when the scalar set payoffs functions and feasible constraint mappings are slightly disturbed, by using Fort theorem and continuity results of set-valued mapping optimal value functions, we obtain a general stability theorem for generalized minimax regret equilibria with scalar set payoffs. At the same time, an example is given to illustrate our result.

Keywords

Minimax Regret Equilibria, Set Payoff, Generic Stability, Vector Optimization

1. Introduction

In the game, individuals are faced with uncertain strategic choices, and they assume that the decisions of other players will form a subjective assessment of probability. Renou and Schlag [1] introduced the concept of minimax regret equilibria, allowing players to be uncertain about the rationality and conjecture of their opponents, and assumed that regret will lead individuals to form probabilistic assessments and ultimately make choices that minimize regret. Yang and Pu [2] obtained the existence and generic stability of minimax regret equilibria. Recently, Zhang and Chen *et al.* [3] studied existence of general n person noncooperative game problems and minimax regret equilibria with set payoff by using Kakutani-Fan-Glicksberg fixed point theorem and a nonlinear scalarization function. For more information, refer to [4] [5] [6] [7].

Under the influence of some uncertain factors, the game model is often disturbed. In 1950, Fort [8] first proposed the concept of essential fixed points when studying the stability of fixed points. Kohlberg and Mertens [9] investigated that every game has at least one equilibrium set with a stable strategy. Wu and Jiang [10] put forward the concept of essential equilibrium of finite noncooperative game, and proved that any finite noncooperative game can be approximated arbitrarily by an essential equilibrium game. In the sense of changing order, Luo [11] studied the existence and general stability of Nash equilibrium points in set-valued games. In recent years, Yu and Peng [12] considered the general stability of Nash equilibria in noncooperative differential games, and proved that equilibrium is the essence of differential games to form dense residual sets by using the theory of set-valued analysis. Liu [13] discussed the existence of Nash equilibrium points of generalized set-valued mapping and the stability of Nash equilibrium point sets of generalized set-valued mapping from the perspective of essential equilibrium points. In real locally convex Hausdorff topological linear spaces, He and Chen *et al.* [14] obtained the general stability of solutions for set-valued generalized strong vector quasi-equilibrium problems when the constraint set-valued mappings are continuous and the target mappings satisfy the cone-true quasi-convex conditions.

Since there are many unpredictable situations in reality, it is very important to study the stability of set-valued game problems. However, to the best of our knowledge, the study of the stability of set-valued game problems is still little. Motivated and inspired by the minimax regret problem in [3], the rest parts are constructed as follows: In Section 2, the generalized minimax regret equilibria problem and some necessary basic knowledge are given. In Section 3, general stability theorem for generalized minimax regret equilibrium with scalar set payoffs is obtained. And a numerical example is given to illustrate our results.

2. Preliminaries

Let Y, K, K_i, $i=1,2,\cdots,n$ are real locally convex Hausdorff topological vector spaces. S is a pointed closed convex cone in Y, and $intS \neq \varnothing$.

Firstly, we introduce the generalized minimax regret equilibrium model in [3].

Let $I=\{1,2,\cdots,n\}$ be a set of players, $X_i \subset K_i$ be the pure strategy set of ith player and $G_i : X=\prod_{i=1}^{n} X_i \to 2^Y$ be the scalar set payoff function of ith player. For each $i \in I$, denote $X_{-i}=\prod_{j\in I\setminus i} X_j$. If $x=(x_1,x_2,\cdots,x_n)\in X$, write $x_{-i}=(x_1,\cdots,x_{i-1},x_{i-1},\cdots,x_n)\in X_{-i}$. Let $S_i \subset X_{-i}$ be subset of ith player beliefs conjectures about the play of his opponents. $P_i : X=\prod_{i=1}^{n} X_i \to 2^R$ be the ith player's ex-post regret function relative $(x_i,x_{-i})\in\prod_{i=1}^{n} X_i$, defined as follows:

$$P_i(x_i,x_{-i})=\sup\bigcup_{u_i\in X_i} G_i(u_i,x_{-i})-G_i(x_i,x_{-i}).$$

A strategy $x^*=(x_1^*,x_2^*,\cdots,x_n^*)\in X$ is said to be a minimax regret equilibria point with scalar set payoffs relative to $(S_1,S_2,\cdots,S_n)$, if, for each $i\in N$,

$$\max \bigcup_{x_{-i}\in S_i} P_i\left(x_i^*, x_{-i}\right) = \min \bigcup_{x_i\in X_i} \max P_i\left(x_i, S_i\right).$$

Definition 2.1. [15] Let $P: K \to 2^Y$ be a set-valued mapping,

(i) A set-valued map P is said to be upper semicontinuous (u.s.c) with nonempty compact valued at $x_0 \in K$, if for any net $x_\alpha \subset K$ with $x_\alpha \to x_0$, and for any $y_\alpha \in P\left(x_\alpha\right)$, there exist $y_0 \in P\left(x_0\right)$ and a subnet $\left\{y_\beta\right\}$ of $\left\{y_\alpha\right\}$, such that $y_\beta \to y_0$;

(ii) A set-valued map P is said to be lower semicontinuous (l.s.c) at $x_0 \in K$, if for any net $x_\alpha \subset K$ with $x_\alpha \to x_0$, and for any $y_0 \in F\left(x_0\right)$, there exist $y_\alpha \in P\left(x_\alpha\right)$, such that $y_\alpha \to y_0$;

(iii) A set-valued map P is said to be continuous at $x_0 \in K$, if P is upper and lower semicontinuous at $x_0 \in K$.

Definition 2.2. Let K_1 and K_2 be true subsets of the metric space, define the Hausdorff distance between K_1 and K_2 as

$$d_H\left(K_1, K_2\right) = \max\left\{\sup_{x\in K_1}\inf_{y\in K_2} d\left(x, y\right), \sup_{y\in K_2} d\left(x, y\right)\right\}.$$

Theorem 2.1. [16] Let K and Y be two Hausdorff topological spaces, $\left\{S^\alpha\right\}$ is a net in X_{-i} with the Vietoris topology, $\left\{y_\alpha\right\}$ is a net of Y and $\left\{f^\alpha\left(x, y\right)\right\}$ is a net of real-valued continuous function defined on $K \times Y$. If $S^\alpha \to S \in X_{-i}$ under the Vietoris topology, $y_\alpha \to y \in Y$ and $\sup_{(x,y)\in K\times Y}\left|f^\alpha\left(x, y\right) - f\left(x, y\right)\right| \to 0$, where $f\left(x, y\right)$ is a real-valued continuous function defined on $K \times Y$, then $\max_{x_{-i}^\alpha \in S^\alpha} f^\alpha\left(x, y^\alpha\right) \to \max_{x_{-i}^0\in S^0} f^0\left(x, y^0\right)$.

Theorem 2.2. [15] Let K and Y be two Hausdorff topological spaces, and Y is compact, if set-valued mapping $P: K \to 2^Y$ is closed, then P is upper semicontinuous.

Cited from the literature [3] Theorem 4.2. For each $i \in I$, $G_i := \prod_{i=1}^n X_i \to 2^R$ and $S_i : X_{-i} \to 2^{X_{-i}}$ are continuous on $\prod_{i=1}^n X_i$ and X_{-i} repectively. For each $x_{-i} \in X_{-i}$, $G_i\left(\cdot, x_{-i}\right)$ is $R_+ - \left(II\right)$ quasiconcave. Then, there exists $x^* = \left(x_1^*, x_2^*, \cdots, x_n^*\right) \in X$ such that for each $i \in I$,

$$\max \bigcup_{u_{-i}\in S_i\left(x_{-i}^*\right)} P_i\left(x_i^*, u_{-i}\right) = \min \bigcup_{u_i\in X_i} \max P_i\left(u_i, S_i\left(x_{-i}^*\right)\right),$$

where P_i is the ith player's ex-post regret function. Remember this generalized minimax regret problem as $\left\{P_i, S_i, G_i\right\}_{i=1}^n$.

Lemma 2.1. [16] Let $P: X \to 2^Y$ be a set-valued mapping with nonempty values. Suppose that

$$\max \bigcup_{x\in X} P\left(x\right) \neq \varnothing \text{ and } \max \bigcup_{x\in X} \max P\left(x\right) \neq \varnothing.$$

Then,

$$\max \bigcup_{x\in X} P\left(x\right) = \max \bigcup_{x\in X} \max P\left(x\right).$$

3. Main Results

In this section, we investigate generic stability theorem of generalized minimax regret equilibria with scalar set payoff when the scalar set payoff functions are disturbed.

For each $i \in I$, let K_i, Y be Banach spaces. suppose Γ be the space composed of $(P_1, \cdots, P_n, S_1, \cdots, S_n)$ and satisfy all assumption of Literature [3] Theorem 4.2. For $m \in \Gamma$, we define the set of all generalized minimax regret equilibria by $\Phi(m)$.

Definition 3.1. Since P be a real set-valued mapping, we define the distance ρ on Γ by

$$\rho(m, m') := \sup_{x \in X} \sum_{i=1}^{n} \left|\max P_i(x), \max P_i'(x)\right| + \sup_{x_{-i} \in X_{-i}} \sum_{i=1}^{n} h_i(S_i, S_i'),$$

where $m = (P_1, \cdots, P_n, S_1, \cdots, S_n) \in \Gamma$, $m' = (P_1', \cdots, P_n', S_1', \cdots, S_n') \in \Gamma$, H_i and h_i are Hausdorff distance on K_i. Obviously, (Γ, ρ) is a complete metric space.

Lemma 3.1. [2] For each $m \in \Gamma$, m is continuous if and only if the set-valued mapping $\Phi : \Gamma \to 2^X$ is lower semicontinuous.

Theorem 3.1. For each $i \in I$, let $X_i \subset K_i$ be nonempty compact convex subset. Assume that $P_i : \prod_{i=1}^{n} X_i \to 2^R$ and $S_i : X_{-i} \to 2^{X_{-i}}$ are continuous with nonempty compact-valued. Then the set-valued mapping $\Phi : \Gamma \to 2^X$ is upper semicontinuous with compact valued.

Proof Since X_i is nonempty compact subset of K_i, by Theorem 2.1, it is sufficient to prove that $Graph(\Phi)$ is closed. *i.e.*, for each $i \in I$, for any $(P^\alpha, S^\alpha) \in \Gamma$ with $(P^\alpha, S^\alpha) \to (P^0, S^0)$, any $x^\alpha \in \Phi(P^\alpha, S^\alpha)$ with $x^\alpha \to x^0$. We will prove $x^0 \in \Phi(P^0, S^0)$.

For any $x^\alpha \in \Phi(P^\alpha, S^\alpha)$, there exists $z_{-i}^\alpha \in S_i^\alpha$ such that

$$\max P_i^\alpha(x_i^\alpha, z_{-i}^\alpha) = \min_{u_i \in X_i} \max_{u_{-i}^\alpha \in S_i^\alpha} P_i^\alpha(u_i, u_{-i}^\alpha),$$

where $P_i^\alpha(x_i, x_{-i}) = \sup_{x_i \in X_i} G_i^\alpha(u_i, x_{-i}) - G_i^\alpha(x_i, x_{-i})$.

Because G_i is continuous and $(P^\alpha, S^\alpha) \to (P^0, S^0)$. We have $P^\alpha \to P^0$ when $G^\alpha \to G^0$. Because S_i is compact-valued, for $z_{-i}^\alpha \in S_i^\alpha$, there exists $z_{-i}^0 \in S_i^0$ such that $z_{-i}^\alpha \to z_{-i}^0$.

Since $\max P_i$ is continuous, for each $i \in I$, then

$$\begin{aligned} &\left|\max P_i^\alpha(x_i^\alpha, z_{-i}^\alpha) - \max P_i^0(x_i^0, z_{-i}^0)\right| \\ &\le \left|\max P_i^\alpha(x_i^\alpha, z_{-i}^\alpha) - \max P_i^0(x_i^\alpha, z_{-i}^\alpha)\right| + \left|\max P_i^0(x_i^\alpha, z_{-i}^\alpha) - \max P_i^0(x_i^0, z_{-i}^0)\right| \\ &\to 0. \end{aligned}$$

By Theorem 2.1 and Lemma 2.1, we have

$$\min_{u_i \in X_i} \max_{u_{-i}^\alpha \in S_i^\alpha} P_i^\alpha(u_i, u_{-i}^\alpha) \to \min_{u_i \in X_i} \max_{u_{-i}^0 \in S_i^\alpha} P_i^0(u_i, u_{-i}^0).$$

Thus, for sufficiently large α, for each $i \in I$, there exists $z_{-i}^0 \in S_i^0$ such that

$$\max P_i^0(x_i^0, z_{-i}^0) = \min_{u_i \in X_i} \max_{u_{-i}^0 \in S_i^0} P_i^0(u_i, u_{-i}^0).$$

Hence, $x^0 = (x_i^0, x_{-i}^0) \in \Phi(P^0, S^0)$, Φ is upper semicontinuous with compact valued. This completes the proof. □

Remark The proof of this theorem is similar to that of Lemma 4 in Reference [2]. If $P: X \to R$, $S: X \to X$, then the proof is the same as Lemma 4 in Reference [2].

Lemma 3.2. [17] Let Γ be a complete metric space and X be a topological space. Suppose that $\Phi: \Gamma \to 2^X$ is upper semicontinuous and nonempty compact valued, there exists a dense residual $Q \subset \Gamma$ such that Φ is lower semicontinuous on Q.

Theorem 3.2. There exists a dense residual set $Q \subset \Gamma$ such that m is continuous for $m \in Q$.

Proof By Theorem 3.1, the set-valued mapping Φ is upper semicontinuous with compact valued. By Lemma 3.2, there exists a dense residual $Q \subset \Gamma$ such that Φ is lower semicontinuous on Q. By Lemma 3.1, m is continuous for any $m \in Q$. This completes the proof. □

The following example illustrates that $Q \neq \Gamma$.

Example 3.1. Consider the generalized minmax regret equilibria problem $\{P_i, S_i, G_i\}_{i=1}^n$. Let $I = \{1,2\}$, $X_1 = X_2 = [0,1]$, and $G_1, G_2: X_1 \times X_2 \to 2^R$ be player's scalar set payoff function,

$$G_1(x_1,x_2) = G_2(x_1,x_2) = 1, \forall (x_1,x_2) \in X_1 \times X_2.$$

P_1, P_2 be player's ex-post regret function,

$$P_1(x_1,x_2) = \sup \bigcup_{x_1 \in X_1} G_1(x_1,x_2) - G_1(x_1,x_2) = 0, \forall (x_1,x_2) \in X_1 \times X_2,$$

$$P_2(x_2,x_1) = \sup \bigcup_{x_2 \in X_2} G_2(x_1,x_2) - G_2(x_1,x_2) = 0, \forall (x_1,x_2) \in X_1 \times X_2.$$

Then $P \in \Gamma$. For each n, define G^n as follows:

$$G_1^n(x_1,x_2) = \frac{1}{n} x_1, \forall (x_1,x_2) \in X_1 \times X_2,$$

$$G_2^n(x_1,x_2) = \frac{1}{n} x_2, \forall (x_1,x_2) \in X_1 \times X_2.$$

And define P^n as follows:

$$P_1^n(x_1,x_2) = \sup \bigcup_{x_1 \in X_1} G_1^n(x_1,x_2) - G_1^n(x_1,x_2) = \frac{1}{n} - \frac{1}{n} x_1, \forall (x_1,x_2) \in X_1 \times X_2,$$

$$P_2^n(x_2,x_1) = \sup \bigcup_{x_2 \in X_2} G_2^n(x_2,x_1) - G_2^n(x_2,x_1) = \frac{1}{n} - \frac{1}{n} x_2, \forall (x_1,x_2) \in X_1 \times X_2.$$

Then $P^n \in \Gamma$, $P^n \to P$. It shows that no point in Q is continuous for $m \in Q$. Similarly, for each n, define G^n as follows:

$$G_1^n(x_1,x_2) = -\frac{1}{n} x_1, \forall (x_1,x_2) \in X_1 \times X_2,$$

$$G_2^n(x_1,x_2) = -\frac{1}{n} x_2, \forall (x_1,x_2) \in X_1 \times X_2.$$

And define P^n as follows:

$$P_1^n(x_1,x_2)=\sup\bigcup_{x_1\in X_1}G_1^n(x_1,x_2)-G_1^n(x_1,x_2)=-\frac{1}{n}+\frac{1}{n}x_1,\forall(x_1,x_2)\in X_1\times X_2,$$

$$P_2^n(x_2,x_1)=\sup\bigcup_{x_2\in X_2}G_2^n(x_2,x_1)-G_2^n(x_2,x_1)=-\frac{1}{n}+\frac{1}{n}x_2,\forall(x_1,x_2)\in X_1\times X_2.$$

Then $P^n\in\Gamma$, $P^n\to P$. It shows that no point in Q is continuous for $m\in Q$. Consequently, $P\notin\Gamma$, So $Q\neq\Gamma$.

Theorem 3.3. There exists a dense residual set $Q\subset\Gamma$ such that Φ is a singleton set for any $m\in Q$.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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