Journal of Applied Mathematics and Physics, 2022, 10, 3072-3082
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Bianchi Type-I Anisotropic Universe with Metric Potential in Saez-Ballester Theory of Gravitation

Md. Rezaul Karim

Department of Mathematics, University of Creative Technology Chittagong (UCTS), Chittagong, Bangladesh
Email: rezakm891@gmail.com

How to cite this paper: Karim, M.R. (2022) Bianchi Type-I Anisotropic Universe with Metric Potential in Saez-Ballester Theory of Gravitation. *Journal of Applied Mathematics and Physics*, **10**, 3072-3082.
https://doi.org/10.4236/jamp.2022.1010205

Received: September 12, 2022
Accepted: October 24, 2022
Published: October 27, 2022

Abstract

Bianchi Type-I cosmological model in the presence of Saez-Ballester theory gravitation is studied. An exact solution of the field equation is given by considering the cosmological model yield a metric potential included with a real number. The relation between the deceleration parameter and Hubble parameter and average scale factor is used in that cosmological model. The effect of the viscosity on the entropy of the universe is utilized by energy momentum tensor with bulk viscous terms in a conservative manner. We obtained a formula for calculating the entropy of the universe in terms of viscosity and used it to compare to the study. Also, various physical and kinematical properties have been discussed.

Keywords

Bianchi Type-I Space Time, Saez-Ballester Theory, Energy Momentum Tensor, Bulk Viscosity, Hubble and Deceleration Parameter

1. Introduction

Friedman Robertson-Walker (FRW) universe is one of the most generalizations of the flat universe. Likewise, the Bianchi Type-1 universe is one of the simple and elegant space times in an anisotropic flat universe. The directional scale factor makes the difference between FRW universe and Bianchi Type-1 universe. That is, FRW universe has the same scale factors. Bianchi Type-1 universe behaves like Kansor universe near the singularity. Since a universe is filled with matter, the initial anisotropy in Bianchi Type-1 universe rapidly dies away and evolves into an FRW universe. Several authors have investigated Bianchi Type-1 universe from different aspects due to its importance.

The present universe is moving through a phase of accelerated expansion that has been supported by a lot of work in astrophysics and cosmology based on observational evidence. The idea, the present universe is expanding with accelerated forward by the recent cosmological observations from a supernova [1] [2]: cosmic microwave background (Emb)) anisotropies [3] [4] [5] and large-scale structure [6] [7]. In the second self-creation theory relativity, Kantowski, R., Sachs, R.K. [8] studied spatially homogeneous and anisotropic FRW space time in the presence of viscous fluid. A five-dimensional Kaluza-Klein cosmological model in the presence of Bulk viscous fluid was investigated by Kumar and Reddy [9] and also by Mohammad Moksud Alam, Mohammad Amjad Hossain, Mohammad Ashraful Islam [10]. Anirudh Pradhan, Anil Kumar Vishwakarma, A. Dolgov [11] and Matjask, J. [12] studied Bianchi Type-1 cosmological model with bulk viscous barotrophic fluid with varying Λ and functional relation on Hubble parameter H with deceleration parameter where the metric potential is taken as a function of x and $\in$ and the coefficient of bulk viscosity is assumed to be a power function of mass density was given by Anirudh Pradhan, Hare Rm Pandey [13]. The significant important is the scalar tensor theories of gravitation generated by [14] [15] [16]. Numerous versions of the scalar tensor theories are based on the introduction of a scalar field $\varnothing$ into the formulation of general relativity and cosmology. In Saez Ballester theory the metric is coupled with a dimensionless scalar field.

The metric [17] field equations are

$$G_{ij} - w\varnothing^{n}\left(\varnothing_{,i}\varnothing_{,j} - \frac{1}{2}g_{ij}\varnothing_{,k}\varnothing^{,k}\right) = -\pi T_{ij} \tag{1.1}$$

where $\varnothing$ satisfies the following conditions

$$2\varnothing^{n}\varnothing^{i}_{,i} + n\varnothing^{n-1}\varnothing^{,k}\varnothing_{,k} = 0 \tag{1.2}$$

$$where\ \ G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} \tag{1.3}$$

Equation (3) is called the Einstein tensor T_{ij} is the stress energy tensor of the matter, w and n are constant. Comma (,) and semicolon (;) denotes partial and co-variant differentiation respectively. A detailed explanation of Saez-Ballester cosmological model is formulated in the work of [18] [19] [20]. In this paper, we obtain Bianchi Type-1 cosmological model in scalar tensor theory of gravitation formulated by [18]. My paper is organized as follows: In Section 2, we derive field equations; in Section 3, we deal with the solution in the presence of Bulk viscous fluid. Section 4 includes the solution for the metric potential. Section 5 is mainly written with physical and kinematical properties. The last section contains the conclusion.

This research was motivated by the influence of the great scientist Albert Einstein who is the author of Relativity and another legendary scientist Stephen Hawking who is regarded as a brilliant theoretical physicist. His works on Black Holes and the Big Bang are the resources of research.

2. The Metric and Field Equations

We consider anisotropic Bianchi Type-1 space time metric is given by [21],

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (2.1)$$

where A, B, C are the directional scale factors and are the functions of cosmic time t. The Bianchi Type-1 space time becomes isotropic if the entire directional scale factor becomes equal and we get the usual FRW space time. The energy momentum tensor is as follows:

$$T_j^i = (\rho + p)u^i u_j + pg_j^i - \varepsilon\theta\left(g_j^i + u^i u_j\right) \quad (2.2)$$

ε is the coefficient of Bulk viscosity, θ is the expansion scalar of the cosmological model, ρ is the energy density and p is the isotropic pressure.

In the commoving coordinates

$$u^4 = -1, u_4 = 1, u^i u_j = -1, u^2 = u^3 = 0 \quad (2.3)$$

Also energy conservation equation

$$T_{,j}^{ij} = 0 \quad (2.4)$$

From Equation (2.2) we can write the component connection,

$$g_{11} = -1,\ g_{22} = A^2,\ g_{33} = B^2,\ g_{33} = C^2$$

$$g^{11} = -1,\ g^{22} = \frac{1}{A^2},\ g^{33} = \frac{1}{B^2},\ g^{44} = \frac{1}{C^2}$$

For this metric, using the definition of affine connection, we compute the following components,

$$\Gamma_{22}^1 = A\dot{A},\ \Gamma_{33}^1 = B\dot{B},\ \Gamma_{44}^1 = C\dot{C},\ \Gamma_{21}^2 = \Gamma_{12}^2 = \frac{\dot{A}}{A},$$

$$\Gamma_{13}^3 = \Gamma_{31}^3 = \frac{\dot{B}}{B},\ \Gamma_{14}^4 = \Gamma_{41}^4 = \frac{\dot{C}}{C} \text{ and other components vanish.}$$

where suffix (.) at the symbol A, B, C denotes ordinary differentiation with respect to t.

Moreover, to compute Ricci scalar, using the definition of Rici tensor, we compute R_{11}, R_{22}, R_{33}, R_{44} as follows by [22].

$$R_{jk} = \frac{\partial}{\partial x^k}\Gamma_{ji}^i - \frac{\partial}{\partial x^i}\Gamma_{jk}^i + \Gamma_{rk}^i\Gamma_{ji}^r - \Gamma_{ri}^i\Gamma_{jk}^r,$$

$$R_{11} = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C},$$

$$R_{22} = -A\ddot{A} - \frac{A\dot{A}\dot{B}}{B} - \frac{A\dot{A}\dot{C}}{C},$$

$$R_{33} = -B\ddot{B} - \frac{B\dot{A}\dot{B}}{A} - \frac{B\dot{B}\dot{C}}{C},$$

$$R_{44} = -C\ddot{C} - \frac{C\dot{A}\dot{C}}{A} - \frac{C\dot{B}\dot{C}}{B}.$$

Therefore the Ricci scalar for this metric by [23].

$$R = g^{ij} R_{ij}$$

$$R = g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33} + g^{44} R_{44}$$
$$= -2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{AC} \right)$$

Using the result derived just above, we have the following set of expressions using Riemann Curvature Tensor by [24].

$$G_{11} = R_{11} - \frac{1}{2} R g_{11} = -\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{AC} \right),$$

$$G_{22} = R_{22} - \frac{1}{2} R g_{22} = -A^2 \left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} \right),$$

$$G_{33} = R_{33} - \frac{1}{2} R g_{33} = -B^2 \left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} \right),$$

$$G_{44} = R_{44} - \frac{1}{2} R g_{44} = -\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{A}}{BA} \right).$$

From Equation (2.2) we get the following expressions:

$$T_1^1 = (\rho + P) u^1 u_1 + p g_1^1 - \varepsilon\theta \left(g_1^1 + u^1 u_1 \right) = -\rho,$$

$$T_2^2 = T_3^3 = T_4^4 = p - \varepsilon\theta .$$

Now, $T^{11} = T_1^1 g^{11} = -\rho$,

$$T^{22} = T_2^2 g^{22} = A^2 \left(p - \varepsilon\theta \right),$$

$$T^{33} = T_3^3 g^{33} = B^2 \left(p - \varepsilon\theta \right),$$

$$T^{44} = T_4^4 g^{44} = C^2 \left(p - \varepsilon\theta \right).$$

In the commoving system, the fluid Equation (1.1) for the metric (2.1) with the help of energy momentum tensor (2.4) can be explicitly written as,

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{AC} - \frac{w\varnothing^n}{2} \dot{\varphi} = 8\pi\rho \tag{2.5}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{w\varnothing^n}{2} \dot{\varphi} = -8\pi\left(p - \varepsilon\theta \right) \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{w\varnothing^n}{2} \dot{\varphi} = -8\pi\left(p - \varepsilon\theta \right) \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{A}}{BA} - \frac{w\varnothing^n}{2} \dot{\varphi} = -8\pi\left(p - \varepsilon\theta \right) \tag{2.8}$$

From Equation (2.4) we can write,

$$T^{ij}_{,j} + \Gamma^i_{kj} T^{kj} + \Gamma^j_{kj} T^{ik} = 0 \tag{2.9}$$

$$\text{or,} \quad -\dot{\rho} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right)\left(p - \varepsilon\theta \right) - \rho\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 .$$

$$\text{Hence,}\quad -\dot{\rho}+\left(p-\varepsilon\theta-\rho\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{2.10}$$

3. Solution of the Field Equations

To obtain the expression of Saez-Ballester scalar field $\varnothing$, we obtain the equation from (1.2).

$$\varnothing^{n}\varnothing^{i}_{,i}+\frac{n}{2}\varnothing^{n-1}\varnothing^{,k}\varnothing_{,k}=0 \tag{3.1}$$

$$\text{or,}\quad \frac{\ddot{\varphi}}{\varphi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n}{2\varphi}\dot{\varphi}=0$$

$$\therefore \dot{\varphi}\left(ABC\right)^{\frac{n}{2}}=k \tag{3.2}$$

where, k is the integrating constant. We denote the average scale factor of the Bianchi-I universe by $a(t)$ which is given by:

$$a\left(t\right)=\left(ABC\right)^{\frac{1}{3}}=V$$

$$\text{or,}\quad a^{3}=ABC$$

$$\therefore \frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)$$

$$So,\quad H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{1}{3}\left(H_{1}+H_{2}+H_{3}\right)$$

where, $\left(H_{1}=\frac{\dot{A}}{A},H_{2}=\frac{\dot{B}}{B},H_{3}=\frac{\dot{C}}{C}\right)$ are the Hubble Parameters in different and usual direction by [25].

So, Equation (3.2) can be written as,

$$\dot{\varphi}\left(a^{3}\right)^{\frac{n}{2}}=k \tag{3.3}$$

$$\therefore \varphi=k_{1}\exp\left[k\int\frac{\mathrm{d}t}{\left(a^{3}\right)^{\frac{n}{2}}}\right] \tag{3.4}$$

where, k_1 is the integrating constant.

Equation (3.3) can be writing,

$$\dot{\varphi}=\frac{k}{\left(a^{3}\right)^{\frac{n}{2}}}$$

$$\text{or,}\quad \log\varphi=\int k\left(a^{3}\right)^{-\frac{n}{2}}\mathrm{d}t+\log k_{1}$$

$$\therefore \varphi=k_{1}\exp\left[k\int\left(a^{3}\right)^{-\frac{n}{2}}\mathrm{d}t\right] \tag{3.5}$$

where, k_1 is the integrating constant.

Equation (2.10) can be written

$$-\dot{\rho}+3\left(p-\varepsilon\theta-\rho\right)H=0$$

$$\text{or,}\quad -\dot{\rho}+\left(p-\varepsilon\theta-\rho\right)\frac{a}{a}=0$$

$$\text{or,}\quad -\frac{\dot{\rho}}{\rho}+\left(\frac{p-\epsilon\theta}{\rho}-1\right)\frac{a}{a}=0$$

$$\text{or,}\quad \log\rho+\log k_2=\left(\frac{p-\epsilon\theta}{\rho}-1\right)\log a$$

$$\therefore a^{\left(\frac{p-\epsilon\theta}{\rho}-1\right)}=\frac{k_2}{\beta} \tag{3.6}$$

where, $\beta=\frac{1}{\rho}$,

In terms of the Hubble parameter in the axial direction the Equation (2.5-2.8) can be expressed by [26].

$$H_1H_2+H_2H_3+H_3H_1-\frac{w\varnothing^n}{2}\dot{\varphi}=8\pi\rho \tag{3.7}$$

$$\dot{H}_2+\dot{H}_3+H_2^2+H_3^2+H_2H_3-\frac{w\varnothing^n}{2}\dot{\varphi}=-8\pi\left(P-\varepsilon\theta\right) \tag{3.8}$$

$$\dot{H}_1+\dot{H}_3+H_3^2+H_1^2+H_1H_3-\frac{w\varnothing^n}{2}\dot{\varphi}=-8\pi\left(P-\varepsilon\theta\right) \tag{3.9}$$

$$\dot{H}_1+\dot{H}_3+H_2^2+H_1^2+H_2H_1-\frac{w\varnothing^n}{2}\dot{\varphi}=-8\pi\left(P-\varepsilon\theta\right) \tag{3.10}$$

Subtract Equation (2.6) from (2.7),

$$\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{C}}{AC}-\frac{\ddot{B}}{B}-\frac{\dot{B}\dot{C}}{BC}=0 \tag{3.11}$$

Again subtract Equation (2.7) from (2.8),

$$\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{C}}{C}-\frac{\dot{A}\dot{C}}{AC}=0 \tag{3.12}$$

Now by adding Equation (3.11) and (3.12),

$$\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}=0$$

$$\text{or,}\quad \frac{\dot{B}}{B}=-\frac{C\ddot{A}-A\ddot{C}}{\dot{A}C-\dot{C}A}$$

By integrating,

$$C\dot{A}-A\dot{C}=\frac{k_3}{B}\text{, where } k_3 \text{ is the integrating constant.}$$

$$\therefore\frac{\dot{A}}{A}-\frac{\dot{C}}{C}=\frac{k_3}{V}\text{, where, } V=ABC.$$

Again integrating both sides

$$\frac{A}{C} = k_3 \int \frac{\mathrm{d}t}{V} + k_4 \quad (3.13).$$

$$\therefore \frac{A}{C} = k_4 \exp\left[k_3 \int \frac{\mathrm{d}t}{V}\right] \quad (3.14)$$

Similarly we can write,

$$\frac{B}{C} = k_6 \exp\left[k_5 \int \frac{\mathrm{d}t}{V}\right] \quad (3.15)$$

$$And \ \frac{A}{B} = k_8 \exp\left[k_7 \int \frac{\mathrm{d}t}{V}\right] \quad (3.16)$$

where, where k_4, k_5, k_6, k_7,k_8 are the integrating constant.

4. Solution of Cosmological Model with Metric Potential

We assume a relation in metric potential by [27].

$$A = B^m \quad (4.1)$$

$$\frac{\dot{A}}{A} = m\frac{\dot{B}}{B} \ \text{ and } \ \frac{\ddot{A}}{A} = m\left[\frac{\ddot{B}}{B} + \frac{m-1}{B^2}\dot{B}^2\right].$$

Equation (3.16) can be written as,

$$B = \left[k_8 \exp\left[k_7 \int \frac{\mathrm{d}t}{V}\right]\right]^{\frac{1}{1-m}} \quad (4.2)$$

We have,

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

$$\text{or,} \quad a^3 = ABC = V$$

$$\text{or,} \quad \frac{\dot{a}}{a} = \frac{1}{3}\left[(m+1)\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]$$

$$\text{or,} \quad a^3 = k_9 B^{m+1} c$$

$$\text{or,} \quad A = k_9 B^m$$

$$\therefore A = k_{10}\left[\exp\left[k_7 \int \frac{\mathrm{d}t}{V}\right]\right]^{\frac{m}{1-m}} \quad (4.3)$$

where, $k_{10} = k_9 \left(k_8\right)^{\frac{m}{m+1}}$ is a constant.

Again,

$$a^3 = ABC$$

$$\text{or,} \quad C = \frac{a^3}{ABC} = \frac{a^3}{B^{m+1}}$$

$$\text{or,} \quad C = a^3\left[k_8 \exp\left[k_7 \int \frac{\mathrm{d}t}{V}\right]\right]^{\frac{m+1}{1-m}}$$

$$\therefore C = Vk_{11}\left[\exp\left[k_7\int\frac{\mathrm{d}t}{V}\right]\right]^{\frac{m+1}{1-m}} \tag{4.4}$$

where, $k_{11} = k_9\left(k_8\right)^{\frac{m}{m+1}}$ is a constant.

Now, $a^3 = ABC = B^{m+1}$

$$a = V^{\frac{1}{3}}k_{12}\left[\exp\left[k_7\int\frac{\mathrm{d}t}{V}\right]\right]^{\frac{2(1+m)}{3(1-m)}} \tag{4.5}$$

5. Physical and Kinematical Properties

Shear Scalar,

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\sum_{i=1}^{3}\left(H_i - H\right)^2 = \frac{1}{2}\left(H_1^2 + H_2^2 + H_3^2 - 3H^2\right)$$

$$\sigma = \frac{1}{\sqrt{2}}\sqrt{\left(H_1^2 + H_2^2 + H_3^2 - 3H^2\right)} \tag{5.1}$$

Deceleration Parameter,

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2} \tag{5.2}$$

Anisotropic Pressure,

$$A = \frac{1}{3}\left(\frac{\nabla H_i}{H^2}\right)^2 = \frac{1}{3}\left[\frac{H_1^2 + H_2^2 + H_3^2 - 3H^2}{H^2}\right] \tag{5.3}$$

Time to Time Component Field Equation.

From Equation (2.6), (5.1) we get,

$$H_1^2 + H_1^2 + H_1^2 - \left(H_1 + H_2 + H_3\right)^2 - \frac{w\varnothing^n}{2}\dot{\varphi} = 8\pi\rho$$

or, $3H^2 = \sigma^2 - \frac{w\varnothing^n}{2}\dot{\varphi} - 4\pi\rho$

$$\therefore 3H^2M_p^2 = \sigma^2M_p^2 - \frac{w\varnothing^n}{4}\dot{\varphi}M_p^2 - \frac{\rho}{2G} \tag{5.4}$$

We know *Plank Mass by* [28].

$$M_p = \frac{1}{\sqrt{8\pi G}} \tag{5.5}$$

By adding Equation (3.8)-(3.10) we get plank mass equation with deceleration and Hubble parameter, Bulk Viscosity and scalar expansion by [29].

$$2\left(\dot{H}_1 + \dot{H}_2 + H_2^2\right) + 2\left(H_1^2 + H_1^2 + H_1^2\right) + \left(H_1H_2 + H_2H_3 + H_3H_1 - \frac{3w\varnothing^n}{2}\dot{\varphi}\right),$$
$$= -24\pi\left(p - \epsilon\theta\right)$$

$$\text{Hence,}\quad \frac{H^2\left(2q-1\right)}{8\pi G} = \sigma^2M_p^2 + \frac{3\left(p - \varepsilon\theta\right)}{G} - \frac{\rho}{G} - w\varphi^n\dot{\varphi}M_p^2 \tag{5.6}$$

6. Conclusions

In this paper, we summarize our findings throughout Sections 1 to 5.

We applied anisotropic Bianchi Type-1 space time metric in the presence of Saez-Ballester's theory of gravitation. By using energy momentum tensor with Bulk Viscosity, energy conservation equation and commoving vector, we get a new cosmological field equation. Moreover, we have considered Hubble parameter, so we get a special solution of the field equation. Furthermore, we have used new metric potential and we determined the time-to-time component equation and Plank Mass.

The obtained result of this paper clearly defines entropy and isotropy of the universe utilizing the new Plank Mass equation with deceleration and Hubble parameter, Bulk Viscosity and scalar expansion. The significant result is that an anisotropic universe with higher anisotropy transits to a late accelerating phase before a universe with lower anisotropy. Numerically we identified that new exponent result plays an important role in identifying the nature of the universe.

In the future, this research work will help to investigate more realistic cosmology. For example, we can take the bilinear deceleration parameter in a suitable form and some other assumptions that may explain the phase transition of the universe [30] more effectively.

Acknowledgements

I am very much thankful to Professor Dr. Mohammed Ashraful Islam, Department of Mathematics, University of Chittagong, Bangladesh. I am giving more thanks to Professor Dr. Mohammed Aman Ullah, Professor and Head, Department of Mathematics, University of Chittagong, Bangladesh, for his great contribution.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Bennet, C.L., Hill, R.S., A Bennett, C.L., Hill, R.S., Hinshaw, G., Nolta, M.R., Odegard, N., Page, L., Spergel, D.N., Weiland, J.L., Wright, E.L., Halpern, M., Jarosik, N., Kogut, A., Limon, M., Meyer, S.S., Tucker, G.S. and Wollack, E. (2003) First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Foreground Emission. *The Astrophysical Journal Supplement Series*, **148**, 97-117. https://doi.org/10.1086/377252

[2] Spergel, D.N., Verde, L., Peiris, H.V., Komatsu, E., Nolta, M.R., Bennett, C.L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S.S., Page, L., Tucke, G.S., Weiland, J.L., Wollack, E. and Wright, E.L. (2003) First-Year Wilkinson Micowave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters. *The Astrophysical Journal Supplement Series*, **148**, 175-194. https://doi.org/10.1086/377226

[3] Perlmutter, S., Gabi, S., Goldhaber, G., Goobar, A., Groom, D.E., Hook, M., Kim,

A.G., Kim, M.Y., Lee, J.C., Pain, R., Pennypacker, C.R., Small, A., Ellis, R.S., McMahon, R.G., Boyle, B.J., Bunclark, P.S., Carter, D., Irwin, M.J., Glazebrook, K., Newberg, H.J.M., Filippenko, A.V., Matheson, T., Dopita, M. and Couch, W.J. (1996) Measurement of the Cosmological Parameters Q and Λ from the First Seven Supernovae at z ≥ 0.35. *The Astrophysical Journal*, **517**, 565-586.

[4] Riess, A.G., Strolger, L.-G., Casertano, S., Ferguson, H.C., Mobasher, B., Gold, B., Challis, P.J., Filippenko, A.V., Jha, S., Li, W.D., Tonry, J., Foley, R., Kirshner, R.P., Dickinson, M., MacDonald, E., Eisenstein, D., Livio, M., Younger, J., Xu, C., Dahlén, T., Stern, D. and Tsvetanov, Z. (2004) Type Ia Supernovae Discoveries at z > 1 from the Hubble Space Telescope: Evidence for First Deceleration and Constraints on Dark Energy Evolution. *The Astrophysical Journal*, **607**, 665-687.

[5] Riess, A.G., Strolger, L.-G., Casertano, S., Ferguson, H.C., Mobasher, B., Gold, B., Challis, P.J., Filippenko, A.V., Jha, S. and Li, W.D. (2007) New Hubble Space Telescope Discoveries Type of Supernovae at z ≥ 1: Narrowing Constraints on the Early Behavior of Dark Energy. *The Astrophysical Journal*, **659**, 98-121. https://iopscience.iop.org/article/10.1086/510378 https://doi.org/10.1086/510378

[6] Sarkar, S. (2014) Holographic Dark Energy with Linearly Varying Deceleration Parameter and Escaping Big Rip Singularity of the Bianchi Type-V Universe. *Astrophysics and Space Science*, **352**, 859-866. https://doi.org/10.1007/s10509-014-1920-0

[7] Tegmark, M. (2004) Cosmological Parameters from SDSS and WMAP. *Physical Review D*, **69**, Article ID: 103501. https://doi.org/10.1063/1.1581768

[8] Kantowski, R. and Sachs, R.K. (1966) Some Spatially Homogeneous Anisotropic Relativistic Cosmological Models. *Journal of Mathematical Physics*, **7**, Article No. 443. https://doi.org/10.1063/1.1704952

[9] Kumar, R. and Reddy, D. (2015) Kaluza-Klein Cosmological Model with Bulk Viscosity in Barber's Second Self Creation Cosmology. *International Journal of Astronomy*, **4**, 1-4.

[10] Alam, M.M., Hossain, M.A. and Islam, M.A. (2017) Super Exponential Expansion for Dark Energy Model with Variable Λ in f(R, T) Gravity. *International Journal of Astrophysics and Space Science*, **5**, 41-46. https://doi.org/10.11648/j.ijass.20170503.11

[11] Pradhan, A., Vishwakarma, A.K. and Dolgov, A. (2002) LRS Bianchi Type-I Cosmological Models in Barber's Second Self Creation Theory. *International Journal of Modern Physics D*, **11**, 1195-1207. https://doi.org/10.1142/S0218271802002207

[12] Matjask, J. (1995) Cosmological Models with a Time-Dependent Λ Term. *Physical Review D*, **51**, Article No. 4154. https://doi.org/10.1103/PhysRevD.51.4154

[13] Pradhan, A. and Pandey, H.R. (2003) Bulk Viscous Cosmological Models in Lyra Geometry. https://arxiv.org/abs/gr-qc/0307038

[14] Hatice, Ö.D. (2021) Gravitational Waves in Brans-Dicke Theory with a Cosmological Constant. *The European Physical Journal C*, **81**, Article No. 326. https://doi.org/10.1140/epjc/s10052-021-09123-7

[15] Sarkar, K. and Bhadra, A. (2006) Strong Field Gravitational Lensing in the Brans-Dicke Theory. *Classical and Quantum Gravity*, **23**, Article No. 6101. https://doi.org/10.1088/0264-9381/23/22/002

[16] Maruya, S.K., Singh, N. and Ray, S. (2021) Anisotropic Stars in Brans-Dicke Gravity. *Chinese Journal of Physics*, **71**, 548-560. https://doi.org/10.1016/j.cjph.2021.03.019

[17] Mete, V.G., Nimkar, A.S. and Elkar, V.D. (1985) Axially Symmetric Cosmological Mod-

el with Bulk Stress in Saez-Ballester Theory of Gravitation. *International Journal of Theoretical Physics*, **55**, 412-420. https://doi.org/10.1007/s10773-015-2675-2

[18] Cole, A.A. (1990) Bianchi V Imperfect Fluid Cosmology. *General Relativity and Gravitation*, **22**, 3-18. https://doi.org/10.1007/BF00769241

[19] Raju, P., Sobhan Babu, K. and Reddy, D.R.K. (2001) Spherically Symmetric Five Dimensional Cosmological Model in Scale Covariant Theory of Gravitation. *Astrophysics and Space Science*, **277**, Article No. 461.

[20] Ghate, H.R. and Sontakke, A.S. (1991) Bianchi Type-IX Magnetized Dark Energy Model in Saez-Ballester Theory of Gravitation. *Astrophysics and Space Science*, **182**, Article No. 289.

[21] Sharma, M. and Sharma, S. (2016) A Study of Bianchi Type-I Cosmological Model with Cosmological Constant. *The African Review of Physics*, **11**, 317-321. https://arxiv.org/pdf/1704.03725

[22] Parker and Christensen (1994) Ricci Curvature Tensor, Wolfram Math World A Wolfram Web Resource. https://mathworld.wolfram.com/RicciCurvatureTensor.html

[23] Yang, R.-J., Zhu, Z.-H. and Feng, Q. (2011) Spatial Ricci Scalar Dark Energy Model. *International Journal of Modern Physics A*, **26**, 317-329. https://doi.org/10.1142/S0217751X11051263

[24] Cox, J. (2019) The Riemann Curvature Tensor. Louisiana Tech University, Ruston. https://digitalcommons.latech.edu/cgi/viewcontent.cgi?article=1008&context=mathematics-senior-capstone-papers

[25] Kassem, A. (2021) Locally Varying Hubble Parameter in Terms of Reduced Friedmann Equation. *International Journal of Astronomy and Astrophysics*, **11**, 175-189. https://doi.org/10.4236/ijaa.2021.112010

[26] Tawfik, A., Mansour, H. and Wahba, M. (2009) Hubble Parameter in Bulk Viscous Cosmology. Egyptian Center for Theoretical Physics (ECTP), Giza.

[27] Reddy, D.R.K., Naidu, R.L. and Rao, S.A. (2007) Axially Symmetric Inflationary Universe in General Relativity. *International Journal of Theoretical Physics*, **47**, 1016-1020. https://doi.org/10.1007/s10773-007-9529-5

[28] Sivaram, C. (2007) What Is Special about the Plank Mass? Indian Institute of Astrophysics, Bangalore.

[29] Li, J.-M. (2005) Modified Hubble Law, the Time-Varying Hubble Parameter and the Problem of Dark Energy. Vol. 2, Cornell University, Ithaca. https://arxiv.org/pdf/physics/0507018

[30] De Vega, H.J., Khalatnikov, I.M. and Sànchez, N.G. (2001) Phase Transitions in the Early Universe: Theory and Observations. Springer Science & Business Media, Berlin. https://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_one.php