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Integrability Tests and Some New Soliton Solutions of an Extended Potential Boiti-Leon-Manna-Pempinelli Equation

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Abstract

This paper is devoted to the study of a (2 + 1)-dimensional extended Potential Boiti-Leon-Manna-Pempinelli equation. Firstly, By means of the standard Weiss Tabor Carnevale approach and Kruskal's simplification, we prove the painlevé non integrability of the equation. Secondly, A new breather solution and lump type solution are obtained based on the parameter limit method and Hirota's bilinear method. Besides, some interaction behavior between lump type solution and N-soliton solutions (N is any positive integer) are studied. We construct the existence theorem of the interaction solution and give the process of calculation and proof. We also give a concrete example to illustrate the effectiveness of the theorem, and some spatial structure figures are displayed to reflect the evolutionary behavior of the interaction solutions with the change of soliton number N and time t.

Keywords

BLMP Equation, Lump Type Solution, Interaction Behavior, Parameter Limit Method, Hirota's Bilinear Method

1. Introduction

Many nonlinear phenomena in nature can be represented by nonlinear partial differential equations (NLPDE). Soliton theory, as one of the important contents in the field of NLPDE, has received extensive attention in recent years. Therefore, it is of great significance to find soliton solutions of nonlinear equations and analyze their structural properties. With the development of soliton theory, many methods of solving soliton are proposed, such as the Darboux transformation method [1] [2], Hirota bilinear method [3] [4], polynomial-expansion method [5] [6], Lie group method [7], wronskian technique [8], etc. A large of exact solutions of nonlinear equations, including soliton solutions, periodic solutions, rational solutions and interaction solutions and so on, have been studied by these methods. In recent years, the research on lump solutions and interaction behavior between solitons, such as rogue wave, the fusion and degeneration of the lump solution, especially the interaction between lump solutions and solitons [9] [10] [11], has become a hot topic because of its application value in nonlinear science.

In this paper, we consider the following (2 + 1)-dimensional extended Potential Boiti-Leon-Manna-Pempinelli (ePBLMP) equation [12],

$$u_{yt} + c_1 u_{xxxy} - 3c_1 \left(u_y u_{xx} + u_x u_{xy} \right) - c_2 u_{xy} - c_3 u_{yy} = 0, \tag{1}$$

where u is a multivariate function about x, y, t, and c_1, c_2, c_3 are arbitrary real parameters. In fact, when $c_1 = 1$, $c_2 = c_3 = 0$, Equation (1) will be simplified as the following standard form of the pBLMP equation.

$$u_{yt} + u_{xxxy} - 3u_y u_{xx} - 3u_x u_{xy} = 0. \tag{2}$$

The (2 + 1)-dimensional BLMP equation [13] was firstly proposed by Boiti *et al.* and based on the relationship of weak Lax pairs. Equation (2) is described the interaction between the long wave propagating along the *x*-axis and the Riemann wave propagating along the *y*-axis. Some exact solutions of the BLMP equation were obtained in works [14] [15] [16] [17]. Recently, some research results have been made on eBLMP, such as Ren studying the D'Alembert wave and soliton molecules of the eBLMP equation [12]. Paliathanasis obtained some new periodic solutions of the eBLMP equation [18]. In addition, there are many methods to study similar equations. For example, Luo obtained the parametric form of soliton solutions according to the solution of the constructed Riemann-Hilbert problem [19], and Zeng obtained a class of soliton solutions of the (2 + 1)-dimensional time fractional equation by using the new mapping method [20].

In this paper, we first give detailed proof of the integrability of Equation (1). Moreover, as far as we know, the breather solutions, lump solutions and some mixed solutions of Equation (1) have not been obtained, and we will get these new results. More importantly, we find that most of the papers only study the interaction solutions between the first-order lump solutions and N-solitons, where the number of solitons N is usually less than 3. We obtain the existence theorem of the interaction solutions between the higher-order lump solutions of Equation (1) and different types of N-soliton solutions. This paper will study these new nonlinear phenomena and dynamic behaviors.

2. Painlevé Test

In this section, we give out the standard test with Kruskal's simplification, which is constituted by three steps, the leading order analysis, the resonance determination and the resonance conditions' verification, respectively. According to the standard WTC approach, we investigated the general solution of Equation (1) of the following form:

$$\begin{cases} u(x,y,t) = \varphi^{-\alpha} \sum_{j=0}^{N} u_j(x,y,t)\varphi^j, \\ \varphi(x,y,t) = x + h(y,t). \end{cases} \tag{3}$$

Step 1: Leading item analysis.

We assume the first item of (3) is

$$u(x,y,t) = \varphi^{-\alpha} u_0. \tag{4}$$

Substituting (4) into (1), comparing the lowest derivative term and the nonlinear term, we get

$$\alpha = 1, \quad u_0(x,y,t) = -2\varphi_x. \tag{5}$$

Step 2: Determine resonance point and recurrence relation.

Substituting $\alpha = 1$ into (3), we obtain the expansion of u, that is

$$u(x,y,t) = \sum_{j=0}^{N} u_j \varphi^{j-1}. \tag{6}$$

Inserting (6) and its derivatives into (1). Comparing the power terms of φ and letting the coefficient be zero, we can get the recurrence relation about u_j,

$$\begin{aligned} & u_{j-3,t}(j-4)\varphi_y + u_{j-2}(j-3)(j-4)\varphi_y + u_{j-3}(j-4)\varphi_{yt} \\ & -c_2 u_{j-2}(j-3)(j-4)\varphi_y - c_3\left(u_{j-2}(j-3)(j-4)\varphi_y + u_{j-3}(j-4)\varphi_{yy}\right) \\ & + c_1 u_j (j-1)(j-2)(j-3)(j-4)\varphi_y \\ & -6c_1 \sum_{i=0}^{j} u_i (i-1) u_{j-i}(j-i-1)(j-i-2)\varphi_y = 0. \end{aligned} \tag{7}$$

Finishing items that contain u_j,

$$\begin{aligned} & c_1 u_j \varphi_y (j-1)(j+1)(j-4)(j-6) \\ & = -u_{j-3,t}(j-4)\varphi_y - u_{j-2}(j-3)(j-4)\varphi_y - u_{j-3}(j-4)\varphi_{yt} \\ & \quad + c_2 u_{j-2}(j-3)(j-4)\varphi_y + c_3\left(u_{j-2}(j-3)(j-4)\varphi_y + u_{j-3}(j-4)\varphi_{yy}\right) \\ & \quad + 6c_1 \sum_{i=1}^{j-1} u_i (i-1) u_{j-i}(j-i-1)(j-i-2)\varphi_y. \end{aligned} \tag{8}$$

It can be found from (8) that the resonance point occurs when

$$j = -1, 1, 4, 6. \tag{9}$$

Step 3: Verify compatibility conditions.

In (7), when $j = 0$,

$$24c_1 u_0 + 12c_1 u_0^2 = 0. \tag{10}$$

Solve (10) and get $u_0 = -2$.

When $j = 1$, we have

$$-6c_1\left(u_0 u_1 \cdot 0 + u_1 u_0 \cdot 0\right) = 0, \tag{11}$$

We get that u_1 is an arbitrary function, so $j = 1$ is the resonance point.

When $j = 2$, we get

$$u_2 = \frac{1 - c_2 - c_3}{6c_1}. \tag{12}$$

When $j = 3$, we get

$$u_3 = \frac{c_1\varphi_{yy} - \varphi_{yt}}{48c_1\varphi_y}. \tag{13}$$

When $j = 4$, we have

$$-6c_1\left(-6u_0u_4 + 6u_4u_0\right) = 0. \tag{14}$$

We can get that u_4 is an arbitrary function, so $j = 4$ is the resonance point.

Substituting $j = 6$ into the left side of (7), we get

$$\begin{aligned} &2u_{3,t}\varphi_y + 6u_4\varphi_y + 2u_3\varphi_{yt} - 6c_2u_4\varphi_y - 6c_3\left(u_4\varphi_y + 2u_3\varphi_{yy}\right) \\ &+120c_1u_6\varphi_y - 6c_1\left(40u_6\varphi_y + 6u_2u_4\varphi_y + 4u_3^2\varphi_y + 10u_0u_6\right). \end{aligned} \tag{15}$$

The items containing u_3, u_4, u_6 in (16) are combined respectively, and the sum of these items is zero. But the left side of (15) is $2u_{3,t}\varphi_y$, which is reduced to $\frac{-\varphi_{ytt}}{24c_1}$. Therefore, when $\varphi_{ytt} = 0$, the (10) is integrable.

In summary, $j = -1, 1, 4$ satisfy the resonance condition, but $j = 6$ does not satisfy the resonance condition. Although it does not pass the Painlevé test, it has many undiscovered mathematical properties. For example, we can obtain the lump solutions and N-solitons by introducing bilinear method.

3. Degradation of Breather Solution and Emergence of Lump Solution

In this section, we study the degeneration of the double breathing solutions and the emergence of lump solutions of the ePBLMP equation. According to Ref. [12], we can assume that

$$u(x, y, t) = -2\left(\ln f(x, y, t)\right)_x, \tag{16}$$

where $f = f(x, y, t)$ represents the real function to be selected. Then substituting Equation (16) into Equation (1), we can obtain the following bilinear form of Equation (1),

$$\left(D_yD_t + c_1D_x^3D_y - c_2D_xD_y - c_3D_y^2\right) f \cdot f = 0, \tag{17}$$

where D denotes the bilinear operators. In Equation (17), through the extended homoclinic test method [21], we choose a new test function, which is first applied to Equation (1).

$$f(x, y, t) = m_1\cosh(\eta_1) + m_2\cos(\eta_2) + m_3\cosh(\eta_3), \tag{18}$$

where $\eta_i = k_i\left(a_ix + b_iy + d_it + r_i\right)$ and $k_i, a_i, b_i, d_i, r_i\ (i = 1, 2, 3)$ are arbitrary parameters. By substituting Equation (18) into Equation (17), the following parameters are obtained:

$$k_1 = -\frac{a_3k_3}{a_1}, a_2 = \frac{a_3k_3^2b_3\left(m_1^2 - m_3^2\right)}{k_2^2b_2m_2^2}, b_1 = -\frac{a_1b_3}{a_3},$$

$$
\begin{aligned}
d_1 &= a_1\left(-a_3^2 c_1 k_3^2 + c_2 - \frac{b_3 c_3}{a_3} + 3\alpha\right),\\
d_2 &= \frac{a_3 b_3 k_3^2\left(m_1^2 - m_3^2\right)}{b_2 k_2^2 m_2^2}\left(-3a_3^3 k_3^2 c_1 - c_2 + \alpha\right) + b_2 c_3,\\
d_3 &= a_3(-a_3^2 k_3^2 c_1 + c_2 + 3\alpha) + b_3 c_3,
\end{aligned}
\tag{19}
$$

where $\alpha = \frac{a_3^3 b_3^2 k_3^4 c_1\left(m_1^2 - m_3^2\right)^2}{b_2^2 k_2^2 m_2^4}$. Plugging Equation (19) into Equation (16), we can obtain the double breathing solution of Equation (1), (See **Figures 1(a)-(c)**).

$$
u = -\frac{2\left(m_1 k_1 a_1 \sinh(\eta_1) - m_2 k_2 a_2 \sin(\eta_2) + m_3 k_3 a_3 \sinh(\eta_3)\right)}{m_1 \cosh(\eta_1) + m_2 \cos(\eta_2) + m_3 \cosh(\eta_3)}
\tag{20}
$$

where

$$
\begin{cases}
\eta_1 = -\dfrac{a_3 k_3}{a_1}\left(a_1 x - \dfrac{a_1 b_3}{a_3} y + a_1\left(-a_3^2 c_1 k_3^2 + c_2 - \dfrac{b_3 c_3}{a_3} + 3\alpha\right)t + r_1\right),\\
\eta_2 = k_2\left(\dfrac{a_3 k_3^2 b_3\left(m_1^2 - m_3^2\right)}{k_2^2 b_2 m_2^2} x + b_2 y + \dfrac{a_3 b_3 k_3^2\left(m_1^2 - m_3^2\right)}{b_2 k_2^2 m_2^2}\left(-3a_3^3 k_3^2 c_1 - c_2 + \alpha\right)t + b_2 c_3 t + r_2\right),\\
\eta_3 = k_3\left(a_3 x + b_3 y + a_3\left(-a_3^2 k_3^2 c_1 + c_2 + 3\alpha\right)t + b_3 c_3 t + r_3\right).
\end{cases}
\tag{21}
$$

When $m_i\,(i = 1,2,3)$ satisfies $\lim_{k_i \to 0}(m_1 + m_2 + m_3) = 0$, $k_2 = pk_3$, $m_1 = \beta\cosh(\rho k_3)$, $m_3 = \gamma\cosh(\sigma k_3)$, $m_2 = -(\beta+\gamma)\cos(nk_3)$, letting $k_3 \to 0$ in Equation (20). We will get the lump type solution of Equation (1), (See **Figure 1(d)**).

$$
u = -\frac{a_3(\gamma\theta_3 - \beta\theta_1) - (\beta+\gamma)p^2\delta\theta_2}{\beta\theta_1^2 + (\beta+\gamma)p^2\theta_2^2 + \gamma\theta_3^2 + \beta\rho^2 + (\beta+\gamma)n^2 + \gamma\sigma^2}
\tag{22}
$$

where $\delta = \lim_{k_3 \to 0} a_2 = \frac{a_3 k_3^2 b_3(\beta-\gamma)}{p^2 b_2(\beta+\gamma)}$,

$$
\begin{cases}
\theta_1 = -a_3 x + b_3 y + a_1\left(c_2 - \dfrac{b_3 c_3}{a_3}\right)t - \dfrac{a_3}{a_1} r_1,\\
\theta_2 = \delta x + b_2 y + \delta c_2 t + b_2 c_3 t + r_2,\\
\theta_3 = a_3 x + b_3 y + a_3 c_2 t + b_3 c_3 t + r_3.
\end{cases}
\tag{23}
$$

In fact, the breather solution (20) represented is double breather solitons which take on the characteristics of a periodic wave with variable η_2, and shows the characteristics of double solitary waves with variable η_1 and η_3, the breather solution with this form is also called double breather solution by some scholars [22] [23]. In **Figure 1**, we give the evolution diagram of spatial structure for the double breather solutions with the change of time *t*. However, the period $\frac{2\pi}{pk_3}$ of $\cos(\eta_2)$ tends to infinity as $k_3 \to 0$, the breather and period of solution (20) is degenerate as a lump type solution (22) when the parametric k_3 tends to 0.

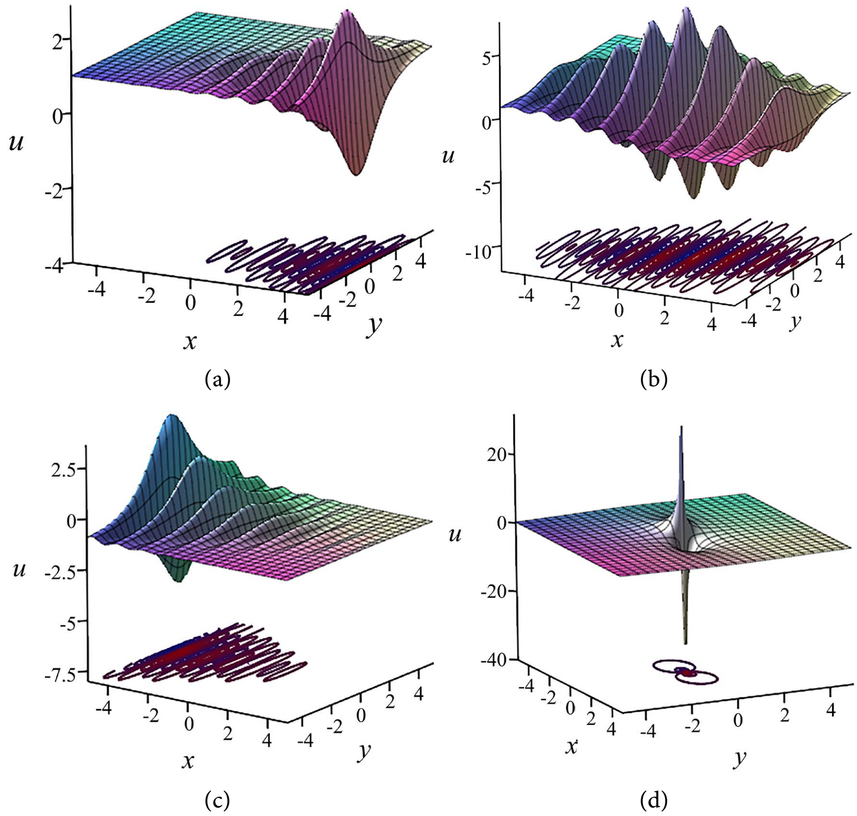

Figure 1. The evolution behavior of the breather solution Equation (20) with time: (a) $t=-0.1$; (b) $t=0$; (c) $t=0.1$, and where the parameters are set as $a_1=-\frac{9}{10}$, $a_3=2$, $b_2=-\frac{1}{2}$, $b_3=3$, $m_1=m_2=\frac{3}{2}$, $m_3=\frac{1}{2}$, $k_2=k_3=\frac{1}{4}$, $c_1=c_3=1$, $c_2=3$. (d) Spatial structures of lump type solution Equation (22): $a_1=a_3=b_3=n=p=2$, $m_2=-\frac{3}{2}$, $m_3=\frac{1}{2}$, $k_2=k_3=\frac{1}{4}$, $m_1=b_2=c_1=c_3=\beta=\rho=\gamma=\sigma=1$, $c_2=3$, $r_1=r_2=r_3=0$, $t=0$.

4. Evolutionary Behavior of Interaction

Now, we study the interaction between lump solutions and N-solitons (where N is any positive integer). Firstly, we give a theorem and prove the calculation process. Here, we choose a new test function:

$$\begin{cases} h(x,y,t)=a_0+\sum_{i=1}^{M}\left(a_ix+b_iy+d_it+m_i\right)^2, \\ g(x,y,t)=\sum_{j=1}^{N}\delta_j \mathrm{e}^{\left(p_jx+q_jy+n_jt+s_j\right)}, \\ f(x,y,t)=h(x,y,t)+g(x,y,t), \end{cases} \tag{24}$$

where $a_0,a_i,b_i,d_i,m_i\left(i=1,2,\cdots,M\right),\delta_j,p_j,n_j,s_j\left(j=1,2,\cdots,N\right)$ are some free real parameters. Obviously, $h(x,y,t)$ is a polynomial function, and $g(x,y,t)$ is an exponential function. They combined to form $f(x,y,t)$. By substituting (24) into (16), we will get a new interaction solution of ePBLMP.

Theorem 1. Let $f(x,y,t)$ satisfies $\sum_{i=1}^{N} a_i b_i = 0$, $d_i = c_2 a_i + c_3 b_i$, $\sum_{i=1}^{N} b_i d_i - c_3 \sum_{i=1}^{N} b_i^2 = 0$, $q_j = 0$, $n_j = c_2 a_j + c_1 p_j^3$, then $f(x,y,t)$ is the solution of Equation (1).

Proof. By direct calculation

$$\begin{aligned} &P(D_x, D_y D_t) f \cdot f \\ &= \left(D_t + c_1 D_x^3 - c_2 D_x - c_3 D_y\right) D_y \left(h \cdot h + h \cdot g + g \cdot h + g \cdot g\right). \end{aligned} \tag{25}$$

We first consider

$$\begin{aligned} &\left(D_y D_t + c_1 D_x^3 D_y - c_2 D_x D_y - c_3 D_y^2\right) h \cdot h \\ &= 2h_{yt} h - h_y h_t + 2c_1 \left(h_{yxxx} h - 3h_{yxx} h_x + 3h_{yx} h_{xx} - h_y h_{xxx}\right) \\ &\quad - 2c_2 \left(h_{yx} h - h_y h_x\right) - 2c_3 \left(h_{yy} h - h_y h_y\right) \\ &= 6c_1 h_{yx} h_{xx} + 2\left(h_{yt} - c_2 h_{yx} - c_3 h_{yy}\right) h + 2\left(-h_t + c_2 h_x + c_3 h_y\right) h_y. \end{aligned} \tag{26}$$

From $h_{yx} = 0$, $h_{yt} - c_2 h_{yx} - c_3 h_{yy} = 0$, $-h_t + c_2 h_x + c_3 h_y = 0$, and relational expression

$$\sum_{i=1}^{N} a_i b_i = 0, \sum_{i=1}^{N} b_i d_i - c_3 \sum_{i=1}^{N} b_i^2 = 0, d_i = c_2 a_i + c_3 b_i. \tag{27}$$

Obviously $g_y = 0$, so $P(D_x, D_y D_t)(g \cdot g) = 0$. So that

$$\begin{aligned} &2\left(D_t + c_1 D_x^3 - c_2 D_x - c_3 D_y\right) D_y (h \cdot g) \\ &= 2\left(D_t + c_1 D_x^3 - c_2 D_x - c_3 D_y\right) D_y \left(h_y g - h g_y\right) \\ &= 2\left(h_{yt} g - h_y g_t\right) + 2c_1 \left(h_{yxx} g - 3h_{yxx} g_x + 3h_{yx} g_{xx} - h_y g_{xxx}\right) \\ &\quad - 2c_2 \left(h_{yx} g - h_y g_x\right) - 2c_3 \left(h_{yy} g - h_y g_y\right) \\ &= 6c_1 h_{yx} g_{xx} + 2\left(h_{yt} - c_2 h_{yx} - c_3 h_{yy}\right) g - 2\left(g_t + c_1 g_{xxx} - c_2 g_x\right) h_y. \end{aligned} \tag{28}$$

From $g_t + c_1 g_{xxx} - c_2 g_x = 0$, we get

$\sum_{j=1}^{N} \delta_j \left(c_2 p_j - c_1 p_j^3\right) \mathrm{e}^{l_j} + c_1 \sum_{j=1}^{N} \delta_j p_j^3 \mathrm{e}^{l_j} - c_2 \sum_{j=1}^{N} \delta_j p_j \mathrm{e}^{l_j} = 0$, then

$$n_j = \left(c_2 p_j - c_1 p_j^3\right), \tag{29}$$

where $l_j = p_j x + \left(c_2 p_j - c_1 p_j^3\right) t + s_j$. Substituting (27), (29) into (25) completes the proof of the theorem.

Corollary 1. Take

$$\begin{aligned} f(x,y,t) = a_0 &+ \sum_{i=1}^{M} \left(a_i x + b_i y + d_i t + m_i\right)^2 \\ &+ \sum_{j=1}^{N} \delta_j \cosh\left(p_j x + \left(c_2 p_j - c_1 p_j^3\right) t + s_j\right) \\ &+ \sum_{j=1}^{N} \beta_l \cos\left(p_l x + \left(c_2 p_l - c_1 p_l^3\right) t + s_l\right), \end{aligned} \tag{30}$$

when (30) satisfies $\sum_{i=1}^{N} a_i b_i = 0$, $d_i = c_2 a_i + c_3 b_i$, $\sum_{i=1}^{N} b_i d_i - c_3 \sum_{i=1}^{N} b_i^2 = 0$, then $h(x,y,t)$ is the solution of (17), and $f(x,y,t)$ is also the solution of (17).

Secondly, we will apply the results of Theorem 1 to the (1). When $M = 2, N = 0$ in (24), two sets of relations can be obtained by direct calculation.

$$a_0 = a_0, a_1 = a_1, a_2 = a_2, b_1 = 0, b_2 = 0, d_1 = d_1, d_2 = d_2, m_1 = m_1, m_2 = m_2, \quad (31)$$

and

$$\begin{aligned} &a_0 = a_0, a_1 = a_1, a_2 = -\frac{a_1 b_1}{b_2}, b_1 = b_1, b_2 = b_2, d_1 = a_1 c_2 + b_1 c_3, \\ &d_2 = -\frac{a_1 b_1 c_2}{b_2} + b_2 c_3, m_1 = m_1, m_2 = m_2. \end{aligned} \quad (32)$$

According to (16), (24) and (32), we obtain the interaction solution between lump type solution and N-soliton solution (See Figure 2 and Figure 3).

$$u = -\frac{-4a_1^2\left(1+\frac{b_1^2}{b_2^2}\right)\left(x+c_2 t\right)+\sum_{j=1}^{N}\delta_j p_j \mathrm{e}^{\left(p_j x+q_j y+n_j t+s_j\right)}}{a_0+\left(1+\frac{b_1^2}{b_2^2}\right)\left(a_1^2\left(x+c_2 t\right)^2+\left(x+c_3 y\right)^2+g\right.}. \quad (33)$$

Figure 2 shows the evolution of the spatial structure of the interaction solution with the increase in the number of solitons. Here, we select parameters in Figure 2, (a): $a_0 = 10$, $a_1 = -2$, $b_1 = 2$, $b_2 = \frac{1}{2}$, $c_1 = 1$, $c_2 = -2$, $c_3 = 4$, $m_1 = m_2 = 0$. (b), (c), (d): $a_0 = 4$, $a_1 = -2$, $b_1 = 2$, $b_2 = \frac{1}{2}$, $c_1 = 1$, $c_2 = -2$, $c_3 = 4$, $p_1 = \frac{1}{4}$,

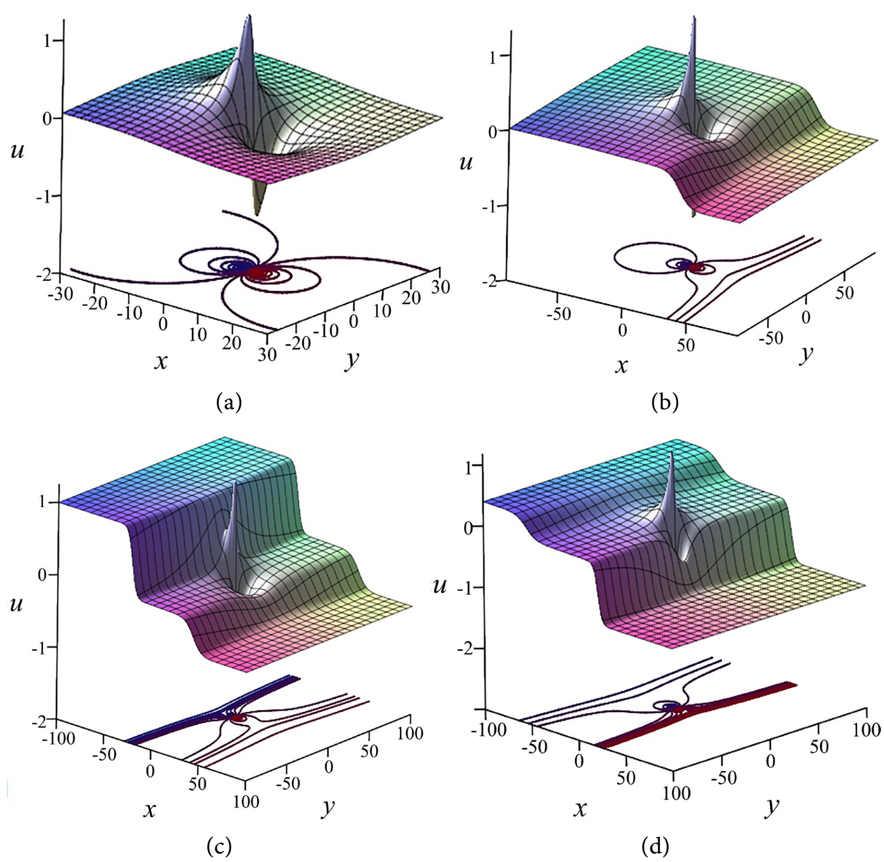

Figure 2. Spatial evolution behavior of interaction solution (33) with soliton number: (a) $N = 0$; (b) $N = 1$; (c) $N = 2$; (d) $N = 3$.

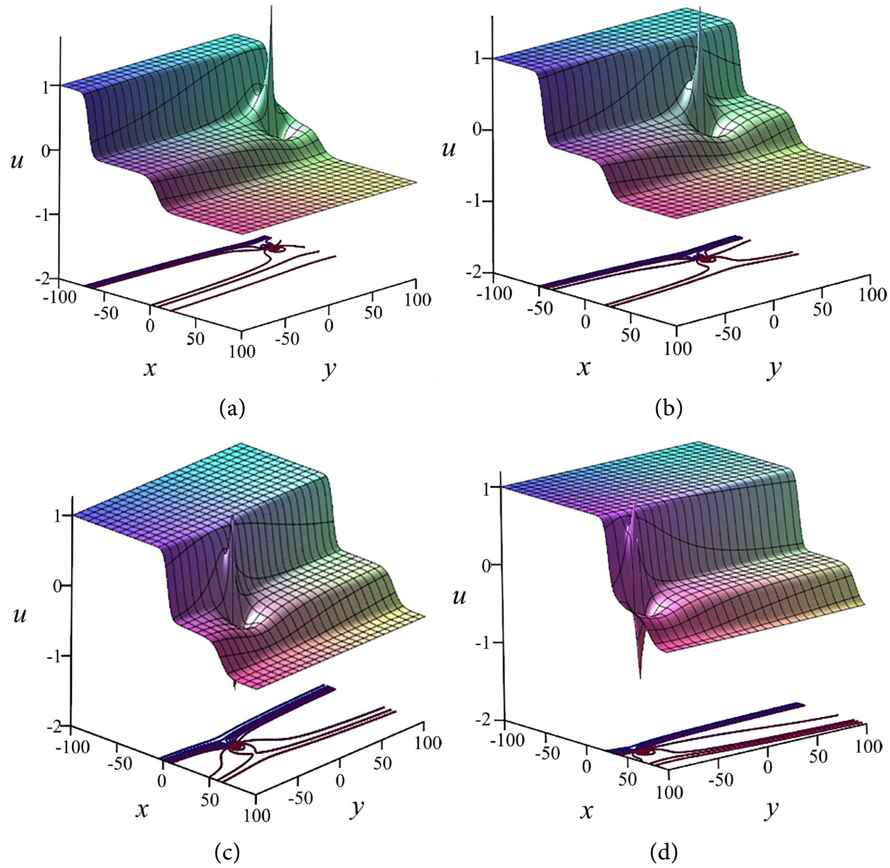

Figure 3. When $N=2$, the evolution behavior of the interaction solution (24) with the change of time *t*. (a) $t=-20$ (b) $t=-10$ (c) $t=10$ (d) $t=20$.

$p_2=\frac{1}{2}$, $p_3=-\frac{1}{5}$, $\delta_1=1$, $m_1=m_2=s_1=s_2=s_3=t=0$. Besides, we also give the evolution behavior of the interaction solution with time when $N=2$, from Figure 3, we find that with the development of time, the lump type solution is swallowed up by 2-solitons.

5. Conclusion

In summary, according to the bilinear form of ePBLMP (1), we constructed different test functions and obtained the double breather solution of the ePBLMP equation. The lump type solution is obtained from the breather solution by using the parameter limit method. Meanwhile, we also studied the interaction between lump type solutions and N-solitons. The spatial evolution behavior is demonstrated by images. More importantly, we give the theoretical calculation process of the interaction solution between lump type solution and N-solitons, and the correctness of the theory is verified by a concrete example. We believe that there are many nonlinear partial differential systems, although not integrable, also have similar dynamic behavior, such as high-dimensional Sine-Gordon equation, Ablowitz-Kaup-Newell-Segur systems, etc. we will focus on this in the future. At the same time, we also hope that the dynamic behavior obtained in this paper can provide new perspectives and useful information for the field of mathematical physics.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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