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Rotating Lepton Model of Pions and Kaons: Mechanics at fm Distances

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Abstract

The present article is a continuation of a recently published paper [1] in which we have modeled the composition and structure of neutrons and other hadrons using the Rotating Lepton Model (RLM) which is a Bohr type model employing the relativistic gravitational attraction between three ultrafast rotating neutrinos as the centripetal force. The RLM accounts for special relativity and also for the De Broglie equation of quantum mechanics. In this way this force was shown to reach the value of the Strong Force while the values of the masses of the rotating relativistic neutrinos reach those of quarks. Masses computed for twelve hadrons and bosons are in very close (~2%) agreement with the experimental values. Here we use the same RLM approach to describe the composition and structure and to compute the masses of Pions and Kaons which are important zero spin mesons. Contrary to hadrons and bosons which have been found via the RLM to comprise the heaviest neutrino eigenmass m_3, in the case of mesons the intermediate neutrino mass eigenstate m_2 is found to play the dominant role. This can explain why the lowest masses of mesons are generally smaller than those of hadrons and bosons. Thus in the case of Pions it is found that they comprise three rotating m_2 mass eigenstate neutrinos and the computed mass of 136.6 MeV/c^2 is in good agreement with the experimental value of 134.977 MeV/c^2. The Kaon structure is found to consist of six m_2 mass eigenstate neutrinos arranged in two parallel pion-type rotating triads. The computed Kaon mass differs less that 2% from the experimental $K^\pm$ and K° values of 493.677 MeV/c^2 and 497.648 MeV/c^2 respectively. This, in conjunction with the experimentally observed decay products of the Kaons, provides strong support for the proposed K structure.

Keywords

Pions and Kaons-Structure and Masses, Gravitational Bohr-de Broglie-Newton-Einstein Type Models, Rotating Lepton Model (RLM), Hadronization, Neutrino Masses, Special Relativity, Gravitational Force, Quantum Mechanics

1. Introduction

The Standard Model (SM) of particle physics has long provided a basis for understanding the fundamental structure of all observable matter in our Universe. Among the indivisable particles it describes are quarks and leptons, which include electrons, positrons and neutrinos. The SM also describes four forces: gravity which plays a very limited role, electromagnetism which regulates interactions between charged particles via photons, the Strong Force which acts between quarks via gluons and the Weak Force which involves exchanges of W and Z boson and plays an important role in radioactive decay. So far the SM has provided an excellent basis for researchers to explain their experimental results [2] [3]. In recent years, however, there is increasing evidence that the SM in its current form is not complete. In fact, the SM does not seem to be compatible with special or general relativity and, in addition, it assumes that neutrinos are massless, despite of subsequently established experimental evidence that neutrinos possess mass and play a paramount dominant role in our Universe [4] [5] [6] [7] [8]. This lack of completeness has led to developing the rotating lepton model (RLM) which is a Bohr-type rotating lepton model [9], combining gravity, special [10] [11] or general relativity [12] and quantum mechanics [13] in a simple manner [1] [14]-[20].

Following the general observation [2] [3] that all composite particles eventually decay to only up to five lepton types (electrons e^-, positrons e^+ and the three neutrinos) the RLM considers these particles as the only truly undivisable, thus fundamental, elementary particles which can synthesize all composite particles [9] [20] [12]-[17]. The RLM also utilizes the three neutrino eigenmass values to show that, due to their very small rest masses m_o, neutrino masses can reach the Planck mass values at modest (up to 313 MeV) energies.

The latter implies that gravitational forces between ultrarelativistic neutrinos, at a distance *d*, can easily reach the value, $\hbar c/d^2$, of the Strong Nuclear Force which is the strongest force for creating composite particles. Here $\hbar$ is the Planck constant and *c* is the speed of light.

Consequently the RLM utilizes only two forces, *i.e.* gravity and electromagnetism and shows that matter is created via the rotation of neutrino triads in circular orbits with rotational speeds near the speed of light c, and corresponding Lorentz factor $\gamma\left(=\left(1-v^2/c^2\right)^{-1/2}\right)$ values up to 10^{10}. In this way it turns out that the new mass created is $(\gamma-1)m_oc^2$. As an example, as shown in the next section, three rotating neutrinos, of rest mass 0.0437 eV/c^2 each, form a rotating triad with a Lorentz factor γ equal to 7.163 × 10^9. The mass of the composite particle formed is $3\gamma m_o$, *i.e.* 939.565 MeV/c^2 which is the rest mass of a neutron. The importance, simplicity and effectiveness of the RLM has been analyzed and discussed recently in Research Features [20].

2. Rotating Lepton Model of the Neutron, Muon and Pion

2.1. Neutron

Within the RLM approach the neutron is modeled as a rotating relativistic neutrino triad of the heaviest neutrino mass eigenstate m_3 (**Figure 1** and **Figure 2**). According to Special Relativity the relativistic mass, m_r, of a neutrino is given by γm_o and its inertial longitudinal mass is given by

$$m_i = \gamma^3 m_o \tag{1}$$

where m_o is the rest mass of the neutrino and γ is the Lorentz factor

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2} \tag{2}$$

In instantaneous reference frames, the above Equation (1), derived initially for linear motions [10] [11], remains valid for arbitrary motion [11] [14]. Therefore, using the equivalence principle, the gravitational mass, m_g, of all particles equals their inertial mass, m_i, thus

$$m_g = \gamma^3 m_o \tag{3}$$

Using the definition of the gravitational mass, m_g, which is the mass value entering Newton's universal gravitational law, *i.e.*

$$m_g^2 = \frac{Fd^2}{G} \tag{4}$$

We obtain the following expression for the gravitational force F

$$F = \frac{G m_o^2 \gamma^6}{d^2} \tag{5}$$

where G (=6.673 × 10^{-11} m^3·kg^{-1}·s^{-2}) denotes the usual gravitational constant. For circular motion of three m_3 mass neutrinos rotating along a circle of radius r, it follows that F is given by

$$F = \frac{G m_3^2 \gamma^6}{3^{1/2} r^2} \tag{6}$$

and therefore the equation of motion of each rotating particle is

$$\gamma m_3 v^2 / r = \frac{G m_3^2 \gamma^6}{3^{1/2} r^2} \tag{7}$$

which in turn, yields

$$r = \frac{G m_3}{3^{1/2} c^2} \gamma^5 \left(\frac{\gamma^2}{\gamma^2 - 1}\right) \tag{8}$$

Solving Equation (8) coupled with the de Broglie quantum mechanics equation, *i.e.* with

$$\gamma m_o v r = \hbar \tag{9}$$

accounting for the number (three) of quarks in a neutron [2] and denoting by m_n the neutron mass, it follows from energy conservation (*i.e.* from $m_n c^2 = 3\gamma m_o c^2$) that $m_n = 3\gamma m_o$ and thus

$$r = 3\hbar/m_n c; \quad m_n = 3^{13/12}\left(m_{Pl} m_o^2\right)^{1/3}; \quad m_o = m_3 = \frac{\left(m_n/3\right)^{3/2}}{3^{1/8} m_{Pl}^{1/2}} \tag{10}$$

Substituting for the neutron mass $m_n = 939.565\ \mathrm{MeV/c^2}$ and for the Planck mass $m_{Pl} = 1.221\times10^{28}\ \mathrm{eV/c^2}$, one obtains $m_o = 0.0437\ \mathrm{eV/c^2}$ and thus, $\gamma\left(= m_n/3m_o\right) = 7.163\times10^9$. The so computed relativistic mass γm_o value is of the order of quark masses (313 MeV/c^2) and the corresponding rest mass m_o value (0.04372 eV/c^2) is in surprisingly good agreement with the heaviest neutrino mass m_3, as shown in **Figure 1**. Since this mass differs less than 2% from the mass, m_3, of the neutrino produced in the W$^\pm$ decay [18] it follows that the W$^\pm$ boson comprises, similarly to neutrons and protons, the heaviest mass neutrinos ν_3 (**Figure 1**).

2.2. Muons

The algebraic expressions for the masses m_1, m_2 and m_3, also shown and compared in **Figure 1** with the Superkamiokande measurements, are obtained by modeling the muon structure ($\mu^\pm$) which is known to comprise an e$^\pm$, a $\bar{\nu}_e$ and a ν_μ [2] [3]. Thus one considers two gravitating neutrinos, ν_1 and ν_3, of masses m_1 and m_3, respectively and, similarly to Equation (7), we have

$$\gamma_1 m_1 v_1^2/r = \frac{G m_1 m_3 \gamma_1^3 \gamma_3^3}{\sqrt{3} r^2} \tag{11}$$

$$\gamma_3 m_3 v_3^2/r = \frac{G m_1 m_3 \gamma_1^3 \gamma_3^3}{\sqrt{3} r^2} \tag{12}$$

Upon multiplying by parts, taking the square root, considering the limit $v_1, v_3 \to c$

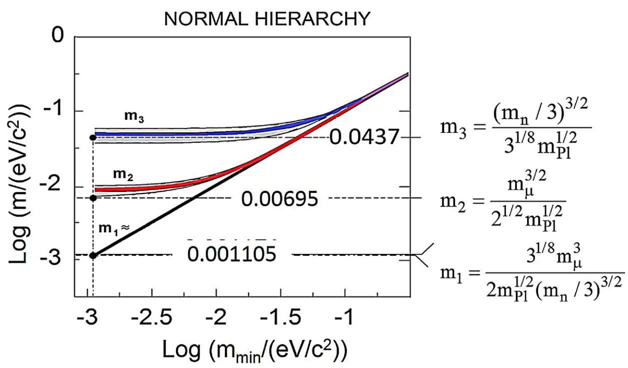

Figure 1. Neutrino mass eigenstates. Comparison of the Superkamiokande neutrino mass measurements with the neutrino mass eigenstate values computed via the corresponding RLM mass expressions shown to the right, (m_n and m_μ are the neutron and muon masses respectively).

and defining $\gamma_{13} = (\gamma_1\gamma_3)^{1/2}$ and $m_{13} = (m_1 m_3)^{1/2}$ one obtains

$$\gamma_{13} m_{13} c^2 / r = G m_{13}^2 \gamma_{13}^6 / 4r^2 \tag{13}$$

Furthermore, by multiplying by parts the two de Broglie wavelength expressions as in Equation (9), we obtain

$$\gamma_{13} m_{13} c r = \hbar \tag{14}$$

From Equations (13) and (14) it follows

$$\gamma_{13}^6 = \frac{4\hbar c}{G m_{13}^2}; \quad \gamma_{13} = 2^{1/3}\left(\frac{m_{Pl}}{m_{13}}\right)^{1/3} \tag{15}$$

where $m_{Pl} = (\hbar c/G)^{1/2}$. Consequently, the muon mass is computed from

$$m_\mu = \gamma_{13} m_{13} = 2^{1/3}\left(m_{Pl} m_{13}^2\right)^{1/3} = \left(2 m_{Pl} m_1 m_3\right)^{1/3} \tag{16}$$

and, therefore, using the experimental muon mass, $m_\mu = 105.66\ \mathrm{MeV/c^2}$ and the m_3 mass eigenstate value of 0.0437 eV/c^2 [14] [15] [17] [18], we obtain

$$m_1 = \frac{m_\mu^3}{2 m_{Pl} m_3} = 0.001105\ \mathrm{eV/c^2} \tag{17}$$

Interestingly the same result for the muon mass obtained from Equation (16) can also be reached by considering two rotating neutrinos, each with rest mass m_2. In this case, similarly to Equation (7), we have

$$\gamma_2 m_2 v^2 / r = \frac{G m_2^2 \gamma_2^6}{4r^2}; \quad \gamma_2 m_2 v r = \hbar \tag{18}$$

resulting to

$$4\hbar c = G m_2^2 \gamma_2^6; \quad \gamma_2 = 2^{1/3}\left(\frac{\hbar c}{G m_2^2}\right)^{1/6} \tag{19}$$

and, therefore, to

$$m_\mu = \gamma_2 m_2 = \left(2 m_{Pl} m_2^2\right)^{1/3} = 105.66\ \mathrm{MeV/c^2} \tag{20}$$

Then, from Equations (16) and (20) it follows that

$$m_2 = (m_1 m_3)^{1/2} = 0.00695\ \mathrm{eV/c^2} \tag{21}$$

The above equation suggests that two neutrinos, of masses m_1 and m_3 each, can hybridize to form two neutrinos with equal masses $m_2 = (m_3 m_1)^{1/2}$. The occurence of neutrino hybridization can be attributed to the need of synchronization when two neutrinos of different initial masses are caught on the same circular orbit in the process of forming a bound rotational state [14] [17] [18]. The phenomenon of hybridization is quite common in chemistry [21]. The phenomenon of neutrino hybridization may be related to the very important phenomenon of neutrino oscillations [4] [5] [6] [7] [8] [22].

2.3. Pions

Interestingly, the same m_1 mass value obtain in Equation (17) can be computed by modeling the structure of the pion, which is a meson, comprising (**Figure 2**) three rotating neutrinos [2] [3] which, as shown by the RLM, form a rotating triad. The analytical computation of the pion mass is shown below.

We consider three rotating neutrinos on a circle of radius r, two of which have the mass m_1 and the third with a mass m_3 as shown in **Figure 2(c)** and **Figure 3**.

The gravitational forces between the three rotating particles are shown in **Figure 3**, *i.e.*

$$F_{11} = \frac{Gm_1^2\gamma_1^6}{4r^2\sin^2\varphi}; \quad F_{13} = F_{23} = \frac{Gm_1m_3\gamma_1^3\gamma_3^3}{r^2\sin^2\varphi}\sin^2(\varphi/2) \tag{22}$$

and thus

$$\frac{F_{11}}{F_{13}} = \frac{m_1\gamma_1^3}{4m_3\gamma_3^3\sin^2(\varphi/2)} \tag{23}$$

From **Figure 3** it also follows

$$\alpha = 2r\sin\varphi \tag{24}$$

$$\frac{\alpha}{\sin\varphi} = \frac{\beta}{\sin(90-\varphi/2)} \tag{25}$$

Thus,

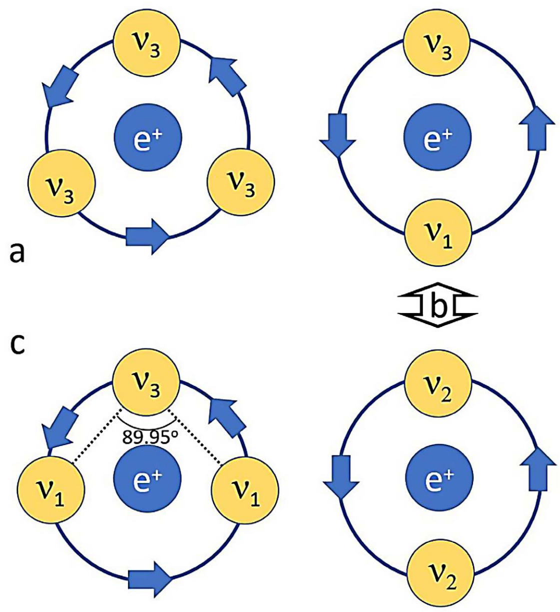

Figure 2. Rotating neutrino model geometry for a proton (a) [14], for a muon μ^+ (b) before (top) and after (bottom) hybridization and for a pion π^+ (c) [16] [17]. The central positron is at rest with respect to the observer ($\gamma = 1$) and, thus, it adds little (0.511 MeV/c^2) to the total mass of the composite state. Similarly to hybridization in Chemistry [21], the rotating ν_1 and ν_3 neutrinos, with masses m_1 and m_3 respectively, become hybridized in the μ^+ and π^+ structures [16] [17] due to rotational synchronization. In this way pairs of m_1 and m_3 mass eigenstate neutrinos produce two m_2 mass eigenstate neutrinos [16] [17] [18], a phenomenon potentially related to neutrino oscillations [4] [5] [6].

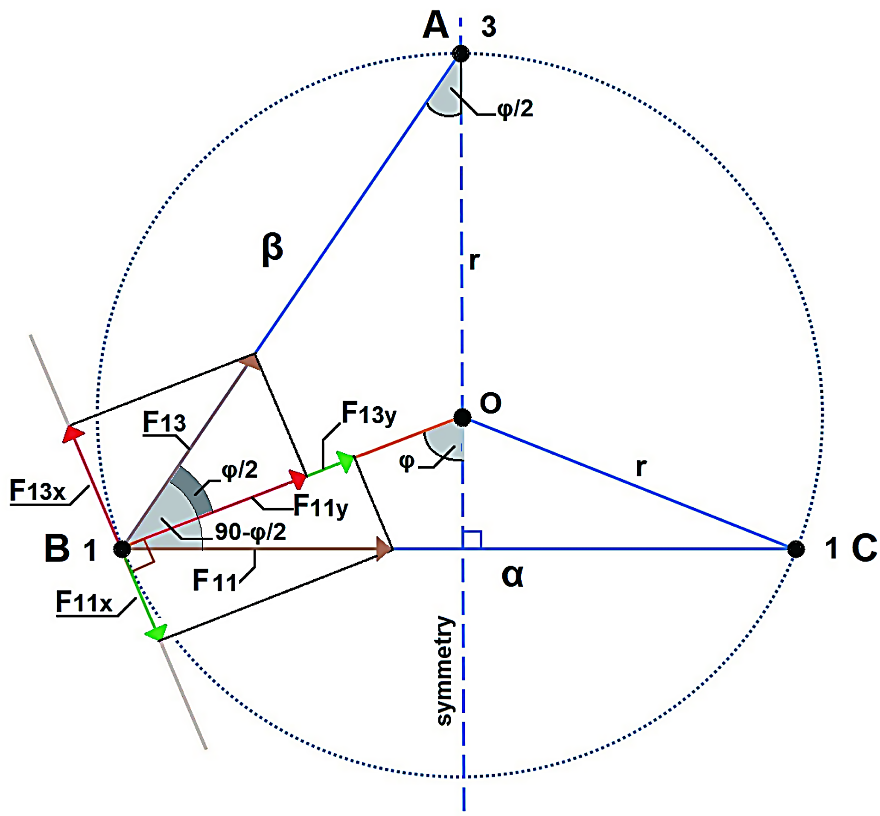

Figure 3. The isosceles triangle geometry of the pion structure.

$$\beta = \alpha \frac{\cos(\varphi/2)}{\sin\varphi} = \frac{\alpha}{2\sin(\varphi/2)} \tag{26}$$

and from Equation (24)

$$\beta = r\frac{\sin\varphi}{\sin(\varphi/2)} \tag{27}$$

Then, the tangential force at B in obtained from

$$F_{11} \cdot \sin(90-\varphi) = F_{13}\sin(\varphi/2) \tag{28}$$

or

$$F_{11} \cdot \cos\varphi = F_{13}\sin(\varphi/2) \tag{29}$$

and, therefore,

$$\frac{F_{11}}{F_{13}} = \frac{\sin(\varphi/2)}{\cos\varphi} = \frac{m_1\gamma_1^3}{4m_3\gamma_3^3\sin^2(\varphi/2)} \tag{30}$$

giving

$$\frac{4\sin^3(\varphi/2)}{\cos\varphi} = \frac{m_1\gamma_1^3}{m_3\gamma_3^3} \tag{31}$$

In view of the de Broglie condition

$$\gamma_1 m_1 cr = \gamma_3 m_3 cr = \hbar \tag{32}$$

we have

$$\gamma_1 m_1 = \gamma_3 m_3 \tag{33}$$

and since

$$m_3/m_1 = \frac{0.0437}{1.174\times10^{-3}} = 37.22 \tag{34}$$

it follows that

$$4\frac{\sin^3(\varphi/2)}{\cos\varphi} = 37.22 \tag{35}$$

which, by trial and error, gives

$$\varphi = 89.95^\circ \tag{36}$$

Considering the equations of motion of the three particles and assuming $v_1 \approx c$, $v_3 \approx c$, it follows that

$$\hbar c = \gamma_3 m_3 c^2 r = \frac{2Gm_1 m_3 \gamma_1^3 \gamma_3^3}{4\cos(\varphi/2)} \tag{37}$$

and also

$$(\hbar c)^2 = (\gamma_1 m_1 c^2 r)^2 = \left[\frac{Gm_1^2\gamma_1^6}{4\sin\varphi} + \frac{Gm_1 m_3\gamma_1^3\gamma_3^3}{4\cos(\varphi/2)}\right]^2 \tag{38}$$

After multiplying by parts, accounting for the fact that

$$\gamma_1 m_1 = \gamma_3 m_3 = \frac{\hbar}{rc} \tag{39}$$

and defining

$$x = \frac{m_1}{m_3}; \quad y = \frac{\gamma_1}{\gamma_3} \tag{40}$$

we obtain

$$\begin{aligned}\gamma_1^2\gamma_3^3 m_1^2 m_3 r^3 c^6 &= \frac{(\hbar c)^3}{G^3 m_3^6 \gamma_3^{18}} = \left(\frac{m_{Pl}}{m_3}\right)^6 \gamma_3^{-18} \\ &= \frac{2}{4^3}\left[\frac{x^5 y^{15}}{\sin^2\varphi\cos(\varphi/2)} + \frac{2x^4 y^{12}}{\cos^2(\varphi/2)\sin\varphi} + \frac{x^3 y^9}{\cos^3(\varphi/2)}\right]\end{aligned} \tag{41}$$

Accounting for the condition $xy = 1$, it follows from Equation (41) that

$$\left[\frac{\hbar c}{Gm_3^2\gamma_3^6}\right]^3 = \frac{1}{32}\left[\frac{y^{10}}{\sin^2\varphi\cos(\varphi/2)} + \frac{2y^8}{\cos^2(\varphi/2)\sin\varphi} + \frac{y^6}{\cos^3(\varphi/2)}\right] \tag{42}$$

and after substitution for $\varphi = 89.95^\circ$, we obtain

$$\left[\frac{\hbar c}{Gm_3^2\gamma_3^6}\right]^3 = 2.37\times10^{14}; \quad \frac{\hbar c}{Gm_3^2\gamma_3^6} = 6.19\times10^4 \tag{43}$$

Thus, it turns out that

$$\gamma_3 = \left[\left(\frac{m_{Pl}}{m_3}\right)^2 \frac{1}{6.19\times 10^4}\right]^{1/6} = 0.159\left(\frac{m_{Pl}}{m_3}\right)^{1/3} \tag{44}$$

and, therefore,

$$m_\pi = 3\gamma_3 m_3 = 3\times 0.159\left(m_{Pl}m_3^2\right)^{1/3} = 0.159 m_n \tag{45}$$

Consequently, it was computed that

$$m_\pi = 136.6\ \text{MeV}/\text{c}^2 \tag{46}$$

in good agreement with $m_{\pi^\circ} = 134.977\ \text{MeV}/\text{c}^2$ which is the experimental value [2] [3] and with the mass value of 137.82 MeV/c^2 computed via the use of equilateral triangular geometry and hybridization between m_1 and m_3 neutrinos leading to m_2 neutrinos [16] [17].

3. Kaon Decay Products

In this section we use the RLM to investigate the structure and mass of the Kaons.

Kaon is a meson, that comes in two types: The charged Kaon (K^+ or K^-), which has a mass of 493.677 MeV/c^2, and the neutral Kaon (K^o), which has a mass of 497.648 MeV/c^2.

Out of these two types, only the charged one is known to decay into a charged pion and a neutral pion. The charge of the pion depends on the charge of the Kaon that decays.

Therefore, for the charged Kaon structure, one can consider six of these hybridized neutrinos (two trios, one staggered onto the other), all rotating around a common axis, as shown in **Figure 4** and **Figure 5**. Moreover, for the charged

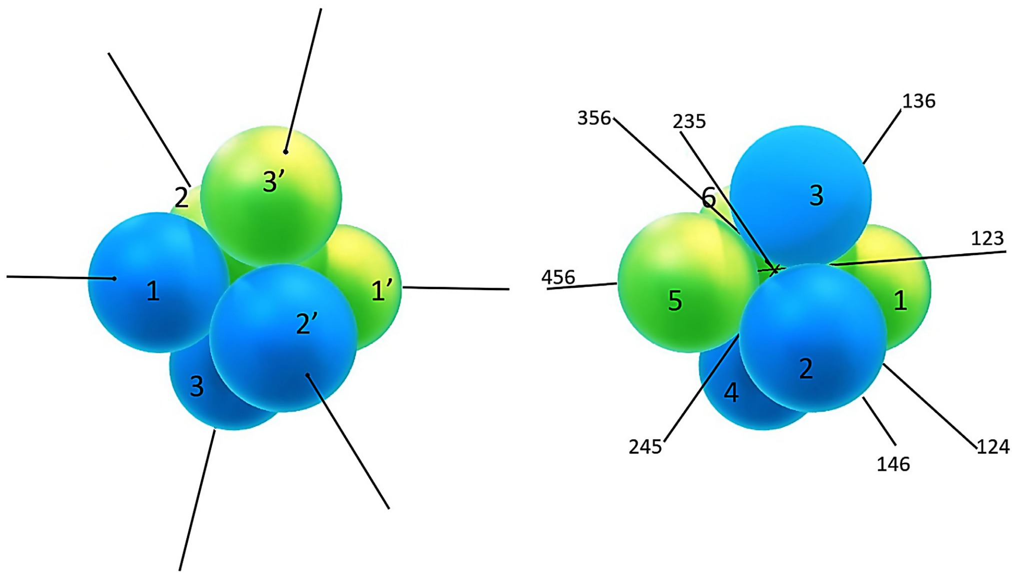

Figure 4. Normal triangular octahedron structure of the Kaon showing (left) the three C4 rotational axes labeled 11', 22' and 33' and (right) the four C3 rotational axes labeled (123)-(456), (124)-(356), (136)-(245) and (146)-(235). Thus there are seven rotational axes. Particle radius r is determined by the particle de Broglie wavelength $\hbar/\gamma_2 m_2 c = 2.38$ fm.

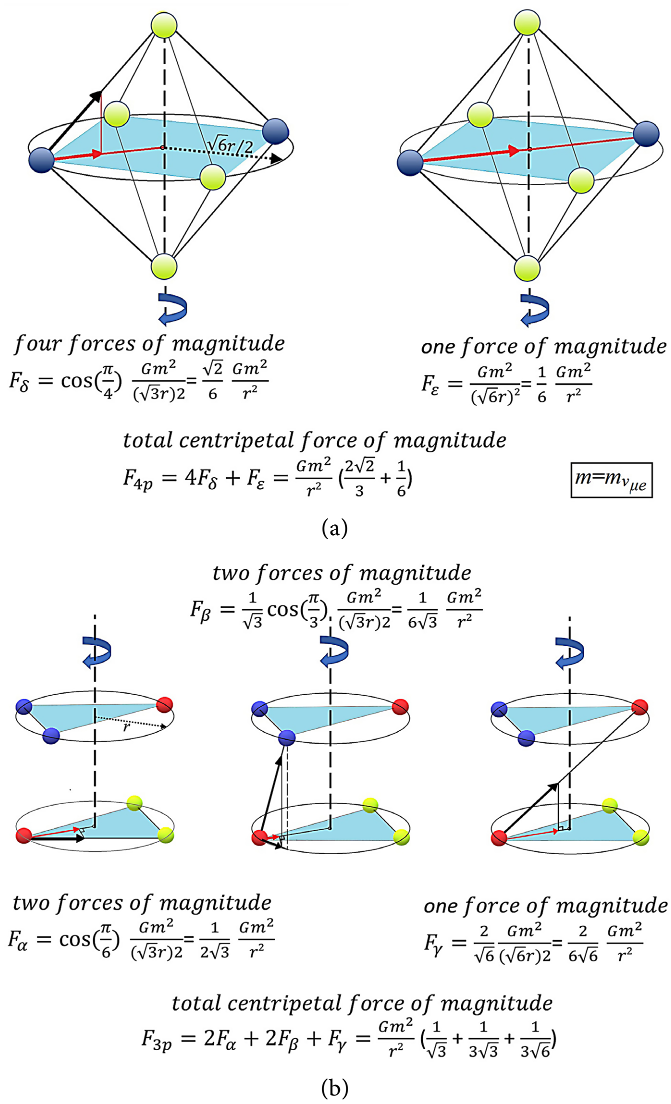

Figure 5. Computation of F_4 (top), and of F_3 (bottom); m denotes m_2.

Kaon (K^+ or K^-) there is a small charged particle (e^+ or e^-) at the center of this configuration, not shown in **Figure 4**, which dictates the Kaon charge. This $e^\pm$ particle does not rotate and, thus, it contributes little by the electron or positron rest mass of (~0.511 MeV/c^2) to the Kaon mass.

The decay products of the K^o and $\bar{K}^o$ (mass 497.65 MeV/c^2) are $\pi\pi$ ($K_o^o = 8.95 \times 10^{-11}$ s) and $\pi e \nu_e$, $\pi\mu\nu_\mu$ and $\pi\pi\pi$ ($K_i^o = 5.11 \times 10^{-8}$ s) [2] [3]. These decay products, *i.e.* the muon and the pion, have recently been shown [16] [17] to contain exclusively the above intermediate mass neutrinos (*i.e.* with mass m_2 (≈6.95 × 10^{-3} eV/c^2) in the normal hierarchy) [14] [15] [16] [17].

From the principal hadronic decay ($\pi^+\pi^o$) of the K^+, since the pion comprises three hybridized neutrinos [16], one may conclude that the Kaons comprise six $\nu_{\mu e}$ hybridized neutrinos. This is also confirmed by the leptonic Kaon decay [2] [3]. According to the RLM [17], it is possible to make an estimate of the order of magnitude of the K mass via the expression

$$m_K \approx 6\left(m_{Pl} m_2^2\right)^{1/3} \tag{47}$$

Using $m_{Pl} = 1.221 \times 10^{28}\ \mathrm{eV/c^2}$ and Equation (21) one obtains $m_K \approx 503.3\ \mathrm{MeV/c^2}$ and this value without any detailed modeling differs already, only approximately 1% and 1.5% from the experimental values [2] [3] of 497.65 MeV/c^2 and 493.57 MeV/c^2, respectively, of the neutral Kaon K^o and of the charged Kaon K^+.

4. RLM Model for the Kaon Structure and Mass Computation

The simplest geometric model for accommodating six rotating neutrinos placed on two equilateral triangles is a normal triangular octahedron (**Figure 4** and **Figure 5**). We therefore hypothesize that the K structure comprises six neutrinos of mass m_2 arranged at the six vertices of a rotating normal triangular octahedron (**Figure 4** and **Figure 5**). This structure corresponds to one of the five platonic solids and belongs to the O_h symmetry point group [21]. Therefore it comprises three C4 rotational axes (**Figure 4**, left) and four C3 axes (**Figure 4**, right).

Consequently, any of the six neutrinos can be rotating on a total of seven different axes. The energy equipartition theorem suggests that all these axial rotations contribute equally to the energy of each Kaon particle.

The total kinetic energy of the six neutrinos with rest mass m_2 each, corresponds to the rest energy, E, of the composite particle and is given by

$$E = 6\bar{\gamma} m_2 c^2 \tag{48}$$

where $\bar{\gamma}$ denotes a mean Lorentz factor value accounting for the rotation of all six neutrinos.

Due to the Kaon particle symmetry, the Lorentz factor γ of each rotating particle can be computed by considering the projection of each force on the seven planes of rotation (**Figure 5**).

Consequently, in order to find first the speeds v_4 and v_3 as well as the corresponding Lorentz factor values γ_3 and γ_4, we compute, from **Figure 5(a)**,

$$F_3 = \frac{Gm_2^2}{r^2}\left[\frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{1}{3\sqrt{6}}\right] \tag{49}$$

where F_3 is the centripetal force exerted on each of the rotating particles in the C4 mode by all the other five particles. Then, similarly as in Equation (52) below, we obtain

$$\gamma_3 m_2 \frac{c^2}{r} = \frac{Gm_2^2 \gamma_3^6}{r^2}\left[\frac{1}{3\sqrt{3}} + \frac{1}{3\sqrt{6}} + \frac{1}{\sqrt{3}}\right] \tag{50}$$

We also have from **Figure 5(b)** that

$$F_4 = \frac{Gm_2^2}{r^2}\left[\frac{2\sqrt{2}}{3} + \frac{1}{6}\right] \tag{51}$$

where F_4 is the centripetal force exerted on each of rotating particles in the 3 mode by all the other five particles.

Accounting for the fact that $F = \gamma m v^2 / r \approx \gamma m c^2 / r$ we obtain for each particle the equation

$$\gamma_4 m_2 \frac{c^2}{\frac{\sqrt{6}r}{2}} = \frac{G m_2^2 \gamma_4^6}{r^2}\left[\frac{2\sqrt{2}}{3} + \frac{1}{6}\right] \quad (52)$$

Using the De Broglie angular momenta quantization equation (*i.e.* the De Broglie equation) (see **Figure 5(a)** & **Figure 5(b)**), we obtain

$$r = \frac{\hbar}{\gamma_3 m_2 c} \quad (53)$$

$$\frac{\sqrt{6}}{2} r = \frac{\hbar}{\gamma_4 m_2 c} \quad (54)$$

and, thus, Equations (52) and (50), yield respectively

$$\frac{\hbar c}{G m_2^2} = \gamma_3^6 \left[\frac{1}{3\sqrt{3}} + \frac{1}{3\sqrt{6}} + \frac{1}{\sqrt{3}}\right] \quad (55)$$

and

$$\frac{\hbar c}{G m_2^2} = \gamma_4^6 \left[\sqrt{2} + \frac{1}{4}\right] \quad (56)$$

Accounting for the fact that $\hbar c / G = m_{Pl}^2$, we establish the following expressions for the two Lorentz factors

$$\gamma_3 = \left(\frac{m_{Pl}}{m_2}\right)^{1/3} \frac{1}{\left[\frac{1}{3\sqrt{3}} + \frac{1}{3\sqrt{6}} + \frac{1}{\sqrt{3}}\right]^{1/6}} \quad (57)$$

$$\gamma_4 = \left(\frac{m_{Pl}}{m_2}\right)^{1/3} \frac{1}{\left[\sqrt{2} + \frac{1}{4}\right]^{1/6}} \quad (58)$$

On using the energy equipartition principle and recalling that there are four 3p axes and three 4p axes, the total energy per particle E is given by the expression

$$E = \left[4\gamma_3 m_2 c^2 + 3\gamma_4 m_2 c^2\right] / 7 \quad (59)$$

where the values of γ_3 and γ_4 are computed from Equations (57) and (58) as

$$\gamma_3 = 1.0166 \left(\frac{m_{Pl}}{m_2}\right)^{1/3} \quad (60)$$

$$\gamma_4 = 0.9186 \left(\frac{m_{Pl}}{m_2}\right)^{1/3} \quad (61)$$

Then, it turns out that

$$\bar{\gamma} = \left[\frac{4}{7}\gamma_3 + \frac{3}{7}\gamma_4\right] \left(\frac{m_{Pl}}{m_2}\right)^{1/3} = 0.9746 \left(\frac{m_{Pl}}{m_2}\right)^{1/3} \quad (62)$$

and, thus, accounting for the six particles in the Kaon, it follows that

$$m_K = 6\bar{\gamma} m_2 = 0.9746 \times 503 \times 10^6 \text{ eV}/\text{c}^2 = 490.5 \text{ MeV}/\text{c}^2 \tag{63}$$

which differs less than 0.7% from the experimental value of 493.677 MeV/c^2 [2] [3].

Consequently, the final formula for the Kaon mass is

$$m_K = 6\left\{\frac{4}{7}\frac{1}{\left[\frac{1}{3\sqrt{3}}+\frac{1}{3\sqrt{6}}+\frac{1}{\sqrt{3}}\right]^{1/6}}+\frac{3}{7}\frac{1}{\left[\sqrt{2}+\frac{1}{4}\right]^{1/6}}\right\}\left(m_{Pl}m_2^2\right)^{1/3} = 490.5 \text{ MeV}/\text{c}^2 \tag{64}$$

If we add the mass of the central e$^+$, then the computed m_K value attains the value $m_K = 491.0 \text{ MeV}/\text{c}^2$ and the deviations from the experimental K$^\pm$ value is 0.55%, while from the experimental K^o value is 1.4%.

5. Conclusions

The present study shows that the Rotating Lepton Model (RLM) which combines the de Broglie wavelength equation, which has been the basis of quantum mechanics, with special relativity and with Newton's Universal gravitational Law, provides, without any adjustable parameters, a very good fit to the masses of Pions and of Kaons. Kaons are found to be three-dimensional rotating structures comprising hybridized rotating relativistic neutrinos with a mass corresponding to the m_2 neutrino mass of the normal hierarchy. So far the RLM has been used to model only one 3D particle structure, *i.e.* that of the Higgs boson [19].

The neutrinos of the Kaon structure result from the hybridization of the m_1 and m_3 flavor type neutrinos, and are the same ones used already to compute the masses of muons [16] and pions [16]. It is worth noting that, similarly to the cases of the proton and the neutron masses, (938.272 MeV/c^2 and 939.565 MeV/c^2 respectively), which comprise only mass m_3 neutrinos and where the neutral composite particle (n) is heavier, here also the neutral composite particle (K^o) has higher mass than the charged one (K$^\pm$). This has been attributed to the fact that electrostatic forces (between quarks [2] [3], or resulting from charge-induced dipole interactions, similar to van der Waals forces in chemistry, [21], enhance the total attraction and thus lower the required γ value for the gravitational confinement of the rotating neutrinos [14].

In conclusion, it should be pointed out that various Yukawa-type potentials [4] [23] have been proposed to model the rapid decay of strong nuclear force at a rate much faster than the inverse square low (~1/r^2) of Coulombic interactions for separation distances at the femtometer range. This is done by solving a "homogeneous" Helmholtz equation involving an additional phenomenological coefficient. An alternative nonlocal gradient intreaction force has been introduced more recently [24] leading to an "in homogeneous" Helmholtz equation, the solution of which also involves an adjustable phenomenological coefficient, which conveniently serves to effectively interpret non-Coulombic internuclear interactions. The RLM free-parameter approach can be directly used to properly determine the values of the aformentioned phenomenological coefficients.

The present study also confirms the conclusion [17] that neutrinos, electrons and positrons are apparently the only undividable elementary particles, as evidenced by examining the decay products of all the Tables of Elementary particles [2] [3]. Consequently these elementary particles are the equivalent of atoms in chemistry.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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