Journal of Applied Mathematics and Physics, 2022, 10, 2527-2537
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Entropy Production and Fractal Dimensions in Heavy Ion Nuclear Reaction at Intermediate Energies

Wenxia Wang[1,2], Yiyan Zhao[1], Yongzhong Xing[1*]

[1]Institute for The Fundamental Physics, Tianshui Normal University, Tianshui, China
[2]School of Nuclear Science and Technology, Lanzhou University, Lanzhou, China
Email: *yzxing@tsnu.edu.cn

How to cite this paper: Wang, W.X., Zhao, Y.Y. and Xing, Y.Z. (2022) Entropy Production and Fractal Dimensions in Heavy Ion Nuclear Reaction at Intermediate Energies. *Journal of Applied Mathematics and Physics*, **10**, 2527-2537.
https://doi.org/10.4236/jamp.2022.108171

Received: February 24, 2022
Accepted: August 28, 2022
Published: August 31, 2022

Abstract

The characteristics of the nonlinear dynamics in the Heavy Ion Collision (HIC) at intermediate energies have been studied by evaluating the productions of the Generalized Entropy (GE) and the Multifragmentation Entropy (ME) as well as the features of the information and fractal dimensions within the Isospin Quantum Molecular Dynamical Model compensated by the lattice methods. Results demonstrate from various views that the existence of deterministic chaos in the dynamical process of reaction.

Keywords

Entropy Production, Fractal Dimensions, Chaotic Behavior, Heavy Ion Nuclear Collision, Intermediate Energy

Multifragmentation is an important phenomenon occuring in the Heavy Ion nuclear Collisions (HICs) at intermediate energies. The dynamical mechanism of the process has been explored for a long time theoretically from both statistics and dynamics aspects [1] [2] and such studies are still a current focus [3] [4] [5]. The liquid-gas phase transition has been revealed in the fragmentation of the hot nuclear matter [6] [7] [8] [9]. Correspondingly, the spinodal instability, non-statistical fluctuation, etc., have been explored in the processes of the HICs intensively in the last decades [10] [11]. In fact, such phenomena link intimately to the non-linear dynamics or deterministic chaos which is considered as one of the possible mechanism of the multifragmentation [12] [13] [14] [15]. The chaos, or nonlinear dynamics, which became an independent science in the 70's of the last century [16], has provided us with new perspectives and ways of understanding the complex world. Therefore, it will make sense for us to reexamine the dynamical process of the HICs from the viewpoints of the nonlinear science. In this letter, we present the production of Generalized Entropy (GE), Multifragmentation Entropy (ME) and the feature of the fractal dimension by simulating the simple realistic collision system, $^{40}Ca+^{40}Ca$, at incident energy within the Isospin Quantum Molecular Dynamical Model (IQMD) compensated by the lattice methods.

The IQMD model is an extended version of Quantum Molecular Dynamics (QMD) model [1] [2] in which the correlations have been kept and thus it is very suitable for studying the multifragmentation of HICs at medium energies. Great successes have been achieved in the interpretation for the HICs induced particularly by the radioactive beams within the IQMD model [17] [18] [19]. This model contains two ingredients: density-dependent mean field containing correct isospin terms including symmetry potential and the in-medium nucleon-nucleon cross sections which are different for neutron-neutron (proton-proton) and neutron-proton collisions. The potential is

$$U(\rho) = U^{Sky} + U^{C} + U^{sym} + U^{Yuk} + U^{MDI} + U^{Pauli}, \tag{1}$$

where U^{C}, U^{Sky}, U^{Yuk}, U^{MDI} and U^{Pauli} are Coulomb potential, Skyrme potential, Yukawa potential, momentum dependent interaction and the Pauli potential, respectively. Their concrete expressions for the potentials and the parameters involved in the formulas are given in Refs. [17] [18] [19] [20] [21].

There are a variety of expressions for the nucleon-nucleon (NN) cross section in the model for studying the intermediate energy nuclear collisions. In the present calculation the formula of the isospin dependent NN cross sections proposed in Ref. [22] are used. They look like

$$\sigma_{nn} = \left(13.73 - 15.08\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{4}\right)\frac{1.0 + 7.772E_{lab}^{0.06}\rho^{1.48}}{1.0 + 18.01\rho^{1.46}} \tag{2}$$

$$\sigma_{np} = \left(-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta\right)\frac{1.0 + 20.88E_{lab}^{0.04}\rho^{2.02}}{1.0 + 25.86\rho^{1.90}} \tag{3}$$

$$\beta = \sqrt{1.0 - \frac{1.0}{\gamma^{2}}}, \gamma = \frac{E_{lab}}{931.5} + 1.0, \tag{4}$$

where β is the ratio of projectile velocity to light velocity and ρ is nuclear matter density in the unit of fm^3. σ_{nn} and σ_{np} are the neutron-neutron (or proton-proton) and the neutron-proton cross sections, respectively. E_{lab} is the incident energy in laboratory frame. The quantum Pauli-blocking effects have been accounted by embedding a novel energy-dependent factor [23]

$$\xi(E) = 0.644 + 0.011E - 1.513E^{2} + 6.214E^{3} \tag{5}$$

which is extracted from the comparison of experiment data and theoretic simulation. The factor ξ in Equation (5) modifies the uncertainty relation $R_r \times R_p \geq \xi h$ which represents the quantum property of the nucleon in the phase space. Here R_r and R_p are the radius of the Fermi sphere occupied by a nucleon in coordinate and momentum space respectively. In the following investigation, our numerical calculations will be performed with these equations and keep the relevant parameters unchanged like in Refs [23] [24].

The entropy is an important thermodynamic quantity. The variation of its' magnitude reflects the confusion in the microscopic state of a system, or the equilibrium of the system under certain macroscopic condition. And thus the entropy production in the HICs have attracted considerable attentions in both nuclear physics and nonlinear dynamics [12] [25]. In 1981, Bertsch and Cugnon [26] studied quantitatively the entropy production in the collision $^{40}Ca+^{40}Ca$ at incident energy $E = 800\ \text{MeV}$ within cascade model and concluded that the generation of the entropy are closely relate to the formation of clusters. In Ref. [27], based on the fireball model, the authors investigated the same issue and speculated that the amount of the entropy production are not the same during the various stage of nuclear reactions and pointed out that there are barely entropy generated in the final stage, expansion stage, since the density of the particles has become so small that they seldom collide and the Liouville's theorem guarantees that the particle's density in phase space remains constant in the absence of collisions. However, S. Das Gupta *et al.*, [28] developed a microscopic model for treating the fragmentation of nuclear matter in which both hard collision and propagation of nucleons are treated. They argued that the mean field can lead to fragmentation of nuclear matter and the entropy increase simultaneously. Therefore, the mechanisms responsible for, and the significance of, the entropy generation in HICs has been a matter of much disputed issue in nuclear physics [25].

The expression of the entropy for a fermi system is defined by [28]

$$S = -\int \mathrm{d}\gamma \left[f \ln(f) - (1-f)\ln(1-f) \right], \quad \mathrm{d}\gamma = g\frac{\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3} \tag{6}$$

where $f = f(\boldsymbol{r}, \boldsymbol{p})$ stands for the occupation probability of nucleons in a phase space volume of h^3 and $g = 4$ is the degeneration of the spin-isospin degree of freedom. In order to avoid the complicated integration which is very time-consuming, we turn to calculate the temporal evolution of the coarse-grained entropy, namely, the so-called generalized entropy (GE) defined in Ref. [29] and adopt the ansantz used in Ref. [28]. Specifically, the available phase space is broken up into cells of volume βh^3 with β is an adjustable parameter. Then the entropy becomes

$$S = -4\beta \sum_i \left[n_i \ln(n_i) - (1-n_i)\ln(1-n_i) \right], \quad n_i = \frac{N}{4R\beta} \tag{7}$$

where N is the number of nucleons in a cell of volume βh^3 and R the number of runs in our numerical simulation. n_i indicates the occupation probability of a phase space of volume h^3.

It's still not easy to perform the calculation of Equation (7) in the six dimension space, in particular for a nucleus with large nucleons. Moreover, there is a set of parameters β, R including the size of the cells which are all adjustable artificially. Fortunately, Ref [29] has proved that the trend of the variation of the

GE does not depend sensitively on the size of the cells although the magnitude of the entropy being determined significantly by the volume of a cell. Coincidentally, our main concern here focus mainly on the tendency of the GE variation reflecting the chaotic behavior of the system. Nevertheless, in order to prevent our calculations from being too rough and to determine an appropriate size of the cells, we first check the results given in the Ref. [26] for the entropy production in the reaction $^{40}Ca+^{40}Ca$ at incident energy 800 MeV/u, and plot our results in **Figure 1**. It shows that the value of the entropy is saturated at 4.0 - 4.4, which is a reasonable value quoted in Ref. [26] for 10 events or 800 nucleons. In the left panel of **Figure 1** the curves from bottom to top correspond respectively to the magnitude of the mesh side-length in six dimension space, (a) $\Delta r = 35$ fm, $\Delta p = 0.29$ GeV/c, (b) $\Delta r = 24$ fm, $\Delta p = 0.19$ GeV/c, (c) $\Delta r = 17$ fm, $\Delta p = 0.15$ GeV/c and (d) $\Delta r = 14$ fm, $\Delta p = 0.15$ GeV/c with $\beta = 0.8$. Comparing the behaviors of each curves we can learn that the sensitivity of the variation of the GE to the mesh size since the function $n_i \ln(n_i)$ depends sensitively on the value of the distribution n_i, as has illustrated in **Figure 1** of Ref. [26]. **Figure 1** here shows that the magnitude of the entropy tends to saturation with decrease of the size of the cell. Of course, it can be expected that the location and the magnitude of the peak of the S will vary with the size of cells if we keep the size going down so that the most cells would be empty. But we are not going to do so because we have come to a reasonable range in the case (d) with the $S \approx 4.0 \sim 4.4$ which is consistent well with the value given in Ref. [26]. We also notice that the increment $\Delta S \approx 0.1 \sim 0.2$ produced in the expansion stage of the reaction [28] has been confirmed in our calculation.

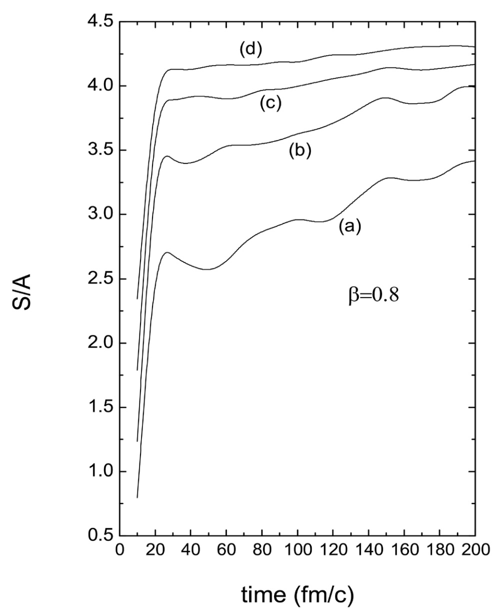

Figure 1. The variation of the Generalized Entropy (GE) with respect to time in the central collision of $^{40}Ca+^{40}Ca$ at incident energy E = 800 MeV/u. Left panel: the GE created in full phase space. Right panel: the GE created in position space (solid line) and in the momentum space (dashed line).

Our results, obtained by using IQMD combining with the lattice method, demonstrate explicitly the production of the GE during the whole reaction process, *i.e.*, the rapid increase in the stage of the collision phase and the slow growth in the expansion stage of the reaction.

As has pointed out by Schuster [30] that entropy increases not only because of the increase of the number of particles, but also because of the dynamical fluctuations. We therefore consider a multifractal description of that fluctuation and focus on the fractal dimension of the system. Fractal geometry is also a new science that comes with chaos which devotes to answer the question how the microscopic behavior is related to what we observe on the macroscopic scale. According to the Fractal theory the information dimension D_I is defined by [31]

$$D_I = -\lim_{\delta \to 0} \frac{S}{\ln(\delta)} \tag{8}$$

with δ being the size of each cell. In **Figure 2** we show the result for the central collision of ^{40}Ca+^{40}Ca at incident energy $E = 800\ \text{MeV/u}$ at time $t = 100\ \text{fm/c}$ when the generalized entropy has come at plateau. In this figure the straight line is the fitted results with our computed results indicated by the scattering points and the extracted information dimension is $D_I = 0.484$. D_I is not a integral meaning that the existence of the self-similarity in the distribution of the available phase space.

In 1999, Y.G. Ma introduced a method [32] to diagnose a nuclear liquid gas phase transition by multiplicity entropy (ME),

$$H = k_b \sum_i p_i \ln(p_i), \tag{9}$$

which determines the critical point by finding the maximum value of multiplicity entropy in a certain state of the system. In this definition k_b is Boltzmann constant and the probability distribution p_i is the ratio of the number of "*i*" particles produced by N_i to the total number of particles produced by *N*, *i.e.* $p_i = N_i/N$ and $\sum_{i=1}^{N} p_i = 1$. He used the ME to study of the liquid gas phase transition of nuclei in the framework of the isospin dependent lattice gas model and the molecular dynamical model and concluded for the first time that the maximum of the ME indicates that the system comes at a largest fluctuation/stochasticity/chaoticity in the event space at this time. It is naturally making sense to check such behavior of the ME and compare of the values of the ME with GE in a given reaction system under the same incident conditions exactly. The calculated results for the ME in the reaction system are plotted in **Figure 3**.

Figure 3 shows clearly that there is a maximum of the ME appearing at the time $t \approx 20\ \text{fm/c}$, in the compressing stage of the reaction. The peak is a significant indicator of phase transition since the maximum of the ME represents the largest fluctuation of the multiplicity probability distribution in the critical point. Comparing the behaviors of the variations of the GE shown in **Figure 1**

and the ME displayed in Figure 3, we can see that both kinds of entropies have a common feature that increase rapidly in the compress stage of the reaction and arrive at their respective maximum values at almost the same moment. Following their peaks, the ME decrease while the GE still increase slowly due to the viscous interaction in the mean field of the nuclear matter. Anyway, the production of both types of the entropy is responsible for the increase in disorder or chaoticity.

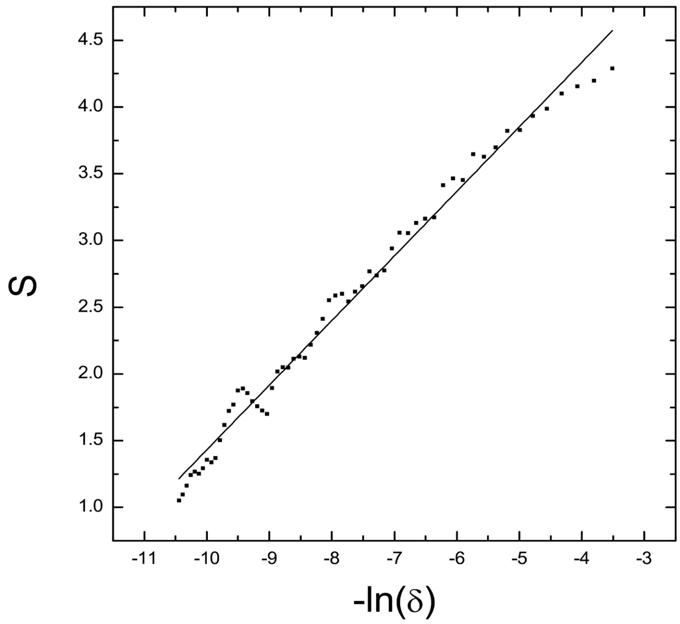

Figure 2. The information dimension corresponding the Generalized Entropy at time $t = 100\,\text{fm/c}$ in the same collision as given in Figure 1.

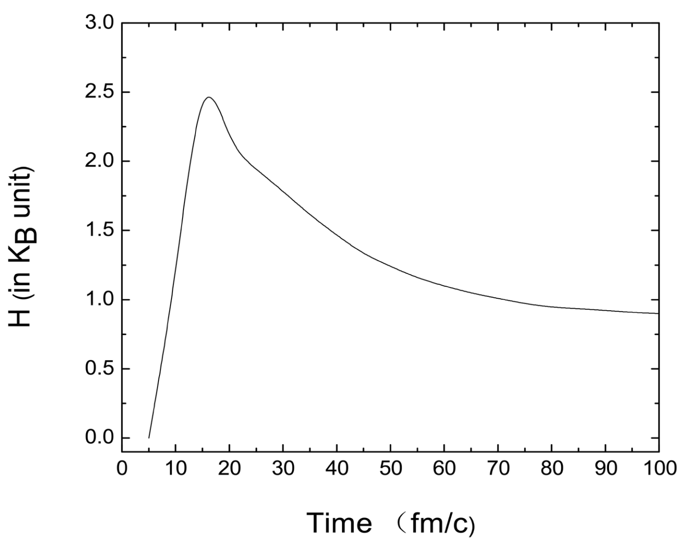

Figure 3. The variation of the Multifragmentation Entropy (ME) in term of the time in the central collision of $^{40}Ca+^{40}Ca$ at incident energy $E = 800$ MeV/u.

Intermittency is a manifestation of the scale invariance of the physical process and randomness of underlying scaling law. According to nonlinear dynamics theory [33], the emergence of the intermittency during the spatial-temporal evolution of a dynamics system is considered as a indicative sign of chaotic behavior, similar to the period-doubling bifurcation. The original idea of studying intermittency in nuclear collisions came from the work of Bialas and Peschanski [34] who looked at the rapidity distributions of produced particles in cosmic ray experiments. They proposed using scaled factorial moments of these rapidity distributions to study the possible appearance of intermittent behavior in such collisions and evidence for non-Poissonian fluctuations. The greatest merit of this method is that the scaled factorial moments filter out the statistical fluctuations and retain only dynamical fluctuations which are just the most essential concerns of the chaotic dynamics. The intermittent behaviour and the self-similarity pattern of the fluctuations in the charge distribution in the breakup of $^{197}_{79}\mathrm{Au}$ nuclei of energy at $E = 1\ \mathrm{GeV/u}$ in a nuclear emulsion [35] have been explained by scaled factorial moments method based on the percolation model [34] [35] [36] and microcanonical model of the thermal breakup of the nucleus [37]. The scaled factorial moments which defined as [34] [37] [38]

$$F_q = M^{q-1} \frac{\sum_{m=1}^{M} n_m (n_m - 1) \cdots (n_m - q + 1)}{N(N-1) \cdots (N-q+1)} \tag{10}$$

The range ΔZ of the distribution of fragment charges Z is divided in to M bins with each of interval $\delta Z = \frac{\Delta Z}{M}$. n_m is the multiplicity of fragments in the mth bin, $(m-1)\delta Z < Z < m\delta Z$. The angle bracket denotes the average over events. We calculate the scaled factorial moments for the central collision of $^{40}\mathrm{Ca}+^{40}\mathrm{Ca}$ at incident energy $E = 800\ \mathrm{MeV/u}$ by using IQMD. **Figure 4** is the results obtained by analyzing the 1000 events.

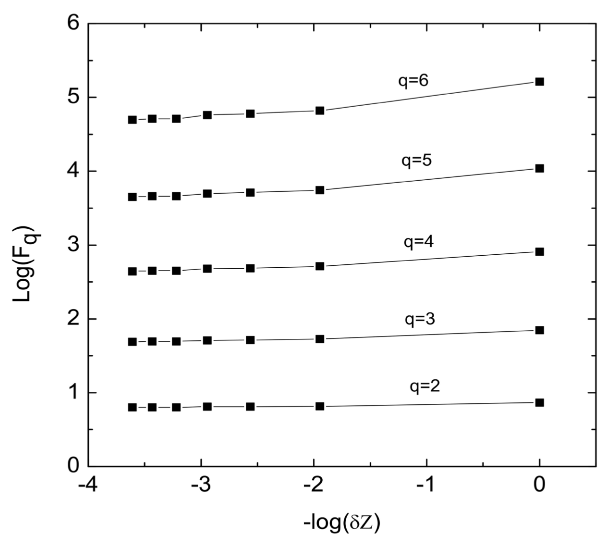

Figure 4. The scaled factorial moments in $^{40}\mathrm{Ca}+^{40}\mathrm{Ca}$ at E = 800 MeV/u.

Table 1. The intermittency exponents and fractal dimensions.

q	2	3	4	5	6
α_q	0.018	0.043	0.074	0.107	0.142
d_q	0.018	0.022	0.025	0.026	0.028

Each curve corresponding to different q in **Figure 4** looks roughly like a straight line indicates that a power-like increase of the scaled factorial moments with decreasing bins size. The slops of these fitted lines, α_q, termed as the intermittency exponent [39], can be express as

$$F_q \propto (\delta Z)^{-\alpha_q}. \tag{11}$$

This is just the typical characteristics of the self-similarity [40] emerged at all scales in the distribution. The fractal dimension can be derived from the formula

$$d_q = \alpha_q / (q-1). \tag{12}$$

Both the intermittency exponent α_q and fractal dimension d_q for various q are listed in **Table 1**.

As shown by **Table 1** the fractal dimension d_q is not integrals this fact indicates that there is scale invariance in the fragmentation pattern.

In summary, we have observed quantitatively the typical characteristics of deterministic chaos in the HICs at intermediate energy with the help of the isospin quantum molecular dynamical model compensated with the lattice methods. The generalized entropy(GE) and multifragmentation entropy (ME), the information dimension and the fractal dimension have been evaluated simulating the central collision of ^{40}Ca+^{40}Ca at incident energy $E = 800\ \mathrm{MeV/u}$. The scale invariance of the fragmentation and the randomness of the scaling law have simultaneously demonstrated in both the phase space and the event space for the certain reaction at exactly the same incident conditions. Comparing the various behaviors of GE in the position space and momentum space, it is clearly shown that the production of GE in the compress stage of the reaction in the phase space comes from the combination of the collisions and attracting mean field. By the way, although the reaction system we have selected here is not too complicated, the features we have obtained are general and universal for such collisions and the extension of the method to other reaction colliding systems, e. g. the heavier and isospin-dependent reaction systems, is straightforward.

Acknowledgements

This work is supported by Natural Science Foundation of Gansu Province under Grant No. 20JR5RA497.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Aichelin, J. (1991) "Quantum" Molecular Dynamics—A Dynamical Microscopicn-Body Approach to Investigate Fragment Formation and the Nuclear Equation of State In-heavy Ion Collisions. *Physics Reports*, **202**, 233-360. https://doi.org/10.1016/0370-1573(91)90094-3

[2] Das Gupta, S. and Mekjian, A.Z. (1998) Phase Transition in a Statistical Model for Nuclear Multifragmentation. *Physical Review C*, **57**, 1361-1365. https://doi.org/10.1103/PhysRevC.57.1361

[3] Liu, H. L., Ma, Y.G., Bonasera, A., Deng, X.G., Lopez, O. and Veselský, M (2017) Mean Free Path and Shear Viscosity in Central ^{129}Xe+^{119}Sn Collisions below 100 MeV/Nucleon. *Physical Review C*, **96**, Article ID: 064604. https://doi.org/10.1103/PhysRevC.96.064604

[4] Lin, W., Ren, P., Zheng, H., Liu, X., Huang, M., Wada, R. and Qu, G. (2018) Sensitivity Study of Experimental Measures for the Nuclear Liquid-Gas Phase Transition in the Statistical Multifragmentation Model. *Physical Review C*, **97**, Article ID: 054615. https://doi.org/10.1103/PhysRevC.97.054615

[5] Liu, H.L., Ma, Y.G. and Fang, D.Q. (2019) Finite-Size Scaling Phenomenon of Nuclear Liquid—Gas Phase Transition Probes. *Physical Review C*, **99**, Article ID: 054614. https://doi.org/10.1103/PhysRevC.99.054614

[6] Bowman, D.R., Peaslee, G.F., DeSouza, R.T., Carlin, N., Gelbke, C.K., Gong, W.G., *et al.* (1991) Multifragment Disintegration of the ^{129}Xe+^{197}Au System at E/A = 50 MeV. *Physical Review Letters*, **67**, 1527-1530. https://doi.org/10.1103/PhysRevLett.67.1527

[7] Minich. R.W., Agarwal. S., Bujak. A., Chuang, J., Finn, J.E., Gutay, L.J., *et al.* (1982) Critical Phenomena in Hadronic Matter and Experimental Isotopic Yields in High Energy Proton-Nucleus Collisions. *Physics Letters B*, **118**, 458-460. https://doi.org/10.1016/0370-2693(82)90224-6

[8] Richert, J. and Wagner, P. (2001) Microscopic Model Approaches to Fragmentation of Nuclei and Phase Transitions in Nuclear Matter. *Physics Reports*, **350**, 1-92. https://doi.org/10.1016/S0370-1573(00)00120-4

[9] Borderie, B. and Rivet, M.F. (2008) Nuclear Multifragmentation and Phase transition for Hot Nuclei. *Progress in Particle and Nuclear Physics*, **61**, 551-601. https://doi.org/10.1016/j.ppnp.2008.01.003

[10] Chomaz, P., Colonna, M. and Rdrup, J. (2004) Nuclear Spinodal Fragmentation. *Physics Reports*, **389**, 263-440. https://doi.org/10.1016/j.physrep.2003.09.006

[11] Jakobsson, B., Jensson, G., Karlsson, L., Kopljar, V., Norén, B., Söderström, K., *et al.* (1990) On the Observation of a Transition from Fusion to Multifragmentation in High Multiplicity ^{16}O Induced Reactions. *Nuclear Physics A*, **509**, 195-220. https://doi.org/10.1016/0375-9474(90)90381-U

[12] Burgio, G.F., Baldo, M. and Rapisarda, A. (1994) Non-Linear Mean Field Dynamics in the Nuclear Spinodal Zone. *Physics Letters B*, **321**, 307-311. https://doi.org/10.1016/0370-2693(94)90247-X

[13] Wang, N., Wu, X.-Z., Li, Z.-X., Wang, N., Zhuo, Y.-Z. and Sun, X.-Q. (2000) Behavior of the Lyapunov Exponent and Phase Transition in Nuclei. *Chinese Physics Letters*, **17**, 711-713. https://doi.org/10.1088/0256-307X/17/10/004

[14] Zhang, Y., Wu, X. and Li, Z. (2004) Connection Between the Largest Lyapunov Exponent, Density Fluctuation and Multifragmentation in Excited Nuclear. *Physical Review C*, **69**, Article ID: 044609. https://doi.org/10.1103/PhysRevC.69.044609

[15] Heiselberg, H., Pethick, C.J. and Ravenhall, D.G. (1988) Instabilities in Hot Nuclear Matter: A Mechanism for Nuclear Fragmentation. *Physical Review Letters*, **61**, 818-821. https://doi.org/10.1103/PhysRevLett.61.818

[16] Schuster, H. G. and Just, W. (2005) Deterministic Chaos. 4th Revised and Enlarged Edition, WILEY-VCH Verlag GmbH and Co. KGaA, Weinheim.

[17] Chen, L.W., Ge, L., Zhang, X. and Zhang, F. (1997) Isospin Equilibrium and Non-Equilibrium in Heavy-Ion Collisions at Intermediate Energies. *Journal of Physics G: Nuclear and Particle Physics*, **23**, 211-218. https://doi.org/10.1088/0954-3899/23/2/008

[18] Liu, J.Y., Guo, W.J., Xing, Y.-Z., Zuo, W., Lee, X.-G. and Li, Z.-H. (2002) Probing the Isospin Dependent In-Medium Nucleon-Nucleon Cross Section by Nucleon Emissions. *Physics Letters B*, **540**, 213-219. https://doi.org/10.1016/S0370-2693(02)02174-3

[19] Xing, Y.-Z., Liu, J.Y. and Guo, W.J. (2003) Influence of Medium Correction of Nucleon-Nucleon Cross-Section on the Fragmentation and Nucleon Emission. *Nuclear Physics A*, **723**, 483-495. https://doi.org/10.1016/S0375-9474(03)01433-7

[20] Liu, J.-Y., Guo, W.-J., Xing, Y.-Z., Zou, W. and Lee, X.-G. (2003) Influence of a Momentum Dependent Interaction on the Isospin Dependence of Fragmentation and Dissipation in Intermediate Energy Heavy Ion Collisions. *Physical Review C*, **67**, Article ID: 024608. https://doi.org/10.1103/PhysRevC.67.024608

[21] Liu, J.Y., Guo, W.-J., Wang, S.-J., Zuo, W., Zhao, Q. and Yang, Y.-F. (2001) Nuclear Stopping as a Probe for In-Medium Nucleon-Nucleon Cross Sections in Intermediate Energy Heavy Ion Collisions. *Physical Review Letters*, **86**, 975-978. https://doi.org/10.1103/PhysRevLett.86.975

[22] Cai, X., Feng, J., Shen, W., Ma, Y., Wang, J. and Ye, W. (1998) In-Medium Nucleon-Nucleon Cross Section and Its Effect on Total Nuclear Reaction Cross Section. *Physical Review C*, **58**, 572. https://doi.org/10.1103/PhysRevC.58.572

[23] Xing, Y.-Z., Zhang, H.F., Liu, X.B. and Zheng, Y.-M. (2017) Pauli-Blocking Effect in Two-Body Collisions Dominates the In-Medium Effects in Heavy-Ion Reactions Near Fermienergy. *Nuclear Physics A*, **957**, 135-143. https://doi.org/10.1016/j.nuclphysa.2016.08.006

[24] Xing, Y.-Z., Liu, X.-X., Liu, X.-B., Zhang, H.F. and Zheng, Y.-M. (2017) Nuclear Stopping Power Evaluated via Free Protons Emitted in Reaction Xe+Sn Near Fermi Energy. *International Journal of Modern Physics E*, **26**, Article ID: 1750010. https://doi.org/10.1142/S0218301317500100

[25] Csernai, L.P. and Kapusta, J.I. (1984) Entropy and Cluster Production in Nuclear Collisions. *Physics Reports*, **131**, 223-318. https://doi.org/10.1016/0370-1573(86)90031-1

[26] Bertsch, G. and Cugnon, J. (1981) Entropy Production in High Energy Collision. *Physical Review C*, **24**, 2514-2520. https://doi.org/10.1103/PhysRevC.24.2514

[27] Kapusta, J. (1981) Viscous Heating of Expanding Fireballs. *Physical Review C*, **24**, 2545-2551. https://doi.org/10.1103/PhysRevC.24.2545

[28] Das Guta, S., Gale, C. and Gallego, J. (1986) Microscopic Model for Nuclear Breakup in Heavy Ion Collision. *Physical Review C*, **33**, 1634-1640. https://doi.org/10.1103/PhysRevC.33.1634

[29] Atalmi, A., Baldo, M., Burgio, G.F. and Rapisarda, A. (1998) Generalized Entropy and Temperature in Nuclear Multifragmentation. *Physical Review C*, **58**, 2238-2248. https://doi.org/10.1103/PhysRevC.58.2238

[30] Schuster, H.G. (1984) Deterministic Chaos. Physik-Verlag, Weinheim.

[31] Feder, J. (1988) Fractals. Plenum Press, New York. https://doi.org/10.1007/978-1-4899-2124-6

[32] Ma, Y.G. (1999) Application of Information Theory in Nuclear Liquid Gas Phase Transition. *Physical Review Letters*, **83**, 3617-3620. https://doi.org/10.1103/PhysRevLett.83.3617

[33] Gutzwiller, M.C. (1990) Chaos in Classical and Quantum Mechanics. Springer-Verlag, New York. https://doi.org/10.1007/978-1-4612-0983-6

[34] Bialas, A. and Peschanski, R. (1986) Moments of Rapidity Distributions as a Measure of Short-Range Fluctuations in High-Energy Collisions. *Nuclear Physics B*, **273**, 703-718. https://doi.org/10.1016/0550-3213(86)90386-X

[35] Ploszajczak, M. and Tucholski, A. (1991) Intermi'Itency in Nuclear Multifragmentation. *Nuclear Physics A*, **523**, 651-676. https://doi.org/10.1016/0375-9474(91)90042-5

[36] Deangelis, A.R., Gross, D.H.E. and Heck, R. (1992) Intermittency in and the Fractal Nature of Nuclear Fragmentation. *Nuclear Physics A*, **537**, 606-630. https://doi.org/10.1016/0375-9474(92)90370-Y

[37] Maruyama, T., Niita, K. and Oyamatsu, K. (1998) Fluctuations and Intermittency in Multifragmentation Processes. *Physical Review C*, **57**, 655-665. https://doi.org/10.1103/PhysRevC.57.655

[38] Ge, L.X., Zhang, X.D. and Zhu, Q.L. (1994) Intermittency in the Heavy Ion Collisions at Intermediate and High Energies. *High Energy Physics and Nuclear Physics*, **18**, 428. (In Chinese)

[39] Gross, D.H.E., De Angelis, A.R., Jaqaman, H.R., Jicai, P. and Heck, R. (1992) Intermittency in the Multifragmentation of Hot Nuclei? *Physical Review Letters*, **68**, 146-149. https://doi.org/10.1103/PhysRevLett.68.146

[40] Lee, S.J. and Mekjiant, A.Z. (1993) Self-Similarity and Scaling Behavior in Nuclear Collision. *Physical Review C*, **69**, 2266-2287. https://doi.org/10.1103/PhysRevC.47.2266