

Gauge Formulation of Heaviside's Equations

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How to cite this paper: Klingman, E.E. (2022) Gauge Formulation of Heaviside's Equations. *Journal of Applied Mathematics and Physics*, 10, 2292-2302.

<https://doi.org/10.4236/jamp.2022.107156>

Received: June 22, 2022

Accepted: July 25, 2022

Published: July 28, 2022

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Abstract

A primordial field *Self-interaction Principle*, analyzed in Hestenes' Geometric Calculus, leads to Heaviside's equations of the gravitomagnetic field. When derived from Einstein's nonlinear field equations Heaviside's "linearized" equations are known as the "weak field approximation". When derived from the primordial field equation, there is no mention of field strength; the assumption that the primordial field was predominant at the big bang rather suggests that ultra-strong fields are governed by the equations. This aspect has physical significance, so we explore the assumption by formulating the gauge field version of Heaviside's theory. We compare with recent linearized gravity formulations and discuss the significance of differences.

Keywords

Gauge Theory of Gravity, Linearized Gravity, Heaviside Equations, Yang-Mills Gauge, Geometric Algebra

1. Introduction

Heaviside in 1893 reformulated Maxwell's equations in 3D vectors and extended Newtonian gravity to include gravitomagnetism [1], effectively removing the "action-at-a-distance" aspect of gravity; however, until *Gravity Probe B* in 2011 there was no experimental proof of the existence of gravitomagnetism [2]. Until 2015, there was no experimental proof that gravity propagates with the same speed as light [3]. Heaviside's equations were later derived as the linearized version of Einstein's nonlinear field equation but erroneously interpreted as the "weak field approximation" to general relativity, thus diminishing the significance of Heaviside's theory with respect to Einstein's theory. Although Weyl in 1929 formulated gravity in gauge field theory, the "weak field" misinterpretation prevented serious use of Heaviside theory. Post-2000 Will and others [4] applied the "linearized equations" to solar system gravity and noted that the "weak field"

theory seemed to apply in strong field situations associated with neutron star collisions.

About the middle of this timescale, 1954, Yang and Mills formulated a non-Abelian version of gauge theory of “isospin” that represents a natural extension of Heaviside; however, they did not have gravity in mind at the time. Our goal in this paper is to reformulate Heaviside as a gauge theory to facilitate its proper use as an “all strength” theory of gravity rather than the erroneous “weak field approximation”. To do so, we begin by deriving the *strong field* version of Heaviside.

2. Derivation of “Full Strength” Heaviside Equations of Gravity

Maxwell’s equations, extended by Hertz and by Heaviside, were based on the electromagnetic field. For at least half a century no other fields were assumed, other than gravity, and no source, other than electric charge. Mid-20th Century, as numerous fundamental particles were discovered to exist, Quantum Field Theory (QFT) changed the field concept to include *one-field-per-particle*, distributed at every spacetime point [5]. Excitation of the electron field by an electron creation operator was assumed to bring an electron into existence; a muon creation operator excited a muon from the muon field, etc. QFT was so entrenched that when Feynman investigated a field theory formulation of gravity [6], he assumed that gravity was the “31st field”. The current theory of particles, the Standard Model of Particle Physics, retains the field-per-particle assumption, but also assumes that all the force fields merge at the big bang, although this has not been demonstrated. In this and other papers we investigate the idea that a single field existed at the Creation and call this field the primordial field.

Physics is largely based on formulating interactions as changes induced by sources, represented as $\nabla\psi = j$, where ∇ is a change operator that generates changes in the field ψ induced by source j , typically separate from field ψ . In the case of primordial field ψ there is nothing separate from ψ , only field ψ exists. Thus, any change operator operating on field ψ must be equivalent to ψ interacting with itself. This *Self-Interaction Principle* [7] is represented by self-interaction Equation:

$$\nabla\psi = \psi\psi \quad (1)$$

Neither field ψ nor operator ∇ have yet been specified. To be meaningful, both must depend on some variable parameter ξ , therefore we extend our formalism via $\psi \rightarrow \psi(\xi)$ and $\nabla \rightarrow \partial_{\xi}$. It is not difficult to exhibit two formal solutions – one for scalar ξ and one for vector ξ .

$$\psi(\xi) = -\xi^{-1}, \quad \psi(\xi) = \xi^{-1} \quad (2)$$

We assign physical meaning to these terms; if scalar $\xi = \text{time}$, then ξ^{-1} is frequency; if vector $\xi = \text{location in space}$, then ξ^{-1} is inverse distance. Corresponding operators are $\nabla_t = \partial/\partial t$ and $\nabla_r = \partial/\partial r$. With these interpreta-

tions we attempt to solve self-interaction Equation (1). Although the choices of mathematical framework were cast in stone by the middle of the 20th century, a new formalism was introduced circa 1965 by David Hestenes; Geometric Algebra, based on an evolution of Clifford algebra. Geometric algebra is the only mathematical framework in which every term has both an algebraic and a geometric interpretation [8]. For 3 spatial dimensions plus time the terms include scalars, vectors, bivectors, trivectors, and pseudoscalars, interpreted as duality operators represented by i , that transform an entity into its dual. The key new relation is the geometric product $uv = u \cdot v + u \wedge v$. Bivector $u \wedge v$ is a directed area representing the rotation of u into v . Duality operator i transforms this bivector into an axial vector: $u \wedge v = iu \times v$. If the vector derivative is substituted for u then the gometric product becomes:

$$\begin{array}{c} \nabla v = \nabla \cdot v + \nabla \wedge v \\ / \quad | \quad \backslash \\ \text{gradient} = \text{div} + \text{curl} \end{array} \tag{3}$$

No other math formalism has this relation. When $\psi = G(r, t) + iC(r, t)$ and $\nabla = \nabla + \partial_t$, then Equation (1) takes the form

$$(\nabla + \partial_t)(G + iC) = (G + iC)(G + iC) \tag{4}$$

Expansion of (4) in terms of geometric products and grouping of like terms yields:

Self-Interaction equations

Heaviside equations

$$\nabla \cdot G = G \cdot G - C \cdot C \qquad \nabla \cdot G = -\rho \tag{5a}$$

$$i\nabla \cdot C = i2G \cdot C \qquad \nabla \cdot C = 0 \tag{5b}$$

$$\partial_t G - \nabla \times C = G \times C \pm C \times G \qquad \nabla \times C = -\rho v + \partial_t G \tag{5c}$$

$$i\nabla \times G + i\partial_t C = 0 \qquad \nabla \times G = -\partial_t C \tag{5d}$$

The equations on the left-hand side of (5) derive from (4) in straightforward fashion. With physical meaning assigned to field ψ , one obtains the equations on the right side, derived by Heaviside, wherein G is gravity and C is the gravitomagnetic field. Decades later Heaviside’s equations were labeled the *weak field approximation* to Einstein’s non-linear field equations.

Self-interaction Equation (5a) yields Newton’s equation. The \pm term in (5c) is “+” in the solution of (4). The Poynting-like $G \times C$ terms are momentum density and can be transported in opposite directions, based on initial and boundary conditions imposed locally; they are represented as ρv in Heaviside (5c), while the field energy density terms, $C \cdot C$ and $G \cdot G$, are represented by ρ in (5a). The time independent gravitational field is irrotational (6d), shown by Michaelson-Gale. From $\nabla \cdot C = 0$, we can use vector identity $\nabla \cdot \nabla \times A = 0$ to replace C with a potential vector $\nabla \times A$. Compatible with Equation (5) are the gauge field equations:

$$C = \nabla \times A, \quad G = -\nabla \phi - \partial_t A, \quad \partial_t \phi + \nabla \cdot A = 0 \tag{6}$$

The first two equations in (6) define the fields in terms of the four-potential A ,

while the last Equation specifies the Lorenz gauge condition, $\partial_\mu A^\mu = 0$. The scalar potential $\phi = -m/r$, and vector potential $\mathbf{A} = \mathbf{v}$. Gauge relations initially held no physical meaning; the electromagnetic \mathbf{E} and \mathbf{B} fields could be measured and exhibited, the gauge field, not. Maxwell used gauge conditions to simplify calculations [via Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$, Lorenz gauge: $\partial_\mu A^\mu = 0$]. Although a full unification of gravitation, electromagnetism, the strong and weak nuclear forces, has not yet been derived; the four fundamental interactions are generated by a single principle, the gauge principle [9], therefore we analyze the gauge aspects of Heaviside's equations.

3. Gauge Formalism of Heaviside's Equations

In analogy with Maxwell's equations, we formulate gauge field four-potential $A = \{\phi, \mathbf{A}\}$. Since $\mathbf{G} = -\nabla\phi + \partial_t \mathbf{A}$ if ϕ is constant then $\mathbf{G} = \partial_t \mathbf{A}$, but since \mathbf{G} is the acceleration of gravity, then $\mathbf{G} = d\mathbf{v}/dt \Rightarrow \mathbf{A} = \mathbf{v}$. Since $\mathbf{C} = \nabla \times \mathbf{A}$ then $\mathbf{C} = \nabla \times \mathbf{v}$ is dimensionally correct; $|\mathbf{C}| \sim t^{-1}$. With gravitational potential $\phi = -M/r$ the \mathbf{G} -field has spatial dependence $|\mathbf{G}| \sim r^{-2}$; correct for Newtonian mass. For the primordial field, as shown in several of the references, $|\mathbf{G}| \sim r^{-1}$. Physically, all Newtonian mass is treated as entirely within the sphere of radius r , whereas the mass of the primordial gravitational field is based only on the portion of the field within the sphere. For local mass density ρ the interaction energy density of the field is $\mathbf{j} \cdot \mathbf{A}$ where $\mathbf{j} = \rho\mathbf{v}$. Heaviside current density \mathbf{j} is momentum density $\mathbf{p} = \rho\mathbf{v}$; the interaction density of the field is $\mathbf{p} \cdot \mathbf{A} = \mathbf{p} \cdot \mathbf{v} = \rho v^2$. Analogous with electrodynamics, momentum $\boldsymbol{\pi}$ is defined in terms of the Lagrangian \mathcal{L} as

$$\boldsymbol{\pi} = \frac{\partial \mathcal{L}}{\partial \frac{d\mathbf{x}}{dt}} = \mathbf{p} + \rho \mathbf{A}(x). \quad (7)$$

such that in $\boldsymbol{\pi} \partial \mathbf{A} = \partial \mathcal{L}(t) = \mathbf{p} \partial \mathbf{A} + \rho \mathbf{A} \partial \mathbf{A}$ every term has dimensions of kinetic energy density.

We are initially interested in gravito-statics, wherein the final state of the particle with fixed mass, charge, and spin, will not change unless affected by external fields. Jefimenko [10] derives relevant expressions for the C-field:

$$\mathbf{C} = -\left(\frac{g}{c^2}\right) \int \left\{ \frac{[\mathbf{p}]}{r^3} + \frac{1}{r^2 c} \frac{\partial [\mathbf{p}]}{\partial t} \right\} \times \mathbf{r} \, dv' \quad (8)$$

where $[\mathbf{p}]$ is the retarded source current density distribution in volume element dv' and r is the distance to the field point at which the field is calculated. Thus, the field \mathbf{C} is produced by continuous mass distributions. For a mass point moving with velocity \mathbf{v} and acceleration \mathbf{a} he obtains:

$$\mathbf{C}_a = -\frac{\mathbf{G} \times [\mathbf{r}]}{c[r]} \quad (9)$$

where $[\mathbf{r}]$ is the retarded position of vector of the moving mass point. For a point mass moving without acceleration

$$\mathbf{C}_v = -\frac{\mathbf{G} \times \mathbf{v}}{c^2}. \tag{10}$$

Heuristically, since $\nabla \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{G}$ we let $\mathbf{G} \approx \nabla$ and $c = 1$ and obtain for the unaccelerated mass the expression $\mathbf{C}_v = -\nabla \times \mathbf{v}$, which, expressed as the gauge field, is $\mathbf{C}_v = -\nabla \times \mathbf{A}$. Observe that

$$\mathbf{C}_v \sim \frac{\partial}{\partial t} \mathbf{C}_a \Rightarrow -\nabla \times \mathbf{A} \sim \frac{\partial}{\partial t} (\nabla \times \mathbf{r}). \tag{11}$$

The gravitomagnetic fields presented in terms of the gauge velocity field, $\mathbf{A} = \mathbf{v}$ yield

$$\nabla \times \mathbf{A} = \mathbf{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ v_x & v_y & v_z \end{bmatrix} = \begin{pmatrix} \hat{i}(\partial_y v_z - \partial_z v_y) \\ \hat{j}(\partial_z v_x - \partial_x v_z) \\ \hat{k}(\partial_x v_y - \partial_y v_x) \end{pmatrix} = \begin{pmatrix} \mathbf{C}_x \\ \mathbf{C}_y \\ \mathbf{C}_z \end{pmatrix}. \tag{12}$$

C-field circulation has angular momentum analogous to the angular momentum contained in the magnetic field, experimentally proved in 1915 by Einstein & deHaas [11]. To demonstrate this for $g = c = 1$: $\nabla \times \mathbf{C} = -(\mathbf{g}/c^2)\rho\mathbf{v} = -\rho\mathbf{v}$ for $\dot{\mathbf{G}} = 0$. Next, multiply both sides by inverse curl operator [12] $(\nabla \times)^{-1} = (\mathbf{r} \times)$ to obtain $(\nabla \times)^{-1}(\nabla \times)\mathbf{C} = -(\mathbf{r} \times)\mathbf{p}$ where $\mathbf{p} = \rho\mathbf{v}$ is momentum density. Hence $\mathbf{C} \sim -\mathbf{r} \times \mathbf{p} = \mathbf{L} =$ angular momentum density. Since $\mathbf{C} \sim \mathbf{p} \times \mathbf{r}$ we find

$$\mathbf{C} \sim \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ p_x & p_y & p_z \\ x & y & z \end{bmatrix} = \begin{pmatrix} \hat{i}(p_y z - p_z y) \\ \hat{j}(p_z x - p_x z) \\ \hat{k}(p_x y - p_y x) \end{pmatrix} \overset{\text{proportional to}}{\hat{p} = -i\hbar\nabla} \begin{pmatrix} \hat{i}(\partial_y z - \partial_z y) \\ \hat{j}(\partial_z x - \partial_x z) \\ \hat{k}(\partial_x y - \partial_y x) \end{pmatrix} \tag{13}$$

From the quantum correspondence principle, $\hat{p} = -i\hbar\nabla$ with $\hbar = 1$, we obtain Equation (13) when the quantum source is accelerated; we find

$\nabla \times \mathbf{A} \approx \frac{\partial}{\partial t} (\nabla \times \mathbf{r})$ where the left-hand term represents constant velocity, while the right side is accelerated. Thus, for the magneto-static analog we use this form to interpret the Abelian form of the field strength:

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} \tag{14}$$

Analogous to electrodynamics we let $A = -\phi + \mathbf{A}$ and $\mathbf{A} = \mathbf{v}$ and evaluate $F_{\mu\nu}$.

$$\begin{aligned} F_{00} &= -\frac{\partial\phi}{\partial t} - \left(-\frac{\partial\phi}{\partial t}\right) \equiv 0, & F_{01} &= -\frac{\partial\phi}{\partial x} - \frac{\partial v_x}{\partial t}, & F_{02} &= -\frac{\partial\phi}{\partial y} - \frac{\partial v_y}{\partial t}, \\ & & F_{03} &= -\frac{\partial\phi}{\partial z} - \frac{\partial v_z}{\partial t} \\ F_{0v} &= F_{00} + F_{0i} \Rightarrow -\nabla\phi - \frac{\partial\mathbf{v}}{\partial t} \Rightarrow \mathbf{G} = -\nabla\phi \\ F_{10} &= \frac{\partial v_x}{\partial t} - \frac{\partial(-\phi)}{\partial x} \Rightarrow (\mathbf{G})_x + (\nabla\phi)_x \end{aligned} \tag{15}$$

$$F_{11} = \frac{\partial v_x}{\partial x} - \frac{\partial v_x}{\partial x} = 0, \quad F_{12} = \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = -C_z, \quad F_{13} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} = C_y$$

$$F_{20} = \frac{\partial v_y}{\partial t} - \frac{\partial(-\phi)}{\partial y} \Rightarrow (\mathbf{G})_y + (\nabla\phi)_y$$

$$F_{21} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = C_z, \quad F_{22} = \frac{\partial v_y}{\partial y} - \frac{\partial v_y}{\partial y} = 0, \quad F_{23} = \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} = -C_x$$

The $F_{3\nu}$ follows cyclically. The field strength matrix constructed from the above is shown:

$$F_{\mu\nu} = \begin{bmatrix} 0 & G_x & G_y & G_z \\ G_x & 0 & -C_z & C_y \\ G_y & C_z & 0 & -C_x \\ G_z & -C_y & C_x & 0 \end{bmatrix} \quad (16)$$

Recall that in classical electromagnetic theory the Lagrangian density in vacuum is given by the term $\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu})$ which yields $\mathcal{L} = E^2 - B^2$, the energy density of the \mathbf{E} and \mathbf{B} fields. Let us calculate the corresponding term for gravitomagnetism for Equation (16):

$$F_{\mu\nu}F^{\mu\nu} = G_x G_x + G_y G_y + G_z G_z - C_x C_x - C_y C_y - C_z C_z \quad (17)$$

Thus, the gravitomagnetism Lagrangian density is

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu}) = \mathbf{G}^2 - \mathbf{C}^2. \text{ Observe that these terms appear in Equation (5a)}$$

as the energy density of the primordial field supplying the mass density for Newton's equation. Rather than work through the indices as above, we also observe that the momentum energy density of the Poynting-like vector $\mathbf{G} \times \mathbf{C} \pm \mathbf{C} \times \mathbf{G}$ appears in (5c) as the source of the C-field circulation.

4. Angular Momentum in Heaviside Gauge Formalism

Linking to our primordial field $\psi = \mathbf{G} + i\mathbf{C}$ gravitomagnetic gauge field ν corresponds to gauge field \vec{A} for electromagnetic field $\mathbf{E} + i\mathbf{B}$. The primary focus on gauge field theory for the last half century has been Yang-Mills 1954-gauge equation treatment [13] in which they introduce and discuss isotopic spin "angular momentum", in quotes because they are unsure what it means physically. They adapt Pauli's $SU(2)$ spin matrices to Heisenberg's *isospin*; a mathematical formalism applied to an abstract *internal* symmetry. The nature of spin, at least classically, is rotation, and rotation in 3D space entails angular momentum. Exactly what is entailed in the space of internal symmetry is unknown. However, the nature of this gauge field is captured by the curl operation, so it must somehow entail an analog of angular momentum, as Einstein and de Haas showed to be possessed by the magnetic field. The Pauli spin matrix algebra is given by

$$\sigma_j \sigma_k = -\delta_{jk} - i\epsilon_{jkl} \sigma_l, \quad (18)$$

where $\{\sigma_x, \sigma_y, \sigma_z\}$ represent the 2×2 Pauli spin matrices of quantum mechanics. In three dimensions we can construct an orthonormal bivector basis based on three orthogonal bivectors, $\{\beta_x, \beta_y, \beta_z\}$. The algebra of the bivectors satisfies $\beta_x \beta_y = -i\beta_z$ specifically, and more generally the bivector algebra,

$$\beta_j \beta_k = -\delta_{jk} - i\epsilon_{jkl} \beta_l \tag{19}$$

with Kronecker delta δ_{jk} and Levi-Civita alternating symbol ϵ_{jkl} . *Bivector algebra is identical to Pauli spin matrix algebra, by inspection.* Since the algebras are identical, their physical implications should be the same. There is no well-defined idea of isotopic spin “angular momentum”, however gravitomagnetic C-field possesses angular momentum; and is proportional to angular momentum: $C = (g/c^2) \mathbf{r} \times \mathbf{p}$ with dimension t^{-1}/l^3 . This is depicted for $F_{\mu\nu}$ in **Figure 1**.

In **Figure 1** we pair C_y with $-C_y$, and cyclical iterations, where the index represents the axis about which these components of the field rotate. We could have labeled the C_z components as $-C_{xy}$ and C_{yx} denoting their position in the (row,col) representation. In other words, the formalism contains the angular momentum aspect of the components. The C-field components are compatible with the three bivectors shown in the 3-space representation at the right, defined by the $x, y,$ and z axes (vectors) and 100% compatible with bivector algebra (Equation (19)) which is identical to the $SU(2)$ Pauli matrix algebra (Equation (18)). It is also compatible with the *Gravity Probe B* experiment that proved the existence of the C-field circulation induced by the rotation of the planet Earth’s mass density. *The nature of C-field circulation, from every perspective, is angular momentum.*

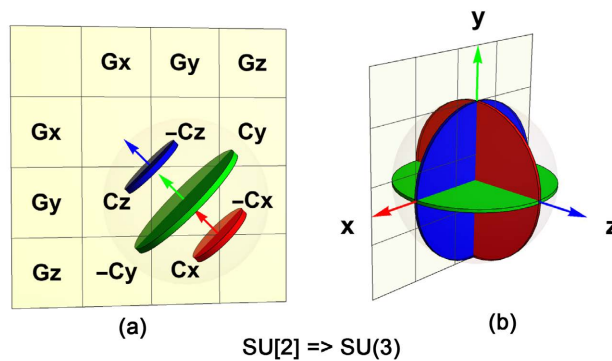


Figure 1. (a) The circulating field, the C-field, can be labeled by the (row,col) component or by the orthogonal axis about which the (row,col) component circulates. For example, the (x,z) element is labeled C_y and the (z,x) element is labeled $-C_y$, since both of these terms rotate about the y -axis; similarly for the other components. These rotations are shown abstractly in the representation of the field strength $F_{\mu\nu}$ matrix on the left. (b) The right-hand illustration maps the three bivector diagrams into 3-space. Colors are used for visual convenience and for suggested correlation with $SU(3) \times SU(2) \times U(1)$ symmetry.

5. Comparison with “Linearized Gravity” Derivations and Other Formulations

A recent article [14] on helicity and spin conservation is based on Barnett’s [15] Maxwell-like formulation of linearized gravity, *i.e.*, the “weak field approximation” to Einstein’s nonlinear field equation. There are times when this is appropriate: in [16] I analyze the Kasner metric, an *exact* solution to Einstein’s equation, in terms of the Heaviside equations derived from the self-interaction equation. As the Kasner metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is (expressed in Narlikar’s and Karmarkar’s formulation)

$$ds^2 = c^2 dt^2 - \sum_{j=1}^{D-1} (1 + nt)^{2p_j} dx_j^2 \quad (20)$$

subject to constraints on p_j , it is necessary to relate Heaviside’s interpretation in flat space to the $g_{\mu\nu}$ representation in curved space, via the linearized expression

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (21)$$

Barnett, in his “*Maxwellian theory of gravitational waves...*” begins with the Riemannian curvature of Einstein’s field equations and presents a Maxwellian theory based on analogies with Maxwell’s theory of electromagnetism. His conclusion is that this analogy is beneficial, however.

“To reach such a description we have had to forego some of the ‘generality’ of general relativity, most particularly in that our description is very much of a gravitational field in a flat background spacetime.”

He asks the reader to judge whether this is a price worth paying. This is conceptually different from our approach: we *assume* a flat background and find that the self-interaction equation leads directly and almost immediately to the Heaviside formulation of gravity which is strongly analogous to Maxwell’s theory. In other words, our derivation is precise, based on an *exact* principle of self-interaction; Barnett’s Maxwellian is a “flat-space approximation” to Einstein’s spacetime curvature, which is based on the *approximate* principle of equivalence.

If physical reality is based on a field in flat space, our approach is not only simpler, but probably much more accurate. In referenced work [17] [18] [19] [20] I argue that curved spacetime geometry is an abstraction, and agree with Feynman, Weinberg, Padmanabhan and others that this concept of curved space geometry is not at all necessary to a theory of gravity. This goes against the grain of modern teaching but may nevertheless be the reality.

The gravitomagnetic approach resembles geometric algebra-based treatments of Maxwell’s equations in (3+1)D. Arthur [21] develops (3+1)D and 4D models in detail. Although Hestenes’ development of Clifford algebra is the most complete and widely used version, alternate formulations exist, such as [22] wherein a Clifford algebra with $i_j^2 = -1$ is used. One issue is the transformation rules of the second-rank antisymmetric electromagnetic field tensor. Our derivation of the gravitomagnetic equations yields the equivalent of a second-rank that is *not* an antisymmetric field tensor, which I believe to be correct.

As an example of advantages to be gained from a powerful new mathematical framework, I briefly follow Arthur’s treatment based on the (3+1)D-Maxwell equation $(\nabla + \partial_t)F = J$ where F is the field tensor and J is the source, $J = \rho(1 + v)$. If we multiply both sides by $(\nabla - \partial_t)$ we obtain

$$(\nabla^2 - \partial_t^2)F = (\nabla - \partial_t)J. \tag{22}$$

In free space, let $J = 0$ and Equation (22) becomes the wave equation, which, in terms of a plane wave, reduces to $(-k^2 + \omega^2)F = 0$. Let us make use of natural units $g = c = \hbar = 1$ and the quantum equivalents: momentum $p = \hbar k$ and $E = \hbar\omega = \hbar C$, in which case

$$(-k^2 + \omega^2) \Rightarrow -p^2 + C^2 = 0 \tag{23}$$

For unit mass this implies

$$\begin{array}{ccc} p^2 = C^2 & & \langle \text{energy densities} \rangle \\ / \quad \backslash & & \\ \text{kinetic} & \text{C-field} & \end{array} \tag{24}$$

In other words, kinetic energy, which is the first thing physicists learn about in high school, and which is never described with more clarity than “*energy of motion*”, appears to be physically represented by the energy of the gravitomagnetic circulation induced by the momentum p . Almost every energy in physics is associated with a potential or energy field—kinetic energy may be unique in having no field correlate. The Heaviside theory of gravitomagnetism appears to imply that the essentially undefined mechanism of storage of *energy of motion* is actually C-field circulation energy, bringing our most basic energy into agreement with all other field energies. This is only one example of physical insight that follows from an appreciation of the fundamental notion of the “all strength” derivation from the *Self-interaction Principle*.

6. Summary

Our goal has been to formulate and interpret the gauge field associated with Heaviside’s equations. In previous papers, I have shown that the derivation of Heaviside’s equations from an exact principle, the *Self-Interaction Principle*, is equivalent to Einstein’s nonlinear field equations derived from the *Equivalence Principle*, and have treated general relativity-based problems such as *Quasi-Local Mass*. The key issue that distinguishes Heaviside from Einstein is that the Heaviside derivation is *field-strength independent*, whereas Einstein’s derivation implies that Heaviside is a “weak field approximation”. If this is an error, as is implied by our derivation, it is an error that misled physicists for a century.

A *mass-based* understanding of gravity, combined with the *weak field approximation* misnomer, has caused physicists to generally ignore gravity in particle physics. A *mass-density-based* understanding of gravity leads to a fuller appreciation of a gravitational basis for particle physics, since mass densities associated with the big bang are effectively limitless. Huang [23], Volovik [24], and others view the primordial field as a *superfluid*. Circa 2006 physicists at the LHC were

expecting a *quark gas* from heavy-ion collisions but instead [25]: “*It is well known that the properties of the Yang-Mills plasma turned out to be unexpected... the plasma is similar rather to an ideal liquid than to a gluon gas interacting perturbatively.*”

This establishes an analogy between phenomena in Yang-Mills theory with physics of *superfluidity*. Our underlying premise has been the *superfluid nature of the primordial field*, with ultra-dense fields. The Heaviside equations are the prototype of Yang-Mills gauge theory. There is no self-interaction of the electromagnetic gauge field, which interacts with charge. If Heaviside had been aware of $E = mc^2$ in 1893, it is possible that he would have invented the Yang-Mills gauge equation 60 years before Yang and Mills, which would probably have led to a very different physics in the 20th century. Instead, gravity theory spent over 100 years in the curved space arena.

In 1919, Einstein wrote “*Do gravitational fields play an essential part in the structure of the elementary particles of matter?*”, showing the possibility of a theoretical construction of matter out of gravitational field and electromagnetic field alone. The implication of the strength-free derivation of Heaviside’s equation, is that we may finally get a chance to answer this question. Since gravity is *density*-based, not *mass*-based, this potentially opens realms of physics to gravitational phenomena that have been overlooked since Newton.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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