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The Mathematical Model of Short-Term Forest Fire Spread

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Abstract

In this paper, we establish a mathematical model of the forest fire spread process based on a partial differential equation. We describe the distribution of time field and velocity field in the whole two-dimensional space by vector field theory. And we obtain a continuous algorithm to predict the dynamic behavior of forest fire spread in a short time. We use the algorithm to interpolate the fire boundary by cubic non-uniform rational B-spline closed curve. The fire boundary curve at any time can be simulated by solving the Eikonal equation. The model is tested in theory and in practice. The results show that the model has good accuracy and stability, and it's compatible with most of the existing models, such as the elliptic model and the cellular automata model.

Keywords

Forest Fire Spread Model, Spatial Velocity Field, Eikonal Equation, Dynamic Simulation, Non-Uniform Rational B-Spline

1. Introduction

Forests are precious resources for human beings and the earth. They can filter water, purify air, and nurture an environment rich in biodiversity. Unfortunately, forest fires have become one of the worst natural disasters [1]. Every year, millions hectares of forests and man-made infrastructures are destroyed by forest fires, and countless lives are threatened and injured [2]. Forest fire prevention has become a global problem. In addition to monitoring methods, it is particularly important to establish prediction models and simulate the dynamic process of forest fire spread so as to make better decisions about forest fire prevention, control, and suppression.

At present, the common forest fire spread models in the world can be divided into three categories: empirical models, statistical models and physical models [3] [4] [5]. And according to the different modeling principles and factor settings, the above three types of spread models can be subdivided into various sub-models.

The elliptical model [6] refers to the ellipse shape of the fire field spread by the fire point under the interaction of wind and terrain. The fire point is located at the focus of the ellipse. The direction of resultant velocity is parallel to the direction of the major axis. American scholar Rothermel [7] [8] used the thermodynamics theory to deduce the differential equation of temperature field. This model is highly theoretical and difficult to calculate as it assumes that combustibles and terrain are continuously distributed in space. Canadian national forest fire spread model [9] [10] [11] does not consider any heat transfer mechanism, but is only a statistical model summarized from the observation of nearly 300 fires of 16 combustibles. It is confirmed that the gain (or loss) effect of terrain on the propagation rate is not a linear relationship of harmonic function. McArthur's model is obtained by many ignition tests according to the actual situation in Australia, but the scope of application of this model is limited to eucalyptus forest and grassland.

In China, Wang [12] conducted a regression analysis on the relevant data of the Liangshan Yi Autonomous Prefecture of Sichuan Province fires. It took into account the influence of slope and wind direction on the spread rate, and the modeling data came from China. Therefore, Wang's model can best fit the real situation of Chinese forest fires. To reduce the model error, Mao [13] [14] used the formula obtained by Lawson in the experiment to replace the terrain correction term in Wang's model, and also derived the calculation formula of the spread rate in the up and down slope, left and right flat slope and wind direction. Tang [15] proposed to combine the above five directions, together with the spreading rates in the upper left, upper right, lower left, and lower right slope directions. Hence, the data structure of the octree can be established, and the cellular automata model can be used to simulate the fire field through the maze algorithm or the boundary interpolation algorithm. Ma [16] worked out with the calculation formulas of Tang's model in the other four directions. Thus, the Wang-Mao combination model (W-M model) has become an advanced model of calculating forest fire spread rate in China, and it may be implemented in business. The realization algorithms are mostly based on the cellular automata algorithms [17]-[25]. The essence of Wang's model is the variation law of the spread rate with the terrain, wind speed and types of combustibles. The prediction and simulation of the fire boundary includes the spread rate and direction, which are the advantages of combining cellular automata and other models with Wang's model.

2. Improvement of Terrain Correction Coefficient

The formula of W-M model is

$$R = R_0 K_S K_\varphi K_W = R_0 K_S \mathrm{e}^{3.533 \tan^{1.2} \varphi} \mathrm{e}^{0.1783V}, \tag{1}$$

where R is the spreading rate and R_0 is the initial spread rate on windless flat ground, K_S is the correction coefficient of combustible configuration pattern, K_φ is the terrain correction coefficient, K_W is the wind speed correction coefficient, φ is the slope angle, and V is the wind speed (m/s).

According to Equation (1), Ma's model projects the slope angle and wind direction to the desired direction to calculate the spread rate in this direction. Although it is reasonable to calculate the projection value of wind speed, it is biased to use $K_\varphi = \mathrm{e}^{3.533 \tan^{1.2} (\varphi \cos \vartheta)}$ to calculate the projection value of slope angle in [16]. Since the effect of the projection transformation on the angle is not linear, the projected image of the angle cannot be expressed as a multiple of the original image, which leads to inaccurate calculation of the angle projection. In order to solve this problem and expand the calculation formula of forest fire spread speed to any direction, we make the following improvements.

We specify that the counterclockwise direction is the positive direction of the oriented angle. The angle between the slope and the horizontal plane is called the main slope angle. And the angle between the tangent vector along the slope and the horizontal plane is called the inclined slope angle. Next, we find the inclined slope angle after rotating $\vartheta \in (-\pi, \pi]$ around the uphill direction.

As shown in **Figure 1**, $\varphi \in [0, \pi/2]$ is the main slope angle, $\overrightarrow{OP}$ is the tangent vector along the uphill direction $\overrightarrow{OU}$ on the slope. Then $\angle P'OQ'$ is the required inclined slope angle, which is denoted as ϕ. We have

$$\tan \phi = \frac{P'Q'}{OQ'} = \frac{A_3 A_1}{\frac{OA_1}{\cos \vartheta}} = \tan \varphi \cos \vartheta.$$

Therefore, it is not appropriate to use the projection of the slope angle in Equation (1), but to use the projection of the slope, namely, $K_\varphi = \mathrm{e}^{3.533 (\tan \varphi \cos \vartheta)^{1.2}}$. When $\vartheta \in (-\pi, -\pi/2) \cup (\pi/2, \pi]$, $\overrightarrow{OQ'}$ points downhill and the exponent part of

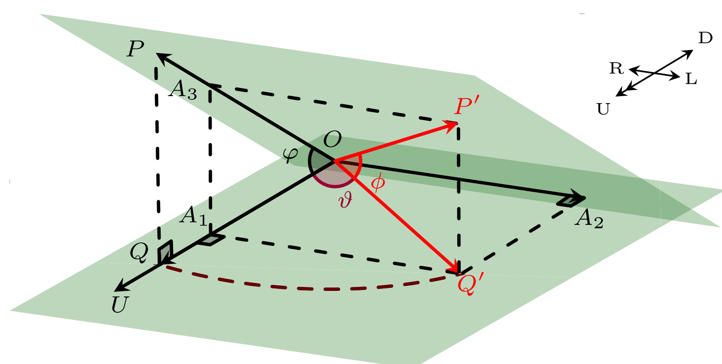

Figure 1. Determination of slope angle.

the Equation (1) should be negative. We take the angle turned counterclockwise in the upslope direction $\overrightarrow{OU}$ to coincide with the wind direction as the wind direction angle. According to Ma's model, the spreading rate of any angle ϑ deviating from the upslope direction can be calculated by

$$R = R_0 K_S K_\varphi K_W = R_0 K_S \mathrm{e}^{3.533(\tan\varphi\cos\vartheta)^{1.2}\cos\vartheta}\mathrm{e}^{0.1783V\cos(\vartheta-\theta)}, \quad (2)$$

where θ is the wind direction angle.

3. Establishment of Space Velocity Field

Since the flame always spreads from the burning area to the unburned area, the boundary of the burning area is a closed curve. It can be assumed that the spread speed at any point always follows the outer normal direction, and its rate is determined by Equation (2). In fact, even if the actual spread direction is not the outer normal direction, only the component in the outer normal direction can really expand the burning area. We can always orthogonally decompose the velocity into normal and tangential component velocities.

As time goes by, the flame boundary continues outward at the rate $v(x,y) \geq 0$. For any time τ, there is always a boundary curve corresponding to it, and the fire area at this moment must include the fire area at the previous moment. Notice that all points (x,y) falling on the same boundary curve burn at the same time. Therefore, the boundary curve is an isochronous line.

Denote the moment when the flame spreads to any point (x,y) in the plane as $u(x,y)$. We say that $u(x,y)$ determines a time field if the equation

$$u(x,y) = \tau \quad (3)$$

represents the boundary curve equation at the moment τ. Before solving this equation, it is necessary for us to study the motion law of points on the boundary.

At the moment τ, a moving point $A = (x,y)$ is taken as the object to study on the boundary curve of the fire area. With the extension of the boundary curve, point A moves based on the following rules:

1) The motion at any time always follows the outer normal direction of the present boundary curve at point A, namely, the direction of $\nabla u(A)$;

2) The rate of spread is determined by Equation (2), namely, $v(A) = R$.

The above rules can be summarized as

$$\boldsymbol{v}(A) = R\boldsymbol{e}_{\nabla u(A)}. \quad (4)$$

Each point $A = (x,y)$ corresponds to a unique speed $\boldsymbol{v}(x,y)$ in the plane, and the direction of $\boldsymbol{v}(x,y)$ is the tangent direction of the trajectory l_A of A, as shown in **Figure 2**. Therefore, there exists a vector field (can be regarded as velocity field) uniquely determined by Equation (4), which describes the velocity distribution of each point on the whole plane.

Since the field line at any point is perpendicular to the tautochrone curve, we

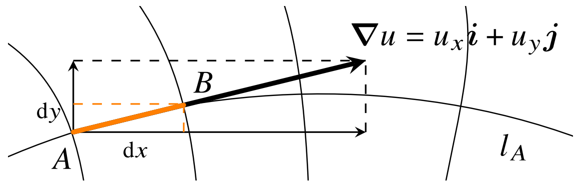

Figure 2. Motion analysis of point A on field line l_A.

assume that A moves along the field line l_A for a short distance $\mathrm{d}s$ and reaches $B=(x+\mathrm{d}x, y+\mathrm{d}y)$ within a small period of time $\mathrm{d}\tau$. Then, we have

$$\frac{\mathrm{d}x}{u_x}=\frac{\mathrm{d}y}{u_y}. \tag{5}$$

Therefore, the equation of the field line l_A can be determined by the field line family given by the Equation (5) and the initial value condition $A\in l_A$.

Take the fire spread rate $v(x,y)$ at $A=(x,y)$ as the average rate of fire spread within the distance of $\mathrm{d}s$. According to the relationship between distance and time, we have

$$\mathrm{d}\tau=\frac{\mathrm{d}s}{v(x,y)}=\frac{\sqrt{(\mathrm{d}x)^2+(\mathrm{d}y)^2}}{v(x,y)}. \tag{6}$$

By Equation (3), the differential of $\tau=u(x,y)$ is

$$\mathrm{d}\tau=u_x\mathrm{d}x+u_y\mathrm{d}y. \tag{7}$$

By Equations (5) to (7), we obtain

$$u_x^2+u_y^2=\frac{1}{v^2(x,y)}. \tag{8}$$

This is an Eikonal equation (see ([26], P111-120) and ([27], P62-101, P203-206)). If it is known that the boundary curve equation at the initial moment $\tau=0$ can be expressed as the parametric equation of the form

$$u[f(r),g(r)]=0, \tag{9}$$

where r is a parameter, then by Equations (8) and (9), a solution of the form Equation (3) can be determined by completing the following steps.

Let $p=u_x$, $q=u_y$, and transform Equation (8) into

$$F(x,y,u,p,q):=\frac{1}{2}p^2+\frac{1}{2}q^2-\frac{1}{2}v^{-2}=0.$$

Without loss of generality, we consider the general form of a non-linear partial differential equation of first order

$$F(x,y,u,p,q)=0. \tag{10}$$

Let $G(r)=u[f(r),g(r)]$, and find the functions $h(r)$ and $k(r)$ such that

$$\begin{cases} F[f(r),g(r),G(r),h(r),k(r)]=0, \\ G'(r)=h(r)f'(r)+k(r)g'(r), \\ F_p[f(r),g(r),G(r),h(r),k(r)]g'(r)\neq F_q[f(r),g(r),G(r),h(r),k(r)]f'(r). \end{cases}$$

For each fixed r, solve the system of characteristic equations

$$\begin{cases} \dfrac{\partial X(r,t)}{\partial t} = F_p[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)], \\ \dfrac{\partial Y(r,t)}{\partial t} = F_q[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)], \\ \dfrac{\partial U(r,t)}{\partial t} = P(r,t)F_p[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)] \\ \qquad\qquad +Q(r,t)F_q[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)], \\ \dfrac{\partial P(r,t)}{\partial t} = -F_x[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)] \\ \qquad\qquad -P(r,t)F_u[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)], \\ \dfrac{\partial Q(r,t)}{\partial t} = -F_y[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)] \\ \qquad\qquad -Q(r,t)F_u[X(r,t),Y(r,t),U(r,t),P(r,t),Q(r,t)], \end{cases}$$

with initial conditions $P(r,0)=h(r)$ and $Q(r,0)=k(r)$, where $X(r,t)$, $Y(r,t)$ and $U(r,t)$ are the solutions with initial conditions $X(r,0)=f(r)$, $Y(r,0)=g(r)$ and $U(r,0)=G(r)$. Then the parametric solution of Equation (10) is $x=X(r,t)$, $y=Y(r,t)$ and $u=U(r,t)$. In particular, when $x=X(r,t)$ and $y=Y(r,t)$ are invertible, we obtain $r=R(x,y)$ and $t=T(x,y)$, and there is an explicit solution

$$u(x,y)=U[R(x,y),T(x,y)]. \tag{11}$$

Hence, the equation of boundary curve at any time τ can be determined by Equations (3) and (11).

4. Empirical Analysis and Results

Example 1. Let the fire spread rate be $v(x,y)=C$ in a certain area, the direction be outward, and the boundary curve of the initial fire area be $x^2+y^2=\varepsilon^2$, where C and ε are constants, and ε is sufficiently small. Find the burning boundary curve equation at any time τ.

Solution. Since $v(x,y)=C$, the partial differential Equation (8) becomes $u_x^2+u_y^2=C^{-2}$. Let $F(x,y,u,p,q)=\frac{1}{2}(p^2+q^2-C^{-2})$. Then the corresponding system of characteristic equations is

$$\begin{cases} X'(t)=P(t), \\ Y'(t)=Q(t), \\ U'(t)=P^2(t)+Q^2(t)=C^{-2}, \\ P'(t)=0, \\ Q'(t)=0. \end{cases}$$

Since the parametric equation of the initial boundary curve $x^2+y^2=\varepsilon^2$ is

$$\begin{cases} x=\varepsilon\cos r=f(r), \\ y=\varepsilon\sin r=g(r), \end{cases}$$

and the initial velocity distribution is $\boldsymbol{v}_0 = h(r)\boldsymbol{i} + k(r)\boldsymbol{j}$. So

$$\begin{cases} h^2(r)+k^2(r) = v^{-2}[f(r), g(r)], \\ h(r)f'(r)+k(r)g'(r) = 0. \end{cases}$$

Then,

$$\begin{cases} h^2(r)+k^2(r) = C^{-2}, \\ h(r)\varepsilon \sin r = k(r)\varepsilon \cos r. \end{cases}$$

We obtain the initial condition

$$\begin{cases} X(r,0) = f(r) = \varepsilon \cos r, \\ Y(r,0) = g(r) = \varepsilon \sin r, \\ U(r,0) = 0, \\ P(r,0) = h(r) = \pm C^{-1}\cos r, \\ Q(r,0) = k(r) = \pm C^{-1}\sin r. \end{cases}$$

Since $P'(t) = 0$ implies $P(t) = C_1(r)$ and notice that $P(r,0) = \pm C^{-1}\cos r = C_1(r)$, we have $P(r,t) = \pm C^{-1}\cos r$. Since $X'(t) = P(t) = \pm C^{-1}\cos r$, it follows that $X(t) = \pm C^{-1}t\cos r + C_2(r)$. By $X(r,0) = \varepsilon\cos r = C_2(r)$, we obtain $X(r,t) = (\pm C^{-1}t + \varepsilon)\cos r$. Similarly, we can prove that $Q(r,t) = \pm C^{-1}\sin r$ and $Y(r,t) = (\pm C^{-1}t + \varepsilon)\sin r$. Since $U'(t) = P^2(t) + Q^2(t) = C^{-2}$, it follows that $U(t) = C^{-2}t + C_3(r)$. As $U(r,0) = 0 = 0 + C_3(r)$, we have $U(r,t) = C^{-2}t$.

To summarize, the parametric solution of $u_x^2 + u_y^2 = C^{-2}$ is

$$\begin{cases} x = X(r,t) = (\pm C^{-1}t + \varepsilon)\cos r, \\ y = Y(r,t) = (\pm C^{-1}t + \varepsilon)\sin r, \\ u = U(r,t) = C^{-2}t. \end{cases}$$

By eliminating the parameters r and t, the burning boundary curve equation at any time τ is obtained as

$$x^2 + y^2 = (C\tau + \varepsilon)^2.$$

Figure 3 is the change of simulated fire field boundary in the situation of $C = \varepsilon = 1$. □

According to **Example 1**, when $\varepsilon \to 0$, the burning boundaries spread by the fire point are concentric circles with the ignition point as the center, which is consistent with the numerical simulation result of Du [28] based on meshing.

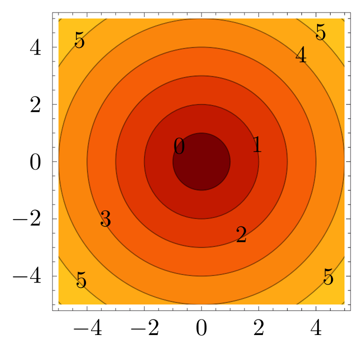

Figure 3. Simulation result of **Example 1**.

Example 2. We selected a coniferous and broad-leaved mixed forest area of 8 km × 10 km in Qingyang County in May 2000, which was studied by Qin [29] [30]. There was no precipitation in the forest area in the past month, the relative humidity was less than 30%, the daily maximum and minimum temperature were 30°C and 14°C respectively, and the herb layer was dry and dense. On a certain day, the wind direction was due south, and the wind speed was 1.8 m/s.

Solution. According to the given data and Wang's initial velocity calculation formula, the initial velocity of forest fire spread is

$$R_0 = 0.0299T_{\max} + 0.047W_{\text{noon}} + 0.009(100 - h_{\min}) - 0.304 = 1.317 \text{ m/s}.$$

By Equation (2), $R = R_0 K_S K_\varphi K_W$, we densely sample the data of forest elevation H and coefficient K_S, and use cubic spline interpolation to construct surface function graphs, as shown in **Figure 4**.

According to the analysis of **Figure 2**, there is $\overrightarrow{OQ'} \parallel (u_x, u_y)$ in **Figure 1**. Let $m = \frac{\partial H}{\partial x}$, $n = \frac{\partial H}{\partial y}$, $p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$ and $\boldsymbol{q}' = (p, q)$. Then $\boldsymbol{p}' \parallel \overrightarrow{OP'}$ with $\boldsymbol{p}' = (p, q, pm + qn)$. Denote $\boldsymbol{e}_3 = (0, 0, 1)$. Since

$$\sin\phi = \cos\langle \boldsymbol{p}', \boldsymbol{e}_3 \rangle = \frac{\boldsymbol{p}' \cdot \boldsymbol{e}_3}{|\boldsymbol{p}'| \cdot |\boldsymbol{e}_3|} = \frac{pm + qn}{\sqrt{p^2 + q^2 + (pm + qn)^2}},$$

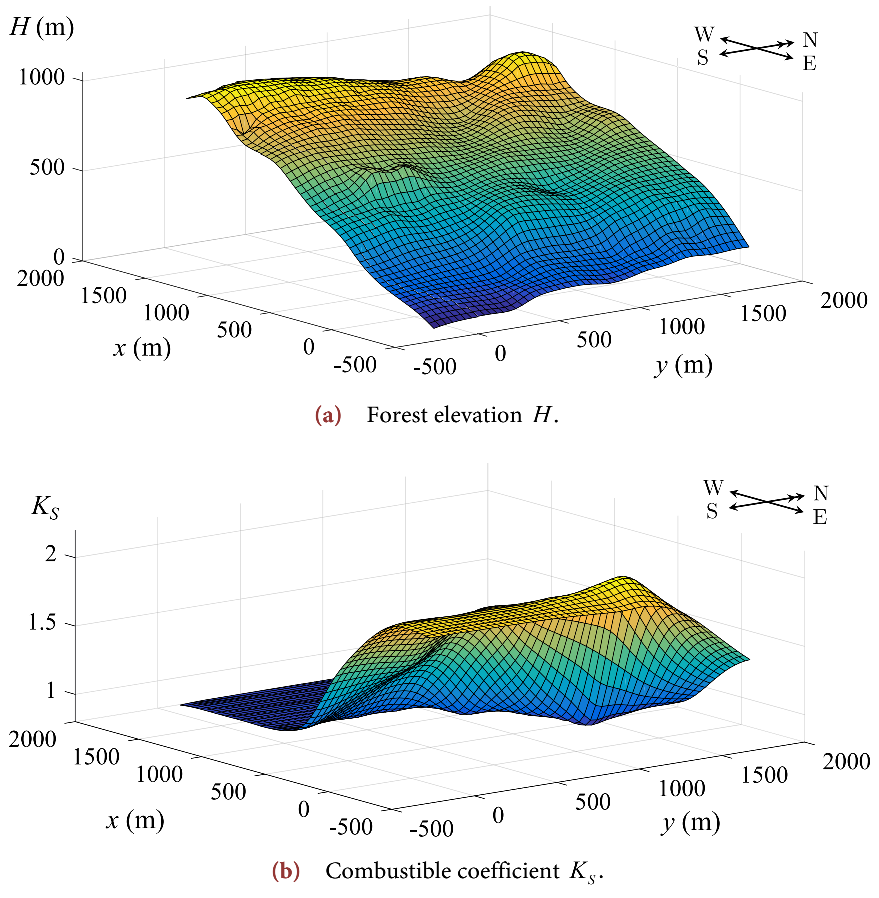

Figure 4. Distribution of coefficients in a forest area of Qingyang County.

we have $\tan\phi = \frac{pm+qn}{\sqrt{p^2+q^2}}$. As $\varphi \in [0, \pi/2]$ in Equation (2), we have $\operatorname{sgn}\cos\vartheta = \operatorname{sgn}\tan\phi = \operatorname{sgn}(pm+qn)$.

Let the coordinates of the wind speed V in the geographic coordinate system (O-NSWE) be (a,b), and the coordinates in the slope coordinate system (O-UDLR) be (a',b'). In the geographic coordinate system, the two orthogonal unit vectors are $\boldsymbol{e}_1 = (1,0)$ and $\boldsymbol{e}_2 = (0,1)$. Since $\overrightarrow{OU} \parallel (m,n)$, the unit vector in the direction of $\overrightarrow{OU}$ is $\boldsymbol{e}_1' = \frac{(m,n)}{\sqrt{m^2+n^2}}$, and the unit vector in the orthogonal direction is $\boldsymbol{e}_2' = \frac{(-n,m)}{\sqrt{m^2+n^2}}$. Hence the transition matrix from geographic coordinate system to slope coordinate system is $\boldsymbol{A} = \frac{1}{\sqrt{m^2+n^2}}\begin{bmatrix} m & -n \\ n & m \end{bmatrix}$. Notice that $\boldsymbol{A}$ is an orthogonal matrix, then $\begin{bmatrix} a' \\ b' \end{bmatrix} = \boldsymbol{A}^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{m^2+n^2}}\begin{bmatrix} m & n \\ -n & m \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$, therefore

$$V\cos(\vartheta-\theta) = |\boldsymbol{V}|\cos\langle \boldsymbol{V}, \boldsymbol{q}'\rangle = \frac{\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} a' \\ b' \end{bmatrix}}{\sqrt{p^2+q^2}} = \frac{\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} m & n \\ -n & m \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}}{\sqrt{m^2+n^2}\sqrt{p^2+q^2}}.$$

For the convenience of computer numerical verification, we use the infinitesimal time slice method to replace the continuous diffusion process. Suppose that the initial burning boundary is a circle with a radius of 5 m, take $i=1$ and collect sample (x_i, y_i) densely. Combined with the above analysis, we solve the equations

$$\begin{cases} pu_y = qu_x \\ \frac{1}{\sqrt{p^2+q^2}} = R_0 K_s \mathrm{e}^{3.533\left(\frac{pm+qn}{\sqrt{p^2+q^2}}\right)^{1.2}\operatorname{sgn}(pm+qn)} \mathrm{e}^{0.1783\frac{\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} m & n \\ -n & m \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}}{\sqrt{m^2+n^2}\sqrt{p^2+q^2}}} \end{cases} \tag{12}$$

obtained from the variation of Equation (2). We can obtain the spread speed $\boldsymbol{v}_i = \frac{(p_i, q_i)}{p_i^2+q_i^2}$ of the *i*-th lap, and the spread boundary coordinates of the $(i+1)$-th lap is $(x_{i+1}, y_{i+1}) = (x_i, y_i) + \boldsymbol{v}_i t$. For the spread speed $\boldsymbol{v}_i$ here, we only select the solution pointing to the outer normal direction, and t is the size of time slice.

The next step is to filter out the singular points in the last lap of spread boundary closure caused by calculation accuracy error, and use cubic non-uniform rational B-spline (NURBS) closed curve [31] [32] to interpolate the new lap of fire boundary to obtain the equation $u(x,y) = i+1$. According to the perimeter of the new boundary, we choose a certain proportion to resample the boundary and calculate the next lap, so as to increase the sampling density of the outer lap. Finally, repeat the above steps and calculate iteratively until the specified number of laps. Mathematical software is used to implement and encapsulate the whole algorithm, as shown in **Algorithm 1**. The burning boundary development law is shown in **Figure 5**. □

Algorithm 1. Iterative algorithm.

Input: Maximum number of laps N
Output: Fire boundary spread map
1: **for** $i = 1, 2, \ldots, N$ **do**
2: Sample coordinates in the i-th lap
3: Solve the Equation (12)
4: Select speed
5: Filter out the singular points
6: Do NURBS interpolation
7: **end for**
8: **return** Fire boundary spread map

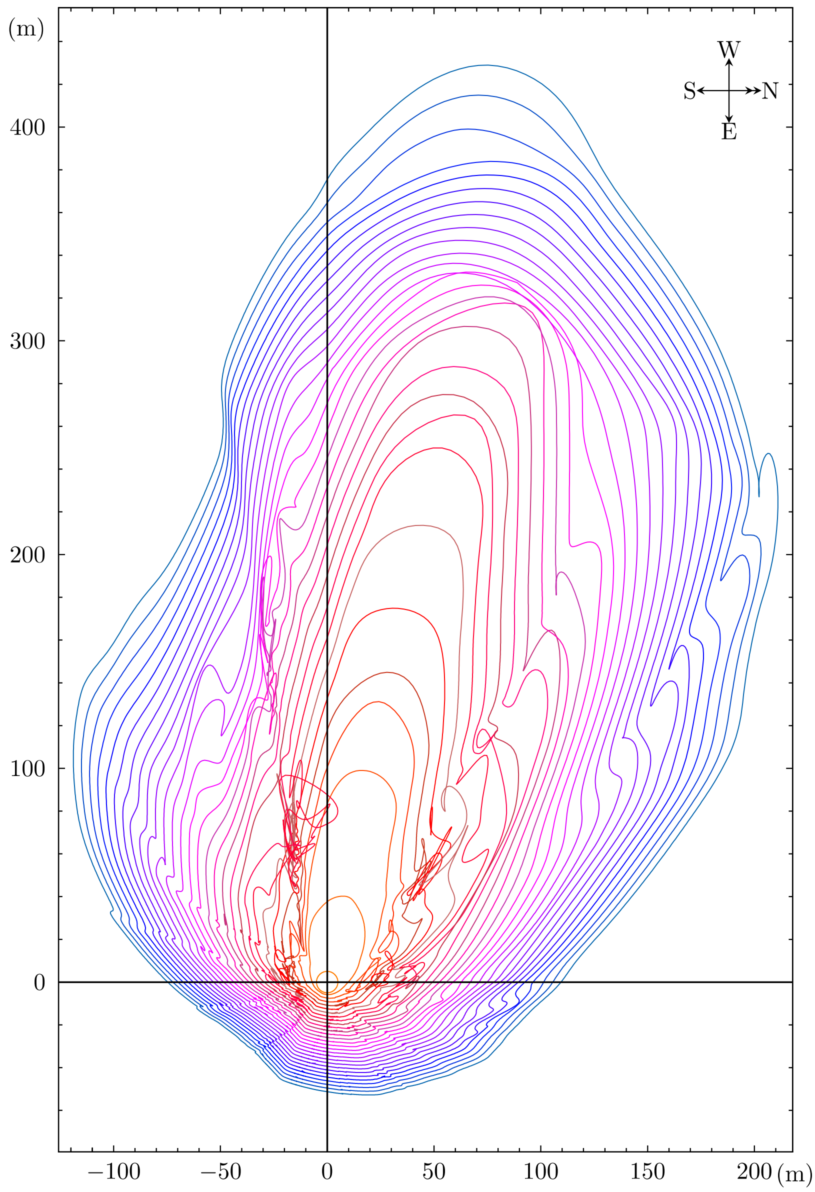

Figure 5. Fire boundary spread simulation.

It can be seen from **Figure 5**, under the condition that the first lap is approximately the fire point, the boundary of the second lap is approximately an ellipse, which is compatible with the elliptic model. After the 13th lap, the rate of upward expansion of the boundary slows down due to the location of the valley, where the slope becomes gentler and the effect of the slope on the rate is weakened, resulting in a decrease in the rate of spread. During this period, the effect of wind on the spread of the fire dominates. After the 26th lap, the speed of the upward spread increases again, but it is obviously slower than the speed before the 13th lap, because although it leaves the valley and goes uphill again, it can be seen from **Figure 4(b)** that the type of vegetation has changed, the combustibles coefficient has dropped significantly, and the slope is gentler, so the upward expansion is slower than it was before the 13th lap.

Figure 5 also shows the chaotic phenomenon of the calculated value of the boundary curve, as the terrain conditions are unstable and the time slice taken is too large, which causes individual points to move outward largely when calculating the next boundary. The step size is too large, and the adjacent points cannot follow this fluctuation, so the boundary curve will occasionally appear cluttered. These singular points can be filtered out in several loops of the **Algorithm 1**, thereby overcoming the instability of discrete terrain conditions, to keep the model stable. Generally, this phenomenon can be alleviated by reducing the time slice.

5. Conclusions

Based on the W-M model, we improve the terrain correction coefficient, and the spread of forest fire in the model is extended from eight directions to any direction. We describe the distribution of the time field and velocity field in the whole two-dimensional space, and develop a continuous algorithm suitable for short-term prediction of the dynamic behavior of forest fire spread. The boundary of fire field at any time can be simulated by calculating the Eikonal equation. When the algorithm is tested by mathematical curve in theory, the result is consistent with the numerical simulation based on meshing. When the algorithm is verified by the actual data, the result meets the expectations, which reveals effectiveness and reliability of the model.

Undoubtedly, there are some disadvantages of the model established in this paper. The spread rate correction coefficient of the model in this paper is determined by the actual environment, not a continuous quantity, nor can it be easily explained by an analytical expression. Therefore, when it is used to predict the approximate direction and speed of fire spread, there are certain error between the predicted result and the actual flame and it will accumulate along with iteration. Hence, this model is suitable for general understanding of the fire spread process, not for actual calculations. And the long-term forecast of forest fire spread is not feasible.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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