

# Nonlocal Theory of the Photon Gas Evolution Beginning from the Planck Time

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**How to cite this paper:** Alexeev, B.V. (2022) Nonlocal Theory of the Photon Gas Evolution Beginning from the Planck Time. *Journal of Applied Mathematics and Physics*, 10, 1443-1460.

<https://doi.org/10.4236/jamp.2022.105102>

**Received:** March 25, 2022

**Accepted:** May 7, 2022

**Published:** May 10, 2022

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## Abstract

Evolution of the photon gas (PG) in the Planck period is considered as a particular case of the physical vacuum (PV) hydrodynamics. Nonlocal quantum hydrodynamic equations are applied for calculation of the photon gas evolution. In general case, PG hydrodynamics contains gravitation in the explicit form. Exact analytical solutions of PG hydrodynamics are obtained. Solutions show the exponential growth of gradient values for internal energy in time and space. In comparison with phenomenological General Relativistic Theory, Nonlocal quantum hydrodynamics (NQH) does not lead to contradictions in all limit cases. Theory of physical vacuum and the theory of photonic gas are related theories. These theoretical (analytical!) results confirm the result of direct observations (Arno Alan Penzias and Robert Woodrow Wilson, Nobel Prize (1978) for their discovery of cosmic microwave background; John C. Mather and George F. Smoot. Nobel Prize (2006) for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation).

## Keywords

Nonlocal Physics, Physical Vacuum, Photon Gas Evolution after Big Bang, Transport Processes in Physical Vacuum, Cosmic Microwave Background, Anisotropy of the Cosmic Microwave Background Radiation

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## 1. Introduction

By definition, a physical vacuum is a state of a physical system in which there is no substance (matter) and all kinds of fields. After the Big Bang background radiation contains the traces of the travelling wave's evolution.

This radiation is known as cosmic microwave background radiation (CMBR) or "relic radiation". With a traditional optical telescope, the space between stars

and galaxies (the background) is completely dark. However, a sufficiently sensitive radio telescope shows a faint background glow, practically isotropic, that is not associated with any star, galaxy, or other object. This glow is strongest in the microwave region of the radio spectrum. Through the 1970s, the radiation was found to be approximately consistent with a black body spectrum in all directions; this spectrum has been redshifted by the expansion of the universe, and today corresponds to approximately 2.725 K.

In 1989, NASA launched the Cosmic Background Explorer satellite (COBE) which made two major advances:

1) In 1990, high-precision spectrum measurements showed the CMBR frequency spectrum is an almost perfect and measured residual temperature of 2.726 K (more recent measurements have revised this figure down slightly to 2.7255 K).

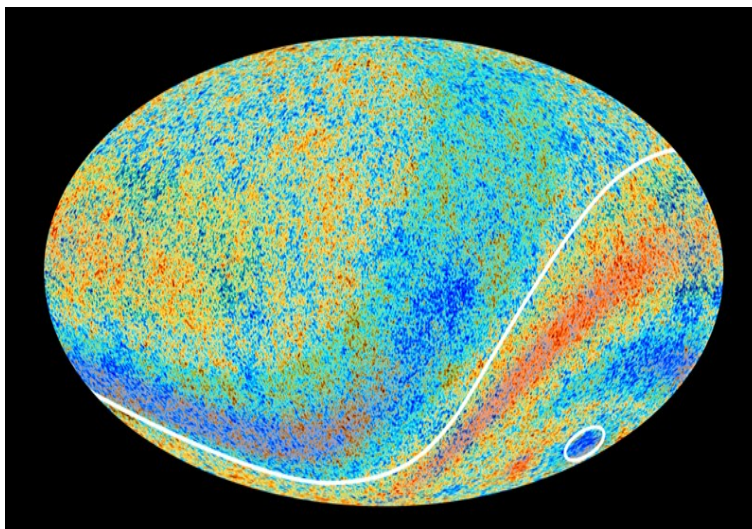
2) In 1992, further COBE measurements discovered *tiny fluctuations (anisotropies)* in the CMBR temperature across the sky, at a level of about one part in  $10^5$  [1] [2]. John C. Mather and George Smoot were awarded the 2006 Nobel Prize in Physics for their leadership in these results.

3) During the following decade, CMBR anisotropies were further investigated by a large number of ground-based and balloon experiments. In 2000-2001, several experiments found the shape of the Universe to be spatially *almost flat* by measuring the typical angular size (the size on the sky) of the anisotropies [3] [4].

In early 2003, the first results of the Wilkinson Microwave Anisotropy Probe (WMAP) were released, yielding what were at the time the most accurate values for some of the cosmological parameters. The Planck space probe was launched in May 2009. Other ground and balloon based cosmic microwave background experiments are ongoing.

Let us look at the measurements realized in the frame of the Planck programme. The temperature variations don't appear to behave the same on large scales as they do on small scales, and there are some particularly large features, such as a hefty cold spot, that were not predicted by basic inflation models. Really, look at the Planck space observatory's map (**Figure 1**) of the universe's cosmic microwave background. This map is in open Internet access (see for example SPACE.com Staff. Date: 21 March 2013 Time: 11:15 AM ET). It was reported that CMBR is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old. It shows tiny temperature fluctuations that correspond to regions of slightly different energy densities, representing the seeds of all future structure: the stars and galaxies of today.

The anomaly of the Cold Spot located in the southern hemisphere; size of the fluctuations  $\sim 10^\circ$ , galactic coordinates  $(b, l) = (-56^\circ, 209^\circ)$ . Anomaly is three-connected cold spots; the distribution of CMBR in this direction has a deep minimum. The map of radio emission at a frequency of 408 Hz has noticeably "warm" spot in a cold spot. Initially, the Spanish group drew the attention of researchers in 2005 [5]. This group found that significant deviation from Gaussian



**Figure 1.** Planck space observatory's map of the universe's cosmic microwave background radiation.

statistics in the southern hemisphere is due to the presence of spots with the mentioned coordinates and the amplitude is lower ( $-4\sigma$ ) where  $\sigma^2$  there is a variance of fluctuations in Gaussian dispersion models.

From the position of the developed nonlocal theory [6] [7] [8] [9] Planck's all-sky map contains the *regular* traces of traveling waves as the alternation of the "hot" (red) and "cold" (blue) strips. In **Figure 1**, the Planck space observatory staff shows the "mysterious" hefty cold spot as the blue small area bounded by the white circle.

From the position of the developed theory, it is the area reflecting the initial explosion of PV. In this case, the center domain of the mentioned hefty cold spot should contain the smallest hot spot as the origin of the initial burst. These effects are considered on the level of the simplified models in [8].

Returning to the model considering above we should constant that homogeneity of the early space can be taken only as a first approximation. The mentioned fluctuations have small amplitudes but the *regular* character.

The first stage of the big Bang corresponds to the Planck époque or to the Planck time. The Planck time  $\tau_p$  is the unit of time in the system of natural units known as Planck units. It is the time required for light to travel, in a vacuum, a distance of unit of Planck length, approximately  $5.391 \times 10^{-44}$  second. The Planck time is defined as:

$$\tau_p = \sqrt{\frac{\hbar \gamma_N}{c^5}}, \quad (1.1)$$

where  $\hbar = \frac{h}{2\pi}$  is the reduced Planck constant (sometimes  $h$  is used instead of  $\hbar$  in the definition),  $\gamma_N$  -gravitational constant,  $c$  is speed of light in vacuum. The Planck time is the unique combination of the gravitational constant  $\gamma_N$ , the special-relativistic constant  $c$ , and the quantum constant  $\hbar$ , to produce a con-

stant with units of time. Because the Planck time comes from dimensional analysis, which ignores constant factors, there is no reason to believe that exactly one unit of Planck time has any special physical significance.

Moreover, the Planck time represents a rough time scale at which gravitational effects ( $\gamma_N$ ) and special relativistic theory ( $c$ ) are not applicable to description of the physical events. Then the Planck time is only the orientation to the time period when no matter ( $\rho = 0$ ) and known fields exist. Obviously in this case we should remove all terms in the classical local transport equations transforming these relations into zero identities.

Before the advent of nonlocal physics (see, for example, [6] [7] [8] [9]), there was no theory that adequately described the evolution of the physical vacuum. Evolution assumes (in a certain sense) a hydrodynamic description, and we immediately fall into the sphere of interests of continuum mechanics. Needless to say, classical (local, in fact) hydrodynamics is powerless in this situation. The introduction of the Einstein cosmological constant into cosmology, and the attempt to express the “density” of the physical vacuum and its evolution through the value of this constant, has no physical meaning.

On the other hand, the lack of an adequate theory leads to paradoxes in the physical field of research—the theory of the evolution of photonic gas in general and the theory of electromagnetic waves in particular. It turns out that the theory of physical vacuum and the theory of photonic gas are related theories. The present work is devoted to the study of this problem.

## 2. Generalized Hydrodynamic Equations

The generalized hydrodynamic equations (GHE) can be obtained from the nonlocal kinetic equation in the frame of the Enskog procedure, [6] [7] [8] [9]:

(Continuity equation for species  $\alpha$ )

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho_\alpha - \tau_\alpha \left[ \frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho_\alpha \mathbf{v}_0 - \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) \right. \right. \\ & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0 \mathbf{v}_0) + \tilde{\mathbf{I}} \cdot \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \right] \right\} = R_\alpha. \end{aligned} \tag{2.1}$$

(Continuity equation for mixture)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho - \sum_\alpha \tau_\alpha \left[ \frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_0 - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) \right. \right. \\ & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0 \mathbf{v}_0) + \tilde{\mathbf{I}} \cdot \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \right] \right\} = 0. \end{aligned} \tag{2.2}$$

(Momentum equation for species  $\alpha$ )

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho_\alpha \mathbf{v}_0 - \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \right] \right\} \\ & - \mathbf{F}_\alpha^{(1)} \left[ \rho_\alpha - \tau_\alpha \left( \frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0) \right) \right] - \frac{q_\alpha}{m_\alpha} \left\{ \rho_\alpha \mathbf{v}_0 - \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) \right. \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{\partial}{\partial \mathbf{r}} \cdot \left[ \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \right] \times \mathbf{B} \\
& + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + p_\alpha \bar{\mathbf{I}} - \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + p_\alpha \bar{\mathbf{I}}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha (\mathbf{v}_0 \mathbf{v}_0) \mathbf{v}_0 \right. \right. \\
& + 2\bar{\mathbf{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p_\alpha \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\bar{\mathbf{I}} p_\alpha \mathbf{v}_0) - \mathbf{F}_\alpha^{(1)} \rho_\alpha \mathbf{v}_0 - \rho_\alpha \mathbf{v}_0 \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha [\mathbf{v}_0 \times \mathbf{B}] \mathbf{v}_0 \\
& \left. \left. - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 [\mathbf{v}_0 \times \mathbf{B}] \right\} = \int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,el} d\mathbf{v}_\alpha + \int m_\alpha \mathbf{v}_\alpha J_\alpha^{st,incl} d\mathbf{v}_\alpha.
\end{aligned} \tag{2.3}$$

(Momentum equation for mixture)

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\{ \rho \mathbf{v}_0 - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \right] \right\} \\
& - \sum_\alpha \mathbf{F}_\alpha^{(1)} \left[ \rho_\alpha - \tau_\alpha \left( \frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\rho_\alpha \mathbf{v}_0) \right) \right] - \sum_\alpha \frac{q_\alpha}{m_\alpha} \left\{ \rho_\alpha \mathbf{v}_0 - \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) \right. \right. \\
& + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial p_\alpha}{\partial \mathbf{r}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 \times \mathbf{B} \left. \left. \right\} \times \mathbf{B} \\
& + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \rho \mathbf{v}_0 \mathbf{v}_0 + p \bar{\mathbf{I}} - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + p_\alpha \bar{\mathbf{I}}) + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha (\mathbf{v}_0 \mathbf{v}_0) \mathbf{v}_0 \right. \right. \\
& + 2\bar{\mathbf{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p_\alpha \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\bar{\mathbf{I}} p_\alpha \mathbf{v}_0) - \mathbf{F}_\alpha^{(1)} \rho_\alpha \mathbf{v}_0 - \rho_\alpha \mathbf{v}_0 \mathbf{F}_\alpha^{(1)} \\
& \left. \left. - \frac{q_\alpha}{m_\alpha} \rho_\alpha [\mathbf{v}_0 \times \mathbf{B}] \mathbf{v}_0 - \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{v}_0 [\mathbf{v}_0 \times \mathbf{B}] \right\} = 0.
\end{aligned} \tag{2.4}$$

(Energy equation for  $\alpha$  species)

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\{ \frac{\rho_\alpha v_0^2}{2} + \frac{3}{2} p_\alpha + \varepsilon_\alpha n_\alpha - \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{\rho_\alpha v_0^2}{2} + \frac{3}{2} p_\alpha + \varepsilon_\alpha n_\alpha \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_\alpha v_0^2 \mathbf{v}_0 \right. \right. \right. \\
& + \left. \left. \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) - \mathbf{F}_\alpha^{(1)} \cdot \rho_\alpha \mathbf{v}_0 \right] \left. \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho_\alpha v_0^2 \mathbf{v}_0 + \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right. \\
& - \left. \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_\alpha v_0^2 \mathbf{v}_0 + \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_\alpha v_0^2 \mathbf{v}_0 \mathbf{v}_0 + \frac{7}{2} p_\alpha \mathbf{v}_0 \mathbf{v}_0 \right. \right. \right. \\
& + \left. \left. \frac{1}{2} p_\alpha v_0^2 \bar{\mathbf{I}} + \frac{5}{2} \frac{p_\alpha^2}{\rho_\alpha} \bar{\mathbf{I}} + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_\alpha \frac{p_\alpha}{m_\alpha} \bar{\mathbf{I}} \right) - \rho_\alpha \mathbf{F}_\alpha^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 - p_\alpha \mathbf{F}_\alpha^{(1)} \cdot \bar{\mathbf{I}} \right. \\
& - \left. \frac{1}{2} \rho_\alpha v_0^2 \mathbf{F}_\alpha^{(1)} - \frac{3}{2} \mathbf{F}_\alpha^{(1)} p_\alpha - \frac{\rho_\alpha v_0^2}{2} \frac{q_\alpha}{m_\alpha} [\mathbf{v}_0 \times \mathbf{B}] - \frac{5}{2} p_\alpha \frac{q_\alpha}{m_\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right. \\
& \left. - \varepsilon_\alpha n_\alpha \frac{q_\alpha}{m_\alpha} [\mathbf{v}_0 \times \mathbf{B}] - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)} \right] \left. \right\} - \left\{ \rho_\alpha \mathbf{F}_\alpha^{(1)} \cdot \mathbf{v}_0 - \tau_\alpha \left[ \mathbf{F}_\alpha^{(1)} \cdot \left( \frac{\partial}{\partial t} (\rho_\alpha \mathbf{v}_0) \right. \right. \right. \\
& + \left. \left. \frac{\partial}{\partial \mathbf{r}} \cdot \rho_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot p_\alpha \bar{\mathbf{I}} - \rho_\alpha \mathbf{F}_\alpha^{(1)} - q_\alpha n_\alpha [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} \\
& = \int \left( \frac{m_\alpha v_\alpha^2}{2} + \varepsilon_\alpha \right) J_\alpha^{st,el} d\mathbf{v}_\alpha + \int \left( \frac{m_\alpha v_\alpha^2}{2} + \varepsilon_\alpha \right) J_\alpha^{st,incl} d\mathbf{v}_\alpha.
\end{aligned} \tag{2.5}$$

(Energy equation for mixture)

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left\{ \frac{\rho v_0^2}{2} + \frac{3}{2} p + \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{\rho_{\alpha} v_0^2}{2} + \frac{3}{2} p_{\alpha} + \varepsilon_{\alpha} n_{\alpha} \right) \right. \right. \\
 & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 + \frac{5}{2} p_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right) - \mathbf{F}_{\alpha}^{(1)} \cdot \rho_{\alpha} \mathbf{v}_0 \right] \right\} \\
 & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{1}{2} \rho v_0^2 \mathbf{v}_0 + \frac{5}{2} p \mathbf{v}_0 + \mathbf{v}_0 \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha} - \sum_{\alpha} \tau_{\alpha} \left[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \right. \right. \right. \\
 & \left. \left. + \frac{5}{2} p_{\alpha} \mathbf{v}_0 + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{v}_0 \mathbf{v}_0 + \frac{7}{2} p_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p_{\alpha} v_0^2 \bar{\mathbf{I}} \right. \right. \\
 & \left. \left. + \frac{5}{2} \frac{p_{\alpha}^2}{\rho_{\alpha}} \bar{\mathbf{I}} + \varepsilon_{\alpha} n_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_{\alpha} \frac{p_{\alpha}}{m_{\alpha}} \bar{\mathbf{I}} \right) - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \mathbf{v}_0 \mathbf{v}_0 - p_{\alpha} \mathbf{F}_{\alpha}^{(1)} \cdot \bar{\mathbf{I}} \right. \\
 & \left. - \frac{1}{2} \rho_{\alpha} v_0^2 \mathbf{F}_{\alpha}^{(1)} - \frac{3}{2} \mathbf{F}_{\alpha}^{(1)} p_{\alpha} - \frac{\rho_{\alpha} v_0^2}{2} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \frac{5}{2} p_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] \right. \\
 & \left. - \varepsilon_{\alpha} n_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} [\mathbf{v}_0 \times \mathbf{B}] - \varepsilon_{\alpha} n_{\alpha} \mathbf{F}_{\alpha}^{(1)} \right] \right\} - \left\{ \mathbf{v}_0 \cdot \sum_{\alpha} \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - \sum_{\alpha} \tau_{\alpha} \left[ \mathbf{F}_{\alpha}^{(1)} \cdot \left( \frac{\partial}{\partial t} (\rho_{\alpha} \mathbf{v}_0) \right. \right. \right. \\
 & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \rho_{\alpha} \mathbf{v}_0 \mathbf{v}_0 + \frac{\partial}{\partial \mathbf{r}} \cdot p_{\alpha} \bar{\mathbf{I}} - \rho_{\alpha} \mathbf{F}_{\alpha}^{(1)} - q_{\alpha} n_{\alpha} [\mathbf{v}_0 \times \mathbf{B}] \right] \right\} = 0.
 \end{aligned} \tag{2.6}$$

The force dimension,  $[F_{\alpha}^{(1)}] = \frac{\text{cm}}{\text{s}^2}$ . Here  $\mathbf{F}_{\alpha}^{(1)}$  are the forces of the non-magnetic origin,  $\mathbf{B}$  -magnetic induction,  $\bar{\mathbf{I}}$  -unit tensor,  $q_{\alpha}$  -charge of the  $\alpha$  -component particle,  $p_{\alpha}$  -static pressure for  $\alpha$  -component,  $\varepsilon_{\alpha}$  -internal energy for the particles of  $\alpha$  -component,  $\mathbf{v}_0$  -hydrodynamic velocity for mixture,  $\tau_{\alpha}$  -non-local parameter.

GHE are extremely important for astrophysics special cases when density  $\rho \rightarrow 0$  (the initial stage of evolution of the Universe, the Big Bang; transport processes in physical vacuum) and when density  $\rho \rightarrow \infty$  (evolution of the black hole). Both limiting cases have no physical or mathematical meaning in “classical” hydrodynamics. Thus, we have a unified statistical theory of dissipative structures, which has a hydrodynamic shape defined by the genesis of GHE. Then we obliged to deliver some comments concerning application of special (SRT) and general (GRT) relativistic theory in theoretical astrophysics.

### 3. Derivation of the Basic Equations

#### Step 1.

Physical vacuum (PV) and photon gas (PG) do not contain matter-density ( $\rho_{\alpha} \rightarrow 0, q_{\alpha} \rightarrow 0$ ). From the system (2.1)-(2.6) we find for PV

(Continuity equation for species  $\alpha$ )

$$\frac{\partial}{\partial \mathbf{r}} \cdot \tau_{\alpha} \left[ \bar{\mathbf{I}} \cdot \frac{\partial p_{\alpha}}{\partial \mathbf{r}} \right] = R_{\alpha}. \tag{3.1}$$

(Continuity equation for mixture)

$$\frac{\partial}{\partial \mathbf{r}} \cdot \sum_{\alpha} \tau_{\alpha} \bar{\mathbf{I}} \cdot \frac{\partial p_{\alpha}}{\partial \mathbf{r}} = 0. \tag{3.2}$$

(Momentum equation for species  $\alpha$ )

$$-\frac{\partial}{\partial t} \left( \tau_\alpha \frac{\partial p_\alpha}{\partial \mathbf{r}} \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ p_\alpha \bar{\mathbf{I}} - \tau_\alpha \left[ \frac{\partial}{\partial t} (p_\alpha \bar{\mathbf{I}}) + 2\bar{\mathbf{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p_\alpha \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\bar{\mathbf{I}} p_\alpha \mathbf{v}_0) \right] \right\} = 0 \quad (3.3)$$

(Momentum equation for mixture)

$$-\frac{\partial}{\partial t} \left( \sum_\alpha \tau_\alpha \frac{\partial p_\alpha}{\partial \mathbf{r}} \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ p \bar{\mathbf{I}} - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} (p_\alpha \bar{\mathbf{I}}) + 2\bar{\mathbf{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p_\alpha \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\bar{\mathbf{I}} p_\alpha \mathbf{v}_0) \right] \right\} = 0. \quad (3.4)$$

(Energy equation for  $\alpha$  species)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \frac{3}{2} p_\alpha + \varepsilon_\alpha n_\alpha - \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{3}{2} p_\alpha + \varepsilon_\alpha n_\alpha \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) \right] \right\} \\ & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 - \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) \right. \right. \\ & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{7}{2} p_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p_\alpha v_0^2 \bar{\mathbf{I}} + \frac{5}{2} \left[ \frac{p_\alpha^2}{\rho_\alpha} \right]_{\rho_\alpha \rightarrow 0} \bar{\mathbf{I}} + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_\alpha \left[ \frac{p_\alpha}{m_\alpha} \right]_{m_\alpha \rightarrow 0} \bar{\mathbf{I}} \right) \right. \right. \\ & \left. \left. - p_\alpha \mathbf{F}_\alpha^{(1)} \cdot \bar{\mathbf{I}} - \frac{3}{2} \mathbf{F}_\alpha^{(1)} p_\alpha - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)} \right] \right\} + \tau_\alpha \mathbf{F}_\alpha^{(1)} \cdot \left( \frac{\partial}{\partial \mathbf{r}} \cdot p_\alpha \bar{\mathbf{I}} \right) = J_{\alpha, en} \end{aligned} \quad (3.5)$$

(Energy equation for mixture)

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \frac{3}{2} p + \sum_\alpha \varepsilon_\alpha n_\alpha - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{3}{2} p_\alpha + \varepsilon_\alpha n_\alpha \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) \right] \right\} \\ & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{5}{2} p \mathbf{v}_0 + \mathbf{v}_0 \sum_\alpha \varepsilon_\alpha n_\alpha - \sum_\alpha \tau_\alpha \left[ \frac{\partial}{\partial t} \left( \frac{5}{2} p_\alpha \mathbf{v}_0 + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \right) \right. \right. \\ & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{7}{2} p_\alpha \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p_\alpha v_0^2 \bar{\mathbf{I}} + \frac{5}{2} \left[ \frac{p_\alpha^2}{\rho_\alpha} \right]_{\rho_\alpha \rightarrow 0} \bar{\mathbf{I}} + \varepsilon_\alpha n_\alpha \mathbf{v}_0 \mathbf{v}_0 + \varepsilon_\alpha \left[ \frac{p_\alpha}{m_\alpha} \right]_{m_\alpha \rightarrow 0} \bar{\mathbf{I}} \right) \right. \right. \\ & \left. \left. - p_\alpha \mathbf{F}_\alpha^{(1)} \cdot \bar{\mathbf{I}} - \frac{3}{2} \mathbf{F}_\alpha^{(1)} p_\alpha - \varepsilon_\alpha n_\alpha \mathbf{F}_\alpha^{(1)} \right] \right\} + \sum_\alpha \tau_\alpha \frac{\partial}{\partial \mathbf{r}} \cdot p_\alpha \bar{\mathbf{I}} = 0 \end{aligned} \quad (3.6)$$

**Step 2.**

For the one species PV we have

(Continuity equation)

$$\frac{\partial}{\partial \mathbf{r}} \cdot \tau \left[ \bar{\mathbf{I}} \cdot \frac{\partial p}{\partial \mathbf{r}} \right] = 0. \quad (3.7)$$

(Momentum equation)

$$-\frac{\partial}{\partial t} \left( \tau \frac{\partial p}{\partial \mathbf{r}} \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ p \bar{\mathbf{I}} - \tau \left[ \frac{\partial}{\partial t} (p \bar{\mathbf{I}}) + 2\bar{\mathbf{I}} \left( \frac{\partial}{\partial \mathbf{r}} \cdot (p \mathbf{v}_0) \right) + \frac{\partial}{\partial \mathbf{r}} \cdot (\bar{\mathbf{I}} p \mathbf{v}_0) \right] \right\} = 0 \quad (3.8)$$

(Energy equation)

$$\begin{aligned}
 & \left. \frac{\partial}{\partial t} \left\{ \frac{3}{2} p + \varepsilon n - \tau \left[ \frac{\partial}{\partial t} \left( \frac{3}{2} p + \varepsilon n \right) + \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{5}{2} p \mathbf{v}_0 + \varepsilon n \mathbf{v}_0 \right) \right] \right\} \right. \\
 & + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \frac{5}{2} p \mathbf{v}_0 + \varepsilon n \mathbf{v}_0 - \tau \left[ \frac{\partial}{\partial t} \left( \frac{5}{2} p \mathbf{v}_0 + \varepsilon n \mathbf{v}_0 \right) \right. \right. \\
 & + \left. \left. \frac{\partial}{\partial \mathbf{r}} \cdot \left( \frac{7}{2} p \mathbf{v}_0 \mathbf{v}_0 + \frac{1}{2} p v_0^2 \tilde{\mathbf{I}} + \frac{5}{2} \left[ \frac{p^2}{\rho} \right]_{\rho \rightarrow 0} \tilde{\mathbf{I}} + \varepsilon n \mathbf{v}_0 \mathbf{v}_0 + \varepsilon \left[ \frac{p}{m} \right]_{m \rightarrow 0} \tilde{\mathbf{I}} \right) \right. \right. \\
 & \left. \left. - p \mathbf{F}^{(1)} \cdot \tilde{\mathbf{I}} - \frac{3}{2} \mathbf{F}^{(1)} p - \varepsilon n \mathbf{F}^{(1)} \right] \right\} + \tau \mathbf{F}^{(1)} \cdot \left( \frac{\partial}{\partial \mathbf{r}} \cdot p \tilde{\mathbf{I}} \right) = 0
 \end{aligned} \tag{3.9}$$

**Step 3.**

Extremely important that for the case of the PG evolution we can realize the following simplifications of the system (3.7)-(3.9). We should take into account:

1) All photons have the constant velocity  $c$ . *This speed does not necessarily coincide with the speed of light in a vacuum in the presence of matter and gravity.*

2) It means that the thermal velocities are absent including  $\left[ \frac{p^2}{\rho} \right]_{\rho \rightarrow 0}$ ,  $\left[ \frac{p}{m} \right]_{m \rightarrow 0}$

and terms containing  $p$ . The corresponding definitions can be found in [6] [7] [8] [9].

As a result we find from Equations (3.7)-(3.9) only energy equation

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left\{ \varepsilon n - \tau \left[ \frac{\partial}{\partial t} (\varepsilon n) + \frac{\partial}{\partial \mathbf{r}} \cdot (\varepsilon n \mathbf{c}) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ \varepsilon n \mathbf{c} - \tau \left[ \frac{\partial}{\partial t} (\varepsilon n \mathbf{c}) \right. \right. \\
 & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot (\varepsilon n \mathbf{c} \mathbf{c}) - \varepsilon n \mathbf{F}^{(1)} \right] \right\} = 0
 \end{aligned} \tag{3.10}$$

In Equation (3.10) the value  $\varepsilon n$  is an internal PG energy per the unit volume. In the following we use the notation  $\varepsilon n \leftrightarrow E$ .

Then

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left\{ E - \tau \left[ \frac{\partial E}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (E \mathbf{c}) \right] \right\} + \frac{\partial}{\partial \mathbf{r}} \cdot \left\{ E \mathbf{c} - \tau \left[ \frac{\partial}{\partial t} (E \mathbf{c}) \right. \right. \\
 & \left. \left. + \frac{\partial}{\partial \mathbf{r}} \cdot (E \mathbf{c} \mathbf{c}) - E \mathbf{F}^{(1)} \right] \right\} = 0
 \end{aligned} \tag{3.11}$$

The nonlocal parameter  $\tau$  plays in GHE the same role as usual transport coefficient (like viscosity) in classical local hydrodynamics. Let us estimate  $\tau$  using the Heisenberg principle

$$\tau h \nu = h, \tag{3.12}$$

where  $\nu$  is frequency. Then

$$\tau = \frac{1}{\nu}. \tag{3.13}$$

Let us add some additional explanation concerning this choice. The appearance of the nonlocal  $\tau$  parameter is consistent with the Heisenberg uncertainty relation. But in principle generalized kinetic nonlocal Equation (and therefore



generalized hydrodynamic equations (GHE)) needn't in using of the "time-energy" uncertainty relation for estimation of the value of the non-locality parameter  $\tau$ . Moreover the "time-energy" uncertainty relation does not produce the exact relations and from position of non-local physics is only the simplest estimation of the non-local effects.

Really, let us consider two neighboring physically infinitely small volumes **PhSV**<sub>1</sub> and **PhSV**<sub>2</sub> in a non-equilibrium system. Obviously the time  $\tau$  should tends to diminish with increasing of the velocities  $u$  of particles invading in the nearest neighboring physically infinitely small volume (**PhSV**<sub>1</sub> or **PhSV**<sub>2</sub>):

$$\tau = H_\tau / u^n. \quad (3.14)$$

But the value  $\tau$  cannot depend on the velocity direction and naturally to tie  $\tau$  with the particle kinetic energy, then

$$\tau = \frac{H_\tau}{mu^2}, \quad (3.15)$$

where  $H_\tau$  is a coefficient of proportionality, which reflects the state of physical system. In the simplest case  $H_\tau$  is equal to Plank constant  $\hbar$  and relation (3.15) became compatible with the Heisenberg relation. Relation (3.13) leads to the absolutely transparent relation  $\tau = \frac{\lambda}{c}$ . It means that the average time of the information transmission in the nearest **PhSV** is equal the wave length divided on the light velocity.

#### 4. Photon Gas in the Non-Stationary 1D System

For the better understanding of general picture we consider the non-stationary 1D system. It should underline that  $\mathbf{F}_\alpha^{(1)}$  is acceleration originated by the forces of the non-electro magnetic origin. In the following the value  $\mathbf{F}^{(1)}$  is acceleration generally speaking, not related to Newtonian gravity. In the following we use notation

$$\mathbf{F}^{(1)} = \mathbf{g}. \quad (4.1)$$

We find

$$\frac{\partial}{\partial t} \left\{ E - \tau \left[ \frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} \right] \right\} + \frac{\partial}{\partial x} \left\{ Ec - \tau \left[ c \frac{\partial E}{\partial t} + c^2 \frac{\partial E}{\partial \mathbf{r}} - Eg \right] \right\} = 0. \quad (4.2)$$

Interesting to notice, that using local hydrodynamics we find from Equation (4.2)

$$\frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} = 0. \quad (4.3)$$

General solution of (4.3) is the general wave solution

$$E = \Phi(ct - x). \quad (4.4)$$

As you see classical hydrodynamics does not contain the gravitation influence in principle. Let us transform (4.2)

$$\frac{\partial E}{\partial t} - \frac{\partial}{\partial t} \left( \tau \frac{\partial E}{\partial t} \right) - c \frac{\partial}{\partial t} \left( \tau \frac{\partial E}{\partial x} \right) + c \frac{\partial E}{\partial x} - c \frac{\partial}{\partial x} \left\{ \tau \left[ \frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} \right] \right\} + \frac{\partial}{\partial x} (\tau E g) = 0 \quad (4.5)$$

If  $\tau = \text{CONST}$  we have

$$\frac{\partial E}{\partial t} - \tau \frac{\partial^2 E}{\partial t^2} - c\tau \frac{\partial}{\partial t} \frac{\partial E}{\partial x} + c \frac{\partial E}{\partial x} - c\tau \frac{\partial}{\partial x} \left[ \frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} \right] + \tau \frac{\partial}{\partial x} (Eg) = 0 \quad (4.6)$$

or

$$\tau \frac{\partial^2 E}{\partial t^2} + c^2 \tau \frac{\partial^2 E}{\partial x^2} = \frac{\partial E}{\partial t} - 2c\tau \frac{\partial}{\partial t} \frac{\partial E}{\partial x} + c \frac{\partial E}{\partial x} + \tau \frac{\partial}{\partial x} (Eg) \quad (4.7)$$

or

$$\frac{\partial^2 E}{\partial t^2} + c^2 \frac{\partial^2 E}{\partial x^2} = \frac{1}{\tau} \frac{\partial E}{\partial t} - 2c \frac{\partial}{\partial t} \frac{\partial E}{\partial x} + \frac{c}{\tau} \frac{\partial E}{\partial x} + \frac{\partial}{\partial x} (Eg). \quad (4.8)$$

Write down the dimensionless form of Equation (4.8) using the scales

$$\tilde{t} = \frac{t}{\tau}, \quad \tilde{x} = \frac{x}{c\tau}. \quad (4.9)$$

We find

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{1}{\tau^2} \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = \frac{1}{\tau^2} \frac{\partial \tilde{E}}{\partial \tilde{t}} - 2 \frac{1}{\tau^2} \frac{\partial}{\partial \tilde{t}} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{1}{\tau^2} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial}{c\tau \partial \tilde{x}} (\tilde{E}g) \quad (4.10)$$

or

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = \frac{\partial \tilde{E}}{\partial \tilde{t}} - 2 \frac{\partial}{\partial \tilde{t}} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\tau}{c} \frac{\partial}{\partial \tilde{x}} (\tilde{E}g). \quad (4.11)$$

Let us introduce the dimensionless “gravitational” acceleration

$$\tilde{g} = \frac{g}{c\nu}. \quad (4.12)$$

Then

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = \frac{\partial \tilde{E}}{\partial \tilde{t}} - 2 \frac{\partial}{\partial \tilde{t}} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial}{\partial \tilde{x}} (\tilde{E}\tilde{g}). \quad (4.13)$$

Equation (4.13) is elliptic equation with the right hand side which is not equal to zero.

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = w(\tilde{E}), \quad (4.14)$$

where

$$w(\tilde{E}) = \frac{\partial \tilde{E}}{\partial \tilde{t}} - 2 \frac{\partial}{\partial \tilde{t}} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial}{\partial \tilde{x}} (\tilde{E}\tilde{g}). \quad (4.15)$$

### 5. Some Important Particular Cases

1) *Effect of “tired light”.*

If  $w(\tilde{E})$  is small value and can be omitted we obtain elliptic equation

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = 0. \quad (5.1)$$

from (4.13). Solution of this equation valid for the case is

$$\tilde{E} = e^{\pm \tilde{t}} \cos(\tilde{x}) \quad (5.2)$$

or in the dimension form

$$E = E_0 e^{\pm \frac{t}{\tau}} \cos\left(\frac{x}{c\tau}\right) \quad (5.3)$$

or

$$E = E_0 e^{\pm i\nu t} \cos\left(\frac{x}{\lambda}\right), \quad (5.4)$$

where  $\lambda$  is the wave length and  $\nu$  is frequency.

$$\lambda = \frac{c}{\nu}. \quad (5.5)$$

Really

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} = e^{\pm \tilde{t}} \cos(\tilde{x}), \quad \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = -e^{\pm \tilde{t}} \cos(\tilde{x}). \quad (5.6)$$

In this particular case we discover the wave regimes of attenuation and explosion. Then effect of tired light can exist *without interaction with the gravitational field*.

### 2) About the connection with local hydrodynamics.

Let us transform the basic Equation (4.13). We have

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} + \frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} = \frac{\partial}{\partial \tilde{t}} \left( \tilde{E} - 2 \frac{\partial \tilde{E}}{\partial \tilde{x}} \right) + \frac{\partial}{\partial \tilde{x}} \left[ \tilde{E} (1 + \tilde{g}) \right] \quad (5.7)$$

or

$$\frac{\partial}{\partial \tilde{t}} \left( \frac{\partial \tilde{E}}{\partial \tilde{t}} - \tilde{E} + \frac{\partial \tilde{E}}{\partial \tilde{x}} \right) + \frac{\partial}{\partial \tilde{x}} \left[ \frac{\partial \tilde{E}}{\partial \tilde{x}} - \tilde{E} (1 + \tilde{g}) + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right] = 0, \quad (5.8)$$

or

$$\frac{\partial}{\partial \tilde{t}} \left[ \tilde{E} - \left( \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right) \right] + \frac{\partial}{\partial \tilde{x}} \left[ \tilde{E} (1 + \tilde{g}) - \left( \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right) \right] = 0 \quad (5.9)$$

It means that the wave relation  $E = \Phi(ct - x)$  of local hydrodynamics takes place if the following relation for the dimensionless internal PG energy is satisfied

$$\tilde{E} \gg \left| \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right|, \quad (5.10)$$

and

$$|\tilde{g}| \ll 1. \quad (5.11)$$

### 3) About the self similar solutions.

Let be

$$A) \quad \tilde{\xi} = \tilde{x} - \tilde{t}. \quad (5.12)$$

From (5.9) one obtains

$$-\frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E} - \left( \frac{\partial \tilde{E}}{\partial \tilde{\xi}} - \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right) \right] + \frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E}(1 + \tilde{g}) - \left( \frac{\partial \tilde{E}}{\partial \tilde{\xi}} - \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right) \right] = 0 \quad (5.13)$$

or

$$-\frac{\partial \tilde{E}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} [\tilde{E}(1 + \tilde{g})] = 0 \quad (5.14)$$

or

$$\frac{\partial}{\partial \tilde{\xi}} [\tilde{E}\tilde{g}] = 0 \quad (5.15)$$

or

$$\tilde{E}\tilde{g} = const. \quad (5.16)$$

or

$$\tilde{E} = \frac{const}{\tilde{g}}. \quad (5.17)$$

Let be

$$B) \quad \tilde{\xi} = \tilde{x} + \tilde{t} \quad (5.18)$$

From (5.9) one obtains

$$\frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E} - \left( \frac{\partial \tilde{E}}{\partial \tilde{\xi}} + \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right) \right] + \frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E}(1 + \tilde{g}) - \left( \frac{\partial \tilde{E}}{\partial \tilde{\xi}} + \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right) \right] = 0 \quad (5.19)$$

$$\frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E} - 2 \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right] + \frac{\partial}{\partial \tilde{\xi}} \left[ \tilde{E}(1 + \tilde{g}) - 2 \frac{\partial \tilde{E}}{\partial \tilde{\xi}} \right] = 0$$

or

$$-4 \frac{\partial^2 \tilde{E}}{\partial \tilde{\xi}^2} + 2 \frac{\partial \tilde{E}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} [\tilde{E}\tilde{g}] = 0 \quad (5.20)$$

or

$$\frac{\partial \tilde{E}}{\partial \tilde{\xi}} = \tilde{E} \left( \frac{1}{2} + \frac{1}{4} \tilde{g} \right) \quad (5.21)$$

or

$$\frac{\partial \ln \tilde{E}}{\partial \tilde{\xi}} = \frac{1}{2} + \frac{1}{4} \tilde{g} \quad (5.22)$$

or

$$\ln \tilde{E} = 0.5\tilde{\xi} + 0.25 \int \tilde{g} d\tilde{\xi} + const \quad (5.23)$$

or

$$\tilde{E} = \tilde{E}_0 \exp \left( 0.5\tilde{\xi} + 0.25 \int \tilde{g} d\tilde{\xi} \right). \quad (5.24)$$

4) *Exact analytical solution of the basic Equation (4.13) ( $\tilde{g} = 0$ ).*

Let us write down Equation (5.9)

$$\frac{\partial}{\partial \tilde{t}} \left[ \tilde{E} - \left( \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right) \right] + \frac{\partial}{\partial \tilde{x}} \left[ \tilde{E} (1 + \tilde{g}) - \left( \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} \right) \right] = 0 \quad (5.25)$$

in the form

$$\frac{\partial}{\partial \tilde{t}} [\tilde{E} - \tilde{Z}] + \frac{\partial}{\partial \tilde{x}} [\tilde{E} (1 + \tilde{g}) - \tilde{Z}] = 0 \quad (5.26)$$

using a new variable

$$\tilde{Z} = \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}}. \quad (5.27)$$

Using (5.27) we find from (5.26)

$$\tilde{Z} - \frac{\partial \tilde{Z}}{\partial \tilde{t}} - \frac{\partial \tilde{Z}}{\partial \tilde{x}} + \frac{\partial \tilde{g} \tilde{E}}{\partial \tilde{x}} = 0. \quad (5.28)$$

If the last term in the left hand side of Equation (5.28) is small we have

$$\frac{\partial \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \tilde{Z}}{\partial \tilde{x}} = \tilde{Z} \quad (5.29)$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1. \quad (5.30)$$

Using new variable  $\tilde{W} = \ln \tilde{Z}$  we obtain for Equation (5.31)

$$\frac{\partial \tilde{W}}{\partial \tilde{t}} + \frac{\partial \tilde{W}}{\partial \tilde{x}} = 1 \quad (5.31)$$

two possible solutions.

The first possible solution

$$\tilde{W}_1 = \tilde{x} + \tilde{\Phi}(\tilde{x} - \tilde{t}), \quad (5.32)$$

Really

$$\frac{\partial \tilde{W}_1}{\partial \tilde{t}} = -\frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad \frac{\partial \tilde{W}_1}{\partial \tilde{x}} = 1 + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad (5.33)$$

and as a result we reach identity

$$\frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} + 1 - \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} = 1. \quad (5.34)$$

The second possible solution is

$$\tilde{W}_2 = \tilde{t} + \tilde{\Phi}(\tilde{x} - \tilde{t}). \quad (5.35)$$

Really

$$\frac{\partial \tilde{W}_2}{\partial \tilde{t}} = 1 - \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad \frac{\partial \tilde{W}_2}{\partial \tilde{x}} = \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad (5.36)$$

and

$$1 - \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} = 1. \quad (5.37)$$

Using the first solution one obtains

$$\ln \tilde{Z}_1 = \tilde{x} + \tilde{\Phi}(\tilde{x} - \tilde{t}) \tag{5.38}$$

or

$$\tilde{Z}_1 = \tilde{E}_1 e^{\tilde{x} + \tilde{\Phi}(\tilde{x} - \tilde{t})} \tag{5.39}$$

or

$$\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} = \tilde{E}_1 e^{\tilde{x} + \tilde{\Phi}(\tilde{x} - \tilde{t})}. \tag{5.40}$$

Analogically we find for the second possible solution

$$\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} = \tilde{E}_0 e^{\tilde{t} + \tilde{\Phi}(\tilde{x} - \tilde{t})}. \tag{5.41}$$

The both solutions lead to the exponential (but wave) growth of the internal energy on the Planck scale.

5) *Exact analytical solution of the basic Equation (4.13) ( $\tilde{g} \neq 0$ ).*

Let us return to Equation (5.9) written as

$$\tilde{Z} - \frac{\partial \tilde{Z}}{\partial \tilde{t}} - \frac{\partial \tilde{Z}}{\partial \tilde{x}} + \frac{\partial \tilde{g} \tilde{E}}{\partial \tilde{x}} = 0 \tag{5.42}$$

using a variable

$$\tilde{Z} = \frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}}, \tag{5.43}$$

The aim is to take into account the gravitation after appearance a matter. We have from (5.42)

$$\frac{\partial \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \tilde{Z}}{\partial \tilde{x}} = \tilde{Z} + \frac{\partial \tilde{g} \tilde{E}}{\partial \tilde{x}} \tag{5.44}$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \frac{\frac{\partial \tilde{g} \tilde{E}}{\partial \tilde{x}}}{\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}}} \tag{5.45}$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \frac{\tilde{g} \frac{\partial \tilde{E}}{\partial \tilde{x}} + \tilde{E} \frac{\partial \tilde{g}}{\partial \tilde{x}}}{\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}}} \tag{5.46}$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \frac{\tilde{g} + \tilde{E} \frac{\partial \tilde{x}}{\partial \tilde{E}} \frac{\partial \tilde{g}}{\partial \tilde{x}}}{1 + \frac{\partial \tilde{x}}{\partial \tilde{E}} \frac{\partial \tilde{E}}{\partial \tilde{t}}} \tag{5.47}$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \frac{\tilde{g} + \tilde{E} \frac{\partial \tilde{g}}{\partial \tilde{E}}}{1 + V} \quad (5.48)$$

or

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \frac{\tilde{g} + \frac{\partial \tilde{g}}{\partial \ln \tilde{E}}}{1 + \tilde{V}} \quad (5.49)$$

Dimensionless velocity  $\tilde{V}$  is the speed of the spreading front. If the dimensionless velocity  $\tilde{V}$  (corresponding the Planck scale) satisfies a relation  $\tilde{V} \gg 1$  ( $V \gg c$ ), we can write (5.49) as

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1, \quad (5.50)$$

see also (5.30).

In the initial period of the explosion corresponding to the Planck scale, *there are no restrictions on the speed of expansion (such as the speed of light  $c$  appeared in special relativity)*. As we see gravitation is not significant in the beginning of explosion on the Planck scale.

The case  $\tilde{V} \ll 1$  corresponds relations

$$\frac{\partial \tilde{x}}{\partial \tilde{t}} \ll 1, \quad \frac{\partial x}{c \partial t} \ll 1 \quad (5.51)$$

or in the dimension form

$$V \ll c. \quad (5.52)$$

Then in the case we have

$$\frac{\partial \ln \tilde{Z}}{\partial \tilde{t}} + \frac{\partial \ln \tilde{Z}}{\partial \tilde{x}} = 1 + \tilde{g} + \frac{\partial \tilde{g}}{\partial \ln \tilde{E}} (1 - \tilde{V}) \quad (5.53)$$

If

$$\frac{\partial \tilde{g}}{\partial \ln \tilde{E}} \ll 1 \quad (5.54)$$

we reach (introducing the variable  $\ln \tilde{Z} = \tilde{W}$ ) the relation

$$\frac{\partial \tilde{W}}{\partial \tilde{t}} + \frac{\partial \tilde{W}}{\partial \tilde{x}} = 1 + \tilde{g}. \quad (5.55)$$

Equation (5.55) has *the first possible solution*

$$\tilde{W}_1 = \tilde{x}(1 + \tilde{g}) + \tilde{\Phi}(\tilde{x} - \tilde{t}), \quad (5.56)$$

Really

$$\frac{\partial \tilde{W}_1}{\partial \tilde{t}} = -\frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad (5.57)$$

$$\frac{\partial \tilde{W}_1}{\partial \tilde{x}} = (1 + \tilde{g}) + \tilde{x} \frac{\partial \tilde{g}}{\partial \tilde{x}} + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad (5.58)$$

or

$$\frac{\partial \tilde{W}_1}{\partial \tilde{x}} = (1 + \tilde{g}) + \frac{\partial \tilde{g}}{\partial \ln \tilde{x}} + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}. \tag{5.59}$$

If

$$\frac{\partial \tilde{g}}{\partial \ln \tilde{x}} \ll 1 \tag{5.60}$$

(see also (5.54)), as a result we find identity

$$-\frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} + 1 + \tilde{g} + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} = 1 + \tilde{g}. \tag{5.61}$$

*The second possible solution is*

$$\tilde{W}_2 = \tilde{t} (1 + \tilde{g}) + \tilde{\Phi} (\tilde{x} - \tilde{t}). \tag{5.62}$$

Really

$$\frac{\partial \tilde{W}_2}{\partial \tilde{t}} = 1 + \tilde{g} - \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \quad \frac{\partial \tilde{W}_2}{\partial \tilde{x}} = \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})}, \tag{5.63}$$

and if  $\left( \frac{\partial \tilde{g}}{\partial \ln \tilde{t}} \ll 1 \right)$  we find identity

$$1 + \tilde{g} - \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} + \frac{\partial \tilde{\Phi}}{\partial (\tilde{x} - \tilde{t})} = 1 + \tilde{g}. \tag{5.64}$$

Using the first solution one obtains

$$\ln \tilde{Z}_1 = \tilde{x} (1 + \tilde{g}) + \tilde{\Phi} (\tilde{x} - \tilde{t}) \tag{5.65}$$

or

$$\tilde{Z}_1 = \tilde{E}_1 e^{\tilde{x}(1+\tilde{g}) + \tilde{\Phi}(\tilde{x}-\tilde{t})} \tag{5.66}$$

or for the first possible solution

$$\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} = \tilde{E}_1 e^{\tilde{x}(1+\tilde{g}) + \tilde{\Phi}(\tilde{x}-\tilde{t})}. \tag{5.67}$$

Analogically, we find for the second possible solution

$$\frac{\partial \tilde{E}}{\partial \tilde{x}} + \frac{\partial \tilde{E}}{\partial \tilde{t}} = \tilde{E}_2 e^{\tilde{t}(1+\tilde{g}) + \tilde{\Phi}(\tilde{x}-\tilde{t})}. \tag{5.68}$$

As we see relations (5.63) and (5.64) reflect even the existence large-scale energy fluctuations of the cosmic microwave background. These theoretical results confirm the result of direct observations, (Arno Alan Penzias and Robert Woodrow Wilson, Nobel Prize (1978) for their discovery of cosmic microwave background; John C. Mather and George F. Smoot. Nobel Prize (2006) for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation).

In the dimension form we have for the first solution (using the scales  $\tilde{t} = \frac{t}{\tau}$ ,

$$\tilde{x} = \frac{x}{c\tau}, \quad \tilde{g} = \tau \frac{g}{c}$$



$$c \frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = \frac{1}{\tau} E_1 \exp \left\{ \frac{x}{c\tau} \left( 1 + \frac{\tau}{c} g \right) + \Phi \left[ \frac{1}{\tau} \left( \frac{1}{c} x - t \right) \right] \right\}. \quad (5.69)$$

Analogically we find for the second possible solution

$$c \frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = \frac{1}{\tau} E_2 \exp \left\{ \frac{t}{\tau} \left( 1 + \frac{\tau}{c} g \right) + \Phi \left[ \frac{1}{\tau} \left( \frac{1}{c} x - t \right) \right] \right\} \quad (5.70)$$

Relations (5.69) and (5.70) can be written as follows (first solution)

$$c \frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = \nu E_1 \exp \left\{ \frac{\nu}{c} x \left( 1 + \frac{\tau}{c} g \right) + \Phi \left[ \nu \left( \frac{1}{c} x - t \right) \right] \right\}. \quad (5.71)$$

Analogically we find for the second possible solution

$$c \frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = \nu E_2 \exp \left\{ t\nu \left( 1 + \frac{1}{c\nu} g \right) + \Phi \left[ \nu \left( \frac{1}{c} x - t \right) \right] \right\} \quad (5.72)$$

For the small dimensionless frequencies ( $\tilde{\nu} \ll 1$ ) we find the known “classical” solution

$$c \frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = 0. \quad (5.73)$$

In Equation (5.71), we use  $x_0 = ct_0$  then

$$\exp \left\{ \Phi \left[ \nu \left( \frac{1}{c} x_0 - t_0 \right) \right] \right\} = e^{\Phi_0(0)} = Q = const. \quad (5.74)$$

As we see if

$$\frac{1}{c} x - t = n \left( \frac{1}{c} x_0 - t_0 \right) = 0 \quad (5.75)$$

for a value  $n$ , then

$$\exp \left\{ \Phi \left[ \nu \left( \frac{1}{c} x - t \right) \right] \right\} = e^{\Phi_0(0)} = Q = const. \quad (5.76)$$

We reach the quantization of the photon field and appearance of the separate objects which can be considered as pseudo particles.

## 6. Conclusions

1) Nonlocal quantum hydrodynamics (NQH) proposed by me obtained from the first principles of physics. NQH allows describing the PV evolution including the Planck time.

2) Evolution of the photon gas (PG) in the Planck period is a particular case of the PV hydrodynamics.

3) In general case, PG hydrodynamics contains gravitation in the explicit form.

4) The exact analytical solutions of PG hydrodynamics are obtained. Solutions show the exponential growth of *gradient values for internal energy in time and space*. The theory reflects even the existence large-scale energy fluctuations of the cosmic microwave background (see also **Figure 1**).

5) In comparison with phenomenological General Relativistic Theory, the

NQH does not lead to contradictions in all limit cases.

### Acknowledgements

I am grateful to Dr. A Fedoseyev for his help and fruitful discussion of the problem.

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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