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Twin Prime Distribution Problem

Dan Liu

Neijiang Teachers College, Neijiang, China
Email: 576672568@qq.com

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Abstract

The distribution of twin prime numbers is discussed. The research method of corresponding prime number distribution is proposed. The distribution of prime numbers corresponding to integers and composite numbers is discussed. Through the corresponding prime distribution rate of integers and composite numbers, it is found that the corresponding prime distribution rate of composite numbers approaches the corresponding prime distribution rate of integers. The distribution principle of corresponding prime number of composite number is proved. The twin prime distribution theorem is obtained. The number of twin prime numbers is thus obtained. It provides a practical way to study the conjecture of twin prime numbers.

Keywords

Prime Distribution, The Distribution of Prime Numbers Corresponding to Integers and Composite Numbers, The Distribution Principle of Prime Numbers Corresponding to Composite Numbers, Twin Prime Distribution Theorem

1. Introduction

In the long process of human understanding the world, there are many problems that attract many scholars and confuse many wise men.

Mathematicians found that the interval between two adjacent prime numbers is 2. This is a prime number, which is called twin prime number [1] [2] [3] [4].

Example 1

Twin prime numbers

11 and 13,

59 and 61,

That is, 11 and 11 + 2,

59 and 59 + 2,

In this way, there are infinite primes, which are called twin prime conjecture.

The twin prime distribution problem is the corresponding prime distribution.

Set the integer k, let's look at the sequence,

k,	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\cdots$ k,
$k+2$,	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\cdots$ $k+2$,

In $k+2$, integers

2, 3, 4, 5, ...

are called the corresponding number of k.

In $k+2$, prime numbers

2, 3, 5, 7, ...

are called the corresponding prime numbers of k. That is, the prime number corresponding to an integer.

In k, the composite numbers

4, 6, 8, 9, ...,

correspond to the prime numbers

11, 17, ...,

which are called the corresponding prime numbers of the composite numbers.

The key to the distribution of twin prime numbers is to study the corresponding prime numbers of integers and composite numbers.

Before studying the twin prime distribution problem, let's look at the prime distribution problem.

Chebyshev function is the focus of mathematicians' research on prime distribution. Let's look at the mathematician's method.

In 1852, the Russian mathematician Chebyshev proposed the function [3] [4] [5]:

$$\theta(x) = \sum_{p \le x} \log p \sim x, \ (x \to \infty)$$

Get

$$\theta(x) = \log 2 + \log 3 + \log 5 + \log 7 + \log 11 + \log 13 + \cdots + \log p$$

Set $x > y$, for large y, $\log p \le \log x$, Obtained by Chebyshev function

$$(\pi(x) - \pi(y))\log y < \theta(x) - \theta(y) < (\pi(x) - \pi(y))\log x$$

Obviously $\theta(x) - \theta(y) < \theta(x)$, can get

$$(\pi(x) - \pi(y))\log y < \theta(x) < \pi(x)\log x$$

Set $y = x^\lambda$, we get [4] [6]:

$$\left(\pi(x) - \pi\left(x^\lambda\right)\right)\log x^\lambda < \theta(x) < \pi(x)\log x$$

Obviously estimate $\pi(x^\lambda) < x^\lambda$, get

$$\lambda\left(\pi(x) - x^\lambda\right)\log x < \theta(x) < \pi(x)\log x \tag{1.1}$$

By (1.1) get

$$\lambda\frac{\pi(x) - x^\lambda}{x}\log x < \frac{\theta(x)}{x} < \pi(x)\frac{\log x}{x}$$

Can get

$$\lambda\left(\frac{\pi(x)}{x}-\frac{x^{\lambda}}{x}\right)\log x<\frac{\theta(x)}{x}<\pi(x)\frac{\log x}{x}$$

$\lambda < 1$, can confirm

$$\frac{x^{\lambda}}{x}\to 0$$

By $\pi(x) > x^{\lambda}$, we can get

$$\lambda\pi(x)\frac{\log x}{x}<\frac{\theta(x)}{x}<\pi(x)\frac{\log x}{x}$$

$\lambda \sim 1$, can get:

$$\lim_{x\to\infty}\pi(x)\frac{\log x}{x}=\lim_{x\to\infty}\frac{\theta(x)}{x}$$

By Chebyshev function

$$\theta(x)\sim x,\ (x\to\infty)$$

The prime theorem is proved

$$\pi(x)\sim\frac{x}{\log x},\ (x\to\infty) \tag{1.2}$$

Example 2

$x = 10^{18}$, calculated by (1.2)

$$\pi\left(10^{18}\right)\approx 24127471216847323$$

$$\pi\left(10^{18}\right)=24739954287740860$$

The method of proof is to construct unequal forms, carry out unequal transformation, delete

$$\frac{x^{\lambda}}{x}$$

by limit, and then obtain the prime theorem.

However, some scholars can't understand it. We can improve the traditional proof method.

2. Improve the Traditional Proof Method

We improve the proof method of (1.1).

Might as well set

$$\lambda=1-\frac{2\log\log x}{\log x}$$

Obviously, x tends to infinity, so we can confirm $\lambda\to 1$, and we get [4] [7]

$$x^{\lambda}=\frac{x}{(\log x)^{2}} \tag{2.1}$$

By (1.1) and (2.1) can get

$$\lambda\left(\pi(x)-\frac{x}{(\log x)^2}\right)\log x<\theta(x)<\pi(x)\log x$$

Now, prove $\pi(x) > x^{\lambda}$, You can delete it x^{λ}/x, however, It's complicated. We use another method to prove the prime theorem.

Obviously $x^{\lambda} < 2x^{\lambda}$, by (1.1) can get

$$\lambda\left(\pi(x)-2x^{\lambda}\right)\log x<\theta(x)<\pi(x)\log x$$

We get

$$\lambda\left(\pi(x)-2x^{\lambda}\right)\log x<\theta(x) \quad \text{and} \quad \theta(x)<\pi(x)\log x$$

Set positive number a and b, can get

$$\pi(x)<\frac{\theta(x)}{\lambda\log x}+2x^{\lambda}=a \quad \text{and} \quad \pi(x)>\frac{\theta(x)}{\log x}=b$$

We can get

$$b<\pi(x)<a$$

Obviously

$$b<\frac{a+b}{2}<a$$

We get the arithmetic mean of $\pi(x)$

$$\pi(x)\approx\frac{a+b}{2}$$

Can get

$$\pi(x)\approx\frac{1}{2}\left(\frac{\theta(x)}{\log x}+\frac{\theta(x)}{\lambda\log x}\right)+x^{\lambda}$$

x tends to infinity, $\lambda\to 1$, and (2.1) get

$$\pi(x)\sim\frac{1}{2}\left(\frac{\theta(x)}{\log x}+\frac{\theta(x)}{\log x}\right)+\frac{x}{(\log x)^2}$$

Can get

$$\pi(x)\sim\frac{\theta(x)}{\log x}+\frac{x}{(\log x)^2}$$

The prime number theorem can be obtained from Chebyshev function

$$\theta(x)\sim x,\ (x\to\infty)$$

We get:

$$\pi(x)\sim\frac{x}{\log x}+\frac{x}{(\log x)^2},\ (x\to\infty) \tag{2.2}$$

Example 3

$x = 10^{18}$, calculated by (2.2)

$$\pi\left(10^{18}\right)\approx 24709606084167120$$

$$\pi\left(10^{18}\right)=24739954287740860$$

The prime theorem can also be obtained in another way.

3. New Research on Prime Theorem

We use a new method to study the prime theorem.

Set prime number $p \le x$,

$$\mu(x)=2^{1/x}+3^{1/x}+5^{1/x}+7^{1/x}+11^{1/x}+13^{1/x}+\cdots+p^{1/x} \tag{3.1}$$

Set e = 2.718…, by (3.1) can get

$$\begin{aligned}&\mathrm{e}^{1/x}+\mathrm{e}^{1/x}+\mathrm{e}^{1/x}+\mathrm{e}^{1/x}+11^{1/x}+13^{1/x}+\cdots+p^{1/x}\\&<2^{1/x}+3^{1/x}+5^{1/x}+7^{1/x}+11^{1/x}+13^{1/x}+\cdots+p^{1/x}\end{aligned}$$

Can get [6] [7]

$$\pi(x)\mathrm{e}^{1/x}<2^{1/x}+3^{1/x}+5^{1/x}+7^{1/x}+11^{1/x}+13^{1/x}+\cdots+p^{1/x}<\pi(x)p^{1/x}$$

$$\pi(x)\mathrm{e}^{1/x}<\pi(x)x^{1/x}$$

Can get

$$\pi(x)\mathrm{e}^{1/x}<\pi(x)x^{1/x}<\pi(x)x^{1/x}+x^{1/x}$$

We get

$$\frac{\pi(x)\mathrm{e}^{1/x}}{x^{1/x}}<\pi(x)<\frac{\pi(x)x^{1/x}+x^{1/x}}{x^{1/x}}$$

Arithmetic mean of $\pi(x)$:

$$\pi(x)\approx\frac{1}{2}\frac{\pi(x)\mathrm{e}^{1/x}+\pi(x)x^{1/x}+x^{1/x}}{x^{1/x}}$$

Can get

$$2\pi(x)x^{1/x}\approx\pi(x)\mathrm{e}^{1/x}+\pi(x)x^{1/x}+x^{1/x}$$

Get

$$2\pi(x)x^{1/x}-\pi(x)x^{1/x}\approx\pi(x)\mathrm{e}^{1/x}+x^{1/x}$$

We can get

$$\pi(x)x^{1/x}\approx\pi(x)\mathrm{e}^{1/x}+x^{1/x}$$

Obviously prove

$$\lim_{x\to\infty}\frac{\pi(x)x^{1/x}}{\pi(x)\mathrm{e}^{1/x}+x^{1/x}}=1$$

From this, we can get a new prime theorem

$$\pi(x)x^{1/x}\sim\pi(x)\mathrm{e}^{1/x}+x^{1/x},\ (x\to\infty)$$

We can get:

$$\pi(x)x^{1/x}-\pi(x)\mathrm{e}^{1/x}\sim x^{1/x},\ (x\to\infty)$$

Can get

$$\pi(x) \sim \frac{x^{1/x}}{x^{1/x} - e^{1/x}}, \ (x \to \infty)$$

That is

$$\pi(x) \sim \frac{1}{x^{1/x} - e^{1/x}}, \ (x \to \infty) \tag{3.2}$$

Example 4

$x = 10^{18}$, calculated by (3.2)

$$\pi(10^{18}) \approx 24723998785919976$$

$$\pi(10^{18}) = 24739954287740860$$

The problem of prime distribution was discussed earlier. We continue to discuss the distribution of twin prime numbers.

4. Corresponding Prime Distribution

The twin prime distribution problem is the corresponding prime distribution.

Set Integer $x = 16$, let's look at the sequence of k and $k + 2$

k,	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14,
$k+2$,	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16,

$k + 2$ is called the corresponding number of k

In $k + 2$, prime

2, 3, 5, 7, 11, 13,

a total of 6. The table shows

$$\pi(16) = 6$$

and the proportion of prime numbers is:

$$\frac{\pi(16)}{16} = \frac{6}{16}$$

Generally [6] [8]:

$$\rho_x = \frac{\pi(x)}{x} \tag{4.1}$$

Here (4.2) is called the prime distribution rate of integers.

Let's look at the prime number corresponding to the composite number.

Set the composite number c. for the convenience of discussion, temporarily set 0 as the composite number to see the sequence of composite numbers c and c + 2

c,	0	4	6	8	9	10	12	14,
$c+2$,	2	6	8	10	11	12	14	16,

In c, there are 8 composite numbers, and the table is $F = 8$.

In $c + 2$, there are 2 primes, 2, 11. The table is $c(16) = 2$.

The proportion of prime numbers is:

$$\frac{c(16)}{F} = \frac{2}{8}$$

Generally:

$$\rho_c = \frac{c(x)}{F} \tag{4.2}$$

Here (4.2) is called the prime distribution rate of composite numbers.

Example 5

(4.1) and (4.2) Partial calculation:

x,	$\pi(x)/x$	$c(x)$	$c(x)/F$
10^4	0.1229	1024	0.116748
10^8	0.05761455	5321143	0.05646461329

A composite number is part of an integer. The greater the proportion of the composite number in the integer, the closer the distribution rate of the corresponding prime number of the composite number is to the distribution rate of the corresponding prime number of the integer.

Now let's discuss the corresponding prime of prime.

Let the prime number p. for the convenience of discussion, temporarily set 1 as the prime number to see the sequence of prime numbers p and $p + 2$

$$\begin{array}{llllllll} p, & 1 & 2 & 3 & 5 & 7 & 11 & 13, \\ p+2, & 3 & 4 & 5 & 7 & 9 & 13 & 15, \end{array}$$

In $p + 2$, there are four primes: 3, 5, 7, 13. The table is

$$L(16) = 4$$

Represents the number of primes in $p + 2$. Can get

$$L(16) = \pi(16) - c(16) = 4$$

According to statistics, if x is larger, then $L(x)$ is more.

If we can prove that $L(x)$ is infinite, we can prove the twin prime conjecture.

5. Principle of Composite Number Corresponding to Prime Number

You can get from the front

$$\pi(16) = c(16) + L(16) = 2 + 4$$

Generally:

$$\pi(x) = c(x) + L(x) \tag{5.1}$$

By (5.1) can get

$$c(x) = \pi(x) - L(x)$$

Set number of set composite numbers

$$F = x - \pi(x)$$

By (4.2) we can get

$$\frac{c(x)}{F}=\frac{\pi(x)-L(x)}{x-\pi(x)}$$

$$\frac{\pi(x)-L(x)}{x-\pi(x)}=\frac{\pi(x)\left(1-\frac{L(x)}{\pi(x)}\right)}{x\left(1-\frac{\pi(x)}{x}\right)}=\frac{c(x)}{F}$$

We get

$$\frac{\pi(x)}{x}\frac{F}{c(x)}=\frac{1-\frac{\pi(x)}{x}}{1-\frac{L(x)}{\pi(x)}}$$

Set x tends to infinity, so we can confirm [4] [8]:

$$\frac{\pi(x)}{x}\to 0 \quad \text{and} \quad \frac{L(x)}{\pi(x)}\to 0$$

We can get

$$\lim_{x\to\infty}\frac{\pi(x)}{x}\frac{F}{c(x)}=1$$

Set λ, can get

$$\frac{\pi(x)}{x}\frac{F}{c(x)}=\lambda$$

Example 6

By λ Partial calculation:

x	$\pi(x)/x$	$c(x)/F$	λ
10^4	0.1229	0.116748	1.0526946928
10^8	0.05761455	0.05646461329	1.02036561738

we get:

$$c(x)=\frac{F\pi(x)}{\lambda x} \tag{5.2}$$

Here (5.2) is called the corresponding prime number principle of composite numbers.

6. Twin Prime Distribution Theorem

We continue to discuss the distribution of twin prime numbers.

The number of twin prime numbers is obtained from (5.1)

$$L(x)=\pi(x)-c(x) \tag{6.1}$$

By (5.2) and (6.1), we get [6] [7]:

$$L(x)=\pi(x)-\frac{F\pi(x)}{\lambda x} \tag{6.2}$$

Here (6.2) is called the twin prime distribution theorem [6] [7].

Example 7

Calculated by part (6.2):

x	π(x)	λ	L(x)
10^4	1229	1.0526946	205
10^8	5761455	1.020365617	440312

7. Discuss the Distribution of Twin Prime Numbers

We confirm the number of twin prime numbers

From (6.2), we can get:

$$L(x)=\pi(x)\left(1-\frac{F}{\lambda x}\right)=\pi(x)\left(\frac{\lambda x-F}{\lambda x}\right) \tag{7.1}$$

Can confirm

$$L(x)>L_\lambda(x)=\pi(x)\left(\frac{\lambda x-F}{(\lambda+1)x}\right)$$

Example 8

By (7.1) Partial calculation:

x	$\pi(x)$	λ	$L(x)$	$L_\lambda(x)$
10^4	1229	1.0526946	205	105
10^8	5761455	1.020365617	440312	222375

x tends to infinity, $\lambda \to 1$, can get [6] [8]

$$L(x)>\pi(x)\left(\frac{x-F}{2x}\right)$$

Set $F=x-\pi(x)$, we get

$$L(x)>\frac{\pi^2(x)}{2x},(x\to\infty)$$

Set $F=x-\pi(x)$, and $\pi(x)>x/\log x$, we get:

$$L(x)>\frac{x}{2(\log x)^2},(x\to\infty) \tag{7.2}$$

From (7.2), $L(x)$ can be confirmed there is an infinite number.

The twin prime conjecture is correct.

8. Conclusion

We discuss the prime distribution

$$\pi(x) \sim \frac{1}{x^{1/x} - e^{1/x}}, (x \to \infty)$$

The twin prime conjecture is discussed

$$L(x) > \frac{x}{2(\log x)^2}, (x \to \infty)$$

The twin prime conjecture is correct.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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