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Mathematical Modeling and Computational Analysis of Underwater Topography with Global Positioning and Echo Sounder Data

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Abstract

This study focuses on change of topography in a water area. Output data from a GPS unit and an echo sounder data were incorporated into analysis for construction of underwater topography. Comparison of two data sets lead to conclusion concerning sedimentation during period from January 2020 to January 2021.

Keywords

Underwater Topography, RTK-GPS, Echo Sounder

1. Introduction

Recent disastrous heavy rain events and floods caused severe damages including human damages and house damages. Those include 119 fatalities and 213 totally destroyed houses due to 2018 Japan floods (July 2018) [1], 104 fatalities and 3308 totally destroyed houses due to Typhoon 19 (Hagibis, October 2019) and subsequent heavy rain events [2], and 84 fatalities and 1621 totally destroyed houses due to July 2020 heavy rain disaster [3]. As the climate change progresses such disastrous heavy rain events and floods may occur more frequently, and it is important to establish reliable sources of information concerning land water areas such as rivers, reservoirs, and coastal areas.

This study focuses on construction of underwater topography based on data obtained in field measurement. Apparatuses including a RTK-GPS (real time kinematic global positioning system) in VRS (virtual reference station) mode and an echo sounder were used in measurement conducted in Kojima Lake, Okayama Prefecture, Japan. Measurement was conducted on September 28th, 2019, October 4th, 2019, December 25th, 2019, January 6th, 2020, December 26th, 2020, January 27th, 2021, March 17th, 2021, and March 20th, 2021 [4] [5] [6] [7]. Previous studies developed numerical techniques to construct surfaces based on data. Those techniques were applied to data sets obtained in the field measurement for construction of surfaces representing underwater topography. Numerical results show sedimentation during period from January 2020 to January 2021.

2. Application of Numerical Techniques to Data Sets

Numerical techniques developed in previous studies [4] [5] [6] [7] were reapplied to two data sets. One data set, which we call data set 1, consisted of results of measurement conducted on September 28th, 2019, October 4th, 2019, December 25th, 2019, and January 6th, 2020. The other data set, which we call data set 2, consisted of results of measurement conducted on December 26th, 2020, January 27th, 2021, March 17th, 2021, and March 20th, 2021.

The Gauss-Krüger projection transformed latitude components and longitude components of GPS data to xy components of a rectangular coordinate. Combination of those components with vertical components including output results from an echo sounder leads to three dimensional data that lay in an underwater topography. In particular, z component of three dimensional data $\left(x_j, y_j, f_j\right), j = 1,2,3,\ldots$ are given by $f_j = h_j - d_j - z_0 - L$, where h_j is the GPS antenna height, d_j is the distance between the oscillator of echo sounder and the bottom, z_0 is the geodetic height of the mean sea level, and L is the distance between the antenna and the oscillator. **Figure 1** shows three dimensional data of Kojima Lake topographic data.

An underwater topography was represented by a piecewise linear function defined on a triangular mesh. An initial triangular mesh T_0 that contains GPS tracks was set in an xy plane. A sequence of triangular meshes $T_0, T_1, T_2, \ldots$ were constructed from the initial mesh. A triangular mesh $T_l (l \geq 1)$ in the sequence was constructed by dividing each element of T_{l-1} into four congruent triangles. **Figure 2** shows an initial triangular mesh T_0. **Figure 2** also shows an approximate outline of Kojima Lake based on data obtained with an online software [8].

Suppose that triangular mesh T_l consists of m elements $E_1, E_2, \ldots, E_m$, and n nodes $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, that elevation of topography z_i at node (x_i, y_i) is given for $i = 1,2,\ldots,n$, and that an element E_k contains p data $\left(x_j, y_j, f_j\right), j = 1,2,3,\ldots,p$, and that coordinates of vertices of E_k are (x_1, y_1), (x_2, y_2), and (x_3, y_3). Note that xy coordinates of the first three data are those of the vertices of E_k, and that f_1, f_2 and f_3 are elevations at the vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3), respectively. Consider a linear function $z = ax + by + c$ such that the values of coefficients a, b, and c are those that minimize the square sum

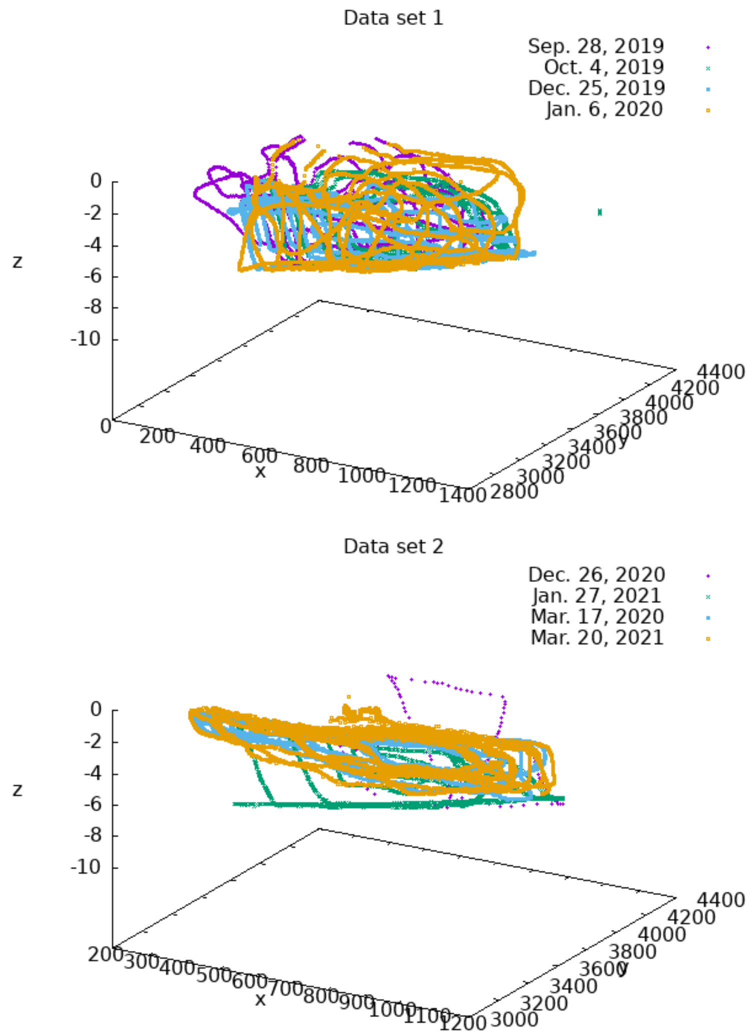
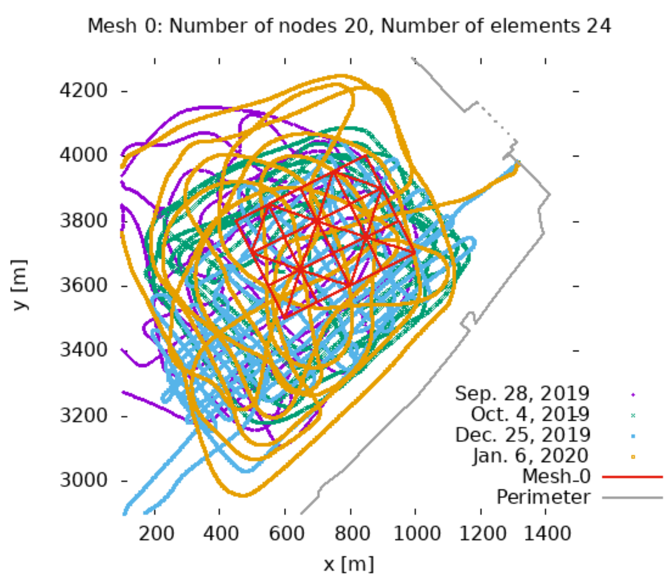


Figure 1. Three dimensional topographic data of Kojima Lake.



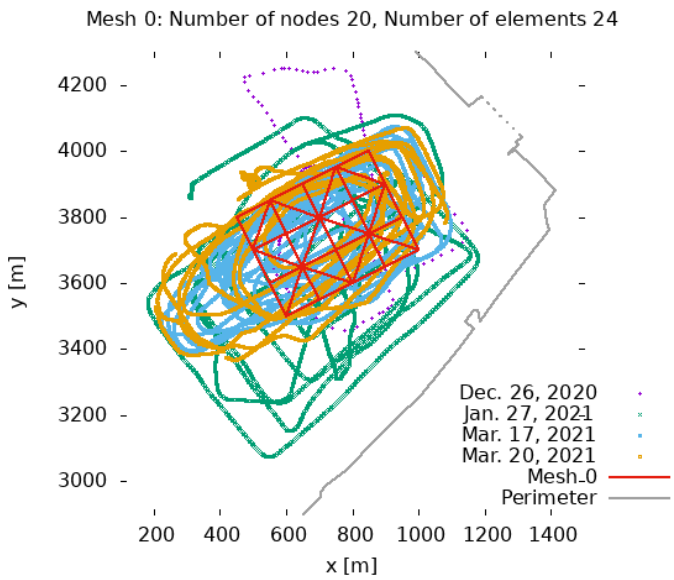


Figure 2. Initial mesh. Three dimensional topographic data are also shown.

$$\left[f_1-\left(ax_1+by_1+c\right)\right]^2+\cdots+\left[f_p-\left(ax_p+by_p+c\right)\right]^2. \tag{1}$$

Once those coefficients are evaluated, value of f_1 is updated, that is, $f_1 = ax_1 + by_1 + c$. With this new value of f_1, values of coefficients a, b, and c that minimize the square sum (1) are updated and the value of f_2 is updated with equation $f_2 = ax_2 + by_2 + c$. With those new values of f_1 and f_2, values of coefficients a, b, and c that minimize the square sum (1) are updated, and the value of f_3 is updated with equation $f_3 = ax_3 + by_3 + c$. After those operations are completed E_k, the operations are repeated for the element E_{k+1}. One cycle of iterations is completed for the triangular mesh T_l when k reaches m, z component or elevation associated with the n nodes, $z_1, z_2, \ldots, z_n$ are obtained.

Denote by $\boldsymbol{Z}_q=\left(z_1^q, z_2^q, \ldots, z_n^q\right)$ the n dimensional vector whose components are elevation associated with n nodes after q iterations. The iteration is terminated when the residual becomes less than ε, that is

$$\left\|\boldsymbol{Z}_q-\boldsymbol{Z}_{q-1}\right\|=\left[\left(z_1^q-z_1^{q-1}\right)^2+\cdots+\left(z_n^q-z_n^{q-1}\right)^2\right]^{1/2}<\varepsilon .$$

Values of initial elevation in T_0 are all set equal to 0, and values of initial elevation for T_l are obtained from values of final elevation for T_{l-1}. **Figure 3** and **Figure 4** show surfaces obtained with $\varepsilon = 0.75$. The results shown in **Figure 3** and **Figure 4** lead to sedimentation during period from January 2020 to January 2021 (**Figure 5**).

3. Discussion

A triangular mesh is set in a part of region covered the triangular mesh shown by Figure by **Figure 2** and numerical procedures described in the previous section were repeated. **Figure 6** shows the initial mesh. **Figure 7** shows the

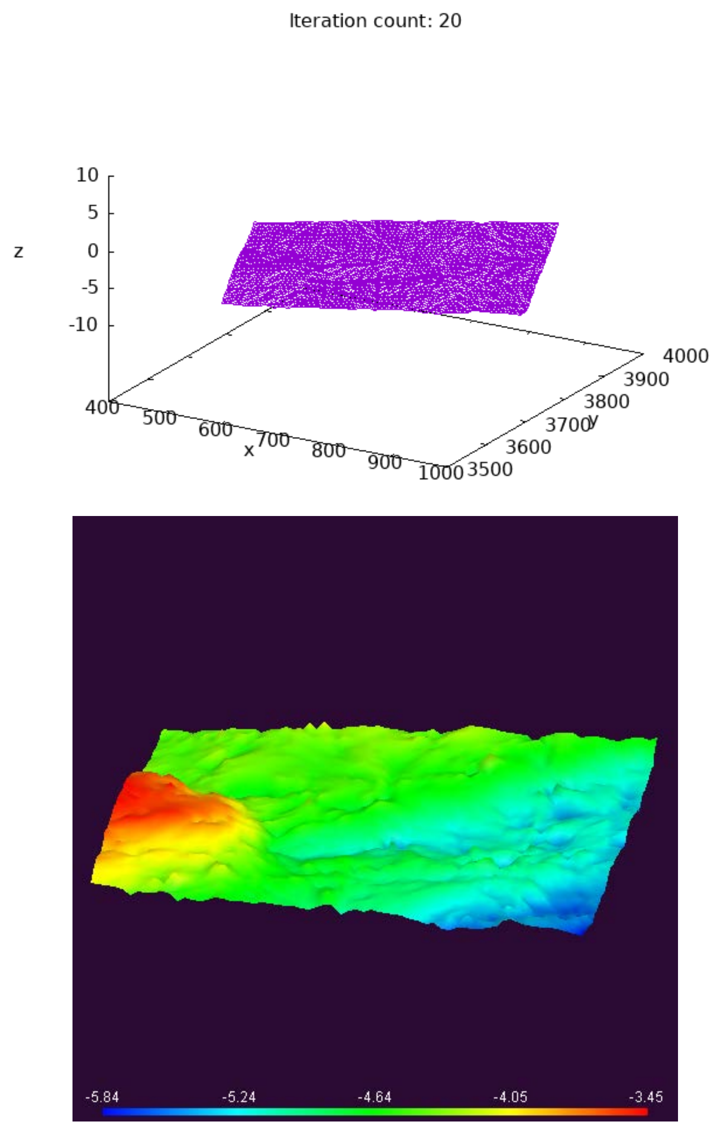
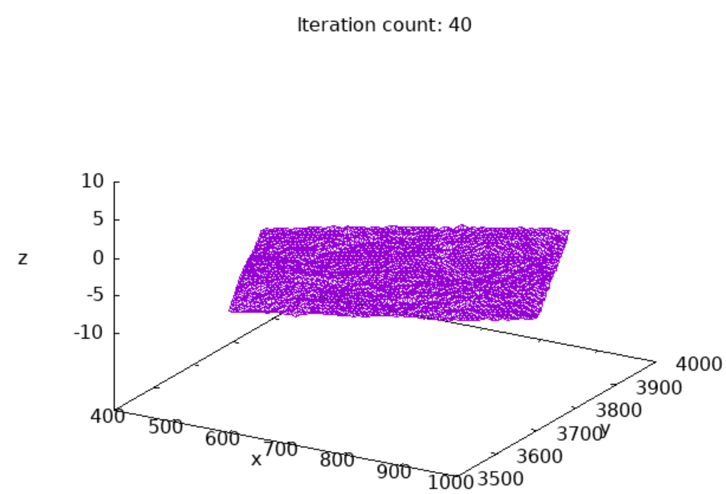


Figure 3. Surface over T_4 based on data set 1 with $\varepsilon = 0.75$, wireframe representation (top), surface with color according to elevation (color).



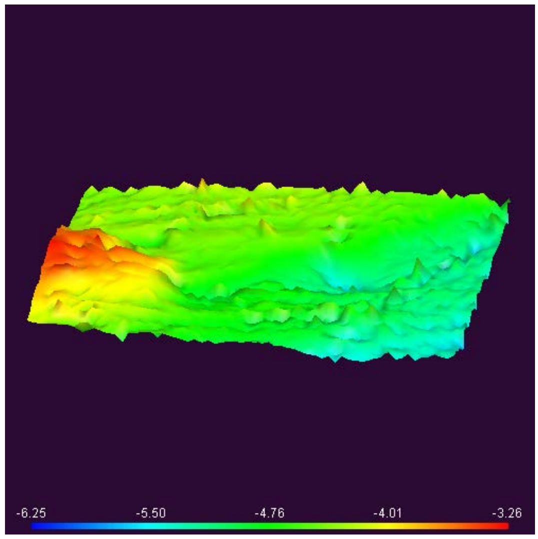


Figure 4. Surface over T_4 based on data set 2 with $\varepsilon = 0.75$, wireframe representation (top), surface with color according to elevation (bottom).

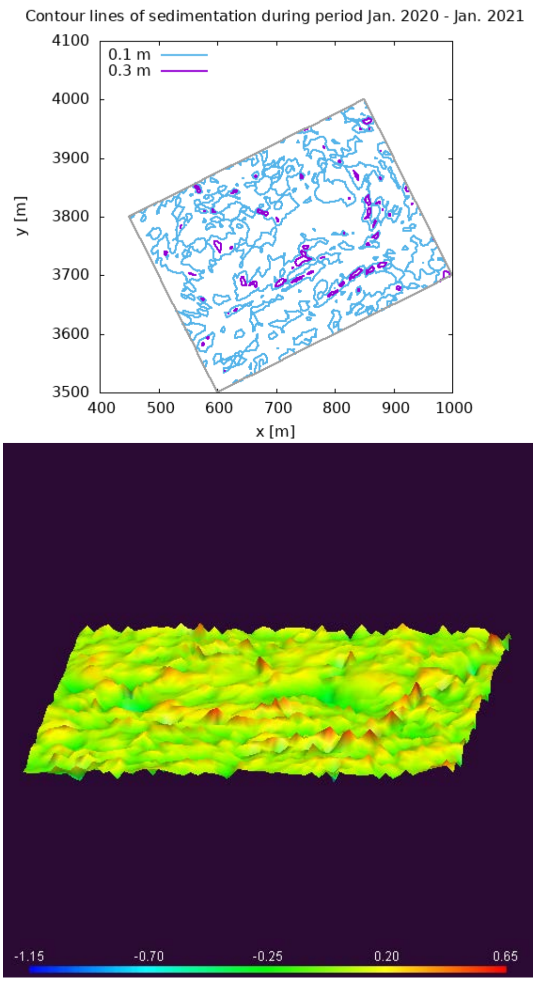


Figure 5. Sedimentation over region over the region covered by the initial mesh shown by **Figure 2** during period from January 2020 to January 2021, contour lines $z = 0.1$ [m] and $z = 0.3$ [m] (top), sedimentation with color according to amount (bottom).

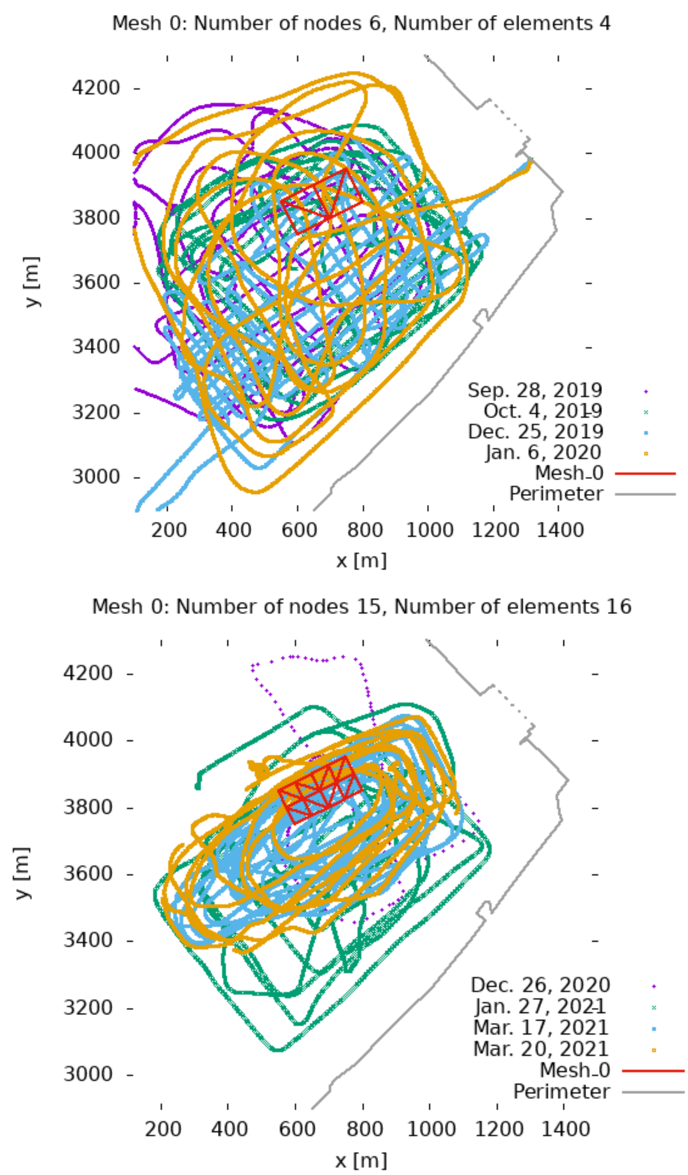


Figure 6. Initial mesh. Three dimensional topographic data are also shown.

sedimentation during period from January 2020 to January 2021.

The area of region covered by the initial mesh shown by **Figure 2** is approximately 150,000 m^2, and the total sedimentation over the equal to region is approximately 5700.569784 m^3. It follows that average increase in elevation of underwater topography over the region is 0.038004 m. The area of region covered by the initial mesh shown by **Figure 6** is approximately 25,000 m^2, and the total sedimentation over the equal to region is approximately 1508.789762 m^3. It follows that average increase in elevation of underwater topography over the region is 0.060352 m.

Major sources of water in Kojima Lake are inflow flow from two rivers Kurashiki River and Sasagase River. Kojima Lake was separated from Kojima Bay by embankment. There are six gates set on the embankment (**Figure 2**). The water

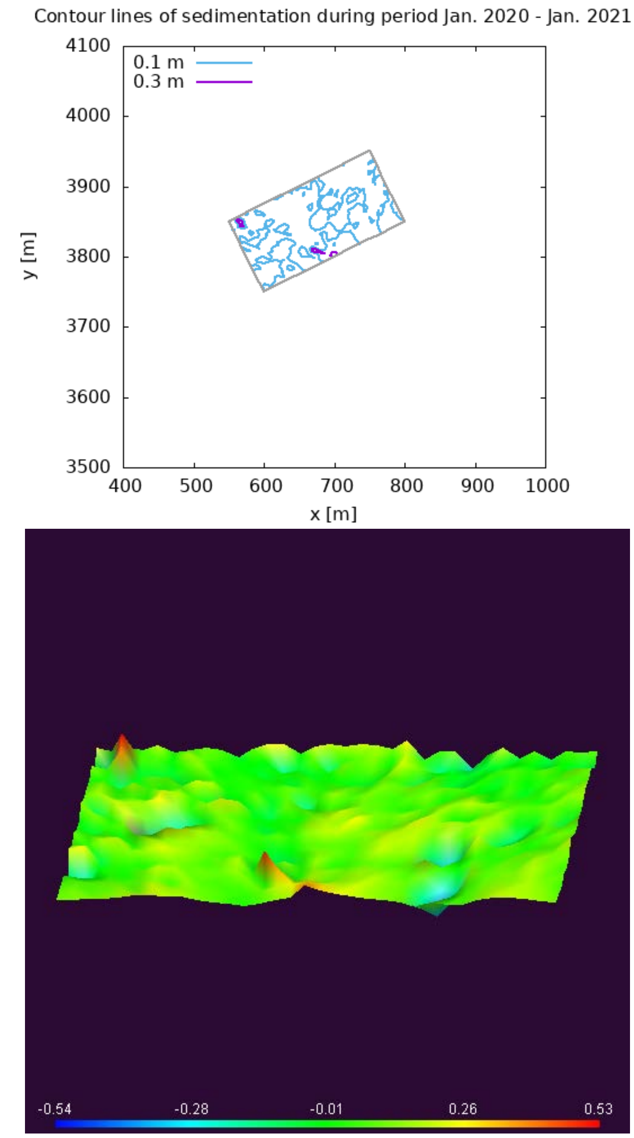


Figure 7. Sedimentation over the region covered by the initial mesh shown by **Figure 6** during period from January 2020 to January 2021, contour lines $z = 0.1$ [m] and $z = 0.3$ [m] (top), sedimentation with color according to amount (bottom).

level of Kojima Lake is controlled by discharge of water through the gates into Kojima bay during low tide. A possible reason for higher sedimentation over the region shown by **Figure 7** is stronger effect of flow generated by the discharge.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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