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Time Varying Deceleration Parameter in *f*(*R*, *T*) Gravity

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Abstract

In the present study, a homogeneous and anisotropic LRS Bianchi type I universe model is considered in $f(R,T)$ theory of gravity. In order to find an exact solution of the field equations of the model, the model presented is based on a unique condition of periodically time varying deceleration parameter. The physical and geometrical characteristics of the universe model have been studied. It has been shown that the model has point-type singularity and all the cosmological parameters possess periodic time behavior. The model has a cyclic expansion history, for example, the model starts with the decelerating expansion, and later it transits to an accelerating phase of expansion and then goes to super-exponential phase of expansion in a period.

Keywords

Periodic Deceleration Parameter, $f(R,T)$ Gravity, Bianchi Type-I Universe

1. Introduction

Several observations such as Super Novae type Ia [1]-[6], Cosmic Microwave Background Radiation [7], Baryon Acoustic Oscillations [8] [9] [10] and PLANK collaborations [11] indicate that our universe is in the phase of an accelerating expansion. It is believed that the reason of the late time acceleration is Dark Energy (DE). However, it is still not completely known what the DE is. According to the abovementioned observations, there is an important known fact that ~70% rate of the energy content of the universe is DE. Scientists generally try to explain dark energy in two ways. One is to adopt some exotic matter sources. The other is to modify the original field equations of Einstein's general theory of relativity. The first and the simple modification of Einstein's field equations (EFEs) as a DE candidate is the cosmological constant Λ. Nevertheless, the cosmological constant is suffered by fine tuning problem and cosmic coincidence problem. Except the cosmological constant Λ, there are several modifications of EFEs. Besides others, the $f(R,T)$ modified gravity theory has become very popular for last ten years [12].

Many researchers have considered many problems of the cosmology in this theory. Adhav [13] has obtained the solutions of locally rotationally symmetric (LRS) Bianchi type-I universe in $f(R,T)$ theory. Houndjo *et al.* [14] have constructed a cosmological scenario in $f(R,T)$ gravity and discoursed a matter dominated era transition. Taking a variable deceleration parameter that depends on the Hubble parameter, Tiwari *et al.* considered the LRS Bianchi type-I model in $f(R,T)$ theory [15]. Sofuoğlu showed that the Gödel universe is a solution of the field equations in $f(R,T)$ theory [16]. In $f(R,T)$ theory of gravity, Tiwari *et al.* examined Bianchi type-I model in which the cosmological term is decaying [17]. Recently, Tiwari *et al.* [18] have examined the $f(R,T)$ gravity using a time dependent cosmological term. Tiwari *et al.* [19] have explained phase transition in expansion of LRS Bianchi type-I universe in $f(R,T)$ theory. Tiwari *et al.* [20] have considered a quadratically varying deceleration parameter in this modified theory.

In this work, adopting a periodically varying deceleration parameter, we investigate LRS Bianchi-type I universe in $f(R,T)$ gravity theory by adopting a particular form of $f(R,T)$ function as $f(R,T)=R+2\lambda T$, where λ is a constant. For finding an exact solution of the field equations of the model, the model presented is based on a unique condition of periodically time varying deceleration parameter.

The outline of this paper as follows: Basic formalism of $f(R,T)$ theory is given in Section 2; the solutions of the field equations for LRS Bianchi-type I universe are obtained and the physical and geometrical characterization of the model is represented in Section 3; and the conclusions are given in Section 4.

Over the study, we use the natural units as $G=1=c$.

2. Basic Formalism of $f(R,T)$ Theory

The action of $f(R,T)$ theory is described by [12]

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi}f(R,T)+L_m\right)\mathrm{d}^4x. \tag{1}$$

Here R is the Ricci scalar and T is the trace of the matter energy-momentum tensor L_m, $f(R,T)$ is an arbitrary function of these two scalars, L_m is the Lagrange density of matter. T_{ij} is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ij}}, \tag{2}$$

where g_{ij} is the covariant components of the metric tensor and g is the determinant of the metric tensor. The variation of the gravitational action (1) with respect to g_{ij}, yields the field equations of this modified theory:

$$\begin{aligned} & f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^k\nabla_k - \nabla_i\nabla_j\right)f_R(R,T) \\ & = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{3}$$

where $f_R \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T \equiv \frac{\partial f(R,T)}{\partial T}$ and ∇_i is the four-dimensional covariant derivative, T_{ij} is defined in Equation (2) and

$\Theta_{ij} \equiv -2T_{ij} + g_{ij}L_m - 2g^{kl}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{kl}}$.

Here we note that if one takes $f(R,T) = f(R)$ then Equation (3) give the field equations of $f(R)$ theory.

The contraction of Equation (3) gives

$$f_R(R,T)R + 3\nabla^k\nabla_k f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)T - f_T(R,T)\Theta. \tag{4}$$

Now, $f(T)$ being any function of T, we take $f(R,T)$ as

$$f(R,T) = R + 2f(T). \tag{5}$$

By putting Equation (5) in Equation (3), we obtain the following field equations of $f(R,T)$ gravity

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} - f(T)g_{ij}, \tag{6}$$

where G_{ij} is the Einstein tensor and $f'(T) = \frac{\mathrm{d}f(T)}{\mathrm{d}T}$.

For a perfect fluid source, T_{ij} may be written as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{7}$$

Here ρ is the matter-energy density of the fluid, p is isotropic pressure, u_i is the four-velocity vector of the observer. In this case the field Equation (6) become [12] [21]

$$G_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + \left[2pf'(T) + f(T)\right]g_{ij}, \tag{8}$$

3. The Model and Solutions

The LRS Bianchi-Type-I universe is defined by the following ansatz

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A(t)^2\,\mathrm{d}x^2 - B(t)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right), \tag{9}$$

where A and B are time-dependent scale factors.

Now, λ being a constant, choosing the function $f(T)$ as

$$f(T) = \lambda T, \tag{10}$$

Equation (8) gives the following field equations for Equation (9)

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}}{A}\frac{\dot{B}}{B} = (8\pi + 3\lambda)\rho - \lambda p, \tag{11}$$

$$\frac{\dot{B}^2}{B^2}+\frac{2\ddot{B}}{B}=\lambda\rho-(8\pi+3\lambda)p, \tag{12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=\lambda\rho-(8\pi+3\lambda)p. \tag{13}$$

Here a dot (·) represents a derivative with respect to time *t*.

Equations (11)-(13) define a system of equations which has four unknowns (A, B, ρ, p) with three independent equations. In order to find the solution such a system, one more relation is required. Hence, we carry out a law of variation of deceleration parameter (DP). The time varying DP is important in evolution of the universe. Its phase transition in expansion may be well explained by the time varying DP. Now, we adopt the following periodic time varying DP [22]

$$q=m\cos(kt)-1. \tag{14}$$

Here m and k are positive real numbers. Figure 1 displays the behavior of DP with time for different values of the constants. One can observe from Figure 1 that the model starts with a decelerating phase of expansion ($q>0$) and later transits to an accelerating phase of expansion ($q<0$) and then goes to super-exponential phase of expansion ($q<-1$) in a cyclic history.

Using the definition of DP as $q=-\frac{\dot{H}}{H^2}-1$, the integration of Equation (14) gives the Hubble parameter H as

$$H=\frac{k}{m\sin(kt)+k_1}, \tag{15}$$

Here k_1 is a constant of integration. Here we may choose $k_1=0$ and then Hubble parameter becomes

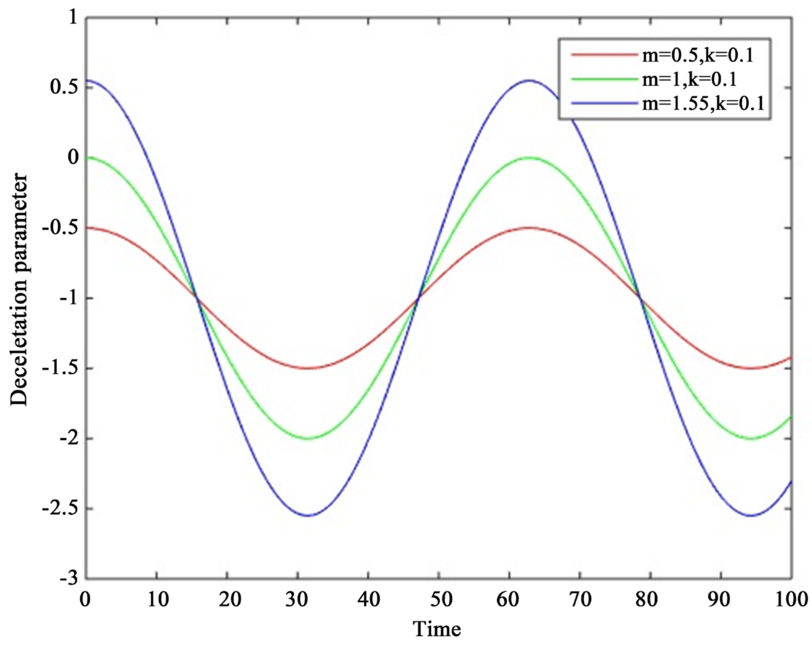

Figure 1. DP vs time in the units of Gyrs.

$$H = \frac{k}{m\sin(kt)}. \tag{16}$$

Using the definition of Hubble parameter as $H = \frac{\dot{a}}{a}$ in Equation (16), the average scale factors a is obtained as

$$a = a_0\left[\tan\left(\frac{1}{2}kt\right)\right]^{1/m}, \tag{17}$$

where a_0 is a constant of integration.

Figures 1-3 show that evolution of the deceleration parameter, the Hubble

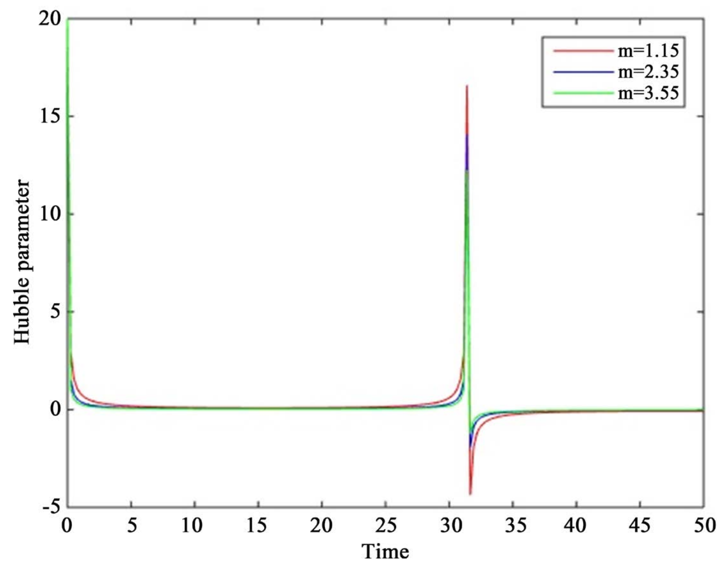

Figure 2. Hubble parameter vs time.

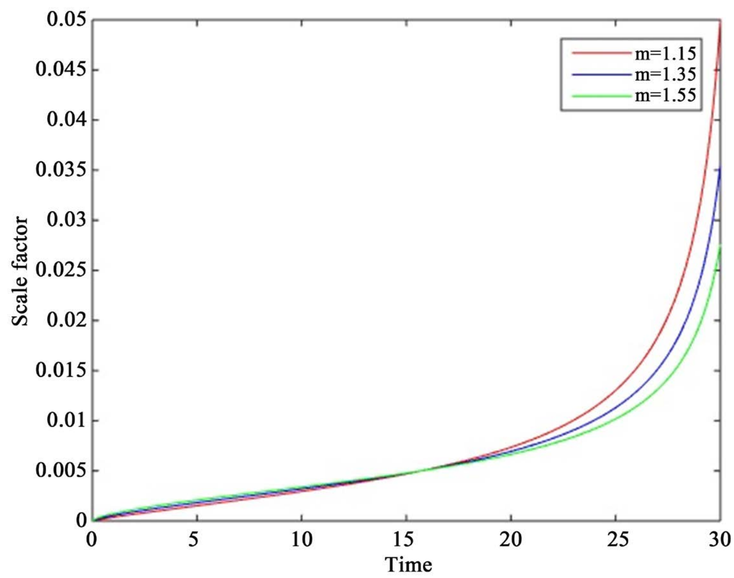

Figure 3. Scale factor vs time.

parameter and the scale factor in time with the unşts of gigayears.

The spatial volume is given by

$$V = a^3 = AB^2, \tag{18}$$

where a is the average scale factor.

The average Hubble parameter H of LRS Bianchi Type-I universe can be written as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{19}$$

The directional Hubble parameters in x, y and z directions are defined as, respectively,

$$H_1 = \frac{\dot{A}}{A} \quad \text{and} \quad H_2 = H_3 = \frac{\dot{B}}{B}, \tag{20}$$

For our model, the directional Hubble parameters are obtained as follows:

$$H_1 = \frac{k}{m\sin(kt)} + \frac{2k_1}{3a_0^3\left[\tan\left(\frac{1}{2}kt\right)\right]^{3/m}}, \tag{21}$$

$$H_2 = H_3 = \frac{k}{m\sin(kt)} - \frac{k_1}{3a_0^3\left[\tan\left(\frac{1}{2}kt\right)\right]^{3/m}}, \tag{22}$$

The anisotropisation in expansion of the model is given by the parameter Δ which is defined and found as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2k_1^2 m^2 \sin^2(kt)}{9k^2 a_0^2\left[\tan\left(\frac{1}{2}kt\right)\right]^{6/m}}, \tag{23}$$

where i runs from 1 to 3.

The expansion scalar $\theta = 3H$ is

$$\theta = \frac{3k}{m\sin(kt)}. \tag{24}$$

The shear scalar σ^2 is defined as $\sigma^2 = \frac{3}{2}\Delta H^2$ and found as

$$\sigma^2 = \frac{k_1^2}{3a_0^6\left[\tan\left(\frac{1}{2}kt\right)\right]^{6/m}}. \tag{25}$$

Using Equations (11)-(13), we obtain the pressure p as

$$p = \frac{-3k^2}{4(2\pi+\lambda)m^2\sin^2(kt)} - \frac{k_1^2}{6(4\pi+\lambda)a_0^6\left[\tan\left(\frac{1}{2}kt\right)\right]^{6/m}} - \frac{(8\pi+3\lambda)k^2\cos(kt)}{4m(2\pi+\lambda)(4\pi+\lambda)\sin^2(kt)} \tag{26}$$

and we obtain the energy density ρ as

$$\rho = \frac{3k^2}{4(2\pi+\lambda)m^2\sin^2(kt)} - \frac{k_1^2}{6(4\pi+\lambda)a_0^6\left[\tan\left(\frac{1}{2}kt\right)\right]^{6/m}} - \frac{\lambda k^2\cos(kt)}{4m(2\pi+\lambda)(4\pi+\lambda)\sin^2(kt)} \quad (27)$$

We see that the average scale factor is zero at initially. It increases in cosmic time and changes periodically. The metric potentials are vanish initially it means our model has point type singularity. Also, the singularities occur periodically at $t = \frac{nr}{k}$ ($n = 0,1,2,3,\cdots$). All the cosmological parameters ρ, p, θ, σ and Δ are infinite initially and they preserve their periodic behavior in time. It is interesting that the density and pressure have large values initially and decrease to a minimum value and then again increase with the evolution of time. So all the quantities are infinite initially and they preserve their periodic behavior against the cosmic time.

4. Conclusion

In the present study, the spatially homogeneous and anisotropic LRS Bianchi type-I universe in $f(R,T)$ modified theory of gravity has been investigated. The gravitational field equations are derived for the function $f(R,T) = R + 2\lambda T$, for a periodically variable DP as a function of cosine trigonometric form. We have held the exact solution of the model. We have shown that the obtained model has a point-type singularity initially and the similar singularities occur periodically. Also, we have explained and discussed the kinematical and dynamical character of the model that all the quantities are infinite initially and they preserve their periodic behavior against the cosmic time.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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