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Diversity of New Three-Wave Solutions and New Periodic Waves for the (3 + 1)-Dimensional Kadomtsev-Petviashvili-Boussinesq-Like Equation

Meiyu Li[1], Sudao Bilige[1*], Runfa Zhang[2], Lihui Han[1]

[1]Department of Mathematics, Inner Mongolia University of Technology, Hohhot, China
[2]School of Software Technology, Dalian University of Technology, Dalian, China

Email: *inmathematica@126.com

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Abstract

Based on the generalized bilinear method, diversity of exact solutions of the (3 + 1)-dimensional Kadomtsev-Petviashvili-Boussinesq-like equation is successfully derived by using symbolic computation with Maple. These new solutions, named three-wave solutions and periodic wave have greatly enriched the existing literature. Via the three-dimensional images, density images and contour plots, the physical characteristics of these waves are well described. The new three-wave solutions and periodic solitary wave solutions obtained in this paper, will have a wide range of applications in the fields of physics and mechanics.

Keywords

KPB-Like Equation, Generalized Bilinear Form, New Three-Wave Solutions, New Periodic Wave

1. Introduction

The nonlinear integrable systems, which are of great importance in the field of physics and mathematics [1], have been paid more and more attention and many exact solutions of the nonlinear integrable systems are obtained by using different effective methods. In recent, experts and scholars have become more interested in the research of solutions to nonlinear partial differential equations (NPDEs). They have extensively expanded the field of NPDEs in many aspects. The Hirota bilinear form plays an important role in presenting soliton solutions, although some intelligent guessing is usually required. In recent years, more exact solutions have been found for solving integrable equations, such as solitary wave [2] [3] [4] [5] [6], rogue waves [7] [8] [9], periodic solitary wave [10] [11] [12], optical soliton [13], lump solution and interaction solution [14]-[34], and cross-kink wave solution [35]. It is natural and interesting to search for new three-wave solutions and new periodic wave to NPDEs taking advantage of hirota bilinear forms.

The (3 + 1)-dimensional Kadomtsev-Petviashvili-Boussinesq-like (KPB-like) equation is usually written as [36]:

$$\mathrm{P}_{\text{KPB-like}}(u): u_{xxxy} + 3(u_x u_y)_x + u_{ty} + u_{xt} + u_{tt} - u_{zz} = 0. \tag{1}$$

The generalized Kadomtsev-Petviashvili-Boussinesq equation is first proposed in [37]. Recently, based on bilinear Bäcklund transformation, some classes of exponential and rational traveling wave solutions of Equation (1) are presented [36]. And by using the Hirota bilinear method, the lump solutions of Equation (1) are obtained [38].

In the present paper, we will give Hirota bilinear from and generalized lilinear from of the KPB-like equation in Section 2. In Section 3, new three-wave solutions of the KPB-like equation will be obtained based on generalized bilinear from. In Section 4, new periodic wave solutions of KPB-like equation will be given, that supplement the existing literature on KPB-like equation. A few conclusion and outlook will be given in Section 5.

2. Bilinear Form

Under dependent variable transformation:

$$u(x,y,z,t) = 2\left[\ln f(x,y,z,t)\right]_x. \tag{2}$$

Equation (1) is transformed into the following generalized bilinear form with $p=3$,

$$\begin{aligned}
&\mathrm{GB}_{\text{KPB-like}}(f)\\
&= \left(D_{p,x}^3 D_{p,y} + D_{p,y} D_{p,t} + D_{p,t} D_{p,x} - D_{p,z}^2 + D_{p,t}^2\right) f \cdot f\\
&= 6 f_{xx} f_{xy} + 2\left(f_{ty} f - f_y f_t\right) + 2\left(f_{xt} f - f_x f_t\right) - 2\left(f_{zz} f - f_z^2\right) + 2\left(f_{tt} f - f_t^2\right)\\
&= 0,
\end{aligned} \tag{3}$$

where p is an arbitrarily given natural number, often a prime number [39],

$$\begin{aligned}
&D_{p,x_1}^{n_1} \cdots D_{p,x_M}^{n_M} a \cdot b(x_1,\cdots,x_M)\\
&= \prod_{i=1}^{M}\left(\frac{\partial}{\partial x_i} + \alpha\frac{\partial}{\partial x_i'}\right)^{n_i} a(x_1,\cdots,x_M) b(x_1',\cdots,x_M') \times \Big|_{x'=x_1,\cdots,x'=x_M},
\end{aligned} \tag{4}$$

where $n_1,\cdots,n_M$ are arbitrary non-negative integers, and for an integer m, the mth power of α is computed as follows,

$$\alpha^m = (-1)^{r(m)},\ m \equiv r(m) \bmod p,\ 0 \le r(m) < p. \tag{5}$$

We can know that the following symbols are required: +1 or −1.

If we take $p=3$, then we have

$$\alpha_3^1 = -1, \alpha_3^2 = 1, \alpha_3^3 = 1, \alpha_3^4 = -1, \alpha_3^5 = 1, \alpha_3^6 = 1, \alpha_3^7 = -1, \alpha_3^8 = 1, \alpha_3^9 = 1, \tag{6}$$

which leads to

$$\begin{aligned}&D_{3,x}D_{3,t}f\cdot f=2f_{x,t}f-2f_xf_t,\\&D_{3,y}^2f\cdot f=2f_{y,y}f-2f_y^2,\\&D_{3,x}^4f\cdot f=6f_{x,x}^2,\\&D_{3,x}^3D_{3,y}f\cdot f=6f_{x,x}f_{x,y},\end{aligned}\tag{7}$$

But if $p=2$, then we can get

$$\begin{aligned}&D_{2,t}D_{2,x}f\cdot f=2f_{xt}f-2f_xf_t,\\&D_{2,y}^2f\cdot f=2f_{yy}f-2f_y^2,\\&D_{2,x}^4f\cdot f=2f_{xxxx}\cdot f-8f_{xxx}f_x+6f_{xx}^2,\\&D_{2,x}^3D_{2,y}f\cdot f=2f_{xxxy}f-6f_{xxy}f_x+6f_{xy}f_{xx}-2f_yf_{xxx}.\end{aligned}\tag{8}$$

We have noticed that when $p=2$, the generalized bilinear form is transformed into Hirota bilinear form. Transform (2) is also a characteristic transformation for establishing Bell polynomial theory of soliton equation [40]. Its exact relation is as follows:

$$\mathrm{GP}_{\text{KPB-like}}(u)=\left[\frac{\mathrm{GB}_{\text{KPB-like}}(\mathrm{u})}{f^2}\right].\tag{9}$$

Via the bilinear transformation (2), Equation (3) is transformed into

$$\begin{aligned}\mathrm{B}_{\text{KPB-like}}(u)=&\frac{3}{4}u_{xx}uv+\frac{9}{4}u_xuu_y+\frac{9}{8}u_xu^2v+u_{yt}+u_{tt}+u_{xt}+\frac{3}{8}u^3u_y\\&+\frac{3}{8}u_xu_{yx}+\frac{3}{2}u_x^2v+\frac{3}{4}u^2u_{yx}+\frac{3}{2}u_{xx}u_y-u_{zz}\\=&0,\end{aligned}\tag{10}$$

where $u_y=v_x$. Hence, if *f* solves the generalized bilinear the KPB-like Equation (3), the dimensionally reduced the KPB-like Equation (10) will be solved.

3. New Three-Wave Solutions of the KPB-Like Equation

To search for the new three-wave solution of the KPB-like Equation (1), we would like to start from an ansatz [41]

$$f=\mathrm{e}^{-g}+k_1\mathrm{e}^{g}+k_2\cos(h)+k_3\sin(n)+a_{16},\tag{11}$$

where $g=a_1x+a_2y+a_3z+a_4t+a_5$, $h=a_6x+a_7y+a_8z+a_9t+a_{10}$, $n=a_{11}x+a_{12}y+a_{13}z+a_{14}t+a_{15}$, $a_i\,(i=1,2,\cdots,16)$ and $k_i\,(i=1,2,3)$ are real parameters to be determined later. Substituting (11) into Equation (3), we obtained a series of periodic wave solutions to the KPB-like equation with the help of Maple.

Case 1:

$$\begin{aligned}&a_1=\frac{a_3^2-a_4^2}{a_4},a_2=0,a_3=a_3,a_4=a_4,a_5=a_5,a_6=0,a_7=\frac{\left(a_3^2-a_4^2\right)a_9}{a_4^2},\\&a_8=\frac{a_3a_9}{a_4},a_9=a_9,a_{10}=a_{10},a_{11}=\frac{\left(a_3^2-a_4^2\right)a_{14}}{a_4^2},a_{12}=0,a_{13}=\frac{a_3a_{14}}{a_4},\\&a_{14}=a_{14},a_{15}=a_{15},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,\end{aligned}\tag{12}$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u=\frac{2}{F}\left[-\frac{a_3^2-a_4^2}{a_4}\mathrm{e}^{-a_4t-\frac{x\left(a_3^2-a_4^2\right)}{a_4}-a_3z-a_5}+\frac{k_1\left(a_3^2-a_4^2\right)\mathrm{e}^{\xi_1}}{a_4}+\frac{k_3\left(a_3^2-a_4^2\right)a_{14}\cos\left(\xi_3\right)}{a_4^2}\right], \quad (13)$$

where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned}
F&=\mathrm{e}^{-a_4t-\frac{\left(a_3^2-a_4^2\right)x}{a_4}-a_3z-a_5}+k_1\mathrm{e}^{a_4t+\frac{\left(a_3^2-a_4^2\right)x}{a_4}+a_3z+a_5}\\
&\quad+k_2\cos\left[a_9t+\frac{\left(a_3^2-a_4^2\right)a_9y}{a_4^2}+\frac{a_3a_9z}{a_4}+a_{10}\right]\\
&\quad+k_3\sin\left[a_{14}t+\frac{\left(a_3^2-a_4^2\right)a_{14}x}{a_4^2}+\frac{a_3a_{14}z}{a_4}+a_{15}\right]+a_{16},\\
\xi_1&=a_4t+\frac{\left(a_3^2-a_4^2\right)x}{a_4}+a_3z+a_5,\\
\xi_2&=a_9t+\frac{\left(a_3^2-a_4^2\right)a_9y}{a_4^2}+\frac{a_3a_9z}{a_4}+a_{10},\\
\xi_3&=a_{14}t+\frac{\left(a_3^2-a_4^2\right)a_{14}x}{a_4^2}+\frac{a_3a_{14}z}{a_4}+a_{15}.
\end{aligned} \quad (14)$$

Case 2:

$$\begin{aligned}
&a_1=\frac{a_3^2-a_4^2}{a_4}, a_5=a_5, a_2=0, a_3=a_3, a_4=a_4, a_6=\frac{\left(a_3^2-a_4^2\right)a_9}{a_4^2}, a_7=0,\\
&a_8=\frac{a_3a_9}{a_4}, a_9=a_9, a_{10}=a_{10}, a_{11}=0, a_{12}=\frac{\left(a_3^2-a_4^2\right)a_{14}}{a_4^2}, a_{13}=\frac{a_3a_{14}}{a_4},\\
&a_{14}=a_{14}, a_{15}=a_{15}, a_{16}=a_{16}, k_1=k_1, k_2=k_2, k_3=k_3,
\end{aligned} \quad (15)$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u=\frac{2}{F}\left[-\frac{a_3^2-a_4^2}{a_4}\mathrm{e}^{-a_4t-\frac{\left(a_3^2-a_4^2\right)x}{a_4}-a_3z-a_5}+\frac{k_1\left(a_3^2-a_4^2\right)\mathrm{e}^{\xi_1}}{a_4}-\frac{k_2\left(a_3^2-a_4^2\right)a_9\sin\left(\xi_2\right)}{a_4^2}\right], \quad (16)$$

where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned}
F&=\mathrm{e}^{-a_4t-\frac{\left(a_3^2-a_4^2\right)x}{a_4}-a_3z-a_5}+k_1\mathrm{e}^{a_4t+\frac{\left(a_3^2-a_4^2\right)x}{a_4}+a_3z+a_5}\\
&\quad+k_2\cos\left[a_9t+\frac{\left(a_3^2-a_4^2\right)a_9x}{a_4^2}+\frac{a_3a_9z}{a_4}+a_{10}\right]\\
&\quad+k_3\sin\left[a_{14}t+\frac{\left(a_3^2-a_4^2\right)a_{14}y}{a_4^2}+\frac{a_3a_{14}z}{a_4}+a_{15}\right]+a_{16},
\end{aligned} \quad (17)$$

$$\xi_1 = a_4 t + \frac{\left(a_3^2 - a_4^2\right)x}{a_4} + a_3 z + a_5,$$
$$\xi_2 = a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 x}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10},$$
$$\xi_3 = a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} y}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}.$$

Case 3:

$$\begin{aligned}
&a_1 = \frac{a_3^2 - a_4^2}{a_4}, a_2 = 0, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = 0, a_7 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2},\\
&a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{13} = \frac{a_3 a_{14}}{a_4},\\
&a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16}, k_1 = k_1, k_2 = k_2, k_3 = k_3,
\end{aligned} \tag{18}$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u = \frac{2}{F}\left[-\frac{a_3^2 - a_4^2}{a_4} \mathrm{e}^{-a_4 t - \frac{\left(a_3^2 - a_4^2\right)x}{a_4} - a_3 z - a_5} + \frac{k_1\left(a_3^2 - a_4^2\right)\mathrm{e}^{\xi_1}}{a_4}\right], \tag{19}$$

where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned}
F &= \mathrm{e}^{-a_4 t - \frac{\left(a_3^2 - a_4^2\right)x}{a_4} - a_3 z - a_5} + k_1 \mathrm{e}^{a_4 t + \frac{\left(a_3^2 - a_4^2\right)x}{a_4} + a_3 z + a_5}\\
&\quad + k_2 \cos\left[a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 y}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}\right]\\
&\quad + k_3 \sin\left[a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} y}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}\right] + a_{16},\\
\xi_1 &= a_4 t + \frac{\left(a_3^2 - a_4^2\right)x}{a_4} + a_3 z + a_5,\\
\xi_2 &= a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 y}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10},\\
\xi_3 &= a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} y}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}.
\end{aligned} \tag{20}$$

Case 4:

$$\begin{aligned}
&a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = 0, a_7 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2},\\
&a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{12} = 0, a_{13} = \frac{a_3 a_{14}}{a_4},\\
&a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16}, k_1 = k_1, k_2 = k_2, k_3 = k_3,
\end{aligned} \tag{21}$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u = \frac{2k_3\left(a_3^2 - a_4^2\right)a_{14}\cos\left(\xi_3\right)}{a_4^2 F}, \tag{22}$$

where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned} F &= e^{-a_4 t - \frac{\left(a_3^2 - a_4^2\right)y}{a_4} - a_3 z - a_5} + k_1 e^{a_4 t + \frac{\left(a_3^2 - a_4^2\right)y}{a_4} + a_3 z + a_5} \\ &\quad + k_2 \cos\left[a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 y}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}\right] \\ &\quad + k_3 \sin\left[a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} x}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}\right] + a_{16}, \\ \xi_1 &= a_4 t + \frac{\left(a_3^2 - a_4^2\right)y}{a_4} + a_3 z + a_5, \\ \xi_2 &= a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 y}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}, \\ \xi_3 &= a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} x}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}. \end{aligned} \tag{23}$$

Case 5:

$$\begin{aligned} &a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, \\ &a_6 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2}, a_7 = 0, a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, \\ &a_{10} = a_{10}, a_{11} = 0, a_{12} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{13} = \frac{a_3 a_{14}}{a_4}, \\ &a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16}, k_1 = k_1, k_2 = k_2, k_3 = k_3, \end{aligned} \tag{24}$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u = \frac{-2k_2 a_9\left(a_3^2 - a_4^2\right)\sin\left(\xi_2\right)}{a_4^2 F}, \tag{25}$$

where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned} F &= e^{-a_4 t - \frac{\left(a_3^2 - a_4^2\right)y}{a_4} - a_3 z - a_5} + k_1 e^{a_4 t + \frac{\left(a_3^2 - a_4^2\right)y}{a_4} + a_3 z + a_5} \\ &\quad + k_2 \cos\left[a_9 t + \frac{\left(a_3^2 - a_4^2\right)a_9 x}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}\right] \\ &\quad + k_3 \sin\left[a_{14} t + \frac{\left(a_3^2 - a_4^2\right)a_{14} y}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}\right] + a_{16}, \end{aligned} \tag{26}$$

$$\xi_1 = a_4 t + \frac{\left(a_3^2 - a_4^2\right) y}{a_4} + a_3 z + a_5,$$

$$\xi_2 = a_9 t + \frac{\left(a_3^2 - a_4^2\right) a_9 x}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10},$$

$$\xi_3 = a_{14} t + \frac{\left(a_3^2 - a_4^2\right) a_{14} y}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}.$$

Case 6:

$$\begin{aligned} &a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = \frac{\left(a_3^2 - a_4^2\right) a_9}{a_4^2}, a_7 = 0, \\ &a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = \frac{\left(a_3^2 - a_4^2\right) a_{14}}{a_4^2}, a_{12} = 0, a_{13} = \frac{a_3 a_{14}}{a_4}, \\ &a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = a_{16}, k_1 = k_1, k_2 = k_2, k_3 = k_3, \end{aligned} \tag{27}$$

with the condition $a_4 \neq 0$. Through the expression (11) and the transformation (2), we get the three-wave solution of the KPB-like equation

$$u = \frac{2}{F}\left[-\frac{k_2\left(a_3^2 - a_4^2\right) a_9 \sin\left(\xi_2\right)}{a_4^2} + \frac{k_3\left(a_3^2 - a_4^2\right) a_{14} \cos\left(\xi_3\right)}{a_4^2}\right], \tag{28}$$

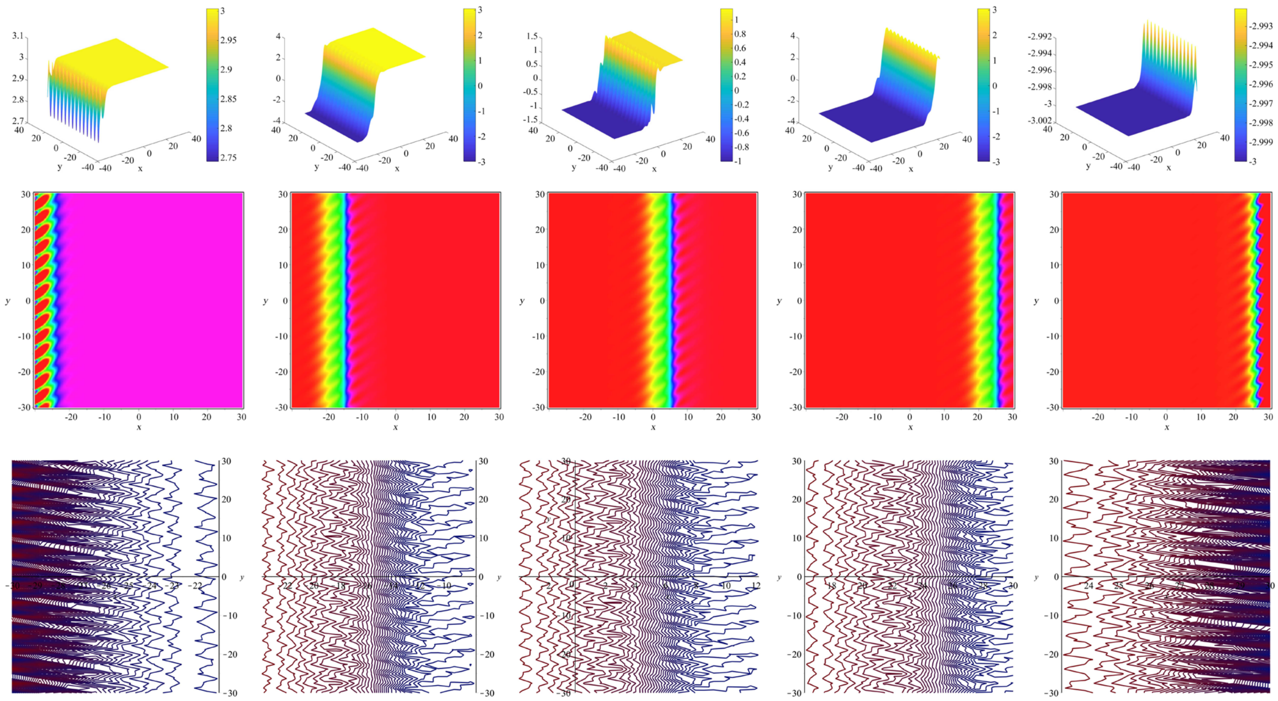
where the functions F, ξ_1, ξ_2 and ξ_3 are given as follows:

$$\begin{aligned} F = {} & \mathrm{e}^{-a_4 t - \frac{\left(a_3^2 - a_4^2\right) y}{a_4} - a_3 z - a_5} + k_1 \mathrm{e}^{a_4 t + \frac{\left(a_3^2 - a_4^2\right) y}{a_4} + a_3 z + a_5} \\ & + k_2 \cos\left[a_9 t + \frac{\left(a_3^2 - a_4^2\right) a_9 x}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}\right] \\ & + k_3 \sin\left[a_{14} t + \frac{\left(a_3^2 - a_4^2\right) a_{14} x}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}\right] + a_{16}, \\ \xi_1 = {} & a_4 t + \frac{\left(a_3^2 - a_4^2\right) y}{a_4} + a_3 z + a_5, \\ \xi_2 = {} & a_9 t + \frac{\left(a_3^2 - a_4^2\right) a_9 x}{a_4^2} + \frac{a_3 a_9 z}{a_4} + a_{10}, \\ \xi_3 = {} & a_{14} t + \frac{\left(a_3^2 - a_4^2\right) a_{14} x}{a_4^2} + \frac{a_3 a_{14} z}{a_4} + a_{15}. \end{aligned} \tag{29}$$

In order to analyze the dynamics properties briefly, we would like to discuss the evolution characteristic. By choosing appropriate values of these parameters in (13), we set

$$\begin{aligned} &z = x, a_3 = 1, a_4 = 2, a_5 = 2, a_9 = 2, a_{10} = 2, a_{14} = 2, \\ &a_{15} = 2, a_{16} = 2, k_1 = 1, k_2 = 1, k_3 = 1. \end{aligned} \tag{30}$$

The three-dimensional dynamic graphs of the wave and corresponding density plots and contour plots were successfully depicted in **Figure 1** with the help of Maple. We can see the exponential function wave and the sine-cosine function wave of KPB-like equation.



(a) t=-10 (b) t=-5 (c) t=0 (d) t=5 (e) t=10

Figure 1. Evolution plots (top), density plot (middle) and contour plots (bottom) of the Equation (13) by choosing $z = x$, $a_3 = 1$, $a_4 = 2$, $a_5 = 2$, $a_9 = 2$, $a_{10} = 2$, $a_{14} = 2$, $a_{15} = 2$, $a_{16} = 2$, $k_1 = 1$, $k_2 = 1$, $k_3 = 1$.

4. The New Periodic Solitary Wave Solutions of the KPB-Like Equation

To search for the new periodic solitary wave solutions of the KPB-like Equation (1), we suppose [11]

$$\begin{aligned} f &= \mathrm{e}^{-G} + k_1 \mathrm{e}^{G} + k_2 \tan(H) + k_3 \tanh(N) + a_{16}, \\ G &= a_1 x + a_2 y + a_3 z + a_4 t + a_5, \\ H &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \\ N &= a_{11} x + a_{12} y + a_{13} z + a_{14} t + a_{15}, \end{aligned} \tag{31}$$

where $a_i\,(i = 1, 2, \cdots, 16)$ and $k_i\,(i = 1, 2, 3)$ are real parameters to be determined later. With the help of Maple, substituting (31) into Equation (3), we obtain a set of algebraic equations in $a_i\,(i = 1, 2, \cdots, 16)$ and $k_i\,(i = 1, 2, 3)$. Solving the set of algebraic equations, we can find the following sets of solutions and these set leads to the corresponding periodic solitary wave solutions of the KPB-like equation.

Case 1:

$$\begin{aligned} & a_1 = \frac{a_3^2 - a_4^2}{a_4}, a_2 = 0, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = \frac{\left(a_3^2 - a_4^2\right) a_9}{a_4^2}, a_7 = 0, \\ & a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = \frac{\left(a_3^2 - a_4^2\right) a_{14}}{a_4^2}, a_{13} = \frac{a_3 a_{14}}{a_4}, \\ & a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3, \end{aligned} \tag{32}$$

with the condition $a_4 \neq 0$.

Case 2:

$$a_1 = \frac{a_3^2 - a_4^2}{a_4}, a_2 = 0, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = 0, a_7 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2},$$
$$a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{12} = 0, a_{13} = \frac{a_3 a_{14}}{a_4}, \tag{33}$$
$$a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3,$$

with the condition $a_4 \neq 0$.

Case 3:

$$a_1 = \frac{a_3^2 - a_4^2}{a_4}, a_2 = 0, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = 0, a_7 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2},$$
$$a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{13} = \frac{a_3 a_{14}}{a_4}, \tag{34}$$
$$a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3,$$

with the condition $a_4 \neq 0$.

Case 4:

$$a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2}, a_7 = 0,$$
$$a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{12} = 0, a_{13} = \frac{a_3 a_{14}}{a_4}, \tag{35}$$
$$a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3,$$

with the condition $a_4 \neq 0$.

Case 5:

$$a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2}, a_7 = 0,$$
$$a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = 0, a_{12} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{13} = \frac{a_3 a_{14}}{a_4}, \tag{36}$$
$$a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3,$$

with the condition $a_4 \neq 0$.

Case 6:

$$a_1 = 0, a_2 = \frac{a_3^2 - a_4^2}{a_4}, a_3 = a_3, a_4 = a_4, a_5 = a_5, a_6 = 0, a_7 = \frac{\left(a_3^2 - a_4^2\right)a_9}{a_4^2},$$
$$a_8 = \frac{a_3 a_9}{a_4}, a_9 = a_9, a_{10} = a_{10}, a_{11} = \frac{\left(a_3^2 - a_4^2\right)a_{14}}{a_4^2}, a_{12} = 0, a_{13} = \frac{a_3 a_{14}}{a_4}, \tag{37}$$
$$a_{14} = a_{14}, a_{15} = a_{15}, a_{16} = 0, k_1 = k_1, k_2 = k_2, k_3 = k_3,$$

with the condition $a_4 \neq 0$.

Case 7:

$$a_1=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2},a_2=0,a_3=\frac{a_4a_8}{a_9},a_4=a_4,a_5=a_5,a_6=\frac{a_8^2-a_9^2}{a_9},$$
$$a_7=0,a_8=a_8,a_9=a_9,a_{10}=a_{10},a_{11}=0,a_{12}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2},\tag{38}$$
$$a_{13}=\frac{a_8a_{14}}{a_9},a_{14}=a_{14},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,$$

with the condition $a_9\neq 0$.

Case 8:

$$a_1=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2},a_2=0,a_3=\frac{a_4a_8}{a_9},a_4=a_4,a_5=a_5,a_6=0,a_7=\frac{a_8^2-a_9^2}{a_9},$$
$$a_8=a_8,a_9=a_9,a_{10}=a_{10},a_{11}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2},a_{12}=0,a_{13}=\frac{a_8a_{14}}{a_9},\tag{39}$$
$$a_{14}=a_{14},a_{15}=a_{15},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,$$

with the condition $a_9\neq 0$.

Case 9:

$$a_1=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2},a_2=0,a_3=\frac{a_4a_8}{a_9},a_4=a_4,a_5=a_5,a_6=0,a_7=\frac{a_8^2-a_9^2}{a_9},$$
$$a_8=a_8,a_9=a_9,a_{10}=a_{10},a_{11}=0,a_{12}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2},a_{13}=\frac{a_8a_{14}}{a_9},\tag{40}$$
$$a_{14}=a_{14},a_{15}=a_{15},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,$$

with the condition $a_9\neq 0$.

Case 10:

$$a_1=0,a_2=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2},a_3=\frac{a_4a_8}{a_9},a_4=a_4,a_5=a_5,a_6=\frac{a_8^2-a_9^2}{a_9},a_7=0,$$
$$a_8=a_8,a_9=a_9,a_{10}=a_{10},a_{11}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2},a_{12}=0,a_{13}=\frac{a_8a_{14}}{a_9},\tag{41}$$
$$a_{14}=a_{14},a_{15}=a_{15},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,$$

with the condition $a_9\neq 0$.

Case 11:

$$a_1=0,a_2=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2},a_3=\frac{a_4a_8}{a_9},a_4=a_4,a_5=a_5,a_6=\frac{a_8^2-a_9^2}{a_9},a_7=0,$$
$$a_8=a_8,a_9=a_9,a_{10}=a_{10},a_{11}=0,a_{12}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2},a_{13}=\frac{a_8a_{14}}{a_9},\tag{42}$$
$$a_{14}=a_{14},a_{15}=a_{15},a_{16}=a_{16},k_1=k_1,k_2=k_2,k_3=k_3,$$

with the condition $a_9\neq 0$.

Case 12:

$$
\begin{aligned}
&a_1=0, a_2=\frac{a_4\left(a_8^2-a_9^2\right)}{a_9^2}, a_3=\frac{a_4 a_8}{a_9}, a_4=a_4, a_5=a_5, a_6=0, a_7=\frac{a_8^2-a_9^2}{a_9},\\
&a_8=a_8, a_9=a_9, a_{10}=a_{10}, a_{11}=\frac{\left(a_8^2-a_9^2\right)a_{14}}{a_9^2}, a_{12}=0, a_{13}=\frac{a_8 a_{14}}{a_9},\\
&a_{14}=a_{14}, a_{15}=a_{15}, a_{16}=a_{16}, k_1=k_1, k_2=k_2, k_3=k_3,
\end{aligned}
\tag{43}
$$

with the condition $a_9 \neq 0$.

To search the periodic solution of the KPB-like equation, for example, let's try to substitute the solution (32) into the expression (31). Then through the expression (31) and the transformation (2), we get the periodic solitary wave solution of the KPB-like equation,

$$
\begin{aligned}
u=\frac{2}{f}&\left[-\frac{a_3^2-a_4^2}{a_4}\mathrm{e}^{-a_4 t-\frac{\left(a_3^2-a_4^2\right)x}{a_4}-a_3 z-a_5}+\frac{k_1\left(a_3^2-a_4^2\right)\mathrm{e}^{\xi_1}}{a_4}\right.\\
&\left.+\frac{k_2\left(a_3^2-a_4^2\right)a_9\left(1+\left(\tan\left(\xi_2\right)\right)^2\right)}{a_4^2}\right],
\end{aligned}
\tag{44}
$$

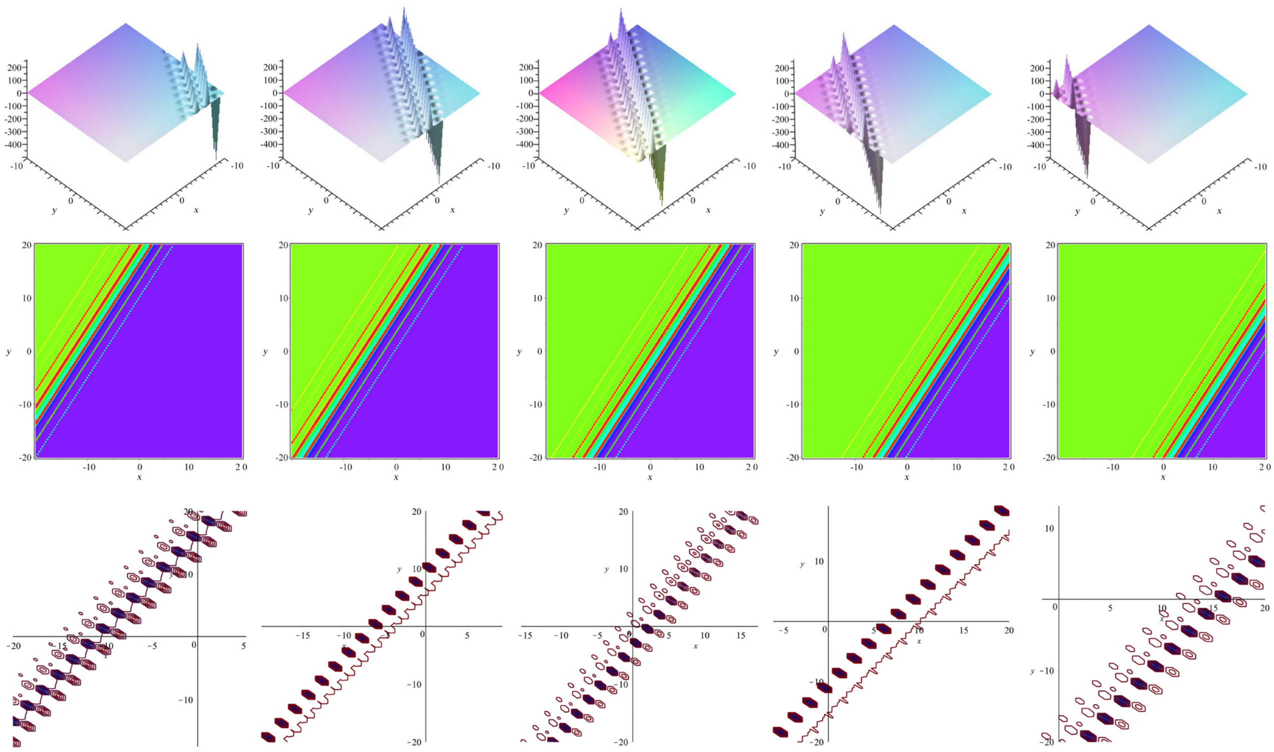
where the functions *F*, ξ_1, ξ_2 and ξ_3 are given as follows:

$$
\begin{aligned}
f=&\,\mathrm{e}^{-a_4 t-\frac{\left(a_3^2-a_4^2\right)x}{a_4}-a_3 z-a_5}+k_1\mathrm{e}^{a_4 t+\frac{\left(a_3^2-a_4^2\right)x}{a_4}+a_3 z+a_5}\\
&+k_2\tan\left[a_9 t+\frac{\left(a_3^2-a_4^2\right)a_9 x}{a_4^2}+\frac{a_3 a_9 z}{a_4}+a_{10}\right]\\
&+k_3\tanh\left[a_{14} t+\frac{\left(a_3^2-a_4^2\right)a_{14} y}{a_4^2}+\frac{a_3 a_{14} z}{a_4}+a_{15}\right]+a_{16},\\
\xi_1=&\,a_4 t+\frac{\left(a_3^2-a_4^2\right)x}{a_4}+a_3 z+a_5,\\
\xi_2=&\,a_9 t+\frac{\left(a_3^2-a_4^2\right)a_9 x}{a_4^2}+\frac{a_3 a_9 z}{a_4}+a_{10},\\
\xi_3=&\,a_{14} t+\frac{\left(a_3^2-a_4^2\right)a_{14} y}{a_4^2}+\frac{a_3 a_{14} z}{a_4}+a_{15}.
\end{aligned}
\tag{45}
$$

In order to analyze the dynamics properties briefly, we would like to discuss the evolution characteristic. By choosing appropriate values of these parameters in (44), we set

$$
\begin{aligned}
&z=y, a_3=1, a_4=2, a_5=2, a_9=2, a_{10}=2, a_{14}=2,\\
&a_{15}=2, a_{16}=2, k_1=1, k_2=1, k_3=1.
\end{aligned}
\tag{46}
$$

The three-dimensional dynamic graphs of the wave and corresponding density plots and contour plots were successfully depicted in **Figure 2** with the help of Maple. In **Figure 2**, we can see that when $t=-10$ to $t=10$, the wave is generated and disappeared, which is the result of the interaction of exponential function wave, tangent function wave and hyperbolic tangent function wave.



(a) t=-10 (b) t=-5 (c) t=0 (d) t=5 (e) t=10

Figure 2. Evolution plots (top), density plot (middle) and contour plots (bottom) of the Equation (44) by choosing $z = y$, $a_3 = 1$, $a_4 = 2$, $a_5 = 2$, $a_9 = 2$, $a_{10} = 2$, $a_{14} = 2$, $a_{15} = 2$, $a_{16} = 2$, $k_1 = 1$, $k_2 = 1$, $k_3 = 1$.

5. Conclusions and Outlook

In this paper, with the help of Maple, we obtained new three-wave solutions and new periodic solitary wave solutions of the (3 + 1)-dimensional KPB-like equation, and successfully depicted the three-dimensional evolution map of the wave and the corresponding density map of the new three-wave solutions and new periodic solitory wave solution, from which we can see the exponential function wave. The new three-wave solutions and new periodic solitary wave solutions obtained in this paper, will have a wide range of applications in the fields of physics and mechanics.

In fact, we can further calculate other exact solutions of the KPB-like equation and other NLEEs; when $p = 5$ and $p = 7$ by generalized bilinear derivative, the new three-wave solutions and new periodic solitary wave solutions of those two classes of equations will be obtained with different calculation.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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