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Fuzzy Foldness of P-Ideals in BCI-Algebras

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Abstract

This paper aims to introduce new notions of (fuzzy) n-fold P-ideals and (fuzzy) n-fold weak P-ideals in BCI-algebras, and investigate several properties of the foldness theory of P-ideals in BCI-algebras. Finally, we construct a computer-program for studying the foldness theory of P-ideals in BCI-algebras.

Keywords

BCK/BCI Algebras, P-Ideals of BCI-Algebras, Fuzzy P-Ideal of BCI-Algebra, Fuzzypoint, (Fuzzy) n-Fold P-Ideals, (Fuzzy) n-Fold Weak P-Ideals

1. Introduction

The study of BCK/BCI-algebras was initiated by Iséki [1] as a generalization of the concept of set-theoretic difference and propositional calculus. Since then, a great deal of theorems has been produced on the theory of BCK/BCI-algebras. In (1965), Zadeh [2] was introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. In 1991, Xi [3] defined fuzzy subsets in BCK/BCI-algebras.

Huang and Chen [4] introduced the notions of n-fold implicative ideal and n-fold (weak) commutative ideals. Y. B. Jun [5] discussed the fuzzification of n-fold positive implicative, commutative, and implicative ideal of BCK-algebras.

In this paper, we redefined a P-ideal of BCI-algebras and studied the foldness theory of fuzzy P-ideals, P-weak ideals, fuzzy weak P-ideals, and weak P-weak ideals in BCI-algebras. This theory can be considered as a natural generalization of P-ideals. Indeed, given any BCI-algebras X, we use the concept of fuzzy point to characterize n-fold P-ideals in X. Finally, we construct some algorithms for studying the foldness theory of P-ideals in BCI-algebras.

2. Preliminaries

Here we include some elementary aspects of BCI that are necessary for this paper. For more detail, we refer to [4] [6].

An algebra $(X;*,0)$ of type (2, 0) is called BCI-algebra if $\forall x,y,z\in X$ the following conditions hold:

BCI-1. $((x*y)*(x*z))*(z*y)=0$;

BCI-2. $(x*(x*y))*y=0$;

BCI-3. $x*x=0$;

BCI-4. $x*y=0$ and $y*x=0\Rightarrow x=y$.

A binary relation $\leq$ can be defined by

BCI-5. $x\leq y\Leftrightarrow x*y=0$.

Then $(X,\leq)$ is a partially ordered set with least element 0.

The following properties also hold in any BCI-algebra [7] [8]:

1) $x*0=x$;

2) $x*y=0$ and $y*z=0\Rightarrow x*z=0$;

3) $x*y=0\Rightarrow(x*z)*(y*z)=0$ and $(z*y)*(z*x)=0$;

4) $(x*y)*z=(x*z)*y$;

5) $(x*y)*x=0$;

6) $x*(x*(x*y))=x*y$; let $(X,*,0)$ be a BCI-algebra.

Definition 2.1. A fuzzy subset of a BCK/BCI-algebra X is a function $\mu:X\to[0,1]$.

Definition 2.2. (C. Lele, C. Wu, P. Weke, T. Mamadou, and C.E. Njock [9]). Let ξ be the family of all fuzzy sets in X. For $x\in X$ and $\lambda\in(0,1]$, $x_\lambda\in\xi$ is a fuzzy point if

$$x_\lambda(y)=\begin{cases}\lambda & \text{if } x=y,\\ 0 & \text{otherwise.}\end{cases}$$

We denote by $\tilde{X}=\{x_\lambda : x\in X,\lambda\in(0,1]\}$ the set of all fuzzy points on X, and we define a binary operation on $\tilde{X}$ as follows

$$x_\lambda*y_\mu=(x*y)_{\min(\lambda,\mu)}$$

It is easy to verify $\forall x_\lambda,y_\mu,z_\alpha\in\tilde{X}$, the following conditions hold:

BCI-1'. $((x_\lambda*y_\mu)*(x_\lambda*z_\alpha))*(z_\alpha*y_\mu)=0_{\min(\lambda,\mu,\alpha)}$;

BCI-2'. $(x_\lambda*(x_\lambda*y_\mu))*y_\mu=0_{\min(\lambda,\mu)}$;

BCI-3'. $x_\lambda*x_\mu=0_{\min(\lambda,\mu)}$;

BCK-5'. $0_\mu*x_\lambda=0_{\min(\lambda,\mu)}$.

Remark 2.3. (C. Lele, C. Wu, P. Weke, T. Mamadou, and C.E. Njock [9]). The condition BCI-4 is not true $(\tilde{X},*)$. So the partial order $\leq(X,*)$ cannot be extended to $(\tilde{X},*)$.

We can also establish the following conditions $\forall x_\lambda,y_\mu,z_\alpha\in\tilde{X}$:

1') $x_\lambda*0_\mu=x_{\min(\lambda,\mu)}$;

2') $x_\lambda*y_\mu=0_{\min(\lambda,\mu)}$ and $y_\mu*z_\alpha=0_{\min(\mu,\alpha)}\Rightarrow x_\lambda*z_\alpha=0_{\min(\lambda,\alpha)}$;

3') $x_\lambda * y_\mu = 0_{\min(\lambda,\mu)} \Rightarrow (x_\lambda * z_\alpha)) * (y_\mu * z_\alpha) = 0_{\min(\lambda,\mu,\alpha)}$ and $(z_\alpha * y_\mu) * (z_\alpha * x_\lambda) = 0_{\min(\lambda,\mu,\alpha)}$;

4') $(x_\lambda * y_\mu) * z_\alpha = (x_\lambda * z_\alpha) * y_\mu$;

5') $(x_\lambda * y_\mu) * x_\lambda = 0_{(\lambda,\mu)}$;

6') $x_\lambda * (x_\lambda * (x_\lambda * y_\mu)) = x_\lambda * y_\mu$.

We recall that if A is a fuzzy subset of a BCK/BCI algebra X, then we have the following:

$$\tilde{A} = \{x_\lambda \in \tilde{X} : A(x) \geq \lambda, \lambda \in (0,1]\}. \quad (1)$$

$$\forall \lambda \in (0,1], \tilde{X}_\lambda = \{x_\lambda : x \in X\}, \text{ and } \tilde{A}_\lambda = \{x_\lambda \in \tilde{X}_\lambda : A(x) \geq \lambda\} \quad (2)$$

We also have $\tilde{X}_\lambda \subseteq \tilde{X}, \tilde{A} \subseteq \tilde{X}, \tilde{A}_\lambda \subseteq \tilde{A}, \tilde{A}_\lambda \subseteq \tilde{X}_\lambda$, and one can easily check that $(\tilde{X}_\lambda; *, 0_\lambda)$ it is a BCK-algebra.

Definition 2.4 (Isèki [10]). A nonempty subset of BCK/BCI-algebra X is called an ideal of X if it satisfies

1) $0 \in I$;

2) $\forall x, y \in X, (x * y \in I \text{ and } y \in I) \Rightarrow x \in I$.

Definition 2.5. A nonempty subset I of BCI-algebra X is P-ideal if it satisfies:

1) $0 \in I$;

2) $\forall x, y, z \in X$

$$((x * z) * (y * z) \in I \text{ and } y \in I) \Rightarrow x \in I$$

Definition 2.6 (Xi [11]). A fuzzy subset A of a BCK/BCI algebra X is a fuzzy ideal if

1) $\forall x \in X, A(0) \geq A(x)$;

2) $\forall x, y \in X, A(x) \geq \min(A(x * y), A(y))$.

Definition 2.7 (Xi [11]). A fuzzy subset A of a BCI-algebra X is called a fuzzy P-ideal of X if.

1) $\forall x \in X, A(0) \geq A(x)$;

2) $\forall x, y, z \in X$

$$A(x) \geq \min(A((x * z) * (y * z)) * A(y))$$

Definition 2.8 [12]. $\tilde{A}$ is a weak ideal of $\tilde{X}$ if

1) $\forall \nu \in \operatorname{Im}(A); 0_\nu \in \tilde{A}$;

2) $\forall x_\lambda, y_\mu \in X$. Such that $x_\lambda * y_\mu \in \tilde{A}$ and $y_\mu \in \tilde{A}$, we have

$$x_{\min(\lambda,\mu)} \in \tilde{A}.$$

Theorem 2.9 [13]. Suppose that A is a fuzzy subset of a BCK-algebra X, then the following conditions are equivalent:

1) A is a fuzzy ideal;

2) $\forall x_\lambda, y_\mu \in \tilde{A}, (z_\alpha * y_\mu) * x_\lambda = 0_{\min(\lambda,\mu,\alpha)} \Rightarrow z_{\min(\lambda,\mu,\alpha)} \in \tilde{A}$;

3) $\forall t \in (0,1]$, the t-level subset $A^t = \{x \in X : A(x) \geq t\}$ in an ideal when $A^t \neq \phi$;

4) $\tilde{A}$ is a weak ideal.

3. Fuzzy n-Fold P-Ideals in BCI-Algebras

Throughout this paper $\tilde{X}$ is the set of fuzzy points on BCI-algebra X and $n \in \mathbb{N}$ (where $\mathbb{N}$ the set of all the natural numbers).

Let us denote $(\cdots((x*y)*y)*\cdots)*y$ by $x*y^n$.

Moreover, $\left(\cdots\left(\left(x_{\min(\lambda,\mu)}*0_\mu\right)*0_\mu\right)*\cdots\right)*0_\mu$ by $x_\lambda * y_\mu^n$ (where y and y_μ occurs respectively n times) with $x, y \in X, x_\lambda, y_\lambda \in \tilde{X}$.

Definition 3.1. A nonempty subset I of a BCI-algebra X is an n-fold P-ideal of X if it satisfies :

1) $0 \in I$;

2) $\forall x, y, z \in X$,

$$((x*z)*(y*z) \in I \text{ and } y \in I) \Rightarrow x*z^n \in I.$$

Definition 3.2. A fuzzy subset A of X is called a fuzzy n-fold P-ideal of X if it satisfies :

1) $\forall x \in X, A(0) \geq A(x)$;

2) $\forall x, y, z \in X$,

$$A(x*z^n) \geq \min\left(A((x*z)*(y*z))*A(y)\right).$$

Definition 3.3. $\tilde{A}$ is P-weak ideal of $\tilde{X}$ if

1) $\forall \nu \in \operatorname{Im}(A), 0_\nu \in \tilde{A}$;

2) $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$,

$$((x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A} \text{ and } y_\mu \in \tilde{A}) \Rightarrow x_{\min(\lambda,\mu)} * z_\alpha \tilde{A}.$$

Definition 3.4. $\tilde{A}$ is an n-fold P-weak ideal of $\tilde{X}$ if

1) $\forall \nu \in \operatorname{Im}(A), 0_\nu \in \tilde{A}$;

2) $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$,

$$((x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A} \text{ and } y_\mu \in \tilde{A}) \Rightarrow x_{\min(\lambda,\mu)} * z_\alpha^n \in \tilde{A}.$$

Example 3.5. Let $X = \{0, a, b, c\}$ with $*$ defined by Table 1.

By simple computations, one can prove that $(X, *, 0)$ is BCI-algebra. Define $\mu : X \to [0,1]$ by $\mu(0) = 1, \mu(a) = \mu(b) = \mu(c) = t$, where $t \in [0,1]$.

Table 1. Example 3.5.

*	0	a	b	c
0	0	0	0	c
a	a	0	0	c
b	b	b	0	c
c	c	c	c	0

One can easily check that for any $n \geq 3$.

Is a fuzzy n-fold P-ideal.

Remark 3.6. $\tilde{A}$ is a 1-fold P-weak ideal of a BCK-algebra $\tilde{X}$ if $\tilde{A}$ is P-weak ideal of $\tilde{X}$.

Theorem 3.7. If *A* is a fuzzy subset of *X*, then *A* is a fuzzy n-fold P-ideal if $\tilde{A}$ is an n-fold P-weak ideal.

Proof. $\Rightarrow$

- Let $\lambda \in \operatorname{Im}(A)$, it is easy to prove that $0_\lambda \in \tilde{A}$;
- Let $(x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A}$ and $y_\mu \in \tilde{A}$
  $A((x*z)*(y*z)) \geq \min(\lambda, \mu, \alpha)$ and $A(y) \geq \mu$.
  Since *A* is a fuzzy n-fold P-ideal, we have

$$A(x*z^n) \geq \min\big(A((x*z)*(y*z))*A(y)\big)$$
$$\geq \min(\min(\lambda,\mu,\alpha),\mu) = \min(\lambda,\mu,\alpha)$$

Therefore $(x*z^n)_{\min(\lambda,\mu,\alpha)} = x_{\min(\lambda,\mu)} * z_\alpha^n \in \tilde{A}$.

$\Leftarrow$

- Let $x \in X$, it is easy to prove that $A(0) \geq A(x)$;
- Let $x, y, z \in X$ and let $A((x*z)*(y*z)) = \beta$ and $A(y) = \alpha$, then
  $((x*z)*(y*z))_{\min(\beta,\alpha)} = ((x_\beta * z_\alpha)*(y_\alpha * z_\alpha)) \in \tilde{A}$ and $y_\alpha \in \tilde{A}$.

Since $\tilde{A}$ is P-weak ideal, we have

$$x_{\min(\beta,\alpha)} * z_\alpha^n = (x*z^n)_{\min(\beta,\alpha)} \in \tilde{A}$$

Thus $A(x*z^n) \geq \min(\beta,\alpha) = \min\big(A((x*z)*(y*z)), A(y)\big)$. □

Proposition 3.8. An n-fold P-weak ideal is a weak ideal.

Proof. $\forall x_\lambda, y_\mu \in \tilde{X}$ let $x_\lambda * y_\mu = (x_\lambda * 0_\mu)*(y_\mu * 0_\mu) \in \tilde{A}$ and $y_\mu \in \tilde{A}$, since $\tilde{A}$ n-fold P-weak ideal, we have

$$x_{\min(\lambda,\mu)} = x_{\min(\lambda,\mu)} * 0_\mu^n \in \tilde{A}$$

Thus $\tilde{A}$ is a weak ideal.

Corollary 3.9. A fuzzy n-fold P-ideal is a fuzzy ideal.

Theorem 3.10. Let $\{\tilde{A}_{i\in I}\}$ be a family of n-fold P-weak ideals and $\{A_{i\in I}\}$ be a family of fuzzy-fold P-ideals. Then: 1) $\bigcap_{i\in I} \tilde{A}_i$ is an n-fold P-weak ideal.

2) $\bigcup_{i\in I} \tilde{A}_i$ is an n-fold P-weak ideal.

3) $\bigcap_{i\in I} A_i$ is a fuzzy n-fold P-ideal.

4) $\bigcup_{i\in I} A_i$ is a fuzzy n-fold P-ideal.

Proof. 1) $\forall \lambda \in \operatorname{Im}\left(\bigcap_{i\in I} \tilde{A}_i\right)$, then $\lambda \in \operatorname{Im}(\tilde{A}_i), \forall i$, so, $0_\lambda \in \tilde{A}_i, \forall i$, *i.e.* $0_\lambda \in \bigcap_{i\in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if $(x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \bigcap_{i\in I} \tilde{A}_i$ and $y_\mu \in \bigcap_{i\in I} \tilde{A}_i$, then

$(x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A}_i$ and $y_\mu \in \tilde{A}_i$ $\forall i$, thus

$$x_{\min(\lambda,\mu)} * z_\alpha^n \in \tilde{A}_i, \forall i$$

So $x_{\min(\lambda,\mu)} * z_\alpha^n \in \bigcap_{i\in I} \tilde{A}_i$. Thus $\bigcap_{i\in I} \tilde{A}_i$ is an n-fold P-weak ideals.

2) $\forall \lambda \in \mathrm{Im}\left(\bigcup_{i\in I} \tilde{A}_i\right)$, then $\exists i_0 \in I$, such that $\lambda \in \tilde{A}_{i_0}$, so, $0_\lambda \in \tilde{A}_{i_0}$, *i.e.* $0_\lambda \in \bigcup_{i\in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if $(x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \bigcup_{i\in I} \tilde{A}_i$ and $y_\mu \in \bigcup_{i\in I} \tilde{A}_i$, then $\exists i_0 \in I$ such that

$$(x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A}_{i_0} \text{ and } y_\mu \in \tilde{A}_{i_0}, \text{ thus } x_{\min(\lambda,\mu)} * z_\alpha^n \in \tilde{A}_{i_0}.$$

So $x_{\min(\lambda,\mu)} * z_\alpha^n \in \bigcup_{i\in I} \tilde{A}_i$. Thus $\bigcup_{i\in I} \tilde{A}_i$ is an n-fold P-weak ideals.

3) Follows from 1) and Theorem 3.7.

4) Follows from 2) and Theorem 3.7.

4. Fuzzy-Fold Weak P-Ideals in BCI-Algebras

In this section, we define and give some characterizations of (fuzzy) n-fold weak P-weak ideals in BCI-algebras.

Definition 4.1. A nonempty subset I of X is called an n-fold weak P-ideal of X if it satisfies

1) $0 \in I$;

2) $\forall x, y, z \in X \left(\left(x * z\right) * \left(y * z^n\right) \in I \text{ and } y \in I\right) \Rightarrow x \in I$.

Definition 4.2. A fuzzy subset A of X is called a fuzzy n-fold weak P-ideal of X if it satisfies

1) $\forall x \in X, A(0) \ge A(x)$;

2) $\forall x, y, z, A(x) \ge \min\left(A\left(\left(x * z\right) * \left(y * z^n\right)\right), A(y)\right)$.

Definition 4.3. $\tilde{A}$ is a weak P-weak ideal of $\tilde{X}$ if

1) $\forall \nu \in \mathrm{Im}(A), 0_\nu \in \tilde{A}$;

2) $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$

$$\left((x_\lambda * z_\alpha)*(y_\mu * z_\alpha) \in \tilde{A} \text{ and } y_\mu \in \tilde{A}\right) \Rightarrow x_{\min(\lambda,\mu,\alpha)} \in \tilde{A}.$$

Definition 4.4. $\tilde{A}$ is an n-fold a weak P-weak ideal of $\tilde{X}$ if

1) $\forall \nu \in \mathrm{Im}(A), 0_\nu \in \tilde{A}$;

2) $\forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}$,

$$\left((x_\lambda * z_\alpha)*(y_\mu * z_\alpha^n) \in \tilde{A} \text{ and } y_\mu \in \tilde{A}\right) \Rightarrow x_{\min(\lambda,\mu,\alpha)} \in \tilde{A}.$$

Example 4.5. Let $X = \{0,1,2,3\}$ in which $*$ is given by Table 2.

Table 2. Example 4.5.

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
c	c	c	c	0

Then $(X;*,0)$ is a BCI-algebra. Let $t_1,t_2 \in (0,1]$ and let us define a fuzzy subset $A: X \to [0,1]$ by

$$t_1 = A(0) = A(a) = A(b) > A(c) = t_2$$

It is easy to check that for any $n > 2$

$$\tilde{A} = \{0_\lambda : \lambda \in (0,t_1]\} \cup \{a_\lambda : \lambda \in (0,t_2]\} \cup \{b_\lambda : \lambda \in (0,t_1]\} \cup \{c_\lambda : \lambda \in (0,t_2]\}$$

Is an n-fold weak P-weak ideal.

Remark 4.6. $\tilde{A}$ is a 1-fold weak P-weak ideal of a BCK-algebra *X* if $\tilde{A}$ is a weak P-weak ideal.

Theorem 4.7. [13] If *A* is a fuzzy subset of *X*, then *A* is a fuzzy n-fold weak P-ideal if $\tilde{A}$ is an n-fold weak P-weak ideal.

Proof. $\Rightarrow$

- Let $\lambda \in \operatorname{Im}(A)$ obviously $0_\lambda \in \tilde{A}$;
- Let $(x_\lambda * z_\alpha) * (y_\mu * z_\alpha^n) \in \tilde{A}$ and $y_\mu \in \tilde{A}$, then

  $A((x_\lambda * z_\alpha) * (y_\mu * z_\alpha^n)) \geq \min(\lambda, \mu, \alpha)$ and $A(y) \geq \mu$.

  Since *A* is a fuzzy n-fold weak P-ideal, we have

$$\forall x, y, z, A(x) \geq \min\left(A((x*z)*(y*z^n)), A(y)\right)$$
$$\geq \min(\min(\lambda, \mu, \alpha), \alpha) = \min(\lambda, \mu, \alpha)$$

Therefore $x_{\min(\lambda,\mu,\alpha)} \in \tilde{A}$.

$\Leftarrow$

- Let $x \in X$, it is easy to prove that $A(0) \geq A(x)$;
- Let $\forall x, y, z, A((x*z)*(y*z^n)) = \beta$ and $A(y) = \alpha$.

  Then $((x*z)*(y*z^n))_{\min(\beta,\alpha)} = ((x_\beta * z_\beta) * (y_\alpha * z_\beta^n)) \in \tilde{A}$ and $y_\alpha \in \tilde{A}$.

  Since $\tilde{A}$ is n-fold weak P-weak ideal, we have

$$x_{\min(\beta,\alpha)} \in \tilde{A}$$

Hence $A(x) \geq \min(\beta, \alpha) = \min\left(A((x*z)*(y*z^n)), A(y)\right)$.

Proposition 4.8. An n-fold weak P-weak ideal is a weak ideal.

Proof. Let $x_\lambda, y_\mu \in \tilde{X}$ and $x_\lambda * y_\mu = (x_\lambda * 0_\mu) * (y_\mu * 0_\mu^n) \in \tilde{A}$ and y_μ.

Since $\tilde{A}$ is n-fold weak P-weak ideal, we have $x_{\min(\lambda,\mu)} \in \tilde{A}$.

Corollary 4.9. A fuzzy n-fold weak P-ideal is a fuzzy ideal.

Theorem 4.10. Let $\{\tilde{A}_{i\in I}\}$ be a family of n-fold weak P-weak ideals and $\{A_{i\in I}\}$ be a family of fuzzy n-fold weak P-ideals. then 1) $\bigcap_{i\in I} \tilde{A}_i$ is an n-fold weak P-weak ideal.

2) $\bigcup_{i\in I} \tilde{A}_i$ is an n-fold weak P-weak ideal.

3) $\bigcap_{i\in I} A_i$ is a fuzzy n-fold weak P-ideal.

4) $\bigcup_{i\in I} A_i$ is a fuzzy n-fold weak P-ideal.

Proof. 1) $\forall \lambda \in \operatorname{Im}\left(\bigcap_{i\in I} \tilde{A}_i\right)$, then $\lambda \in \operatorname{Im}\left(\tilde{A}_i\right), \forall i$, so, $0_\lambda \in \tilde{A}_i, \forall i$, *i.e.* $0_\lambda \in \bigcap_{i\in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if

$\left(x_\lambda * z_\alpha\right) * \left(y_\mu * z_\alpha^n\right) \in \bigcap_{i\in I} \tilde{A}_i$ and $y_\mu \in \bigcap_{i\in I} \tilde{A}_i$, then

$\left(x_\lambda * z_\alpha\right) * \left(y_\mu * z_\alpha^n\right) \in \tilde{A}_i$ and $y_\mu \in \tilde{A}_i$ $\forall i$, thus

$$x_{\min(\lambda,\mu,\alpha)} \in \tilde{A}_i, \forall i$$

So $x_{\min(\lambda,\mu,\alpha)} \in \bigcap_{i\in I} \tilde{A}_i$. Thus $\bigcap_{i\in I} \tilde{A}_i$ is an n-fold weak P-weak ideal.

2) $\forall \lambda \in \operatorname{Im}\left(\bigcup_{i\in I} \tilde{A}_i\right)$, then $\exists i_0 \in I$, such that $\lambda \in \tilde{A}_{i_0}$, so, $0_\lambda \in \tilde{A}_{i_0}$, *i.e.* $0_\lambda \in \bigcup_{i\in I} \tilde{A}_i$. For every $x_\mu, y_\lambda, z_\alpha \in \tilde{X}$, if $\left(x_\lambda * z_\alpha\right) * \left(y_\mu * z_\alpha^n\right) \in \bigcup_{i\in I} \tilde{A}_i$ and $y_\mu \in \bigcup_{i\in I} \tilde{A}_i$, then $\exists i_0 \in I$ such that

$$\left(x_\lambda * z_\alpha\right) * \left(y_\mu * z_\alpha^n\right) \in \tilde{A}_{i_0} \text{ and } y_\mu \in \tilde{A}_{i_0}, \text{ thus } x_{\min(\lambda,\mu,\alpha)} \in \tilde{A}_{i_0}.$$

So $x_{\min(\lambda,\mu,\alpha)} \in \bigcup_{i\in I} \tilde{A}_i$. Thus $\bigcup_{i\in I} \tilde{A}_i$ is an n-fold weak P-weak ideal.

3) Follows from 1) and Theorem 4.7.

4) Follows from 2) and Theorem4.7.

5. Algorithms

Here we give some algorithms for studding the structure of the foldness of (fuzzy) P-ideals In BCI-algebras

Algorithm for AP-Ideals of BCI-Algebra

```
Input(X: BCI-algebra, * : binary operation, I: the subset of X);
Output("I is aP-ideal of X or not");
Begin
  If I = φ then
     go to (1.);
  EndIf
  If 0 ∉ I then
     go to (1.);
  EndIf
  Stop:=false;
i :=1;
  While i ≤ |X| and not (Stop) do
j :=1;
    While j ≤ |X| and not (Stop) do
k :=1;
      While k ≤ |X| and not (Stop) do
        If (x_i * z_k)*(y_j * z_k) ∈ I and y_j ∈ I then
          If x_i ∉ I
            Stop:=true;
          EndIf
```

```
            EndIf
          Endwhile
        Endwhile
      Endwhile
      If Stop then
        Output ("I is aP-ideal of X")
      Else
        (1.) Output ("I is not aP-ideal of X")
    EndIf
End
```

Algorithm for n-fold P-Ideals of BCI-Algebra

Input(*X*: *BCI-algebra*, $*$: binary operation, *I*: a subset of *X*);

Output("*I* is n-fold P-ideal of *X* or not");

Begin

If $I = \phi$ then

go to (1.);

EndIf

If $0 \notin I$ then

go to (1.);

EndIf

Stop:=false;

$i := 1$;

While $i \leq |X|$ and not (*Stop*) do

$j := 1$;

While $j \leq |X|$ and not (*Stop*) do

$k := 1$;

While $k \leq |X|$ and not (*Stop*) do

If $(x_i * z_k) * (y_j * z_k) \in I$ and $y_j \in I$ then

If $x_i * z_k^n \notin I$

Stop:=true;

EndIf

EndIf

Endwhile

Endwhile

Endwhile

If *Stop* then

Output ("*I* is ann-fold P-ideal of *X*")

Else

(1.) Output ("*I* is not ann-fold P-ideal of *X*")

EndIf

End

Algorithm for Fuzzy P-Ideals of BCI-Algebra

Input(*X*: *BCI-algebra*, $*$: binary operation, *A*: the fuzzy subset of *X*);

```
Output("A is a fuzzy P-ideal of X or not");
Begin
  Stop:=false;
i := 1;
  While  i ≤ |X|  and not (Stop) do
     If  A(0) < A(x_i)  then
        Stop:=true;
     EndIf
j := 1;
     While  j ≤ |X|  and not (Stop) do
k := 1;
        While  k ≤ |X|  and not (Stop) do
           If  A(x_i * z_k) < min(A((x_i * z_k) * (y_j * z_k)), A(y_j))  then
              Stop:=true;
           EndIf
         Endwhile
        Endwhile
     Endwhile
   If Stop then
     Output ("A is not a fuzzyP-ideal of X")
   Else
     Output ("A is a fuzzyP-ideal of X")
   EndIf
End
```

Algorithm for Fuzzy n-fold P-Ideals of BCI-Algebra

```
Input(X: BCI-algebra, * : binary operation, A: the fuzzy subset of X);
Output("A is a fuzzy n-fold P-ideal of X or not");
Begin
  Stop:=false;
i := 1;
  While  i ≤ |X|  and not (Stop) do
     If  A(0) < A(x_i)  then
        Stop:=true;
     EndIf
j := 1;
     While  j ≤ |X|  and not (Stop) do
k := 1;
        While  k ≤ |X|  and not (Stop) do
           If  A(x_i * z_k^n) < min(A((x_i * z_k) * (y_j * z_k)), A(y_j))
              Stop:=true;
           EndIf
         Endwhile
        Endwhile
     Endwhile
```

```
    If Stop then
      Output ("A is not a fuzzy n-fold P-ideal of X")
    Else
      Output ("A is a fuzzy n-fold P-ideal of X")
    EndIf
End
```

Algorithm for N-Fold weak P-Ideals of BCI-Algebra

```
Input(X:BCI-algebra, I: subset of X, n ∈ ℕ);
Output("I is ann-fold weak P-ideal of X or not");
Begin
  If I = ϕ then
     go to (1.);
  EndIf
  If 0 ∉ I then
     go to (1.);
  EndIf
  Stop:=false;
i := 1;
  While i ≤ |X| and not (Stop) do
j := 1;
    While j ≤ |X| and not (Stop) do
k := 1;
       While k ≤ |X| and not (Stop) do
         If (x_i * z_k)*(y_j * z_k^n) ∈ I and y_j ∈ I then
           If x_i ∉ I
             Stop:=true;
           EndIf
         EndIf
       Endwhile
    Endwhile
  Endwhile
  If Stop then
    Output ("I is ann-fold weak P-ideal of X")
  Else
   (1.) Output ("I is not ann-fold weak P-ideal of X")
  EndIf
End
```

Algorithm for Fuzzy n-Fold weak P-Ideals of BCI-Algebra

```
Input(X: BCI-algebra, ∗: binary operation, A fuzzy subset of X);
Output("A is a fuzzy n-fold weak P-ideal of X or not");
Begin
  Stop:=false;
i := 1;
    While i ≤ |X| and not (Stop) do
```

```
        If $A(0) < A(x_i)$ then
          Stop:=true;
        EndIf
    $j := 1$;
        While $j \le |X|$ and not (Stop) do
    $k := 1$;
          While $k \le |X|$ and not (Stop) do
            If $A(x_i) < \min\left(A\left((x_i * z_k) * (y_j * z_k^n)\right), A(y_j)\right)$ then
              Stop=true;
            EndIf
          Endwhile
        Endwhile
      Endwhile
      If Stop then
        Output ("A is not a fuzzy n-foldweakP-ideal of X")
      Else
        Output ("A is a fuzzy n-foldweakP-ideal of X")
      EndIf
    End
```

6. Conclusions and Future Research

In this paper, we introduce new notions of (fuzzy) n-fold P-ideals, and (fuzzy) n-fold weak P-ideals in BCI-algebras. Then we studied relationships between different type of n-fold P-ideals and investigate several properties of the foldness theory of P-ideals in BCI-algebras. Finally, we construct some algorithms for studying the foldness theory of P-ideals in BCI-algebras.

In our future study of foldness ideals in BCK/BCI algebras, maybe the following topics should be considered:

1) Developing the properties of foldness of implicative ideals of BCK/BCI algebras.

2) Finding useful results on other structures of the foldness theory of ideals of BCK/BCI algebras.

3) Constructing the related logical properties of such structures.

4) One may also apply this concept to study some applications in many fields like decision making, knowledge base systems, medical diagnosis, data analysis and graph theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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