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Event-Triggered Finite-Time H_∞ Control for Switched Stochastic Systems

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Abstract

This paper investigates the problem of event-triggered finite-time H_∞ control for a class of switched stochastic systems. The main objective of this study is to design an event-triggered state feedback H_∞ controller such that the resulting closed-loop system is finite-time bounded and satisfies a prescribed H_∞ level in some given finite-time interval. Based on stochastic differential equations theory and average dwell time approach, sufficient conditions are derived to ensure the finite-time stochastic stability with the prescribed H_∞ performance for the relevant closed-loop system by employing the linear matrix inequality technique. Finally, the desired state feedback H_∞ controller gain matrices can be expressed in an explicit form.

Keywords

Average Dwell Time, Event-Triggering Scheme, Finite-Time Stochastic Stability (FTSS), Linear Matrix Inequalities (LMIS), Switched Stochastic Systems

1. Introduction

In the last few decades, switched systems have attracted much attention in the field of control systems [1] [2]. This is mainly due to the fact that switched system is an important subclass of hybrid systems and has found many practical and broad applications [3] [4] [5] [6]. A switched system is composed of a family of interconnected subsystems, featured with continuous and discrete-time dynamics, appropriately described by differential or difference equations, respectively, along with a switching law governing the switching among the subsystems. Many practical systems exist that can be well modeled as switched systems, which motivated a large number of researchers to investigate it widely. Quantities of important conclusions have been developed in the literature [7]-[12]. Stability criteria were presented for switched and hybrid system in [7]. In [8] [9], switching stabilization and robust stabilization problems were investigated respectively for a class of slowly switched systems and linear switched systems. Some valuable results have been derived for switched time-delay systems and switched nonlinear systems [10] [11] [12].

On the other hand, the periodic and aperiodic control strategies are presented on digital platforms. The conventional sampled-data scheme is the so-called periodic sampling or time-triggered control (TTC) mechanism. In the time-triggered control scheme, all the sampled data are transmitted and updated and the actuator state is adjusted at each sampling instant which leads to some unnecessary sampling and inefficient waste of communication resource. To this end, the event-triggered control (ETC) is introduced which is a typical aperiodic sampling scheme and capable of efficiently utilizing the communication bandwidth and significantly reducing the number of unnecessary data transmission for some networked control systems with limited communication bandwidth. Over the past few years, many worthy results have been provided for event-triggered control of switched systems [13]-[23]. In [13] [14], event-triggered control problems were discussed for continuous-time switched linear systems, in which the switching only occurred at the triggering instants. The closed-loop system is modeled as a switched system with delayed state and augmented switching signal to receive exponential stability conditions in [15]. Switching event-triggered control problems for a class of uncertain nonlinear systems were studied in [16] [17]. [18] designed H_∞ controller for uncertain switched linear systems. Fault detection filtering, H_∞ filtering and finite-time asynchronous filtering were investigated respectively for nonlinear switched systems and switched linear systems via event-triggered control scheme in [19] [20] [21]. In [22] [23], finite-time event-triggered control problems were respectively considered for switched systems.

Recently, increasing attention has been paid to the ETC of stochastic systems due to their significance in science and engineering applications. In the past few years, some contribution has been reported for ETC of stochastic systems in the literature [24]-[31]. [24] [25] investigated the problem of event-triggered stabilization for a class of stochastic nonlinear systems. Applying fuzzy logic systems to approximate stochastic nonlinear systems with actuator faults, adaptive event-triggered controller was designed in [26]. Based on event-triggered control strategy, the consensus tracking problem was investigated for a class of continuous switched stochastic nonlinear multiagent systems in [27]. Event-based recursive filtering, event-trigger-based finite-time, fault detection problem, finite-time fault detection filter were addressed for stochastic nonlinear systems respectively [28] [29] [30] [31]. However, the above results were all focused on non-switched stochastic systems. As far as we know, event-triggered finite-time control for switched stochastic systems has not been fully studied, which motivates us to investigate the present study.

This paper will study the event-triggered finite-time H_∞ control problem for switched stochastic systems. The main contributions can be summarized as follows. The coupling between the switching signals and triggered signals is analyzed. A novel framework of finite-time stability for augmented closed-loop switched stochastic system is established. The sufficient condition for event-triggered finite-time H_∞ controller of switched stochastic systems is obtained by adopting the average dwell time technique and multiple Lyapunov-Krasovskii functional method with LMIs. The design of controller parameters are presented which can guarantee the mentioned system is finite-time bounded and satisfies a weighted H_∞ disturbance attenuation performance, which can avoid some unnecessary data transmission.

The rest of this paper is arranged as follows. In Section 2, the problem formulation and necessary preliminaries are presented. We give a sufficient condition for finite-time ETC of the mentioned augmented system in terms of LMIs in Section 3. Moreover, a designing approach of an event-triggered finite-time H_∞ controller is presented. Finally, some conclusions are summarized in Section 4.

Notation: The notations used in this paper are quite standard. R^n stands for the n-dimensional Euclidean space. The notation $X > Y$ (respectively, $X \geq Y$, where X and Y are real symmetric matrices) means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I and 0 denote the identity and zero matrices with appropriate dimensions. $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denotes the maximum and the minimum of the eigenvalues of a real symmetric matrix Q. The superscript T denotes the transpose for vectors or matrices. The symbol * in a matrix denotes a term that is defined by symmetry of the matrix.

2. Problem Formulation and Preliminaries

Consider the following continuous-time switched stochastic system:

$$\begin{aligned} \mathrm{d}x(t) = {} & \left[A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + D_{1\sigma(t)}v(t)\right]\mathrm{d}t \\ & + \left[A_{1\sigma(t)}x(t) + B_{1\sigma(t)}u(t) + D_{2\sigma(t)}v(t)\right]\mathrm{d}\omega(t) \end{aligned} \tag{1}$$

$$z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t),\ x(0) = x_0 \in R^n \tag{2}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $v(t) \in R^p$, $z(t) \in R^q$ are state vector, control input vector, external disturbance, and controlled output respectively, where $v(t)$ satisfies the constraint condition with respect to the finite-time interval $[0, T]$

$$\int_0^T v^{\mathrm{T}}(t)v(t)\mathrm{d}t \leq d, \quad d > 0 \tag{3}$$

and $\omega(t) \in R$ is a standard Wiener process satisfying $\Xi\{d\omega(t)\} = 0$, $\Xi\{d\omega^2(t)\} = dt$, where Ξ is the expected value, which is assumed to be independent of the system mode $\sigma(t)$. $\sigma(t): [0, +\infty) \to S = \{1, 2, 3, \cdots, p\}$ is the switching signal which is a piecewise constant function depending on t, and p is the number of subsystems. $A_{\sigma(t)}, B_{\sigma(t)}, D_{1\sigma(t)}, A_{1\sigma(t)}, B_{1\sigma(t)}, D_{2\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}$ are known constant matrices of appropriate dimensions.

In this paper, the finite-time H_∞ controller is event-triggered, and the state-feedback sub-controllers are determined on the sampled states of the sub-system.

Assume that $\{t_k\}_{k\in N}$ denotes the triggered instants and there is no time-delay in sampler and actuator. Then, the state is sampled and the control input is computed at instant t_k simultaneously such that

$$u(t) = K_{\sigma(t)}x(t_k), t \in [t_k, t_{k+1}) \tag{4}$$

where $K_{\sigma(t)}$ are the sub-controller gains, $x(t_k)$ is the current sampled system state, t_{k+1} is the next sampled instant, which can be determined by the event-trigger, and $x(t_0) = x_0$ is chosen as the initial sampled state.

Remark 2.1 At sampling time instant t_k, the controller (4) will receive the sampled state $x(t_k)$, which will be held constant until next event is generated at time instant t_{k+1}. The sampled state $x(t_k)$ is used to update the control input in (4) which keep the control signal continuous on the interval $[t_k, t_{k+1})$ by zero order holder.

In this paper, the event-triggering schemes are described by

$$t_{k+1} = \inf\left\{t > t_k \mid e_{t_k}^{\mathrm{T}}(t)\Omega e_{t_k} \geq \mu^2 x^{\mathrm{T}}(t)\Omega x(t)\right\} \tag{5}$$

where

$$e_{t_k}(t) = x(t_k) - x(t) \tag{6}$$

μ is a constant and Ω is a symmetric and positive definite matrix with appropriate dimension to be determined.

Remark 2.2 when the equality $e_{t_k}^{\mathrm{T}}(t)\Omega e_{t_k} = \mu^2 x^{\mathrm{T}}(t)\Omega x(t)$ is satisfied, the sampler will be triggered to sample the system state immediately. Then the sampled data is transmitted to the subcontroller for calculating the control input which will be further used by the subsystem.

Remark 2.3 It should be pointed out that the parameter μ has great influence on the event-trigger instants, *i.e.* different values of μ correspond to different event-trigger frequencies. The less μ is selected, the shorter the event-trigger period is. Hence, μ should be selected in accordance with the specific control requirement and control capacity.

Let $\{r_q\}_{q\in N}$ be a given time sequence satisfying $r_1 < r_2 < \cdots < r_p$, where r_q is the switching instant. Meanwhile, define the event-triggered time instants as $t_0 < t_1 < t_2 \cdots < t_k < \cdots$.

Assumption 2.1 There is a number $\tau_s > 0$ such that any two switches are separated by at least τ_s to evade zeno phenomena, which means $r_{q+1} - r_q \geq \tau_s$ for any $q > 0$ [32].

Substituting the state-feedback controllers (4) into (1) (2), the event-triggered switched stochastic closed-loop system is obtained for $t \in [t_k, t_{k+1})$ as follows:

$$\begin{aligned} \mathrm{d}x(t) = & \left[\bar{A}_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}e_{t_k}(t) + D_{1\sigma(t)}v(t)\right]\mathrm{d}t \\ & + \left[\bar{A}_{1\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}e_{t_k}(t) + D_{2\sigma(t)}v(t)\right]\mathrm{d}w(t) \end{aligned} \tag{7}$$

$$z(t)=\bar{C}_{\sigma(t)}x(t)+D_{\sigma(t)}K_{\sigma(t)}e_{t_k}(t) \tag{8}$$

where $\bar{A}_{\sigma(t)}=A_{\sigma(t)}+B_{\sigma(t)}K_{\sigma(t)}$, $\bar{A}_{1\sigma(t)}=A_{1\sigma(t)}+B_{1\sigma(t)}K_{\sigma(t)}$, $\bar{C}_{\sigma(t)}=C_{\sigma(t)}+D_{\sigma(t)}K_{\sigma(t)}$.

We introduce the following definitions and lemmas, which will be useful in the succeeding discussion.

Definition 2.1 (Average dwell time [33]) Given time instants t and T such that $0\le t\le T$, let $N_\sigma(t,T)$ denote the switching number of $\sigma(t)$ over (t,T), if $N_\sigma(t,T)\le N_0+\frac{T-t}{\tau_a}$ holds for $\tau_a>0$ and an integer $N_0\ge 0$ (N_0 is called the chatter bound), then τ_a is called an average dwell time.

Definition 2.2 (Finite-time stochastic stabilizable [34]) The system (7) (8) with event-triggered control input (4) is said to be finite-time stochastic stable (FTSS) with respect to $(c_1,c_2,T,d,R,\sigma(t))$, where $R>0$, $0<c_1<c_2$, a switching signal $\sigma(t)$, if for a given time-constant $T>0$, the following relation holds:

$$\Xi\left[x^{\mathrm{T}}(0)Rx(0)\right]<c_1\Rightarrow\Xi\left[x^{\mathrm{T}}(t)Rx(t)\right]<c_2,\quad\forall t\in[0,T]. \tag{9}$$

Definition 2.3 (Finite-time H_∞ stochastic stabilization [34]) The switched system (7) (8) with event-triggered control input (4) is said to be H_∞ finite-time stabilization with respect to $(c_1,c_2,T,d,R,\sigma(t))$, here $R>0$, $0<c_1<c_2$, a switching signal $\sigma(t)$, if for a given time-constant $T>0$, the following conditions holds:

1) Switched system (7) (8) with control input (4) is finite-time stochastic stabilizable.

2) Under the zero initial condition, there is

$$\Xi\int_0^T z^{\mathrm{T}}(t)z(t)\mathrm{d}t\le\gamma^2\Xi\int_0^T v^{\mathrm{T}}(t)v(t)\mathrm{d}t, \tag{10}$$

where the prescribed value γ is the attenuation level.

Lemma 2.1 [35] For any real matrices X,Y with appropriate dimensions and a positive scalar $\varepsilon>0$, one has

$$X^{\mathrm{T}}Y+Y^{\mathrm{T}}X\le\varepsilon X^{\mathrm{T}}X+\varepsilon^{-1}Y^{\mathrm{T}}Y. \tag{11}$$

Lemma 2.2 [36] Let $W\in R^{n\times n}$ be a symmetric matrix, and let $x\in R^n$, then the following inequality holds:

$$\lambda_{\min}(W)x^{\mathrm{T}}x\le x^{\mathrm{T}}Wx\le\lambda_{\max}(W)x^{\mathrm{T}}x. \tag{12}$$

Lemma 2.3 (Schur complement [37] [38]) Given a symmetric matrix $\phi=\begin{bmatrix}\phi_{11}&\phi_{12}\\\phi_{21}&\phi_{22}\end{bmatrix}$, the following three conditions are equivalent to each other:

1) $\phi<0$;

2) $\phi_{11}<0$, and $\phi_{22}-\phi_{12}^{\mathrm{T}}\phi_{11}^{-1}\phi_{12}<0$;

3) $\phi_{22}<0$, and $\phi_{11}-\phi_{12}\phi_{22}^{-1}\phi_{12}^{\mathrm{T}}<0$.

3. Main Results

In this section, we focus on the finite-time stabilization of the switched stochastic system (7) (8) with event-triggered control input (4), and some sufficient conditions which can ensure the switched stochastic system (7) (8) is finite-time H_∞ stochastic stabilizable are given by the following theorem.

Theorem 3.1 For any $\sigma(t)=i\in S=\{1,2,\cdots,p\}$, a given positive definite matrix *R*, if there exist positive constants ε,γ, $\eta>1$ $\alpha>0$ and symmetric positive definite matrices $P_i, i\in S$ satisfying

$$P_i=R^{\frac{1}{2}}Q_iR^{\frac{1}{2}}=R^{\frac{1}{2}}H_i^{\mathrm{T}}H_iR^{\frac{1}{2}} \tag{13}$$

$$P_i\le\eta P_j \tag{14}$$

such that the following LMIs hold

$$\Theta<0 \tag{15}$$

where

$$\Theta=\begin{bmatrix}\mu^2\Omega-\alpha P_i & 0 & 0 & \Pi_1 & P_i & \Pi_2 & \Pi_3\\ 0 & -\Omega & 0 & 0 & 0 & 0 & 0\\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0\\ * & * & * & -\varepsilon^{-1}I & 0 & 0 & 0\\ * & * & * & * & -\varepsilon I & 0 & 0\\ * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & -I\end{bmatrix} \tag{16}$$

$$\Pi_1=\begin{bmatrix}A_i+B_iK_i & B_iK_i & D_{1i}\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \Pi_2=\begin{bmatrix}C_i+D_iK_i & D_iK_i & 0\end{bmatrix},$$

$$\Pi_3=\begin{bmatrix}H_i\left(A_{1i}+B_{1i}K_i\right) & H_iB_{1i}K_i & H_iD_{2i}\end{bmatrix}.$$

Then, under the event-triggering strategy (5), the event-triggered state-feedback controllers (4) and any switching signal with the average dwell time satisfying

$$\tau_a\ge\frac{T\ln\eta}{\ln\dfrac{\inf_{i\in S}\left[\lambda_{\min}\left(Q_i\right)\right]c_2}{\sup_{i\in S}\left[\lambda_{\max}\left(Q_i\right)\right]c_1+\gamma^2 d}-\alpha T} \tag{17}$$

the switched closed-loop stochastic system (7) (8) is finite-time H_∞ stochastic stable (FTSS) with respect to $\left(c_1,c_2,T,d,R,\sigma(t)\right)$.

Proof Assume that subsystem $\sigma(t)=i\in S=\{1,2,\cdots,p\}$ is active on the interval $\left[r_q,r_{q+1}\right)$. When $t\in\left[r_q,r_{q+1}\right)$, define the following stochastic Laypunov-Krasovskii functional candidate:

$$V_i\left(x(t)\right)=x^{\mathrm{T}}(t)P_ix(t) \tag{18}$$

By Itô formula, define a weak infinitesimal operator *L*, then, the stochastic derivative of $V_i\left(x(t)\right)=x^{\mathrm{T}}(t)P_ix(t)$ is given by

$$\mathrm{d}V_i\left(x(t)\right)=LV_i\left(x(t)\right)\mathrm{d}t+2x^{\mathrm{T}}(t)P_i\left[\bar{A}_{1i}x(t)+B_{1i}K_{1i}e_{t_k}(t)+\bar{D}_{1i}v(t)\right]\mathrm{d}w(t) \tag{19}$$

We have the weak infinitesimal operator of $LV_i\left(x(t)\right)$ as follows:

$$\begin{aligned}&LV_i\left(x(t)\right)\\&=x^{\mathrm{T}}(t)P_i\left[\bar{A}_i x(t)+B_iK_ie_{t_k}(t)+\bar{D}_{1i}v(t)\right]\\&\quad+\left[\bar{A}_i x(t)+B_iK_ie_{t_k}(t)+\bar{D}_{1i}v(t)\right]^{\mathrm{T}}P_i x(t)\\&\quad+\left[\bar{A}_{1i}x(t)+B_{1i}K_ie_{t_k}(t)+\bar{D}_{2i}v(t)\right]^{\mathrm{T}}P_i\left[\bar{A}_{1i}x(t)+B_{1i}K_ie_{t_k}(t)+\bar{D}_{2i}v(t)\right]\end{aligned}\tag{20}$$

The relationship between the switching instants and event-triggered instants will be discussed as the following two cases.

Case 1 Suppose that there is no sampling in $\left[r_q,r_{q+1}\right)$, *i.e.* $t_k\le r_q<r_{q+1}\le t_{k+1}$

In view of event condition (5), together with Lemma 2.3, the following inequality can be deduced:

$$\begin{aligned}&LV_i\left(x(t)\right)-\alpha V_i\left(x(t)\right)+\mu^2x^{\mathrm{T}}(t)\Omega x(t)-e_{t_k}^{\mathrm{T}}\Omega e_{t_k}+z^{\mathrm{T}}(t)z(t)-\gamma^2v^{\mathrm{T}}(t)v(t)\\&\le\xi^{\mathrm{T}}(t)\Theta_i\xi(t)\end{aligned}\tag{21}$$

where $\xi(t)=\left[x^{\mathrm{T}}(t)\quad e_{t_k}^{\mathrm{T}}\quad v^{\mathrm{T}}(t)\right]^{\mathrm{T}}$,

$$\Theta_i=\begin{bmatrix}\Theta_{11} & P_iB_iK_i+\left(A_{1i}+B_{1i}K_i\right)^{\mathrm{T}}P_iB_{1i}K_i+\left(C_i+D_iK_i\right)^{\mathrm{T}}D_iK_i & P_iD_{1i}+\left(A_{1i}+B_{1i}K_i\right)^{\mathrm{T}}P_iD_{2i}\\ * & \left(B_{1i}K_i\right)^{\mathrm{T}}P_i\left(B_{1i}K_i\right)+\left(D_iK_i\right)^{\mathrm{T}}D_iK_i-\Omega & \left(B_{1i}K_i\right)^{\mathrm{T}}P_iD_{2i}\\ * & * & D_{2i}^{\mathrm{T}}P_iD_{2i}-\gamma^2I\end{bmatrix}\tag{22}$$

$$\begin{aligned}\Theta_{11}=&P_i\left(A_i+B_iK_i\right)+\left(A_i+B_iK_i\right)^{\mathrm{T}}P_i+\left(A_{1i}+B_{1i}K_i\right)^{\mathrm{T}}P_i\left(A_{1i}+B_{1i}K_i\right)\\&+\left(C_i+D_iK_i\right)^{\mathrm{T}}\left(C_i+D_iK_i\right)+\mu^2\Omega-\alpha P_i\end{aligned}$$

From (15) and (21), there is

$$LV_i\left(x(t)\right)\le\alpha V_i\left(x(t)\right)+\gamma^2v^{\mathrm{T}}(t)v(t)-z^{\mathrm{T}}(t)z(t)\tag{23}$$

Integrate both sides of the inequality (23) from r_q to *t*, and obtain

$$V_i\left(x(t)\right)\le\mathrm{e}^{\alpha\left(t-r_q\right)}V_i\left(x(t)\right)+\int_{r_q}^{t}\mathrm{e}^{\alpha(t-s)}\left[\gamma^2v^{\mathrm{T}}(s)v(s)-z^{\mathrm{T}}(s)z(s)\right]\mathrm{d}s,\quad t\in\left[r_q,r_{q+1}\right)\tag{24}$$

Case 2 For any $t\in\left[r_q,r_{q+1}\right)$, there are *n* sampling, *i.e.* $t_k\le r_q<t_{k+1}<t_{k+2}<\cdots<t_{k+n}<r_{q+1}$, $\forall n\in N$, where $t_{k+1},t_{k+2},\cdots,t_{k+n}$ is the updating sequence of the event-triggered controller.

On the intervals $\left[r_q,t_{k+1}\right),\left[t_{k+1},t_{k+2}\right),\cdots,\left[t_{k+n},r_{q+1}\right)$, (23) (24) can also be similarly received respectively. Then, the following inequalities can be established.

$$V_i\left(x(t)\right)\le\begin{cases}\mathrm{e}^{\alpha\left(t-r_q\right)}V_i\left(x\left(r_q\right)\right)+\int_{r_q}^{t}\mathrm{e}^{\alpha(t-s)}\left[\gamma^2v^{\mathrm{T}}(s)v(s)-z^{\mathrm{T}}(s)z(s)\right]\mathrm{d}s, & t\in\left[r_q,t_{k+1}\right)\\ \mathrm{e}^{\alpha\left(t-t_{k+1}\right)}V_i\left(x\left(t_{k+1}\right)\right)+\int_{t_{k+1}}^{t}\mathrm{e}^{\alpha(t-s)}\left[\gamma^2v^{\mathrm{T}}(s)v(s)-z^{\mathrm{T}}(s)z(s)\right]\mathrm{d}s, & t\in\left[t_{k+1},t_{k+2}\right)\\ \vdots & \\ \mathrm{e}^{\alpha\left(t-t_{k+n}\right)}V_i\left(x\left(t_{k+n}\right)\right)+\int_{t_{k+n}}^{t}\mathrm{e}^{\alpha(t-s)}\left[\gamma^2v^{\mathrm{T}}(s)v(s)-z^{\mathrm{T}}(s)z(s)\right]\mathrm{d}s, & t\in\left[t_{k+n},r_{q+1}\right)\end{cases}\tag{25}$$

On the other hand, it can be derived from (14) that

$$V_{\sigma\left(r_q\right)}\left(x\left(r_q\right)\right)\le\eta V_{\sigma\left(r_q^-\right)}\left(x\left(r_q^-\right)\right)\tag{26}$$

Suppose that $0=r_1<r_2<\cdots<r_q<T$, where $r_1,r_2,\cdots,r_q$ are the switching

sequence. Correspondingly, from (25) (26) and definition 2.1, we have

$$
\begin{aligned}
V_i\left(x(t)\right) &\le \eta e^{\alpha(t-r_q)} V_i\left(x\left(r_q^-\right)\right) + \int_{r_q}^{t} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\le \eta e^{\alpha(t-r_q)}\left\{e^{\alpha(r_q-r_{q-1})} V_i\left(x\left(r_{q-1}\right)\right) + \int_{r_{q-1}}^{r_q} e^{\alpha(r_q-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds\right\} \\
&\quad + \int_{r_q}^{t} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&= \eta e^{\alpha(t-r_{q-1})} V_i\left(x\left(r_{q-1}\right)\right) + \eta\int_{r_{q-1}}^{r_q} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\quad + \int_{r_q}^{t} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\le \eta^2 e^{\alpha(t-r_{q-1})} V_i\left(x\left(r_{q-1}^-\right)\right) + \eta\int_{r_{q-1}}^{r_q} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\quad + \int_{r_q}^{t} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\le \eta^2 e^{\alpha(t-r_{q-1})} V_i\left(x\left(r_{q-2}\right)\right) + \eta^2\int_{r_{q-2}}^{r_{q-1}} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\quad + \eta\int_{r_{q-1}}^{r_q} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\quad + \int_{r_q}^{t} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\le \cdots \\
&\le \eta^{N_\sigma(0,T)} e^{a(t-r_1)} V_i\left(x(0)\right) + \int_{r_1}^{t} \eta^{N_\sigma(s,t)} e^{\alpha(t-s)}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&= e^{\alpha t + N_\sigma(0,T)\ln\eta} V_i\left(x(0)\right) + \int_{0}^{t} e^{\alpha(t-s)+N_\sigma(s,t)\ln\eta}\left[\gamma^2 v^{\mathrm{T}}(s)v(s) - z^{\mathrm{T}}(s)z(s)\right]ds \\
&\le e^{\alpha T + \frac{\ln\eta}{\tau_a}T} V_i\left(x(0)\right) + \int_{0}^{T} e^{(\alpha+\ln\eta)T}\gamma^2 v^{\mathrm{T}}(s)v(s)ds
\end{aligned}
\tag{27}
$$

Furthermore, by using lemma 2.2, it follows from (3) (12) (27) that:

$$
\Xi\left[V_i\left(x(t)\right)\right] \le e^{\left(\alpha+\frac{\ln\eta}{\tau_a}\right)T}\left[\sup_{i\in S}\lambda_{\max}\left(Q_i\right)\right]x_0^{\mathrm{T}} R x_0 + e^{\left(\alpha+\frac{\ln\eta}{\tau_a}\right)T}\gamma^2 d \tag{28}
$$

and

$$
\Xi\left[V_i\left(x(t)\right)\right] \ge \inf_{i\in S}\left[\lambda_{\min}\left(Q_i\right)\right]x^{\mathrm{T}}(t)Rx(t) \tag{29}
$$

Taking (27) (28) (29) and (17) into account, the following conclusion is obtained

$$
\Xi\left[x^{\mathrm{T}}(t)Rx(t)\right] \le \frac{e^{\left(\alpha+\frac{\ln\eta}{\tau_a}\right)T}\left\{\left[\sup_{i\in S}\lambda_{\max}\left(Q_i\right)\right]c_1 + \gamma^2 d\right\}}{\inf_{i\in S}\left[\lambda_{\min}\left(Q_i\right)\right]} < c_2 \tag{30}
$$

Therefore, the H_∞ control performance is obtained from Definition 2.3. This completes the proof.

Theorem 3.2 For any $\sigma(t) = i \in S = \{1, 2, \cdots, p\}$, given positive definite matrix R, and positive constants ε, γ, $\eta > 1$, $\alpha > 0$ consider the switched closed-loop stochastic system (7) (8) with the event-triggering strategy (5), the event-triggered state-feedback controllers (4) can be obtained, if there exist symmetric positive definite matrices $X_i, Y_i, i \in S$ with appropriate dimensions satisfying

$$\tilde{\Theta}=\begin{bmatrix} \mu^2 X_i^{\mathrm{T}}\Omega X_i-\alpha X_i^{\mathrm{T}}Y_i^{\mathrm{T}}Y_iX_i^{\mathrm{T}} & 0 & 0 & \tilde{\Pi}_1 & Y_i^{\mathrm{T}}Y & \tilde{\Pi}_2 & \tilde{\Pi}_3 \\ 0 & -\Omega & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon^{-1}I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \tag{31}$$

where $\tilde{\Pi}_1=\begin{bmatrix} X_i^{\mathrm{T}}A_iX_i+X_i^{\mathrm{T}}B_iY_i & X_i^{\mathrm{T}}B_iYX_i & X_i^{\mathrm{T}}D_{1i}X_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$\tilde{\Pi}_2=\begin{bmatrix} X_i^{\mathrm{T}}C_iX_i+X_i^{\mathrm{T}}D_iY_i & D_iY_i & 0 \end{bmatrix}$,

$$\tilde{\Pi}_3=\begin{bmatrix} X_i^{\mathrm{T}}Y_iR^{-\frac{1}{2}}\left(A_{1i}X_i+B_{1i}Y_i\right) & X_i^{\mathrm{T}}Y_iR^{-\frac{1}{2}}B_{1i}Y_i & X_i^{\mathrm{T}}Y_iR^{-\frac{1}{2}}D_{2i}X_i \end{bmatrix}$$

then the corresponding controller gains of the event-triggered H_∞ controllers (5) can be obtained as

$$K_i=Y_iX_i^{-1} \tag{32}$$

Proof Let $P_i^{\frac{1}{2}}=Y_i$, pre- and post-multiplying both sides of the inequality (15) by $\mathrm{diag}\left\{X_i^{\mathrm{T}},I,\cdots,I\right\}$ and $\mathrm{diag}\left\{X_i,I,\cdots,I\right\}$ respectively. By Schur complement, the proof can be completed.

4. Conclusion

The event-triggered finite-time H_∞ control problem has been investigated for switched stochastic system with exogenous disturbance. For the proposed event-triggering schemes, the prescribed H_∞ performance level of the switched stochastic system has been guaranteed by adopting Lyapunov-Krasovski function method and average dwell time method in a given finite-time interval. In order to avoid the Zeno behavior, a lower bound on the triggered inter-event intervals has been estimated. Furthermore, sufficient conditions for H_∞ control performance analysis and control design have been provided in terms of LMIs technique, respectively.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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