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Existence of Infinitely Many High Energy Solutions for a Fourth-Order Kirchhoff Type Elliptic Equation in $\mathbb{R}^3$

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Abstract

In this paper, we consider the following fourth-order equation of Kirchhoff type

$$\Delta^2 u - \left(a + b\int_{\mathbb{R}^3} |\nabla u|^2 dx\right)\Delta u + V(x)u = |u|^{p-1}u, \quad x \in \mathbb{R}^3,$$

where $a, b > 0$ are constants, $3 < p < 5$, $V \in C(\mathbb{R}^3, \mathbb{R})$; $\Delta^2 := \Delta(\Delta)$ is the biharmonic operator. By using Symmetric Mountain Pass Theorem and variational methods, we prove that the above equation admits infinitely many high energy solutions under some sufficient assumptions on $V(x)$. We make some assumptions on the potential $V(x)$ to solve the difficulty of lack of compactness of the Sobolev embedding. Our results improve some related results in the literature.

Keywords

Fourth-Order Kirchhoff Type Elliptic Equation, Infinitely Many Solutions, Symmetric Mountain Pass Theorem, Variational Methods

1. Introduction

Consider the following fourth-order Kirchhoff type elliptic equation

$$\Delta^2 u - \left(a + b\int_{\mathbb{R}^3} |\nabla u|^2 dx\right)\Delta u + V(x)u = |u|^{p-1}u, \quad x \in \mathbb{R}^3, \quad (1.1)$$

where $a, b>0$ are constants; $3<p<5$, $V(x)$ is a continuous function.

Since problem (1.1) involves the term $\int_{\mathbb{R}^3} |\nabla u|^2 dx$, it is no longer a local problem, which gives rise to some analytical difficulties. Moveover, the term $\int_{\mathbb{R}^3} |\nabla u|^2 dx$ involving in problem (1.1) implies that the equation is not a pointwise identity.

In the recent years, in purely mathematical research and practical applications, non-local operators have appeared in the description of various phenomena, such as fractional quantum mechanics [1], physics and chemistry [2], obstacle problems [3], etc.

If we let $V(x) = 0$, replace $\mathbb{R}^3$ with a bounded smooth domain $\Omega \subset \mathbb{R}^3$ and set $u = \Delta u = 0$ on $\partial\Omega$, then problem (1.1) would be reduced to

$$\begin{cases} \Delta^2 u - \left(a + b\int_\Omega |\nabla u|^2\right)\Delta u = |u|^{p-1}u, & \text{in } \Omega, \\ u = 0, -\Delta u = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

Problem (1.2) is the special case of the following Kirchhoff-type equation

$$\begin{cases} \Delta^2 u - \left(a + b\int_\Omega |\nabla u|^2\right)\Delta u = f(x,u), & \text{in } \Omega, \\ u = 0, \Delta u = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.3}$$

It is known that problem (1.3) is connected with the stationary analogue of the following fourth-order Kirchhoff type equation

$$\Delta^2 u + u_{tt} - \left(a + b\int_\Omega |\nabla u|^2\right)\Delta u = f(x,u), \quad \text{in } \Omega. \tag{1.4}$$

It was proposed by Kirchhoff as an extension of the classical D'Alembert's wave equation for free vibrations of elastic strings. The early classical research of Kirchhoff equations is dedicated by Bernstein [4] and Pohožaev [5]. However, (1.4) was greatly brought into focus only after Lions [6] investigated problem (1.4) involving an abstract framework.

Recently, more and more researchers began to focus on studying fourth-order Kirchhoff type problems, for instance, see [7–12] and references therein. Meanwhile, many researchers pay attention to the Kirchhoff type problems when the domain is unbounded or is the whole space $\mathbb{R}^N$. For the existence and infinitely many solutions for Kirchhoff type problems in $\mathbb{R}^N$, please see the interesting results in [13–15] and the references therein.

Now, we mention some recent works related to the Kirchhoff-type problem. By the Mountain Pass Lemma and the local minimization, Mao [16] obtained two type of nontrivial solutions for the following nonlocal problem

$$\Delta^2 u - \left(1 + \lambda\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + V(x)u = f(x,u), \quad \text{in } \mathbb{R}^3, \tag{1.5}$$

where λ is a parameter, and $V(x)$ is a continuous function. In [17], Wu obtained the existence and infinitely many solutions for the following fourth-order Kirchhoff type elliptic equation

$$\Delta^2 u - M(\|\nabla u\|_2^2)\Delta u + V(x)u = f(x,u), \quad \text{in } \mathbb{R}^N, \tag{1.6}$$

where $1 < N < 8$, $M \in C([0,+\infty),\mathbb{R})$ is a Kirchhoff-type function. Via variational methods, the authors proved the existence and infinitely many solutions for the above fourth-order Kirchhoff type elliptic equation. In [18], Zhang and Jia studied the equation

$$\Delta^2 u - \left(a + b\int_{\mathbb{R}^N} |\nabla u|^2 dx\right)\Delta u - \frac{1}{2}\Delta(u^2)u = f(x,u), \quad \text{in } \mathbb{R}^N, \tag{1.7}$$

where $N \leq 5$. Zhang and Jia applied the Fountain theorem and variational methods to establish the multiplicity of solutions for problem

(1.7) by making many reasonable hypotheses on the potential $V(x)$ and the nonlinearity $f(x,u)$. In [19], Almuaalemi, Chen and Khoutir combined the variational methods and Symmetric Mountain Pass Theorem to research the existence of high energy solutions for problem (1.1) with a nonlinearity $f(x,u)$. In [20], Xu obtained positive solutions for a class of second-order nonlinear Kirchhoff type equations in $\mathbb{R}^N$.

Motivated primarily by the above-mentioned results, we will investigate infinitely many high energy solutions for problem (1.1) and obtain some results by variational methods.

For the sake of convenience, we shall state some appropriate assumptions as follows:

(V0) $V \in C(\mathbb{R}^3, \mathbb{R})$, and there exists a positive constant V_0 such that $\inf_{x\in\mathbb{R}^3} V(x) \geq V_0$.
(V1) there exists a constant $r > 0$ such that

$$\lim_{|z|\to+\infty} \operatorname{meas}\{y \in \mathbb{R}^3 : |y - z| \leq r, V(y) \leq C\} = 0, \quad \forall\ C > 0, \tag{1.8}$$

where $\operatorname{meas}(\cdot)$ denotes the Lebesgue measure in $\mathbb{R}^3$.

Our main theorem is given in the following.

Theorem 1.1. *Suppose that (V0) and (V1) hold. Then problem (1.1) has an unbounded sequence of nontrivial solutions $\{u_n\}$ such that when $n \to \infty$,*

$$\begin{aligned} &\frac{1}{2}\int_{\mathbb{R}^3}(a|\Delta u_n|^2 + |\nabla u_n|^2 + V(x)u_n^2)dx + \frac{b}{4}\left(\int_{\mathbb{R}^3}|\nabla u_n|^2 dx\right)^2 \\ &- \frac{1}{p+1}\int_{\mathbb{R}^3}|u|^{p+1}dx \to \infty. \end{aligned} \tag{1.9}$$

Remark 1.1. It is known to all that we meet the difficulty that is short of compactness of the Sobolev embedding, because we consider problem (1.1) on the whole space $\mathbb{R}^3$. To get over the difficulty, we suppose that the potential $V(x)$ satisfies the conditions (V0) and (V1). The result obtained in this paper can be seemed as a generalization of the related result obtained in [20] when $N = 3$.

In the present paper, under the conditions of (V0) and (V1), we prove the boundedness of (PS) sequence (Palais-Smale sequence) and the infinitely many high energy solutions for a fourth-order Kirchhoff type elliptic equation, which extend the related results in the literature.

The rest of this paper is organized as follows: in Section 2, some framework are demonstrated. In Section 3, the proof of the main result is given. In Section 4, the conclusion is given.

2. Preliminaries

In the following, first of all, we shall introduce some properties of the weighted Sobolev space E and then present the concept of the nontrivial solutions for problem (1.1). On the workspace E, the certain variational functional related to (1.1) is defined and the nontrivial solutions are the critical points of the certain functional.

Let $1 \leq p < \infty$ and $L^p(\mathbb{R}^3) = \{u : \mathbb{R}^3 \to \mathbb{R} | u$ is measureable and

$\int_{\mathbb{R}^3} |u|^p dx < \infty\}$ with the norm

$$\|u\|_{L^p} := |u|_p = \left(\int_{\mathbb{R}^3} |u|^p dx\right)^{\frac{1}{p}},$$

where := denote "defined as".

Let

$$H := H^2(\mathbb{R}^3) = \{u \in L^2(\mathbb{R}^3) : \nabla u, \Delta u \in L^2(\mathbb{R}^3)\},$$

equipped with the inner product and norm

$$\langle u, v\rangle_H = \int_{\mathbb{R}^3} (\Delta u \Delta v + \nabla u \nabla v + uv) dx, \quad \|u\|_H = \langle u, u\rangle_H^{\frac{1}{2}}.$$

We consider the working space as a weighted Sobolev space E defined by

$$E = \left\{u \in H : \int_{\mathbb{R}^3} V(x) u^2 dx < +\infty\right\},$$

endowed with the inner product and norm

$$\langle u, v\rangle = \int_{\mathbb{R}^3} (\Delta u \Delta v + a\nabla u \nabla v + V(x) uv) dx, \quad \|u\| = \langle u, u\rangle^{\frac{1}{2}},$$

where $\|.\|$ is equivalent to the norm $\|.\|_H$. And then, due to the continuity of the embedding $E \hookrightarrow L^p(\mathbb{R}^3)$ for any $p \in [2, 2_*]$, there exists a constant $\varepsilon_p > 0$ such that

$$|u|_p \leq \varepsilon_p \|u\|, \quad \forall \ u \in E. \tag{2.1}$$

Moreover, similar to [21, Lemma 3.1], we have the following lemma.

Lemma 2.1. *Suppose that (V0) and (V1) hold, the embedding from E to $L^p(\mathbb{R}^3)$ is compact for all $p \in [2, 2_*]$.*

Define $\Phi : E \to \mathbb{R}$ by

$$\Phi(u) = \frac{1}{2}\|u\|^2 + \frac{b}{4}\left(\int_{\mathbb{R}^3} |\nabla u|^2 dx\right)^2 - \frac{1}{p+1}\int_{\mathbb{R}^3} |u|^{p+1} dx, \tag{2.2}$$

which is called the energy functional corresponded to problem (1.1) and is of C^1, and whose critical points are the weak solutions of (1.1). For all $u, v \in E$, we have

$$\begin{aligned}\langle \Phi'(u), v\rangle = &\int_{\mathbb{R}^3} (\Delta u \Delta v + a\nabla u \nabla v + V(x) uv) dx \\ &+ b\left(\int_{\mathbb{R}^3} |\nabla u|^2 dx\right)\int_{\mathbb{R}^3} \nabla u \nabla v dx - \int_{\mathbb{R}^3} |u|^{p-1} uv dx. \end{aligned} \tag{2.3}$$

As is known that $u \in E$ is a weak solution of (1.1), if

$$\int_{\mathbb{R}^3} (\Delta u \Delta v + a\nabla u \nabla v + V(x) uv) dx + b\left(\int_{\mathbb{R}^3} |\nabla u|^2 dx\right)\int_{\mathbb{R}^3} \nabla u \nabla v dx - \int_{\mathbb{R}^3} |u|^{p-1} uv dx = 0, \quad \forall \ v \in E.$$

Definition 2.1. *Let $\varphi \in C^1(E, \mathbb{R})$, accordding to Ekeland's variational principal, we say that a sequence $\{u_n\}$ is a Palais-Smale sequence at level c, if the sequence $\{u_n\}$ satisfying $\varphi(u_n) = c, \varphi'(u_n) \to 0$ has a* *convergent subsequence. The functional φ meets the $(PS)_c$ condition, if any Palais-Smale sequence at level c has a convergent subsequence. If φ satisfies $(PS)_c$ for any $c \in \mathbb{R}$, φ satisfies (PS)-condition.*

We shall use the following Symmetric Mountain Pass Theorem to prove the main theorem.

Proposition 2.1. [22]. *Let E be an infinite dimensional Banach space and let $\varphi \in C^1(E, \mathbb{R})$ be even, satisfy the (PS)-condition and $\varphi(0) = 0$. If $E = Y \oplus Z$, where Y is finite dimensional, and φ satisfies*

(i) *there exist constants ρ, $\alpha > 0$ such that $\varphi_{\partial B_\rho \cap Y} \geq \alpha$.*

(ii) *for each finite dimensional subspace $\tilde{E} \subset E$, there exists $R = R(\tilde{E})>0$ such that $\varphi \leq 0$ on $\tilde{E}\backslash B_R$.*

Then φ admits an unbounded sequence of critical value.

Lemma 2.2. *If $V(x)$ satisfies conditions (V0) and (V1). Then any (PS)-sequence $\{u_n\}$ is bounded.*

Proof. Let $\{u_n\} \subset E$ be such that

$$\Phi(u_n) = c, \quad \Phi'(u_n) \to 0. \tag{2.4}$$

For any $n \in \mathbb{N}$, we have

$$\begin{aligned} 1 + c &\geq \Phi(u_n) - \frac{1}{4}\langle \Phi'(u_n), u_n\rangle \\ &= \frac{1}{4}\|u_n\|^2 + \int_{\mathbb{R}^3} \left(\frac{1}{4} - \frac{1}{p+1}\right) |u|^{p+1} dx \\ &\geq \frac{1}{4}\|u_n\|^2, \end{aligned} \tag{2.5}$$

which implies that $\{u_n\} \subset E$ is bounded. □

Lemma 2.3. *Assume that (V0) and (V1) hold, then any (PS)-sequence $\{u_n\} \subset E$ defined by (2.4) has a convergent subsequence in E.*

Proof. By Lemma 2.2, the (PS)-sequence $\{u_n\} \subset E$ is bounded. Then by Lemma 2.1, passing to a subsequence, $u_k \to u$ in E, $u_k \to u$ in $L^p(\mathbb{R}^3)$, $2 \leq p < 2_*$. By (2.3), we get

$$\begin{aligned} &\langle \Phi'(u_k) - \Phi'(u), u_k - u\rangle \\ = &\int_{\mathbb{R}^3} |\Delta(u_k - u)|^2 dx - \int_{\mathbb{R}^3} (|u_k|^{p-1}u_k - |u|^{p-1}u)(u_k - u)dx \\ &-b\left(\int_{\mathbb{R}^3} |\nabla u|^2 dx - \int_{\mathbb{R}^3} |\nabla u_k|^2 dx\right) \int_{\mathbb{R}^3} \nabla u \nabla(u_n - u)dx \\ &+\int_{\mathbb{R}^3} V(x)|u_k - u|^2 dx + \left(a + b\int_{\mathbb{R}^3} |\nabla u_k|^2 dx\right) \\ &\int_{\mathbb{R}^3} |\nabla(u_k - u)|^2 dx \\ \geq &\|u_k - u\|^2 - \int_{\mathbb{R}^3} (|u_k|^{p-1}u_k - |u|^{p-1}u)(u_k - u)dx \\ &-b\left(\int_{\mathbb{R}^3} |\nabla u|^2 dx - \int_{\mathbb{R}^3} |\nabla u_k|^2 dx\right) \int_{\mathbb{R}^3} \nabla u \nabla(u_k - u)dx. \end{aligned} \tag{2.6}$$

We can obtain

$$\begin{aligned}\|u_k-u\|^2 &\le \langle\Phi'(u_k)-\Phi'(u),u_k-u\rangle\\ &+\int_{\mathbb{R}^3}(|u_k|^{p-1}u_k-|u|^{p-1}u)(u_k-u)dx\\ &+b\left(\int_{\mathbb{R}^3}|\nabla u|^2dx-\int_{\mathbb{R}^3}|\nabla u_k|^2dx\right)\int_{\mathbb{R}^3}\nabla u\nabla(u_k-u)dx. \qquad (2.7)\end{aligned}$$

Due to the boundedness of $\{u_n\}$, we have

$$|u_k|^{p-1}u_k\to|u|^{p-1}u,\quad \text{as}\quad k\to+\infty.$$

Then, we can get

$$\int_{\mathbb{R}^3}(|u_k|^{p-1}u_k-|u|^{p-1}u)(u_k-u)dx\to 0,\quad \text{as}\quad k\to+\infty. \qquad (2.8)$$

Furthermore, we define functional $L_u:E\to\mathbb{R}$ by

$$L_u(v)=\int_{\mathbb{R}^3}\nabla u\nabla v dx,\quad \forall\ \ v\in E.$$

Obviously, L_u is linear and

$$|L_u(v)|=\int_{\mathbb{R}^3}|\nabla u\nabla v|dx\le\|u\|\|v\|,$$

which shows the boundedness of L_u on E, *i.e.*, $L_u\in E^*$. Therefore, we obtain

$$L_{u_k}(u_k)\to L_u(u),\quad \text{as}\quad k\to\infty.$$

By the above and $u_k\rightharpoonup u$, we gain that $\int_{\mathbb{R}^3}\nabla u\nabla(u_k-u)dx\to 0$ as $k\to\infty$. Since $\{u_k\}$ is bounded, then we have

$$b\left(\int_{\mathbb{R}^3}|\nabla u|^2dx-\int_{\mathbb{R}^3}|\nabla u_k|^2dx\right)\int_{\mathbb{R}^3}\nabla u\nabla(u_k-u)dx\to 0 \quad \text{as}\quad k\to+\infty. \qquad (2.9)$$

Hence, $\langle\Phi'(u_k)-\Phi'(u),u_k-u\rangle\to 0$ is obvious. Hence, combining (2.8) and (2.9), we can conclude that $\|u_k-u\|\to 0$ as $k\to+\infty$. The proof is completed. □

3. Proof of the Main Result

In this section, in order to prove Theorem 1.1, we shall use the Symmetric Mountain Pass Theorem. We illustrate that Φ satisfies (PS)-condition and will prove that Φ satisfies conditions (i)-(ii) of Proposition 2.1.

Proof of Theorem 1.1. The proof of Theorem 1.1 is divided into two steps. Firstly, we prove that (i) of Proposition 2.1 holds. Set

$$\eta_n(s)=\sup_{u\in Z_n,\|u\|=1}|u|_s,\quad \forall\ \ n\in\mathbb{N},2\le s<2_*. \qquad (3.1)$$

Due to the compactness of the embedding from E to $L^s(\mathbb{R}^N)$ for $2\le s<2_*$, we have

$$\beta_n(s)\to 0\quad \text{as}\quad n\to\infty. \qquad (3.2)$$

The proof of (3.2) is similar to [21, Lemma 8.18], so we omit it. By (2.1) and (2.2), we can get

$$\begin{aligned}\Phi(u) &= \frac{1}{2}\|u\|^2+\frac{b}{4}\left(\int_{\mathbb{R}^3}|\nabla u|^2dx\right)^2-\frac{1}{p+1}\int_{\mathbb{R}^3}|u|^{p+1}dx\\ &\geq \frac{1}{2}\|u\|^2-\frac{1}{p+1}|u|_{p+1}^{p+1}\\ &\geq \frac{1}{2}\|u\|^2-\frac{\beta_n(s)}{p+1}\|u\|^{p+1}. \end{aligned} \tag{3.3}$$

By (3.2), there exists $m\geq 1$, $m\in\mathbb{N}^+$ such that

$$\beta_n^p(p)\leq\frac{p}{2},\quad \forall\ n\geq m. \tag{3.4}$$

Consequently, there exists $\xi\in(0,1)$ such that

$$\Phi(u)\geq\frac{1}{2}\xi^2(1-\xi^{p-1})=\alpha>0,$$

where $\|u\|=\xi,\quad p\geq 3$.

Secondly, we prove that (ii) of Proposition 2.1 holds. We now prove that for any subspace $\tilde{E}\subset E$ with finite dimension, there holds

$$\Phi(u)\to-\infty\ \text{ as }\ \|u\|\to+\infty,\quad u\in\tilde{E}. \tag{3.5}$$

Using an indirect method, assume that for $\{u_n\}\subset\tilde{E}$ with $\|u_n\|\to\infty$, there exists $C>0$ such that $\Phi(u)\geq -C$ for all $n\in\mathbb{N}$. Let $v_n=\frac{u_n}{\|u_n\|}$, then $\|v_n\|=1$. Passing to a subsequence, we can assume that $v_n\rightharpoonup v_0$ in $\tilde{E}$. By the fact that the dimension of $\tilde{E}$ is finite, we have $v_n\to v_0$ in $\tilde{E}$, $v_n(x)\to v_0(x)$ a.e. on $\mathbb{R}^3$, and $\|v_0\|=1$. From the definition of the norm on E, we know that $|v_0|_2\neq 0$. Moreover, we get $|v_n|_s\leq\varepsilon_s\|v_n\|\leq\varepsilon_s$ for $s\in(2,2_*)$.

Set $\Omega_n=\{x\in\mathbb{R}^3:|u_n(x)|\leq\delta\}$ and $A_n=\{x\in\mathbb{R}^3:v_n\neq 0\}$, then we have that $\text{meas}(A_n)>0$. Moreover, due to the assumption that $\|u_n\|\to\infty$, as $n\to\infty$, we obtain

$$|u_n(x)|\to\infty\ \text{ as }\ n\to\infty,\quad\forall\ \ x\in A_n. \tag{3.6}$$

Hence, for sufficiently large n, we have $A_n\subset\mathbb{R}^3\backslash\Omega_n$. Therefore, it follows from (2.1), (2.2), (2.4), (3.6) and Fatou's Lemma that

$$\begin{aligned}0&\leq\lim_{n\to\infty}\frac{\Phi(u_n)}{\|u_n\|^4}\\ &=\lim_{n\to\infty}\left[\frac{1}{2\|u_n\|^2}+\frac{b}{4\|u_n\|^4}\left(\int_{\mathbb{R}^3}|\nabla u_n|^2dx\right)^2-\int_{\mathbb{R}^3}\frac{|u_n|^{p+1}}{(p+1)\|u_n\|^4}dx\right]\\ &\leq\frac{b}{4}+\lim_{n\to\infty}\left[\int_{\Omega_n}\frac{|u_n|^{p+1}}{(p+1)|u_n|^4}v_n^4dx-\int_{\mathbb{R}^3\backslash\Omega_n}\frac{|u_n|^{p+1}}{(p+1)|u_n|^4}v_n^4dx\right]\\ &\leq\frac{b}{4}+\frac{L^{p-3}}{p+1}\varepsilon_2^4-\liminf_{n\to\infty}\int_{\mathbb{R}^3\backslash\Omega_n}\frac{|u_n|^{p-3}}{(p+1)}v_n^4dx\\ &\leq\frac{b}{4}+\frac{L^{p-3}}{p+1}\varepsilon_2^4-\int_{\mathbb{R}^3\backslash\Omega_n}\liminf_{n\to\infty}\frac{|u_n|^{p-3}}{(p+1)}v_n^4dx\\ &=\frac{b}{4}+\frac{L^{p-3}}{p+1}\varepsilon_2^4-\frac{1}{(p+1)}\int_{\mathbb{R}^3}\liminf_{n\to\infty}|u_n|^{p-3}[\chi A_n(x)]v_n^4dx\\ &=-\infty.\end{aligned} \tag{3.7}$$

Clearly, this is a contradiction. Therefore (3.5) holds. From (3.5), we know that for any subspace $\tilde{E} \subset E$ with finite dimension, there exists $C = C(\tilde{E}) > 0$ such that

$$\Phi(u) \leq 0, \quad \forall \quad u \in \tilde{E} \backslash B_C.$$

Hence, (i)-(ii) of Proposition 2.1 hold. By (2.1), we can get that $\Phi \in C^1(E, \mathbb{R})$ and $\Phi(0) = 0$. And it is easy to know that Φ is even. By Lemma 2.2 and Lemma 2.3, we know that Φ satisfies (PS)-condition. Therefore, Φ has a sequence of nontrivial critical points $\{u_n\} \subset E$ such that

$$\lim_{n \to \infty} \|u_n\| = +\infty \quad \text{and} \quad \lim_{n \to \infty} \Phi(u_n) = +\infty.$$

Consequently, problem (1.1) has infinitely many nontrivial solutions.

4. Conclusion

In this paper, we firstly obtained a (PS) sequence by compactness conditions, and then prove that the (PS) sequence has a convergent sequence. Finally, the existence of the infinitely many high energy solutions is proved by Symmetric Mountain Pass Theorem and variational methods. It is obviously that the compactness conditions have been successfully applied to find the infinitely many high energy solutions of fourth-order Kirchhoff-type elliptic systems. We hope the result can be widely used in elliptic systems.

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Conflict of Interest

The authors declare that they have no competing interests.

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