

Criterion for the Emergence of Meta-Stable States in Traffic Systems

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How to cite this paper: Zhu, L.H. (2020) Criterion for the Emergence of Meta-Stable States in Traffic Systems. *Journal of Applied Mathematics and Physics*, 8, 976-982. <https://doi.org/10.4236/jamp.2020.86076>

Received: May 11, 2020

Accepted: May 28, 2020

Published: June 1, 2020

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Abstract

The measurements on actual traffic have revealed the existence of meta-stable states with high flow. Such nonlinear phenomena have not been observed in the classic Nagel-Schreckenberg traffic flow model. Here we just add a constraint to the classic model by introducing a velocity-dependent randomization. Two typical randomization strategies are adopted in this paper. It is shown that the Matthew effect is a necessary condition to induce traffic meta-stable states, thus shedding a light on the prerequisites for the emergence of hysteresis loop in the fundamental diagrams.

Keywords

Traffic Flow, Cellular Automaton, Matthew Effect, Hysteresis Loop
Meta-Stable State

1. Introduction

In the past decades, a lot of attention has been devoted to the study of traffic flow. Since the seminal work of Nagel and Schreckenberg in the early 1990s [1], a number of cellular automata models describing traffic flow have been proposed in order to consider the real traffic scenes such as road blocks, intersections, adverse weather conditions, and so on. These cellular automata models can be used in real-time simulation very effectively, and they successfully replicate many nonlinear phenomena which are consistent with the actual traffic.

In recent years, the cellular automata models have been extended to investigate the meta-stable states in traffic systems [2]-[8]. The meta-stable states are usually represented as a hysteresis loop in the fundamental diagram. The latter is a consequence of phase separation within a certain density range. The slow-to-start rule can reduce the outflow of the traffic congestion area and keep

the inflow unchanged, so the congestion area gradually expands, resulting in phase separation [9] [10]. The slow-to-start rule was once considered as a necessary condition to induce traffic meta-stable states.

The study of meta-stable states is of great practical significance. On the smooth road, using cruise constant velocity can keep the vehicles running at a constant velocity and reduce fuel consumption. Based on this fact, D. Chowdhury *et al.* proposed a cruise-control limit model, which successfully reproduced the meta-stable states in traffic system [11]. This meta-stable state strategy has been applied to Lincoln and Holland tunnels in New York City to reduce frequent traffic congestions. Both the slow-to-start and the cruise-control limit can keep the vehicles in a meta-stable state with high flow, rather than transition to a congestion state, which effectively relieves the traffic pressure.

2. Model

For the sake of completeness, let us briefly recall the evolution rules of the classic Nagel-Schreckenberg model. This set of rules describes the principles that the vehicles must follow when driving on a one-dimensional ring road. The road is divided into a series of cells. Each cell is either empty or occupied by just one vehicle with a discrete velocity $v_i(t) \in [0, v_{\max}]$. Here v_{\max} is the velocity limit of the vehicles. The density ρ of the road is defined as the ratio of the number N of vehicles to the length L of the road, *i.e.*, $\rho = N/L$.

The configurations of the vehicles are updated in parallel according to the following four rules. **R1:** Acceleration, $v_i(t+1/3) = \min(v_i(t) + 1, v_{\max})$; **R2:** Braking, $v_i(t+2/3) = \min(v_i(t+1/3), d_i(t))$; **R3:** Randomization (with probability p), $v_i(t+1) = \max(v_i(t+2/3) - 1, 0)$; **R4:** Location updating, $x_i(t+1) = x_i(t) + v_i(t+1)$. Here $x_i(t)$ and $v_i(t)$ are the position and the velocity of i th vehicle at time t . The parameter $d_i(t)$ is the empty cells between vehicle i and the nearest neighbor vehicle $i+1$ in front of it.

Although these rules seem simple, they can simulate some complex traffic phenomena such as the free flow and ghostly congestion. Rule 1 characterizes a driver's trait to drive as fast as possible without exceeding the maximum velocity limit. Rule 2 is designed to avoid collisions between vehicles. Rule 3 requires drivers to slow down randomly and change their visual angle, so as to effectively alleviate visual fatigue. The randomization is crucial for the spontaneous emergence of traffic jams.

In the classic Nagel-Schreckenberg traffic flow model, the meta-stable states will not occur, because the indiscriminate randomization makes the homogeneous structure of the system difficult to maintain. In this paper, we do not modify the evolution rules of Nagel-Schreckenberg model, but add a specific function to control the random slowing probability of each vehicle. The determination of random slowing probability is placed before the acceleration step. **R0:** $p_i(t) = p_0 g(v_i(t))$. Here $g(x)$ is a bounded sine or cosine function.

3. Main Results

For simplicity, only one type of vehicles is considered in this paper and therefore the same maximum velocity $v_{\max} = 5$ is applied to all vehicles. For a realistic description of highway traffic, the length of a cell is set to 7.5 m, which is interpreted as the length of one vehicle plus the average gap between two adjacent vehicles in a jam.

First, we show the differences of fundamental diagrams for three different control strategies, as shown in **Figure 1(a)**. For the sine or cosine control strategy, its peak value p_0 is set to 0.35. As a contrast, the middle curve in **Figure 1(a)** is controlled by a constant control strategy, *i.e.*, $p = 0.25$, because the effective value of the sine or cosine wave is 0.707 times of its peak value.

The cosine control strategy mainly limits the low-velocity vehicles, but has no limit to the vehicles with maximum velocity. At the low density, only vehicles with the cosine control strategy can drive at maximum velocity, as shown in **Figure 1(b)**.

Under the control of the sinusoidal law, the vehicles traveling at maximum velocity are most restricted, while the stationary vehicle can start at any time as long as there is free space in front of it. Therefore, in the middle and high density areas, the traffic flow under the control of sine function is significantly higher than the other two strategies, as shown in **Figure 1(a)**.

On one hand, the cosine control strategy requires that the vehicles running at the maximum velocity do not slow down randomly, which corresponds to the phenomenon of “rich get richer” in life. On the other hand, it demands the stationary vehicles to slow start, which is equivalent to the phenomenon of “poor get poorer” in life. In this sense, the cosine control strategy is equivalent to Matthew effects, which is conducive to the separation of phases and the emergence of meta-stable states.

In **Figure 2**, p_0 is set to 0.3 because the maximum flows obtained from the two different control strategies are 1872 vehicles per hour and 2160 vehicles per hour respectively, which is consistent with the current single lane capacity of expressways. In **Figure 2(a)**, we obtain two different fundamental diagrams starting with two different initial configurations, labeled as homogeneous and random initializations, respectively. The upper branch corresponds to the calculation starting with a homogeneous initialization, while the lower branch is obtained starting from a random configuration. In certain regions, the traffic flow is no longer a single value function of density, which is known as meta-stable regions. It is obvious that Matthew effects induce meta-stable states in traffic systems. However, for the sine control strategy the two curves in **Figure 2(b)** show a typical second-order phase transition with a perfect match between homogeneous and random initializations.

4. Minor Perturbation

In this section, we show the effect of a minor perturbation on the traffic system.

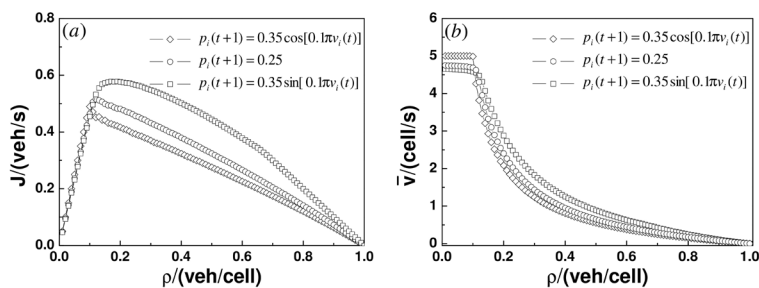


Figure 1. Flow-density and average velocity-density diagrams under three different control strategies (a) The fundamental diagrams for different randomization functions. As a contrast, the middle curve adopts a constant control strategy. (b) Under the same conditions, the evolution of average velocity with density.

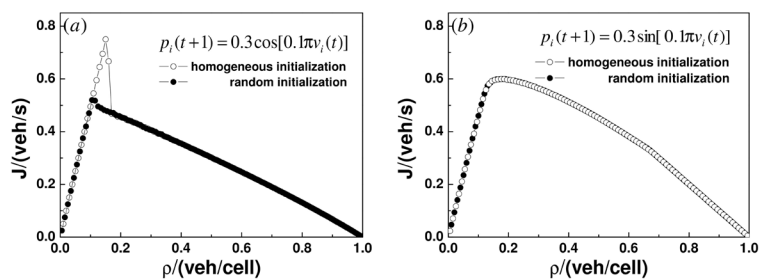


Figure 2. The fundamental diagrams obtained from two different control strategies. (a) the cosine control strategy; (b) the sine control strategy. The hollow dots are obtained from a homogeneous initial state, while the solid dots from a random initial configuration. The parameter p_0 is set to 0.3.

Figure 3(a) shows that when there is no perturbation the queue of vehicles is orderly and runs at the maximum velocity. A minor perturbation leads to the spontaneous emergence and cascading effects of traffic congestions. In this uniform and dense region, the gap between adjacent vehicles just meets the demand of maximum velocity driving. In the region all following vehicles have to slow down to avoid rear end collision once a vehicle in front is disturbed. After being disturbed, the orderly pace of the vehicles is broken. This kind of damage is irreversible, and cannot be repaired by itself with the passage of time, resulting in a sharp decline in traffic flow, as shown in **Figure 3(b)**. According to the control function of randomization, the slower the velocity is, the greater the random slowing probability is. The chain reaction of following vehicles eventually leads to the expansion of traffic congestion.

For the random initial configuration, at the same traffic density there are already traffic congestions in the lane. After being disturbed, the chain reaction of subsequent vehicles leads to new traffic congestions. The new traffic congestions here relieve the traffic pressure elsewhere, so the width of the early traffic congestions in the system becomes narrow as shown in **Figure 4(a)**. This mechanism of “as one falls, another rises” leads to the fluctuation of traffic flow, as shown in **Figure 4(b)**.

For the sine control strategy, at the same traffic density some small traffic

congestions are randomly scattered on the lane, as shown in **Figure 5(a)**. According to the control function of randomization, the slower the velocity is, the smaller the random slowing probability is. Static vehicles start fast, high-velocity vehicles slow down fast, so traffic congestions happen and disappear from time to time. **Figure 5(a)** is also disturbed after 100 evolution time steps. However, we can not even find the disturbed position accurately. This kind of spatial-temporal distribution, which has little difference before and after the perturbation, will not cause the great change of traffic flow naturally, as shown in **Figure 5(b)**. Compared with **Figure 4(b)**, the traffic flow here is more stable and the anti-interference ability is stronger.

5. Conclusion

In this paper, the main factors and limiting conditions of meta-stable and hysteretic phenomena are explored. Although the Nagel-Schreckenberg model itself cannot reflect the meta-stable and hysteretic phenomena found in real traffic, it can capture more complex traffic phenomena with a little modification. For any traffic flow model, it is a challenge to describe the possibility of hysteresis loop. Our study generalizes some of the previous results and extends the possibility of

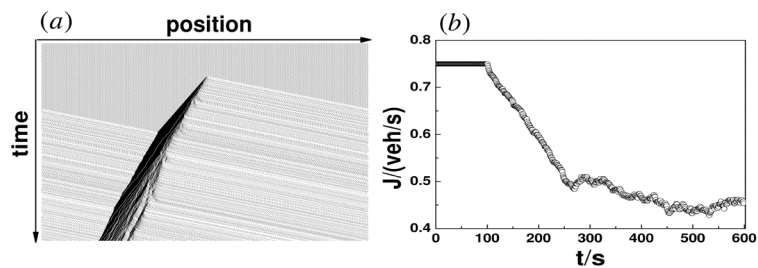


Figure 3. Spatiotemporal evolutions under a minor disturbance and corresponding change of flow. (a) The spatial-temporal diagrams at $\rho=0.15$ for the upper branch of the cosine control strategy. The vehicles are moving from left to right. A vehicle is plotted by a black dot. The time axis is vertical down. After the system reaches a steady state, a randomly selected vehicle is forbidden to move within 5 time steps. (b) Under the same conditions, the evolution of traffic flow with time.

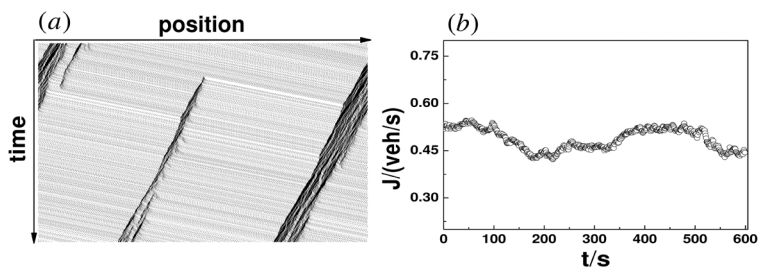


Figure 4. Spatiotemporal evolutions under a minor disturbance and corresponding change of flow. (a) The spatial-temporal diagrams at $\rho=0.15$ for the lower branch of the cosine control strategy. The vehicles are moving from left to right. A vehicle is plotted by a black dot. The time axis is vertical down. After the system reaches a steady state, a randomly selected vehicle is forbidden to move within 5 time steps. (b) Under the same conditions, the evolution of traffic flow with time.

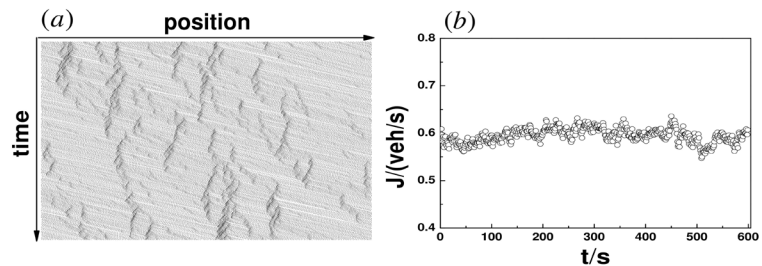


Figure 5. Spatiotemporal evolutions under a minor disturbance and corresponding change of flow. (a) The spatial-temporal diagrams at $\rho = 0.15$ for the sine control strategy. The vehicles are moving from left to right. A vehicle is plotted by a black dot. The time axis is vertical down. After the system reaches a steady state, a randomly selected vehicle is forbidden to move within 5 time steps. (b) Under the same conditions, the evolution of traffic flow with time.

meta-stable states in traffic systems to a general criterion. Only when the Matthew effect is embedded in the evolution rules, the meta-stable state will appear as scheduled. Our results will pave the way for the research with the same dynamic background in the real traffic systems.

Acknowledgements

This work is supported by the National Natural Science Foundation of China under Grant Nos. 61563054 and 10562020, the Natural Science Foundation of Guangxi under Grant No. 2019GXNSFAA245023, the open fund of Guangxi Colleges and Universities Key Laboratory of Complex System Optimization and Big Data Processing under Grant No. 2015CSOBDP0101, Initiation Fund of Doctoral Research of Yulin Normal University under Grant No. G20150003.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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