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A New Binomial Tree Method for European Options under the Jump Diffusion Model

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Abstract

In this paper, the binomial tree method is introduced to price the European option under a class of jump-diffusion model. The purpose of the addressed problem is to find the parameters of the binomial tree and design the pricing formula for European option. Compared with the continuous situation, the proposed value equation of option under the new binomial tree model converges to Merton's accurate analytical solution, and the established binomial tree method can be proved to work better than the traditional binomial tree. Finally, a numerical example is presented to illustrate the effectiveness of the proposed pricing methods.

Keywords

Option Pricing, Binomial Tree, Jump-Diffusion Process, Moment Estimation

1. Introduction

Since the establishment of Black-Scholes formula for the European call option in 1973, lots of economic models have been reported, such as Heston's model, jump diffusion model, constant elasticity of variance (CEV) model, Cox-Ingersoll-Ross (CIR) [1] model and so on. Among them, the jump diffusion model has stirred a great deal of research interest, as the jump diffusion processes more in line with the actual market law. In 1976, Merton [2] has set up a discontinuous model firstly to describe the stock returns process and analyze the option pricing problem. Jump diffusions with positive exponential jumps have been studied by Mordecki in 1999, who obtained the analytical solution of some optimal stopping problems. A double exponential jump diffusion model has been introduced to study the option pricing by Kou in [3], and the model produced analytical solutions for a variety of option-pricing problems, including call and put options, interest rate derivatives, and path-dependent options. Boyarchenko and Levendorskii have proposed the expected present value (EPV) pricing model in [4] according to the Lévy process, and studied the mean return of stochastic volatility.

The binomial tree method, first proposed by Cox, Ross and Rubinste [5] in 1979, is one of the most popular approaches to price options in diffusion models. Nowadays, the binomial tree pricing method has been well studied in the past few years. For example, Amin [6] first generalized Cox, Ross and Rubinstein's binomial tree method to jump diffusion models for vanilla options. Alfredo Ibonez [7] discussed the algorithm of American put options and obtained the optimal execution boundary by Newton interpolation method. Hou and Zhou [8] analyzed the currently used option pricing binary tree by using random error, the correction method promoted a binary tree parameter model, but it is limited to the analysis of European options and lacks practical examples. Zhang and Yue [9] discussed the no-arbitrage conditions of binary tree option pricing, and obtained a single time period and European call option pricing formula for multi-period market. More relevant studies on the various kinds option pricing and binary tree methods were investigated in [10] [11] [12] [13] and [14] and Liu [15], Song [16] and Lian [17].

In view of the above discussion, although the previous studies have their own characteristics, there are still some shortcomings: 1) The price of the underlying asset is subject to the majority of the Black-Scholes model, which does not fully reflect the characteristics of the market price. 2) Simply consider Binary tree pricing, without combining the binary tree pricing with the analytical pricing of the model. 3) When calculating the binary tree parameters, most literatures default the condition $ud = 1$ or $p = 1/2$. In this paper, following the idea of Merton (1976) [2] and Zhang (2000) [18], we try to use the binary tree method to analyze the pricing problem of European options based on a class of jump diffusion model in this paper. The rest of the paper is organized as follows. In Section 2, the jump-diffusion model is introduced and the problem under consideration is formulated. In Section 3, the binomial tree is constructed to design the pricing formula for European options under the jump-diffusion model. In Section 4, a simulation example is given to demonstrate the main results obtained. Finally, we conclude the paper in Section 5.

2. Model Formulation and Preliminaries

Assume all the work following is performed in a given risk-neutral probability space $(\Omega, \mathcal{F}, P)$. Consider following class of stochastic differential equation:

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N(t)} (Y_i - 1)\right), \tag{1}$$

where S_t denotes the stock price at time t, μ is the expectation yield rate, W_t is a standard Brownian motion, $N(t)$ is a poisson process with rate λ, and Y_i is a sequence of independent identically (iid) nonnegative random variable such that $\gamma = \ln(Y)$ has a normal distribution with following density:

$$f\left(y\right)=\frac{1}{\sqrt{2\pi}\sigma_y}\mathrm{e}^{\frac{-\left(y-u_y\right)^2}{2\sigma_y^2}}. \tag{2}$$

According to the Itô formula for the stochastic differential equation with jump diffusion, we can get the solution of the Equation (1) as follows:

$$S_t=S_0\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W(t)}\prod_{i=1}^{N(t)}Y_i. \tag{3}$$

Then, we can furthermore rewrite the Equation (3) as follows:

$$S_t=S_0\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)+\sigma W(t)+\sum_{i=1}^{N(t)}\left(\ln Y_{(i)}\right)}. \tag{4}$$

Noting that $E\left(\gamma\right)=u_y, Var\left(\gamma\right)=\sigma_y^2$ and $J=E\left(Y_i\right)=E\left(\mathrm{e}^{\gamma}\right)=\mathrm{e}^{u_y+\frac{1}{2}\sigma_y^2}$.

Taking the mathematical expectation on both sides of Equation (4), we have

$$E\left(\frac{S_t}{S_0}\right)=E\left(\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)+\sigma W(t)+\sum_{i=1}^{N(t)}\left(\ln Y_{(i)}\right)}\right). \tag{5}$$

Because of the independence of $W\left(t\right), N\left(t\right)$ and Y_i, one has

$$\begin{aligned}
&E\left(\frac{S_{(t)}}{S_0}\right)=E\left(\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)t}\right)E\left(\mathrm{e}^{\sigma W(t)}\right)E\left(\mathrm{e}^{\sum_{i=1}^{N(t)}\gamma_i}\right),\\
&E\left(\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)t}\right)=\mathrm{e}^{\left(\mu-\frac{1}{2}\sigma^2\right)t},\\
&E\left(\mathrm{e}^{\sigma W(t)}\right)=E\left(\mathrm{e}^{\sigma Z\sqrt{t}}\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\sigma Z\sqrt{t}}\mathrm{e}^{-\frac{Z^2}{2}}\mathrm{d}Z=\mathrm{e}^{\frac{1}{2}\sigma^2 t},
\end{aligned} \tag{6}$$

where $Z\sim N\left(0,1\right)$, before calculating the mathematical expectation of $\mathrm{e}^{\sum_{i=1}N(t)\gamma}$, we shall introduce following lemma.

Lemma 2.1 If $Q\left(t\right)=\sum_{i=1}^{N(t)}Y_i$ and $E\left(Y_i\right)=\beta$, then, we have

$$\begin{aligned}
E\left(Q\left(t\right)\right)&=\sum_{k=0}^{\infty}E\left[\sum_{i=1}^{k}Y_i\mid N\left(t\right)=k\right]\left(P\left\{N\left(t\right)=k\right\}\right)\\
&=\sum_{k=0}^{\infty}\beta k\frac{\left(\lambda t\right)^k}{k!}\mathrm{e}^{-\lambda t}\\
&=\beta\lambda t\mathrm{e}^{-\lambda t}\sum_{k=1}^{\infty}\frac{\left(\lambda t\right)^k-1}{\left(k-1\right)!}=\beta\lambda t.
\end{aligned} \tag{7}$$

Let $\varphi_Y\left(x\right)=E\left(\mathrm{e}^{xY_i}\right)$ be the moment generation function of random variable Y_i, then, the moment generation of a compound poisson process is

$$\begin{aligned}
\varphi_{Q(t)}\left(x\right)&=E\left(\mathrm{e}^{x\sum_{i=1}^{N(t)}Y_i}\right)\\
&=P\left\{N\left(t\right)=0\right\}+\sum_{k=1}^{\infty}E\left[\mathrm{e}^{x\sum_{i=1}^{k}Y_i}\mid N\left(t\right)=k\right]P\left\{N\left(t\right)=k\right\}\\
&=P\left\{N\left(t\right)=0\right\}+\sum_{k=1}^{\infty}E\left[\mathrm{e}^{x\sum_{i=1}^{k}Y_i}\right]P\left\{N\left(t\right)=k\right\}\\
&=\mathrm{e}^{-\lambda t}+\sum_{k=1}^{\infty}E\left(\mathrm{e}^{xY_1}\right)E\left(\mathrm{e}^{xY_2}\right)\cdots E\left(\mathrm{e}^{xY_k}\right)\frac{\left(\lambda t\right)^k}{k!}\mathrm{e}^{-\lambda t}\\
&=\mathrm{e}^{\lambda t\left(\varphi_Y(x)-1\right)}.
\end{aligned} \tag{8}$$

Therefore, we have

$$E\left(\mathrm{e}^{\sum_{i=1}^{N(t)}\gamma}\right)=\mathrm{e}^{\lambda t\left(\varphi_\gamma(x)-1\right)}, \tag{9}$$

$$\varphi_\gamma(x)=E\left(\mathrm{e}^{x\gamma}\right)=\int_{-\infty}^{\infty}\mathrm{e}^{xZ}\frac{1}{\sqrt{2\pi}\sigma_y}\mathrm{e}^{-\frac{Z-u_y^2}{2\sigma_y^2}}\mathrm{d}Z=\mathrm{e}^{xu_y+x\frac{\sigma_y^2}{2}}. \tag{10}$$

When $x=1$ and $\varphi_\gamma(x)=J$, we obtain

$$E\left(\frac{S(t)}{S(0)}\right)=\mathrm{e}^{\mu t}\mathrm{e}^{\lambda t(J-1)}=\mathrm{e}^{(\mu+\lambda J-\lambda)t}. \tag{11}$$

3. Binomial Tree Model

Assume the stock price jumps n times in $[0,T]$, let $P\{N(t)=k\}=\frac{\mathrm{e}^{-\lambda T}(\lambda T)^k}{k!}$, divide the period into m parts. Set $\Delta t=T/m$ and $t_k=k\Delta t,(k=1,2,\cdots,m)$. Assume that the stock price will move to two new values Su and Sd with probability pand $1-p$. If the initial stock price is $S_{0,0}$ at the current time $t=0$. The stock price will become $S_{0,0}u$ or $S_{0,0}d$ after Δt. Thus, there will be 3 values $S_{0,0}u^2$, $S_{0,0}ud$ and $S_{0,0}d^2$. Repeat the above operation and we can achieve the binomial tree shown in **Figure 1**.

Theorem 3.1 Consider the stock price model as Equation (1), the corresponding pricing formula can be designed as following equation by using the binomial tree method:

$$\begin{aligned}C_m(S,0)&=\sum_{n=0}^{\infty}P\{N(t)=n\}C_{0,0}\\&=\mathrm{e}^{-r_fT}\sum_{n=0}^{\infty}\frac{\mathrm{e}^{-\lambda T}(\lambda T)^n}{n!}\left[SB_1(m,k,u,d,p)-KB_2(m,k,p)\right].\end{aligned} \tag{12}$$

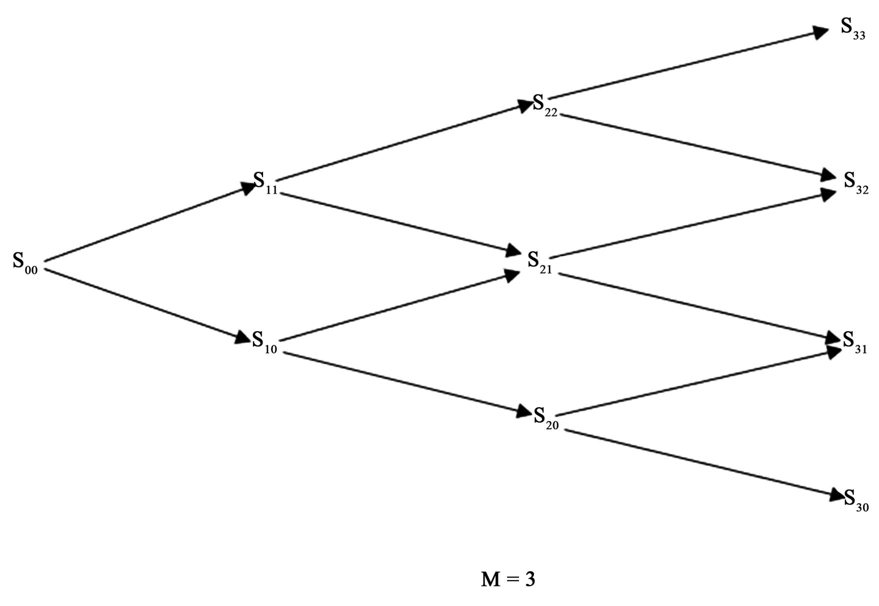

Figure 1. Binary tree.

Proof. Let $C_{k,j}\left(j=0,1,\cdots,k\right)$ be the option price at node $\left(k,j\right)$ after $k\Delta t$, r_f is the interest rate. The price can be recursively computed by the backward induction algorithm.

$$C_{k,j}=\mathrm{e}^{-r_f\Delta t}\left[PC_{k+1,j}+\left(1-P\right)C_{k+1,j+1}\right]. \tag{13}$$

The proof of (13) can be easily given, we assume at anytime the price of option is C and at the next step. The price of option will become Cu with probability P or Cd with probability 1 - P like Figure 1. So we have

$$C=\mathrm{e}^{-r_f\Delta t}\left[PCu+\left(1-P\right)Cd\right]$$

By the method of induction, at the any node in a binary tree, the formula of (13) is always correct.

Therefore, the price of European options at the initial time can be given as follows:

$$\begin{aligned}C_{0,0}&=\mathrm{e}^{-r_fT}\sum\nolimits_{j=0}^{m}C_m^j\left[p^j\left(1-p\right)^{m-j}\max\left(S_{m,j}-K,0\right)\right]\\&=\mathrm{e}^{-r_fT}\sum\nolimits_{j=0}^{m}C_m^j\left[p^j\left(1-p\right)^{m-j}\max\left(Su^jd^{m-j}-K,0\right)\right]\\&=\mathrm{e}^{-r_fT}\sum\nolimits_{j=k}^{m}C_m^j\left[p^j\left(1-p\right)^{m-j}\left(Su^jd^{m-j}-K\right)\right]\\&=\mathrm{e}^{-r_fT}\sum\nolimits_{j=k}^{m}C_m^j\left\{S\left(pu\right)^j\left[\left(1-p\right)d\right]^{m-j}-Kp^j\left(1-p\right)^{m-j}\right\}\\&=\mathrm{e}^{-r_fT}\left[SB_1\left(m,k,u,d,p\right)-KB_2\left(m,k,p\right)\right],\end{aligned} \tag{14}$$

where $B_1\left(m,k,u,d,p\right)=\sum\nolimits_{j=k}^{m}C_m^j\left(pu\right)^j\left[\left(1-p\right)d\right]^{m-j}$, $B_2\left(m,k,p\right)=\sum\nolimits_{j=k}^{m}C_m^jp^j\left(1-p\right)^{m-j}$ and $k=\min\left\{j:Su^jd^{m-j}-K>0,0<j<m\right\}$. According to the law of the total expectation, the European call option at the initial time should obey the following equation:

$$\begin{aligned}C\left(S,0\right)&=\sum\nolimits_{n=0}^{\infty}P\left\{N\left(t\right)=n\right\}\left(E\left[C_{0,0}\mid N\left(t\right)=n\right]\right)\\&=\mathrm{e}^{-r_fT}\sum\nolimits_{n=0}^{\infty}\frac{\mathrm{e}^{-\lambda T}\left(\lambda T\right)^n}{n!}\left[SB_1\left(m,k,u,d,p\right)-KB_2\left(m,k,p\right)\right].\end{aligned} \tag{15}$$

To compute the $C\left(S,0\right)$, we must confirm the parameter p, u and d first. Next, we shall calculate p, u and d by using the moment estimation theory.

$$\begin{aligned}E\left(\frac{S_t}{S_{t-1}}\right)&=pu+\left(1-p\right)d,\\E\left(\frac{S_t}{S_{t-1}}\right)^2&=pu^2+\left(1-p\right)d^2.\end{aligned} \tag{16}$$

In the traditional model, $u=1/d$ usually supply and we have following equation:

$$\begin{cases}pu+\left(1-p\right)d=\mathrm{e}^{\left(\mu+\lambda J-\lambda\right)\Delta t},\\pu^2+\left(1-p\right)d^2=\mathrm{e}^{\left(2\mu+\sigma^2\right)\Delta t}\mathrm{e}^{\lambda\Delta t\left(J^2\mathrm{e}^{\sigma_y^2}-1\right)},\\u=1/d.\end{cases} \tag{17}$$

However, there is a fault in this method, because when $\sigma \to 0$, p may be zero or a negative number which obviously contradicts with the reality. In this paper, we restructure the formula to calculate the parameter of the binomial tree by introducing the third moment:

$$E\left(\frac{S_t}{S_{t-1}}\right)^3 = pu^3 + (1-p)d^3. \tag{18}$$

Then, we set up the following equation groups:

$$\begin{cases} pu + (1-p)d = \mathrm{e}^{(\mu+\lambda J-\lambda)\Delta t}, \\ pu^2 + (1-p)d^2 = \mathrm{e}^{\left(2\mu+\sigma^2\right)\Delta t}\mathrm{e}^{\lambda\Delta t\left(J^2\mathrm{e}^{\sigma_y^2}-1\right)}, \\ pu^3 + (1-p)d^3 = \mathrm{e}^{3\left(\mu+\sigma^2\right)\Delta t}\mathrm{e}^{\lambda\Delta t\left(J^3\mathrm{e}^{3\sigma_y^2}-1\right)}, \end{cases} \tag{19}$$

Thus, we can get the solution of p, u and d as follows:

$$\begin{cases} u = \dfrac{XY - H + \sqrt{(H-XY)^2 - 4\left(X^2-Y\right)\left(Y^2-HX\right)}}{2\left(X^2-Y\right)}, \\ d = \dfrac{Y - Xu}{x-u}, \\ p = \dfrac{X-d}{u-d}. \end{cases} \tag{20}$$

where $X = \mathrm{e}^{(\mu+\lambda J-\lambda)\Delta t}$, $Y = \mathrm{e}^{\left(2\mu+\sigma^2\right)\Delta t}\mathrm{e}^{\lambda\Delta t\left(J^2\mathrm{e}^{\sigma_y^2}-1\right)}$ and

$$H = \mathrm{e}^{3\left(\mu+\sigma^2\right)\Delta t}\mathrm{e}^{\lambda\Delta t\left(J^3\mathrm{e}^{3\sigma_y^2}-1\right)}.$$

Substitute the three parameters into the Equation (14) and the according to option price with a jump diffusion model can be calculated. Now, we want to calculate Europe option with a continuous model and compare the result with binomial tree. While, in the risk-neutral world, the asset grows at the market risk-less instant interest rate r_f. Thus, the asset price would satisfy $E\left(S(T)\right) = S(0)\mathrm{e}^{r_f T}$.

Let $E\left(S(T)\right) = S(0)\mathrm{e}^{r_f T} = S(0)\mathrm{e}^{(\mu+\lambda J-\lambda)t}$ and $\mu = r_f - \lambda J + \lambda$.

Then, a new real-valued measure Q is absolutely continuous with respect to P, and can be defined as follows:

$$\frac{\mathrm{d}Q}{\mathrm{d}P} = \xi \text{ with } \xi = \mathrm{e}^{(\lambda J-\lambda)T}.$$

In fact, the new measure Q can be seen as the risk-neutral measure. Consequently, under the risk-neutral measure Q, the price C of European call option with expiry date T and strike price K can be formulated as the discounted expectation of the payoff $\max\left[S(T)-K\right]$.

$$
\begin{aligned}
C &= \mathrm{e}^{-r_f T} E\left[\max\left\{S(T)-K,0\right\}\right] \\
&= \mathrm{e}^{-r_f T} E\left[\max\left(S_0 \mathrm{e}^{(r_f-\lambda J+\lambda)T-\frac{1}{2}\sigma^2 T+\sigma z\sqrt{T}+\sum_{i=1}^{k}\gamma_i}-K,0\right)\right] \\
&= \mathrm{e}^{-r_f T}\sum_{k=0}^{\infty} P\left(N(t)=k\right) E\left[\max\left(S_0 \mathrm{e}^{(r_f-\lambda J+\lambda)T-\frac{1}{2}\sigma^2 T+\sigma z\sqrt{T}+\sum_{i=1}^{k}\gamma_i}-K,0\right) \mid N(t)=k\right] \\
&= \mathrm{e}^{-r_f T}\sum_{k=0}^{\infty} P\left(N(t)=k\right) E\left[\max\left(S_0 \mathrm{e}^{(r_f-\lambda J+\lambda)T-\frac{1}{2}\sigma^2 T+\sigma z\sqrt{T}+\sum_{i=1}^{k}\gamma_i}-K,0\right) \mid N(t)=k\right] \\
&= \mathrm{e}^{-r_f T}\sum_{k=0}^{\infty} P\left(N(t)=k\right)\int_{z(k)}^{\infty}\left(S_0 \mathrm{e}^{(r_f-\lambda J+\lambda)T-\frac{1}{2}\sigma^2 T+z}-K\right)\phi\left(z,u_1,\sigma_1^2\right)\mathrm{d}z \\
&= S(0)\mathrm{e}^{\left(-\lambda J+\lambda-\frac{1}{2}\sigma^2\right)T}\sum_{k=0}^{\infty} P\left(N(t)=k\right)\int_{z(k)}^{\infty}\mathrm{e}^{z}\phi\left(z,u_1,\sigma_1^2\right)\mathrm{d}z \\
&\quad -\sum_{k=0}^{\infty} P\left(N(t)=k\right)\int_{z(k)}^{\infty}\left(K\mathrm{e}^{-r_f T}\phi\left(z,u_1,\sigma_1^2\right)\right)\mathrm{d}z,
\end{aligned}
\tag{21}
$$

where

$$
\phi\left(z,u_1,\sigma_1^2\right)=\frac{1}{\sqrt{2\pi}\sigma_1}\mathrm{e}^{-\frac{(z-u_1)^2}{2\sigma_1^2}},\ \tilde{Z}=\sigma z\sqrt{T}+\sum_{i=1}^{k}\gamma_i,
$$

$$
u_1=E\left[\tilde{Z}\right]=ku_y,\ \sigma_1^2=V\left(\tilde{Z}\right)=\sigma^2 T+k\sigma_y^2
$$

$$
z(k)=\ln\left(K/S(0)\right)-\left(r_f-\lambda J-\frac{1}{2}\sigma^2+\lambda\right)T..
$$

Meanwhile, we know following equations hold

$$
\int_{z(k)}^{\infty}\mathrm{e}^{z}\phi\left(z,u_1,\sigma_1^2\right)\mathrm{d}z=\int_{z(k)}^{\infty}\mathrm{e}^{z}\frac{1}{\sqrt{2\pi}\sigma_1}\mathrm{e}^{-\frac{(z-u_1)^2}{2\sigma_1^2}}\mathrm{d}z=\mathrm{e}^{u_1+\frac{1}{2}\sigma_1^2}\Phi\left(\frac{u_1+\sigma_1^2-z(k)}{\sigma_1}\right),
$$

$$
\int_{z(k)}^{\infty}K\mathrm{e}^{-r_f T}\Phi\left(z,u_1,\sigma_1^2\right)\mathrm{d}z=K\mathrm{e}^{-r_f T}\Phi\left(\frac{u_1-z(k)}{\sigma_1}\right).
$$

Thus, after some simple calculation, we have following theorem.

Theorem 3.2 According to the considered jump-diffusion model (1), the analytical solution of the European call option can be designed as follows:

$$
\begin{aligned}
C=\sum_{k=0}^{\infty}P\left(N(t)=k\right)&\left[S(0)\mathrm{e}^{-\lambda JT+ku_y+\frac{1}{2}\sigma_y^2}\Phi\left(\frac{u_1+\sigma_1^2-z(k)}{\sigma_1}\right)\right. \\
&\left.-K\mathrm{e}^{-r_f T}\Phi\left(\frac{u_1-z(k)}{\sigma_1}\right)\right].
\end{aligned}
\tag{22}
$$

Until now, we have obtained the European call option through two methods. In fact, when $m\to 0$, the option price given by the binomial tree model will tend to the value aboving because of the Central limit theorem and we will use a numerical analysis to prove it in the next section.

4. Numerical Analysis

In this section, we aim to demonstrate the effectiveness and applicability of the proposed methods. Assume $S_0 = 40, \mu = 0.1, \sigma = 0.5, u_y = 0.2, \sigma_y = 0.25, \lambda = 3$. **Figure 2** shows the change process of the option price when given the different steps $\Delta t = T/m$. Compared with continuous analytic result, we can see that the options price is equal to the continuous model with m tending up. When $m = 100$, they are almost coincided. It proves that the options pricing formula established by the proposed new binomial tree model based on jump diffusion model (1) is convergent.

We also wonder realize the options price changing in maturity [0, T] with stable steps M.

As it shows in **Figure 3** and **Figure 4**, at the same maturity, when binary tree steps M is becoming greater. The difference between analytical solution and our binary tree methods get small at time T. It is consistent with the real finance market and also proves the correction of our methods.

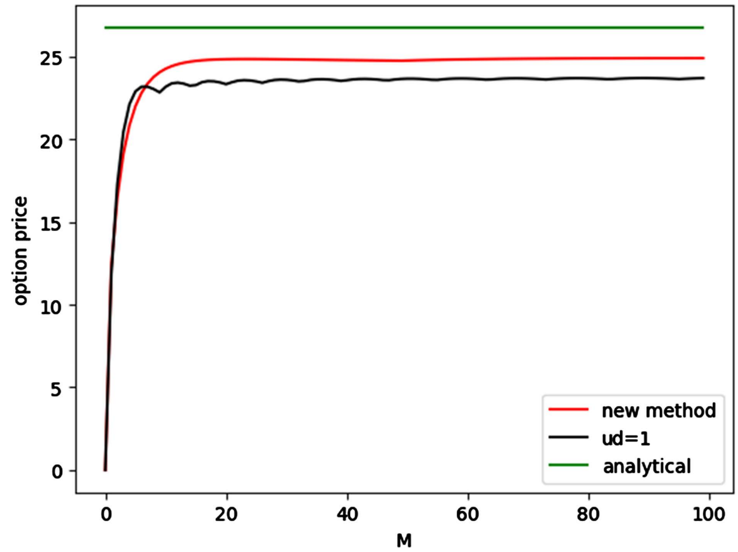

Figure 2. Option price.

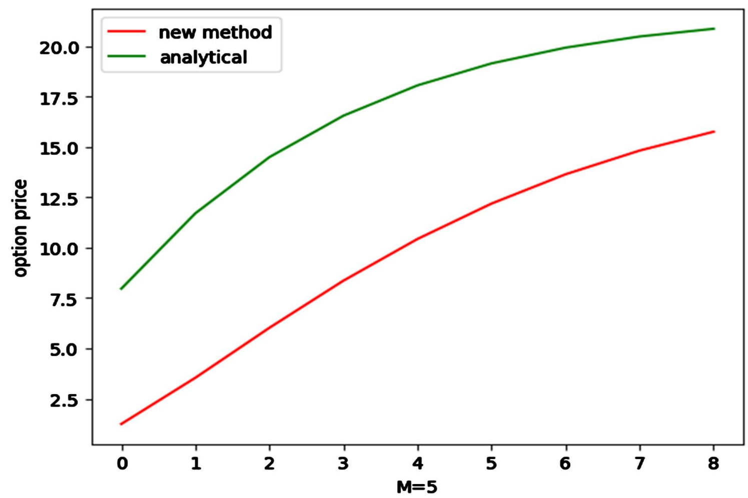

Figure 3. Option price when steps $M = 5$.

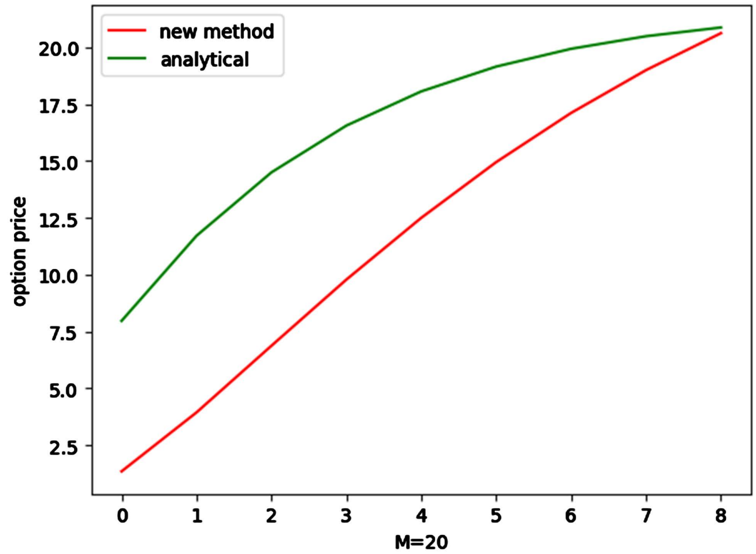

Figure 4. Option price when steps $M = 20$.

5. Conclusion

In this paper, we have dealt with the pricing problem for European option based on a class of jump diffusion model. A new binomial tree has been constructed and the corresponding pricing scheme for European option has been proposed. We have adopted the third moment to calculate the parameter of binomial tree. Compared with the analytic solutions, a numerical example has shown that the proposed binomial tree model with jump-diffusion can approximate the Merton model, and can be proved that the established binomial tree method works better than traditional binomial tree. When nodes m tends to be infinite, both of them are the same as analytic solutions. Furthermore, the model can be extended to the pricing of exotic options such as American options, lookback options and butterfly options based on the jump diffusion process.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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