

A Hysteretic Description of Non-Kepler's Orbit in Two-Body Celestial Mechanics

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Abstract

Tycho Brahe was known for his comprehensive and remarkably accurate astronomical observations, and was considered one of the greatest astronomers before the invention of the telescope. However, Johannes Kepler, using conic sections, formulated three laws of planetary motion based on Tycho's observations. The formulas for circles and ellipses thus derived, and the traditional formula for ellipses based on a single focus of an elliptical orbit, were impractical and led to large and small errors. Because astronomy at that time was based on observations and mathematical formulas derived from them, these laws are still considered valid today. Unfortunately, Kepler's laws are not really "laws" of the laws of physics, but rather trends that Kepler noticed and calculated using astronomical observations of the planets. This paper describes the eccentricity, amplitude, phase shift, angular momentum, polarization, radial path, and orbital energy of two-body orbital mechanics simultaneously, and then presents a wave function formula that avoids the above-mentioned difficulties. The results are in good agreement with the observational data. This paper contains 26 new equations and 11 figures, and it is hoped that the findings and results will contribute to the progress of the theory of celestial mechanics.

Keywords

Celestial Mechanics, Orbital Mechanics, Kepler's Laws, Wave Function, Planetary Motion

1. Introduction

Two-body orbital mechanics is an old field of science.

The derivation of the two-body problem yields the well-known Kepler laws.

Kepler empirically formulated these laws by analyzing Tycho Brahe's planetary observations. Although Kepler recognized the elliptical orbit, his laws can easily be generalized to other conic sections. Newton's calculations show that motion occurs in a plane, and its trajectory is a conic section that depends on the system's mechanical energy (*i.e.*, the sum of the kinetic and potential energies). If the mechanical energy is negative (*i.e.*, the system is closed), then the motion trajectory will be an ellipse; in the case of positive energy (open system), it will be a hyperbola. In the limiting case between the two, at zero energy, we see a parabolic trajectory. For example, planets move in elliptical orbits, while some comets follow parabolic orbits. In special cases, the motion trajectory can also be straight [1] [2].

The usual mathematical representations of circles and ellipses are related to Euclidean geometry, specifically conic sections. This makes them more difficult to use in astronomy, astrophysics, and celestial mechanics.

We disagree with Kepler's conic section because the 2D section is not elliptical, more oval, and the perihelion is sharper than the aphelion. In other words, the shape of the ellipse near perihelion is more ellipsoidal, and the aphelion is a paraboloid. To the best of my knowledge, such a phenomenon has not been observed in astronomy to date.

"Included should be the understanding that we're talking about soft limits here; no orbit is exactly Keplerian because gravity goes everywhere." Strictly speaking, no orbits are in perfect accordance with Kepler's laws. Kepler's laws aren't really "laws" in terms of physical laws, but are instead trends that Kepler noticed and calculated using astronomical observations of the planets. A "non-Keplerian" orbit is an orbit in which Kepler's laws lack predictive and descriptive power. Suppose a question about an orbit requiring a specified accuracy can't be answered with the required accuracy using Kepler's laws. In that case, the orbit is "Non-Keplerian" in the context of that question" [3].

For us, it was wonderful to derive the formula for an ellipse centered at a circle using a cylindrical coordinate system. However, modern astrophysics requires, in addition to orbital parameters, a wave function formula that simultaneously describes the energy, force, amplitude, and momentum of the motion of a celestial body orbiting one focus of an ellipse. In other words, I believe that there should be a set of wave parameters expressed in trigonometry of motion, not a simple linear relationship expressed in terms of one or two single parameters. This is the view that reality isn't fundamentally a collection of objects—particles, atoms—spread out in three-dimensional space or even four-dimensional spacetime, but instead, reality is fundamentally a wave function, a field-like concept that exists in some higher-dimensional quantum reality.

In this article, a new and efficient method for determining the main parameters of elliptical orbits of two bodies is shown based on cylindrical sections instead of Kepler's conic sections. As a result, the eccentricity, amplitude, phase shift, angular momentum, polarization, radial trajectory, and orbital energy are shown, respectively.

2. Geometry of Circle and Ellipse

2.1. Conventional Formulas of the Circle and the Ellipse

2.1.1. Classical Formula of a Circle

The traditional equation of the circle:

$$(x-a)^2 + (y-b)^2 = r^2 \quad [4] \quad (1)$$

where the center of the circle is located at $(a \text{ and } b)$, and r is the radius.

2.1.2. Classical Formula of an Ellipse

$$\frac{(x-a)^2}{h^2} + \frac{(y-b)^2}{k^2} = 1 \quad [4] \quad (2)$$

where the center is (a, b) , the length of the major axis is $2h$, and the length of the minor axis is $2k$.

The distance between the center and either focus is

$$c^2 = h^2 - k^2, \\ h > k > 0$$

2.2. New Formulas of Circle and Ellipse in the Cylinder

In this section, the circle and ellipse formulas derive from the cylindrical section's trigonometric functions.

2.2.1. The Hysteretic Formula of the Circle

Based on **Figure 1**, the two transversal axes of the circular motion are shown in **Figure 2**.

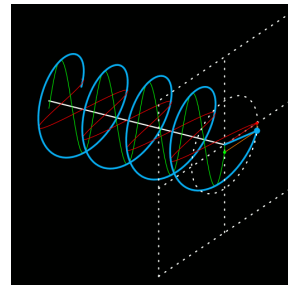


Figure 1. The 3D motions of the two axes of a circle [5].

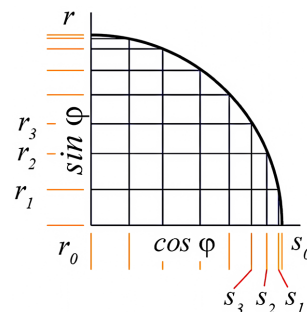


Figure 2. Formulation of the unit circle, which is a cross-section of the cylinder.

The unit circle formula equals:

$$\sin^2 \varphi + \cos^2 \varphi = 1 \quad (3)$$

where 1 is the radius of a unit circle, and φ is the angle of the circle oscillating from 0° to 360° , which is a real number measured in radians. 1 is a number that can represent all radii of the circle from subatomic size to the Universe-scale (**Figure 1**, **Figure 2**, and Formula (3)). Nobody has called Equation (3) the formula of the unit circle to date.

The harmonic ratio of the semi-major and semi-minor axes of a circle can play an important role in the description of the hysteretic oscillation (**Figure 1**).

2.2.2. The Hysteretic Formula of the Ellipse

Based on the eccentricity of the ellipse and the barycenter of the interacting two celestial bodies, we can describe the semi-major axis (a) and semi-minor axis (b) of the ellipse.

To determine the formula of the ellipse in the cylindrical coordinate system, we apply the polar coordinates, which are:

- Semi-major axis (a);
- Semi-minor axis (b);
- Angle (φ) is the angle within the circle;
- β is the angle of the ellipse on the circle (0° to 360°).

In **Figure 3** a is the semi-major axis of the ellipse, b is the semi-minor axis, $r = b$ is the radius of the cylinder, β is the angle of the ellipse relative to the base circle, h is the amplitude of the sine wave, $2\pi r$ is the period of sine wave [6] [7] (Equation (4) and Equation (5)).

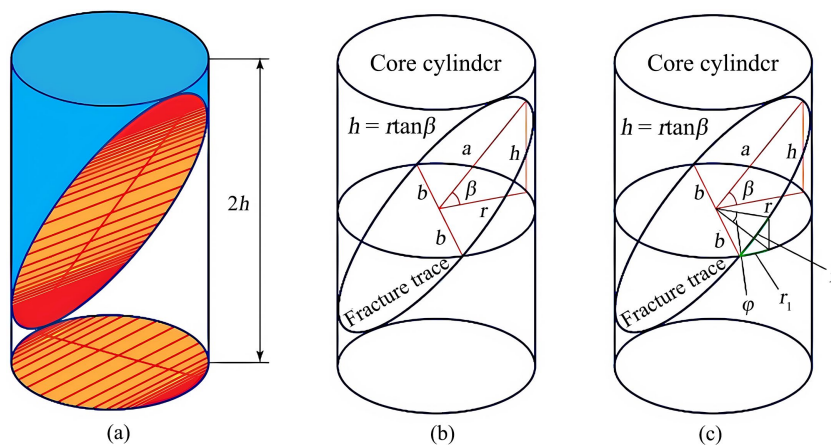


Figure 3. The scales of projections of the circle and ellipse on the diameter of the circle.

Based on **Figure 1** and **Figure 2**, we can describe the ellipse parameters.

$$\frac{h}{a} = \sin \beta \quad (4)$$

$$e = \sin \beta \quad (5)$$

Here e is the eccentricity of the ellipse.

When $0^\circ \leq \beta \leq 90^\circ$ and $0 \leq e \leq 1$ the semi-major axis is $r \leq a \leq \infty$ (Equation (4) and Equation (6)), but the semi-minor axis ($r = b$) is unchangeable.

In the case of an ellipse, the ratio of the semi-minor and semi-major axes is written in the next form:

$$e = \sqrt{1 - \cos^2 \beta} \quad (6)$$

or

$$\begin{aligned} \cos \beta &= \sqrt{1 - \sin^2 \beta} = \sqrt{1 - e^2} \\ \frac{b}{a} &= \cos \beta, \quad a = \frac{b}{\cos \beta} = \frac{r}{\cos \beta} \end{aligned} \quad (7)$$

The semi-major axis (a) is $\frac{1}{\cos \beta}$ times larger than the semi-minor axis (b).

Like the radius (r) of the circle shown in Equation (3), the radial trajectory (L_{ell}) of the ellipse (Equations (8) and (9)) is:

$$\left(\frac{r \cdot \sin \varphi}{\cos \beta} \right)^2 + r^2 \cos^2 \varphi = L_{ell}^2 \quad (8)$$

It is the formula of the ellipse (Equation (8)). Then, L_{ell} is the radial trajectory of a given angle (φ) from the center of the ellipse:

$$L_{ell} = r \sqrt{\left(\frac{\sin \varphi}{\cos \beta} \right)^2 + \cos^2 \varphi} = r \sqrt{\left(\frac{\sin \varphi}{\sqrt{1 - e^2}} \right)^2 + \cos^2 \varphi} \quad (9)$$

The radial trajectory (L_{ell}) or distance measured from the ellipse's center is symmetrical at 0° and 180° , as well as 90° and 270° (Figure 4).

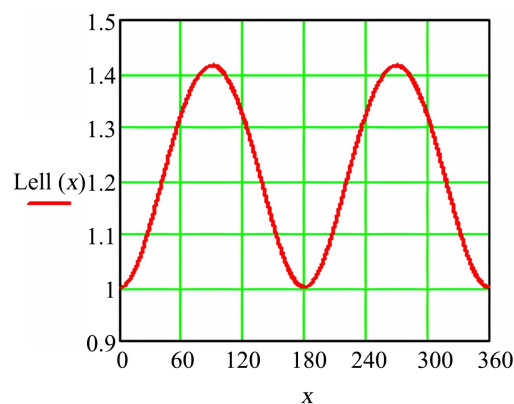


Figure 4. Radial trajectory of an ellipse ($r = 1$ and $e = 0.5$).

From Figure 4, we see that the major axis of the ellipse (90° and 270°) is 1.43 units away from its center, while the minor axis (0° and 180°) is 1.0 units away.

The eccentricity and radial trajectories of planets and their calculations have long been one of the most difficult topics in astronomy and celestial mechanics. To this day, they are based on Kepler's laws [8]-[11].

3. Two-Body Elliptical Orbit in Non-Kepler Section

Section II manifests the structure of the ellipse based on the sum of squares of the semi-major and semi-minor axes.

The definitions of the circle and ellipse in Formulas (1)-(3) and (9) are strict trigonometric wave equations that describe their structure. Now here comes the cool part of this paper. In other words, we need a living mathematical description of the momentum, time, position, and rotation of a celestial body in an elliptical orbit.

3.1. Non-Keplerian Orbit of a Celestial Body

The Fermi-Dirac distribution approximates the distribution of such dynamic expressions, but it does not fully represent reality. Because the Fermi-Dirac distribution is written in exponential terms, while the motions of a celestial body in an orbit must be expressed in terms of periodicity (See next Subsection 3.1.1). Therefore, to accurately describe these properties, we have chosen a cylindrical section and have developed a new Alternative Fermi-Dirac Distribution.

3.1.1. Fermi-Dirac Distribution and Alternative Fermi-Dirac Distribution

From quantum physics, the Fermi-Dirac Distribution is in [12]-[15].

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \quad (10)$$

where E is the probability energy, energy at the Fermi level, k is the Boltzmann constant, and T is the absolute temperature in Kelvin.

The problem of analytical integration involving powers and derivatives of the Fermi function is frequently encountered in many theoretical analyses. To tide us over this difficulty, an alternative model for the Fermi-Dirac function has been proposed [16].

We see the Fermi-Dirac distribution is somewhat exponential but needs a periodic (Equation (10) and **Figure 5(a)**). For this reason, based on the open hysteresis law, the Alternative Fermi-Dirac Distribution [16]-[18] (**Figure 5(b)**) is processed (Equation (10)) and it is used in many papers.

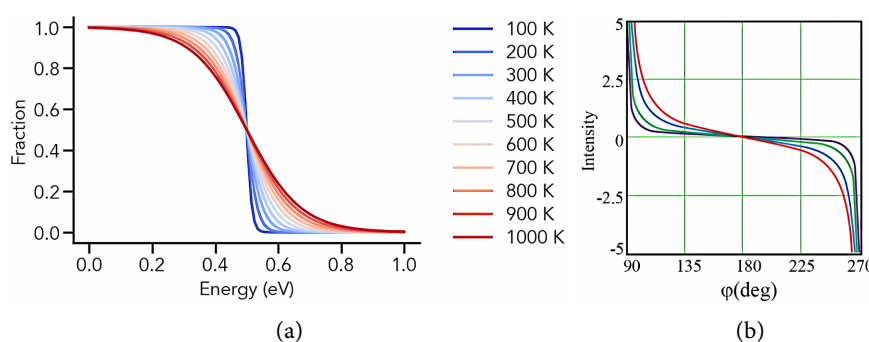


Figure 5. (a) The Fermi-Dirac distribution is plotted for a few temperatures, (b) The Alternative Fermi-Dirac distribution [16]-[18].

$$f(\varphi) = \frac{1}{\sqrt{1-e^2}} \cdot \frac{\sin(\varphi)}{|\cos \varphi|} \quad (11)$$

We used the Alternative Fermi-Dirac Distribution in the next Subsections.

3.1.2. Parameters of Hysteretic Circular Orbit

The formula derivation of the hysteresis formula is published in papers [19]-[21] (Figure 6).

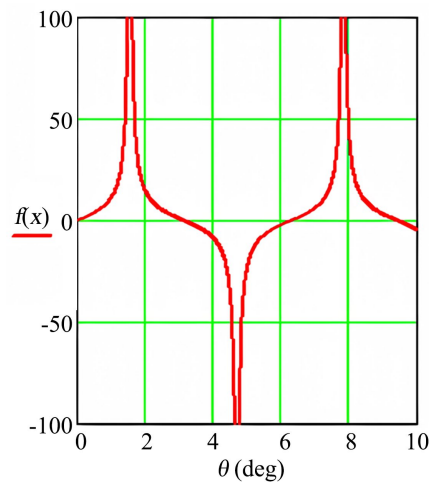


Figure 6. Hysteretic circle and circular polarization.

From Figure 2, we can write the next ratio:

$$\tan(\varphi) = \frac{r_{\varphi}}{s_{\varphi}} = \frac{\sin(\varphi)}{\cos(\varphi)} \quad (12)$$

In mathematics, a real number is a value of a continuous quantity that can represent a distance along a line (or a quantity that can be represented as an infinite decimal expansion). The refraction light is positive, due to only a period. True that only space is always absolute; everything else is relative. For this reason, there is no negative distance or space, and the denominator of Equation (12) must be written in absolute value. [19]-[22]

$$f(\varphi) = \frac{r_{\varphi}}{s_{\varphi}} = \frac{\sin(\varphi)}{|\cos(\varphi)|} \quad (13)$$

Its first definition, shown in 2018, Formula (13), has become the basis for explaining many physical phenomena. [16]

Have you noticed something in the above equation? The peculiarity of this circle formula seems to be that it is not directly dependent on space or time. Does the angle involve time or space? Does the angle contain time or space? No, it's just an angle. However, it is possible to connect space and time by force or indirect means. In astronomy, distance is determined by angle and time. In astronomy, position angle (usually abbreviated PA) [23] is the convention for measuring angles in the sky.

The variations of Formula (13) by amplitude, eccentricity, phase shift, and polarization are used in many research papers [7] [16]-[21] [24]-[39].

Figure 6 shows the coincident plot of $f(x)$ of the circle orbit.

3.1.3. Parameters of Hysteretic Elliptical Orbit

We continue the description of the elliptic orbit based on the Formulas written in Subsection 2.2.2.

The amplitude (h) of the ellipse in a cylinder with different eccentricities in **Figure 3**:

$$\frac{h}{a} = e, \quad h = a \cdot e \quad (14)$$

The c is the distance of the center of ellipse to the foci, then eccentricity $e = c/a$ [8] (Equation (6) and Equation (7)).

Since,

$$\frac{h}{a} = \frac{c}{a}, \quad h = c \quad (15)$$

$$c = a \cdot e \quad (16)$$

$$c = \frac{r \cdot e}{\cos \beta} = r \cdot \tan \beta \quad (17)$$

If $e \approx 1$ the ellipse becomes very tapered in Equations (16) and (17).

The distance of the perihelion:

$$d_{peri} = a - c = r \cdot \left(\frac{1}{\cos \beta} - \tan \beta \right) \quad (18)$$

The distance of the aphelion:

$$d_{apeli} = a + c = r \cdot \left(\frac{1}{\cos \beta} + \tan \beta \right) \quad (19)$$

Now, we should discuss the hysteresis formulas of the circle and the ellipse.

In summary, the relationship between the semi-major and semi-minor axes of an ellipse is the basis of most laws of nature. Circle and ellipse formulas are defined as sums of squares of sine and cosine functions, while their hysteresis formulas are expressed as ratios of those functions because the eigenfunction of the division operation is a ratio.

Let's consider this separately.

3.1.4. Polarized Ellipse Orbit

The polarization of the ellipse is:

$$f(\varphi) = \frac{1}{\cos \beta} \cdot \frac{\sin(\varphi \pm u)}{|\cos(\varphi)|} = \frac{1}{\sqrt{1-e^2}} \cdot \frac{\sin(\varphi \pm u)}{|\cos(\varphi)|} \quad (20)$$

Suppose $e = 0.99$ and $u = 0$ the hysteresis of the ellipse orbit is:

$$f(\varphi) = \frac{1}{\sqrt{1-0.99^2}} \cdot \frac{\sin(\varphi \pm 0)}{|\cos(\varphi)|}$$

The $f(\varphi)$ graph overlaps in **Figure 7**.

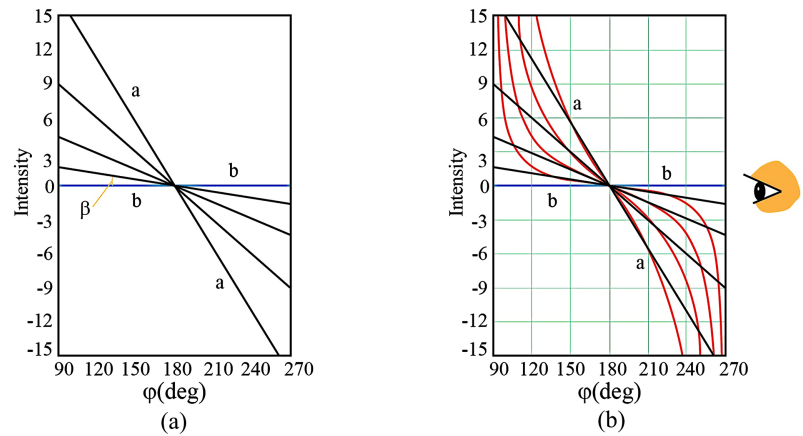


Figure 7. Hysteretic eccentricity. The locations of the ellipse in the cylinder are shown in (a), (b) Dynamic intensities (amplitude) of hysteretic ellipses with their strengths described by the Alternative Fermi-Dirac Distribution (b).

The direction of electric field of an electromagnetic wave, and how that direction varies in time or space, is called its polarization. The simplest type of polarization is called linear polarization (**Figure 7**).

If we use the Formula (20) of an elliptical orbit the ellipse polarization is described by the phase shift (u).

The phase shifting needs to account for the hysteresis formula of the ellipse (**Figure 8**).

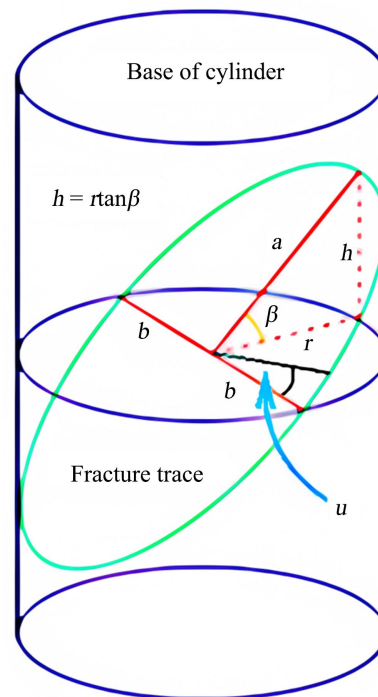


Figure 8. Polarization angle.

Let $e = 0.999$ and $u = \frac{\pi}{1.8}$

$$f3(x) = \frac{1}{\sqrt{1-0.999^2}} \cdot \frac{\sin\left(\varphi - \frac{\pi}{1.8}\right)}{|\cos \varphi|};$$

$$f4(x) = \frac{1}{\sqrt{1-0.999^2}} \cdot \frac{\sin\left(\varphi + \frac{\pi}{1.8}\right)}{|\cos \varphi|}$$

According to the hysteresis law, the phase shift simultaneously expresses the polarization ellipse. (Figure 9)

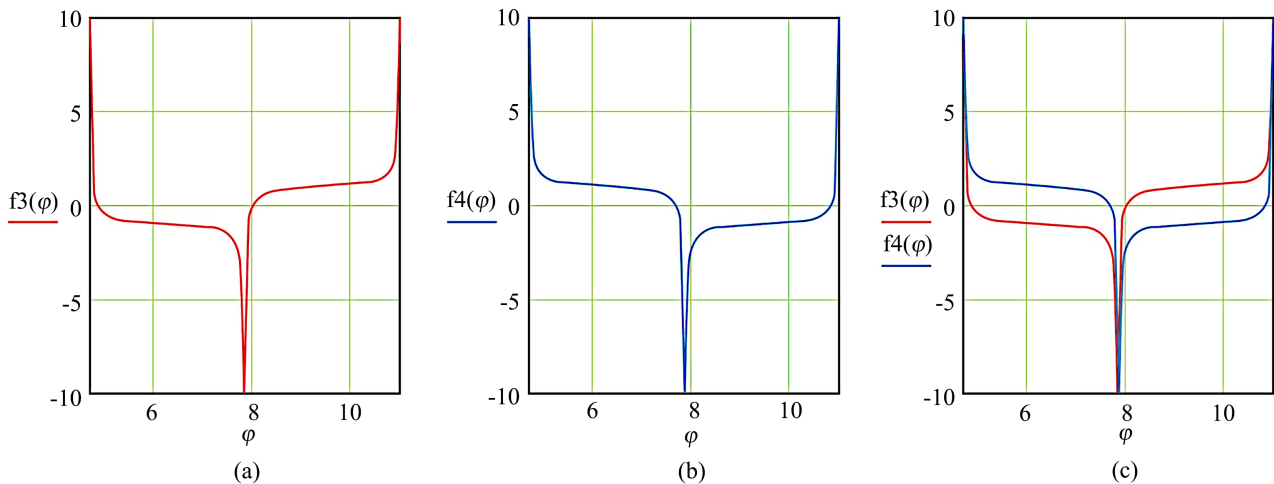


Figure 9. Hysteresis from Ellipse Orbit. (a) Left open hysteresis, (b) Right open hysteresis, (c) Closed hysteresis.

Polarization angle is the incident angle and also phase shift as same (δ) in **Figure 10.**

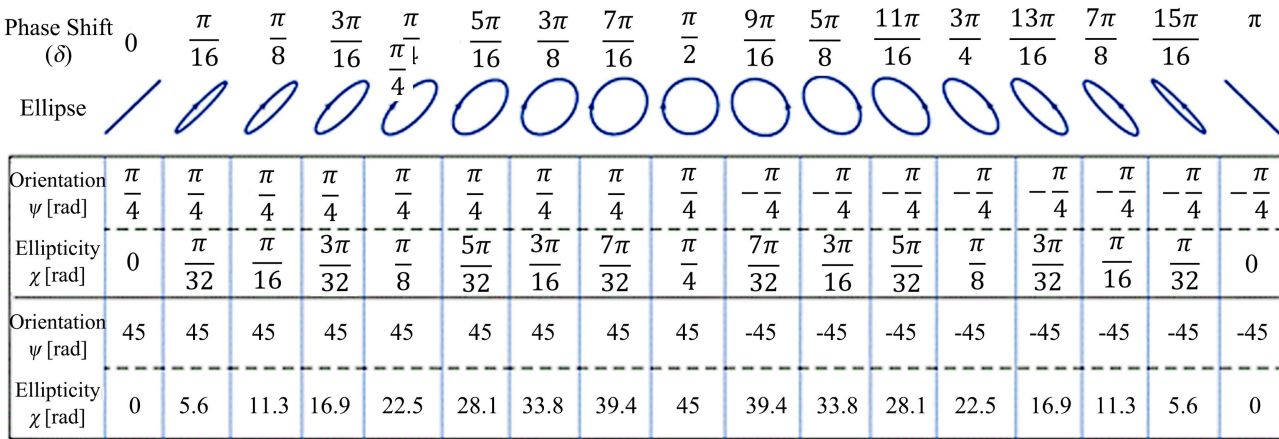


Figure 10. The ellipticity and orientation of the polarization ellipse provide information about the phase shift (δ) between the E_x and E_y components of the electric field. The ellipses shown above result when the peak amplitudes of both components are the same. The direction of the E vector's rotation is indicated by the direction of the arrow on the polarization ellipse [40]. It is in our case, $\chi = \beta$ and $\psi = u$.

If we lengthen the cylinder that includes the ellipse, the whole physical and geometrical picture of the fiber optic cable appears before us (**Figure 7**). It is written in the following section.

3.2. Main Parameters of the Elliptical Orbit

The most stunning parameters of celestial mechanics and orbital mechanics happen in elliptical structures and orbits. For this reason, we will discuss the main events in detail.

Let's calculate the next parameters of the ellipse orbit:

- 1) the radial trajectories at a given angle;
- 2) angles of the trajectory in relation to the major axis of ellipse;
- 3) angle between the adjacent two trajectories;
- 4) the length of the arc on the elliptical orbit;
- 5) the sector areas between the two trajectories;
- 6) The velocity of the orbiting object;
- 7) the angular momentum.

3.2.1. Radial Trajectory of the Celestial Body on the Elliptical Orbit

Based on **Figure 11** we can calculate the radial trajectory from a focus of the ellipse as follows:

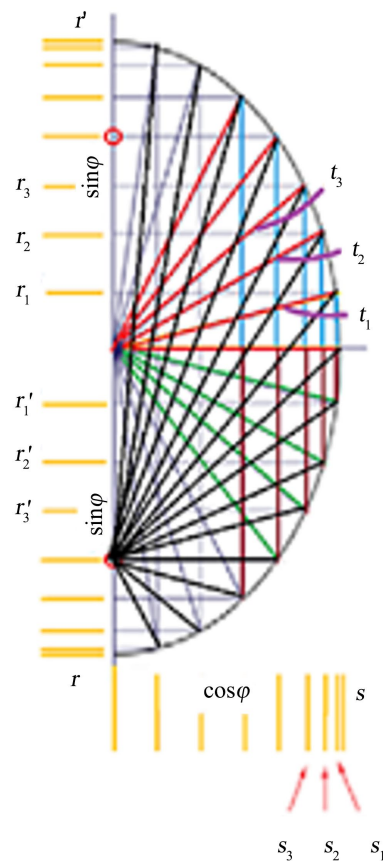


Figure 11. Radial trajectories.

$$\begin{aligned}
t_0^2 &= s_0^2 + (c + r_0^1)^2 = r^2 + (r \tan \beta + r \cdot 0)^2 = r^2 (1 + \tan^2 \beta) \\
t_1^2 &= s_1^2 + (c + r_1')^2 = (r \cdot \cos 1)^2 + \left(r \tan \beta + r \frac{1}{\cos \beta} \cdot \sin 1 \right)^2 \\
t_2^2 &= s_2^2 + (c + r_2')^2 = (r \cdot \cos 2)^2 + \left(r \tan \beta + r \frac{1}{\cos \beta} \cdot \sin 2 \right)^2 \\
t_3^2 &= s_3^2 + (c + r_3')^2 = (r \cdot \cos 3)^2 + \left(r \tan \beta + r \frac{1}{\cos \beta} \cdot \sin 3 \right)^2 \\
&\dots \\
t_\varphi^2 &= s_\varphi^2 + (c + r_\varphi')^2 = (r \cdot \cos \varphi)^2 + \left(r \tan \beta + r \frac{1}{\cos \beta} \cdot \sin \varphi \right)^2 \\
t_\varphi &= r \sqrt{\cos^2 \varphi + \left(\tan \beta + \frac{1}{\cos \beta} \cdot \sin \varphi \right)^2} \quad (21)
\end{aligned}$$

where t_φ is the radial trajectory in φ -th degree, r is the average distance between foci and celestial body or semi-minor axis, which is constant in the given orbit (radius of the cross-section of the cylinder).

It is the formula of the radial trajectory in the ellipse (**Figure 11** and Formula (21)).

Let's r is 10 units, β is 85° and e is 0.996, the ellipse trajectories vary from 0 (perihelion) to ~230 units (aphelion).

When the ellipse's semi-minor axis is 10 units, the distance from the focus to the aphelion is approximately 230 units (229.0), and the perihelion reaches too close (0.43 units) (**Figure 12**).

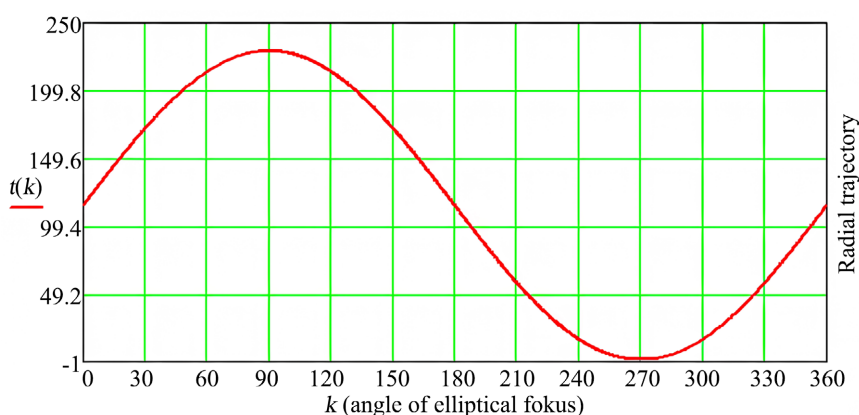


Figure 12. The ellipse trajectories along the orbit (the semi-major axis (a) and eccentricity (e)).

For another example, in the relation of Earth-Sun, e is 0.01671, and r is b and approximately 149.6 million kilometers. If we use Equation (20), and Equation (21), the perihelion is 147,120,725 km (147,100,000 km [41]), and the aphelion is 152,121,055 km (152,100,000 km [41]).

How far can an elliptical orbit last? Tapping of the ellipse is infinite due to description of ellipse based on the circle.

10 units, β is 89° .

3.2.2. Angles of the Trajectory in Relation to the Major Axis of Ellipse

Based on **Figure 11**, we can calculate the angle of the trajectory in relation to the major axis of the ellipse (Equation (22)):

$$\begin{aligned}\tan(\gamma_0) &= \frac{r}{a \cdot e} = \frac{1}{\tan \beta} \\ \tan(\gamma_1) &= \frac{\cos 1}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 1} \\ \tan(\gamma_2) &= \frac{\cos 2}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 2} \\ &\dots \\ \tan(\gamma_\varphi) &= \frac{\cos \varphi}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin \varphi}\end{aligned}\quad (22)$$

3.2.3. Angle between the Adjacent Two Trajectories

$$\begin{aligned}\Delta\gamma_{1,0} &= \gamma_1 - \gamma_0 \\ &= \tan^{-1}\left(\frac{\cos 1}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 1}\right) - \tan^{-1}\left(\frac{\cos 0}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 0}\right) \\ \Delta\gamma_{2,1} &= \tan^{-1}\left(\frac{\cos 2}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 2}\right) - \tan^{-1}\left(\frac{\cos 1}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin 1}\right) \\ &\dots \\ \Delta\gamma_{\varphi,\varphi-1} &= \tan^{-1}\left(\frac{\cos \varphi}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin \varphi}\right) - \tan^{-1}\left(\frac{\cos(\varphi-1)}{\tan \beta + \frac{1}{\cos \beta} \cdot \sin(\varphi-1)}\right)\end{aligned}\quad (23)$$

The most important parameter in celestial mechanics is the angle ($\Delta\gamma_{\varphi,\varphi-1}$) of the two adjacent trajectories (Equation (23)).

3.2.4. The Length of the Ellipse Orbit's Arc

Based on Formula (21) of the radial trajectory the length of the arc ($z_{k,k-1}$) is determined by the cosine theorem (Formula (24)):

$$z_{\varphi-1,\varphi} = t_\varphi^2 + (t_{\varphi-1})^2 - 2t_\varphi \cdot t_{\varphi-1} \cdot \cos(\gamma) \quad (24)$$

where γ is the angle between two adjacent trajectories ($t_{\varphi-1}$ and t_φ) described in Equation (21).

If the step of the angle (γ) is too large, the length of the ellipse orbit's arc becomes incorrect. For this reason, the length of the arc ($z_{\varphi-1,\varphi}$) must be calculated from each angle to the next angle.

3.2.5. The Sector Areas between Two Radial Trajectories

Area (A) of the triangle with 3 sides ($t_{\varphi-1}$ and t_{φ} and $z_{\varphi-1,\varphi}$) is calculated by the next Formula (25).

$$A = \sqrt{s(s-t_{\varphi})(s-t_{\varphi-1})(s-z_{\varphi-1,\varphi})} \quad (25)$$

where s is the semi-perimeter of the triangle given by

$$s = (t_{\varphi} + t_{\varphi-1} + z_{\varphi-1,\varphi})/2$$

Kepler's second law of planetary motion describes the speed of a planet traveling in an elliptical orbit around the Sun. It states that a line between the Sun and the planet sweeps equal areas at equal times. Thus, the speed of the planet increases as it nears the Sun and decreases as it recedes from the Sun. [42]

3.2.6. The Angular Momentum

The velocity (V) of the celestial body in the orbit is changed degree by degree. For example, the velocity of the first angle (Equation (26)):

$$V_{0-1} \cdot T = t_0^2 + t_1^2 - 2t_0 \cdot t_1 \cdot \cos(1) \quad (26)$$

Since velocities are determined by the sum of arcs according to each degree, the consequence of this requires difficult mathematical calculation. For example, we see it from the next series from 0 to 5 degrees (Equation (27)):

$$\begin{aligned} V_{0-5} &= z_{k,k-1}/T \\ &= \frac{t_0^2 + t_1^2 - 2t_0 \cdot t_1 \cdot \cos(1)}{T_1} + \frac{t_1^2 + t_2^2 - 2t_1 \cdot t_2 \cdot \cos(1)}{T_2} \\ &\quad + \frac{t_2^2 + t_3^2 - 2t_2 \cdot t_3 \cdot \cos 1}{T_3} + \frac{t_3^2 + t_4^2 - 2t_3 \cdot t_4 \cdot \cos 1}{T_4} \\ &\quad + \frac{t_4^2 + t_5^2 - 2t_4 \cdot t_5 \cdot \cos 1}{T_5} \\ V_{i,k} &= \frac{t_i^2 + t_{i+1}^2 - 2t_i \cdot t_{i+1} \cdot \cos(1)}{T_{i+1}} + I + \frac{t_{k-1}^2 + t_k^2 - 2t_{k-1} \cdot t_k \cdot \cos 1}{T_k} \end{aligned} \quad (27)$$

I left it to the researchers and others because the calculation is too long.

If the mass of the orbiting celestial body is m the angular momentum (Equation (29)) is equal to:

$$mV_{i,k} = m \left\{ \frac{t_i^2 + t_{i+1}^2 - 2t_i \cdot t_{i+1} \cdot \cos(1)}{T_{i+1}} + I + \frac{t_{k-1}^2 + t_k^2 - 2t_{k-1} \cdot t_k \cdot \cos 1}{T_k} \right\} \quad (28)$$

$$L = I\omega = mV_{i,k}R \quad (29)$$

3.3. Orbital Eccentricity in the Universe

The influences of eccentricities are estimated based on Formula (17).

$$c = r \cdot \tan \beta$$

β changes only in the interval between 0° and 90° .

As the eccentricity increases, the c -parameter increases, so the celestial bodies can approach the perihelion of the elliptical orbit (**Figure 13**). This curve shows firstly the Stellar Populations. Low-eccentricity Milky Way, called Population I stars. In contrast, high and most highly eccentric stars are located in the halo (extreme) and nuclear bulge (intermediate) regions, called Population II stars [43]-[45].



Figure 13. The disc of the Milky Way [46].

How far can travel the celestial body in elliptical orbit last if no any external influences?

Suppose the minor axis of the ellipse is 10 unit of semi-minor axis.

In this case, we can calculate the distances of aphelion from focus for different eccentricities (**Table 1**).

Table 1. The radial trajectory of a celestial body from the focus for different eccentricities.

φ , degrees of circle	$\beta = 85^\circ$	$\beta = 89^\circ$	$\beta = 89.99^\circ$
0, Spring Equinox	114.7	572.98	57295.77
90, Aphelion	229.0	1145.88	114591.56
180, Autumn Equinox	114.7	572.98	57295.77
270, Perihelion	0.43	0.087	0.0008726

Table 1 shows that the stability of the ellipse is valid for any eccentricities. Even when the eccentricity is equal to 90, the elliptical structure is still valid. In other words, if two objects are entangled, the traveling object will return and the law of recurrence will apply.

The disk of our galaxy is very thin, about 100 times wider than its height (**Figure 13**). It contains almost all the gas and dust in our galaxy, as well as all the hot young stars and regions of star formation [46].

Population I stars are younger stars found in the disk of the galaxy that contain lots of atoms heavier than helium (metals), while population II Stars are older, metal-poor stars found in a galaxy's nuclear bulge, halo, and globular clusters [45].

The members of these stellar populations differ from each other in various ways, most notably in age, chemical composition, and location within galactic sys-

tems [43]-[46].

Since eccentricity affects stellar astronomy and orbital mechanics, it is more convenient to use the ratio $1/\cos \beta$ instead of eccentricity for precise calculations.

For example, the eccentricity of the hysteresis equals 0 ($e = 0$) $e = 0.99$, and $e = 0.9999$.

$$f_0(x) = \frac{1}{\sqrt{1-0^2}} \cdot \frac{\sin x}{|\cos x|}$$

$$f_{0.99}(x) = \frac{1}{\sqrt{1-0.99^2}} \cdot \frac{\sin x}{|\cos x|}$$

$$f_{0.9999}(x) = \frac{1}{\sqrt{1-0.9999^2}} \cdot \frac{\sin x}{|\cos x|}$$

Suppose you ask what forces cause stars and astronomical objects to clump together and twist into the disk-like spiral structure of the galaxy. I'm also referring to the effect of the electromagnetic field that causes the hysteresis force. Therefore, the study of electromagnetic fields is very important (Figure 14).

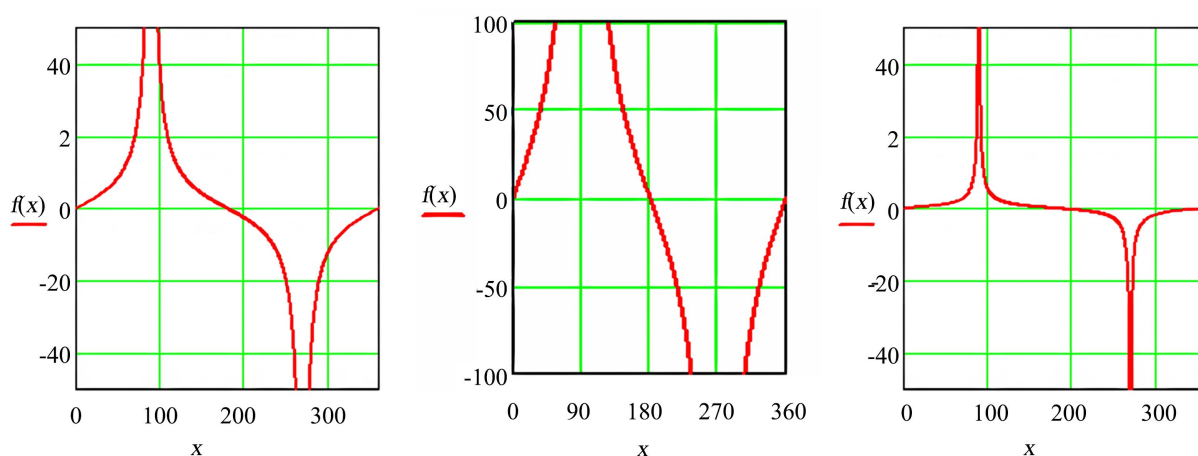


Figure 14. The influences of the eccentricity on the hysteresis.

Looking back, do the parabolic and hyperbolic structures stand out in the orbit formulation? Unfortunately, there is no word that it is.

Most comets orbit the Sun in elongated elliptical orbits. They are classified into two groups according to their orbits: short and long frequency. Some examples are shown in Table 2.

Table 2. Some data on comets [46].

	Short periodicity	Long periodicity
Period	Average 7 years	Average 10^6 years
Eccentricity	0.2 - 0.9	0.99999
Orbital plain	Approximately near the symmetric plane	Random
Aphelion		>100,000 AU

Some comets wander as far as 2.4 light-years from the Sun.

Now, an ancient, big interesting problem of astronomy remains unsolved. It is a question of what kind of force can organize all motions in the Universe.

Here, we can explain a lot of the rotation curve of a disc galaxy [47], Modified Newtonian Dynamics (MOND) [48], missing baryon, and Lambda-CDM. MOND's primary motivation is to explain galaxy rotation curves without invoking dark matter and is one of the most well-known theories of this class. However, it has not gained widespread acceptance, with the majority of astrophysicists supporting the Lambda-CDM model as providing a better fit for observations [47]-[51].

Several independent observations suggest that the visible mass in galaxies and galaxy clusters is insufficient to account for their dynamics when analyzed using Newton's laws. This discrepancy—known as the “missing mass problem”—was first identified for clusters by Swiss astronomer Fritz Zwicky in 1933 (who studied the Coma cluster), [52] [53] and subsequently extended to include spiral galaxies by the 1939 work of Horace Babcock on Andromeda [51].

The universe favors going from an ellipse to a circle because it prefers to expend the least amount of energy. In other words, it strives for less eccentricity.

3.4. Orbital Energy

According to the orbital energy conservation equation (also referred to as the *vis-viva* equation), it does not vary with time [54]:

$$\epsilon = \epsilon_k + \epsilon_p = \frac{v^2}{2} - \frac{\mu}{r} = \frac{1}{2} \frac{\mu^2}{h^2} (1 - e^2) = -\frac{\mu}{2a} \quad (30)$$

where

- v is the relative orbital speed;
- r is the orbital distance between the bodies;
- $\mu = G(m_1 + m_2)$ is the sum of the standard gravitational parameters of the bodies;
- h is the specific relative angular momentum in the sense of relative angular momentum divided by the reduced mass;
- e is the orbital eccentricity;
- a is the semi-major axis.

The orbital motion is very easy to see, as the orbital motion of an electron will produce a magnetic field because the orbit of an electron around the nucleus serves as a closed current loop, in which the current doesn't vanish.

3.4.1. Energy of Orbit in Classical Physics

Let's think a bit about the total energy of orbiting objects. Suppose an object with mass (m) is doing a circular orbit around a much heavier object with mass M . Now we know its potential energy. It's:

$$U = -\frac{GMm}{R}$$

How about it is kinetic energy? The fact that $v = \omega R$ have

$$v^2 = \omega^2 R^2 = G \cdot \frac{M}{R}$$

so that

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{GMm}{R}$$

Notice that $K = -U/2$ and that

$$E = K + U = U/2 = -\frac{GMm}{2R} \quad (31)$$

So, the total energy is always negative (Formula (31)). In the same way that electrons in an atom are bound to their nucleus, we can say that a planet is bound to the sun. Its energy is negative, so it doesn't have enough power to escape to infinity. But what if the energy were positive? In that case, the trajectories are no longer elliptical; instead, you get hyperbolic orbits!

The object comes in from interstellar space, almost going in a straight line, then cruises around the sun and is finally deflected in a straight line off into never-never land, never to be seen by us again!

Orbits are conic sections with the force center at the focus. Kepler's Laws of orbital mechanics were published in 1618 by Johannes Kepler, who deduced them from reams of astronomical data. Kepler's Laws follow mathematically from Newton's Laws of motion and his formula for the gravitational force [55] [56].

3.4.2. Quantum Mechanical Description of the Elliptical Orbit Energy

Quantum mechanics asserts that the total energy flow (E_n) is linear and probabilistic. Harmonic oscillator energy for stable orbital conditions.

$$E_n = n\hbar\omega \quad (32)$$

It was first deduced from Planck's hypothesis.

Let us denote the coordinates of the oscillator by q and the momentum by p . Then the energy of the oscillator is [57].

$$E_n = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} = n\hbar\omega \quad (33)$$

So, that becomes the formula of an ellipse

$$\frac{q^2}{2qn\hbar/m\omega} + \frac{p^2}{2mn \cdot \hbar\omega} = 1 \quad (34)$$

The coordinate plane (p, q) is called the phase plane. The curve representing the relationship between p and q on this plane is called a phase trajectory. From Formula (33), it is obvious that the phase trajectory of the harmonic oscillator is an ellipse [57]. In other words, the shape of the total energy is an ellipse.

From Formula (1), the semi-axes of the ellipse:

$$a = \sqrt{2qn\hbar/m\omega}, \quad b = \sqrt{2mn\hbar\omega}$$

Quantum mechanics proves that the quantization of elliptical orbits is cyclic (Formula (33) and **Figure 15**).

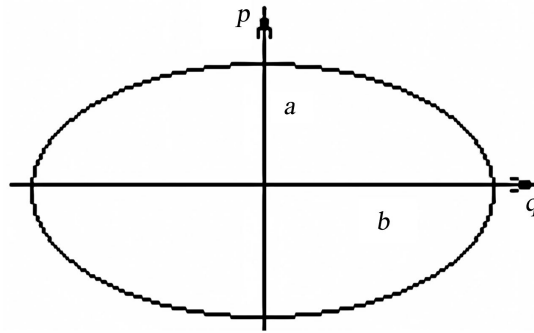


Figure 15. The ellipse-shaped total energy.

3.4.3. Hysteresis Description of the Elliptical Orbit Energy

The hysteretic description of the elliptical orbit energy is based on the Formula of the ellipse in the cylinder.

Suppose that the ellipse's semi-major axis (a) represents kinetic energy and the semi-minor axis represents the potential energy. In that case, the total energy is described in Formula (34):

$$(KE)^2 + (PE)^2 = (E_{total})^2$$

If the shape is elliptical it is written in the next form:

$$\left(\frac{1}{\sqrt{1-e^2}} \cdot \frac{KE}{E_{total}} \right)^2 + \left(\frac{PE}{E_{total}} \right)^2 = 1 \quad (35)$$

It is formula of the elliptical orbit energy as same Equation 34. It means the shape of Universe is elliptical in aspect of energy.

Considering **Figure 3** and Equation (9), the two kinds of total energy are described by the following form:

$$E_{total} = \sqrt{(KE)^2 + (PE)^2} = \sqrt{\left(\frac{1}{\sqrt{1-(\cos \beta)^2}} \cdot \sin(\varphi) \right)^2 + (\cos(\varphi))^2} \quad (36)$$

If $\varphi = 0^\circ$, then total energy equals stable potential energy and no kinetic energy. At this point, the body rotates smoothly without expending energy. However, when $\varphi = 90^\circ$, the total energy is the same as the kinetic energy, it is maximum at perihelion and minimum at aphelion. At this moment, the maximum energy is required to move in orbit. The potential energy is zero.

In summary, the total energy of the system is determined in 3 ways. It includes:

1) Classic physics formula

$$E = K + U = -\frac{GMm}{2R}.$$

2) Quantum Mechanical Description

$$\frac{q^2}{2qn\hbar/m\omega} + \frac{p^2}{2mn \cdot \hbar\omega} = 1$$

3) Hysteretic Description

$$(E_{total})^2 = \left(\frac{1}{\sqrt{1 - (\cos \beta)^2}} \cdot \sin \varphi \right)^2 + \cos^2 \varphi$$

$$E_{total} = \sqrt{(KE)^2 + (PE)^2} = \sqrt{\left(\frac{1}{\sqrt{1 - (\cos \beta)^2}} \cdot \sin \varphi \right)^2 + \cos^2 \varphi}$$

In conclusion, the three expressions above of total energy are described by an ellipse. The semi-major axis (a) indicates kinetic energy, whereas the semi-minor axis (b) is equal to potential energy.

Thus, the structure of the Universe, from an energy perspective, is likely to be an ellipsoid of rotation.

4. Conclusion

The conventional formulas of the circles and ellipses generated from the Keplerian conic section are strict mathematical expressions, but some fail in practice. They can only show their locations in the coordinate systems and some parameters of ellipses or circles. So, they became more difficult to use. We have needed a living, spiritual formula that represents the values in between, not just the binary numbers 0 and 1 (or ON and OFF). This is the hysteresis formula. We have created the wave function formula that simultaneously describes eccentricity, amplitude, phase shift, angular momentum, polarization, radial trajectory, and orbital energy in two-body orbital mechanics. The results of the formula align closely with observational data. We use a hysteresis model that can be solved exactly with mathematical methods, so neither approximations, renormalizations, nor computer simulations are required. It means that we may use the cylindrical section instead of Kepler's conic section.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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