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Mathematics' Limitation Modelling Universal Structures: The "Not" Function—Paradox's Mechanism in Linguistics, Mathematics, and Physics

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Abstract

The 1-*D* geometric model studies the structure of states universally closed to the discrete delineation of their properties and defined as infinities. The closure mechanism is the logical "not" function attached to the named property, creating a paradoxical relationship between segments. Two correlated fundamental reference frames are identified. In the first framework, the paradox mechanism prohibits the discrete enumeration of the state's internal structure. In the second, segments share property for the same infinity but are excluded from common membership due to their paradoxical relationship across the boundary that divides them. The geometric model analyzes the role of the "not" function in linguistics, mathematics, physics, and the generic structure of dimensional development across the quantum to classical platforms. Logical formalisms necessarily discount paradoxes as anomalies open to more advanced understanding, worked around by restrictions to logic or ignored as nonsensical. The 1-*D* geometric model takes an opposing analytical perspective, considering paradox a fundamental mechanism. The geometric proof examines two constructions of the right triangle within the unit circle. Although the two formats are paradoxical, with the second having no rational basis, the cosine squared calculations agree. Two paradoxical frameworks cohabit within a universal state defined by the cosine squared function. The 1-*D* model identifies the power function's systemic limit in modelling universal states that inherently contain the paradox mechanism in their segment relationship.

Keywords

Paradox, Universality, Bell's Inequality, Russell's Paradox, Cantor's Diagonal

Slash Argument, Gödel's Incompleteness Theorem, Infinity

1. Introduction

Examples of universal structures with paradoxical frameworks exist in logic, mathematics, and physics. The key distinction between paradox's mathematical and linguistic structure lies in the difference between extension and circularity in their respective properties.

Mathematical symbology possesses the property of infinite extension (e.g., the natural numbers). Cantor's diagonal slash argument exemplifies the paradoxical incompleteness that such extension introduces, and Gödel's incompleteness theorems demonstrate the equivalent limitation encountered when attempting to formulate absolute truth statements.

Linguistic structures exhibit an opposing characteristic of incompleteness, where the symbology lacks a basis for linear extension (e.g., dog, cat). Linguistic arguments with absolute self-reference result in nonresolvable infinite regressions. The Russell and Liar paradoxes serve as two of many such examples.

1.1. Paradoxical Structures in Mathematics

Cantor's diagonal slash argument: Cantor's argument exposes the paradox in mathematical logic when defining infinity as a unitary state [1]. The argument first constructs a partial random listing of the real numbers between zero and one in a vertical column. Then the number that appears on the diagonal of that listing is reconstructed. by a fixed operation (e.g. add 1 to each digit after the decimal). Paradoxically, the new number is not included in the vertical column, even though it is correctly constructed as infinite.

Gödel's incompleteness theorems: "Among the things that Gödel indisputably established was that no system of sound mathematical rules of proof can ever suffice, even in principle to establish all the true propositions of ordinary arithmetic [2] (pp. 64-65)."

In other words, for any inclusive set of all possible true propositions, a second set must exist that is paradoxically conjoined and cannot be neatly confined to form a single basis of all true propositions.

1.2. The Complementarity of Linguistic Paradox

Linguistic statements constructed to universally contain all logical reference internally to an argument's property devolve into self-circular infinite regressions, prohibiting conclusion. The logical "not" attached to the state's property prevents its identification as a discrete entity. Whatever property is named, the segments sharing membership do "not" contain that property [3].

Russell's paradox [4]: Russell's paradox is the prime example of applying the logical "not" to a membership property, resulting in self-circular infinite regression. The Russell set (R) is the set of all sets that are not members of each other.

It is not possible to determine whether the "set of all the sets" (R) shares membership in its collection of segments. If (R) is placed within itself, it represents an error since it should "not" have a common property with its members. However, if it is not placed within itself, it constitutes an error because it shares the property designation with its members.

The only resolution for the logical paradox created in arguments such as Russell's paradox is to apply set theory's Zermelo-Fraenkel rule, which includes the axiom of choice (ZFC) and prohibits the existence of universal sets [5]. This rule restores consistency to the logic of set theory but does not address the underlying paradoxical issue that infinity introduces; instead, it merely avoids it.

The liar paradox [6]: "I am telling a lie." The argument is the most concise format of a universal linguistic statement, devolving into entangled self-circularity. The logical "not" function is implicit because the statement and its intention are "not" consistent with truth. The statement is "not" true if it intends to be false, and it is "not" false if it intends to be true. The speaker's self-reference creates an entanglement between two conclusions comparable to the entanglement structure in quantum states (discussed in Section 4). If the statement's structure were (he is telling a lie), the argument would have a discrete classical format between the speaker and the subject.

2. The 1-*D* Geometric Model

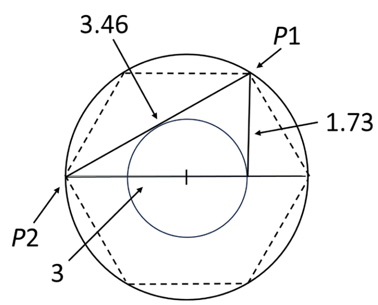

Figure 1. Formal geometry.

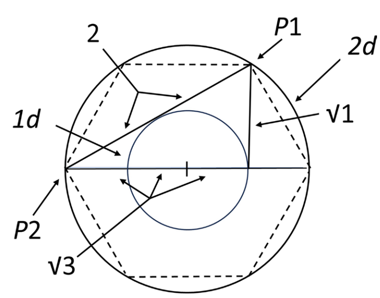

Figure 2. 1-*D* geometric model.

In **Figure 1**, the diameter of the outer circumference is assigned a value of 4, while the relevant portion for calculating the cosine squared value is 3. The sides of the

30-60-90 triangle measure 3, 1.732, and 3.464 [7]. In **Figure 2**, each vector segment represents a unitary state with an entangled identity/value 1. Dimensional complexity transitions from the inner circumference's one-dimensional boundary to the outer circumference's two-dimensional boundary.

Vectors that converge eccentrically relative to the inner circumference carry the square root. The hypotenuse consists of two vectors that start and end concentrically on the outer two-dimensional circumference, and together, the square roots cancel each other out.

3. Calculating the Cosine Squared Function in the Two Geometries

The 1-D geometry calculates the formal cosine squared value based on **Figure 1** and the nonformal (rationally nonsensical) value based on **Figure 2**. In **Figure 2**, the inner, one-dimensional circumference (labelled 1d) is preemergent to the outer circumference's two-dimensional level (labelled 2d).

Because the 1-D geometry's dimensional structure develops outward from the inner to the outer circumference, the formal calculation rules of the Cartesian plane's fixed dimensional framework do not apply. Each vector represents a contained infinity at its boundaries.

3.1. Formal Calculations in Figure 1

$$P1 - \text{Cos}^2\,(60) = (1.73205/3.4641)^2 = 0.25 \tag{1}$$

$$P2 - \text{Cos}^2\,(30) = (3/3.4641)^2 = 0.75 \tag{2}$$

3.2. Nonformal Calculations in Figure 2

$$P1 - \text{Cos}^2\,(60) = (\surd 1/2)^2 = 0.25 \tag{3}$$

$$P2 - \text{Cos}^2\,(30) = (\surd 3/2)^2 = 0.75 \tag{4}$$

3.3. Interpreting the Results of 3.1 and 3.2

The 1-D geometric model is constructed on the two-dimensional flat plane of classical space. In **Figure 1**, the linear sides of the right triangle have consistent and discrete values for calculating the cosine square. In contrast, **Figure 2** shows the model's dimensional structure as outwardly developmental across the two circumferences. Consequently, the geometry is inconsistent with the fixed dimensional basis of the Cartesian plane.

The proof of the equivalence between **Figure 1** and **Figure 2**, albeit in a paradoxical relationship, is the calculation of the cosine squared value for both figures.

A parent/sibling analogy is employed. Two paradoxical "sibling" geometries are conjoined in a "not" structure relationship by the "parent" cosine squared values.

The counter-rational relationship of the geometries is comparable to that created by the "not" function in the mathematical and linguistic examples above.

4. The Logical "Not" Across Its Dimensional Platforms

In the (x, y) coordinate structure of the two-dimensional classical plane (**Figure**

1), each vector contributes one dimension to the space, and the classical state extends in the "eye of the observer". The operations of formal mathematics specifically require that the two dimensions of the structure have a fixed common basis rather than emerging as in **Figure 2**.

The half-silvered mirror experiment provides important context for the difference between a one-dimensional space, wherein the y-axis is imaginary (with the symbol iy), and a two-dimensional classical space, in which the higher complexity of the classical plane allows the y-axis to be incorporated as a "real" dimension [2] (pp. 259-263).

<u>The half-silvered mirror experiment</u>: In the classical framework of the experiment, when any measurement occurs, the photon is always found to occupy one of the paths within the structure. Upon exiting the apparatus, it is similarly obeys classical probability that it will occupy either the x-axis or the y-axis.

However, if no measurement occurs within the apparatus, the paths interfere at a quantum level. The crucial distinction for the classical description is that the photon exits the apparatus only in the original direction of projection upon entry, and never in the orthogonal direction. The quantum evolution of the state across the apparatus is easily calculated and is categorically paradoxical from a classical perspective [2] (p. 262).

In the quantum basis, the orthogonal structure of the classical state from the first mirror down-converts to (x, iy), where (i) represents the imaginary square root of minus one. The two-path structure in the apparatus is simultaneously quantum entangled, causing the axes to lose their discrete basis of time-sequenced separation.

Of the two orthogonal vectors, x and iy (as siblings), only the x-axis can be considered real in contributing "dimension" to the space, and the x- and iy-axes share a sibling "not" relationship. They are "not" discrete members of each other on the plane they share.

In reversing the quantum state to its classical format (by collapsing the wavefunction), the path entanglement mechanism of the "not" function (at the one-dimensional level) transforms to accommodate the addition of the classical plane's second level of dimension. In its new format, the photon displays occupation on one of the exit paths and "not" the other.

The classical plane "fractures" the entanglement property of the quantum state, allowing sufficient dimensional complexity so that both coordinate vectors, the x and y axes, are separately "real" and "discrete." In other words, the dimensional level at which the photon displays occupation determines how the "not" function exhibits its dualistic property—quantum or classical.

5. Interpreting the Cosine Squared Function across the Dimensional Structures of Figure 1 and Figure 2

Although the geometric and mathematical basis of **Figure 2** is formally nonsensical, the cosine squared calculations yield the same results in Sections 3.1 and 3.2.

This can be understood by examining the relative dimensional framework for each figure.

In the formal basis of **Figure 1**, the linear ratio of the sides forming the cosine is squared, and the value takes on the property of an "area".

Figure 2, on the other hand, presents an inconsistent basis for linear structure since "linearity" and "object" identity are entangled as a unitary state. The rotationally enclosed circle is the simplest format of a state containing a region absolutely (infinitely) closed for its membership property.

In a nonclassical format, the vectors share a common property with the unit circle as "area-like" states. The boundaries of each vector form absolute limits to extension (as infinities). Accordingly, the square root function is applied to down-convert each vector's "area-like" basis to "linear-like". The ratio of the terms is then squared.

6. The Dimensional Platform of the Classical Observer

A significant ongoing question in the foundations of quantum structure is the role of the "observer" at the classical level in the collapse of the wavefunction. The term has been applied in various ways, leading to confusion regarding its intended context. In the 1-D geometric model, the generalized term "observer" refers to the collapse of the quantum state through detection by an apparatus and represents a two-part procedure.

First, a down-conversion apparatus opens the lower-dimensional platform of a quantum structure. Then, through a measurement disturbance at the classical level, the state is instantaneously reinterpreted in its higher-dimensional classical format.

The difference in dimensional complexity between correlated quantum and classical frameworks (lower and higher, respectively) determines the basis for the "not" function operation. At both levels, the paradox mechanism plays a role, not as an anomaly but as the foundation of the segment relationship.

7. Bell's Inequality

The experimental confirmation of Bell's inequality demonstrates that classical particles separated in space and time obey quantum, not classical probability rules [8] (pp. 211-227). The unavoidable conclusion is that the experiment contradicts the absolute limit of communication at the speed of light, as posited by relativity theory. However, it does not directly invalidate the classical basis.

"As in the case of the EPR paradox, it's important to realize what Bell did not do. He did not discover an experimental situation in which non-local interactions are directly observed. Instead, he invented a simple argument based on experimental results that indirectly demonstrated the necessary existence of non-local connections [8] (p. 220)."

The 1-D geometric model's basis applies to Bell's inequality, wherein classical and quantum states coexist simultaneously in a "not" function relationship. Their structures are not members of each other across the boundary between the two inconsistent dimensional platforms.

8. Conclusions

Formal logic and its applications in physics rely on the principle that rationalism constitutes the intrinsic foundation of Nature's structure. It includes the implicit assumption that paradoxes are anomalies. The geometric model presents the opposing perspective that paradoxes are mechanisms rooted in a more fundamental foundation than can be defined within a consistent system of propositions.

The geometric model demonstrates that the two paradoxically conjoined geometries, with categorically inconsistent formats, are linked within a common foundation by the cosine squared calculations. This is the same framework identified in Sections 1.1 and 1.2 for the paradox mechanism.

Cantor's diagonal slash argument demonstrates that the infinity of the real numbers between zero and one cannot be contained as a unitary state. Gödel's incompleteness theorems illustrate the same principle for the rules that would apply to such a collection.

Cantor's and Gödel's arguments point to the existence of a boundary that prohibits the internal absolute completeness for the membership property of unitary states. The 1-D model completes the argument by first illustrating the mechanism that links members as inconsistent for their common property, and second, identifying the unitary state in which the segment structures cohabit. The mechanism of incompleteness is the paradox mechanism of the "not" function.

8.1. Paradox's Expression in Static and Emerging Frameworks

The geometric model defines paradox's role in two frameworks, static and emerging:

Static: The inconsistent relationship between vector segments in **Figure 2** (each representing a self-contained infinite state) is the paradox mechanism's static format.

Emerging: The structural complexity of the inner and outer circumference is dimensionally emergent. Vectors pointing eccentrically to the inner circumference necessitate taking the square root, which indicates that the region of the inner circumference is one-dimensional. In contrast, the hypotenuse connected across the outer circumference does not require the square root function, signifying that the outer circumference has a two-dimensional, classical basis.

Although the 1-D geometry has a limit based on the two-dimensional flat plane, the further development of dimensional complexity in a higher-dimensional model is theoretically not restricted, all within a boundary defined as infinity.

8.2. Detecting Quantum States from the Classical Platform

Formal mathematics' inherent and unavoidable bias is that it is restricted to interpreting dimensional structure on a consistent basis, applying the power function.

However, raising a quantum state to the classical platform by a squaring operation conceals its native format, having only one dimension.

The quantum state examined in Section 4 for the half-silvered mirror experiment is represented on the two-dimensional classical plane, despite the vertical orthogonal axis (iy) being imaginary. In other words, on a classical basis, the vertical axis does not exist, and the representation in two dimensions, although necessary from a geometric perspective, is misleading for the native structure of the quantum state.

A deductive mathematical proof for the role of paradox is not possible because paradox is viewed as an anomaly in formal logic from the outset. Instead, an inductive argument is needed to reveal the relationship structure obscured from direct formal interpretation.

The only exception to the prohibition on measuring a quantum state in its native dimensional framework is the "weak measurement" technique [9]. When a quantum state is only "weakly" disturbed, data collected from a large sampling can replicate the native quantum basis of phenomena without causing a complete collapse of the state.

Nevertheless, the quantum-level statistical results from these experiments do not conform to classical probability rules. The negative probability values produced in the experiments hold no meaning in classical terms.

Note: This paper further discusses the 1-D geometric model introduced in previous papers in the series [10]-[13].

Conflicts of Interest

The author declares no conflicts of interest in the publication of this paper.

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