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Dynamic Event-Triggered and Impulsive Control for Nonlinear Impulsive Systems

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Abstract

In this paper, the problem of dynamic event-triggered impulsive control for a class of nonlinear systems is studied. Based on the input-to-state stability results of nonlinear systems, a dynamic event-triggered control strategy is designed to stabilize the nonlinear system, and the lower bound of triggering is set. By using Lyapunov method, a sufficient condition for the stabilization of nonlinear systems is obtained. The conservation of the original theorem is maintained while the number of transmissions is greatly reduced. Numerical simulations show the effectiveness of the theoretical results.

Keywords

Dynamic Event-Triggered Impulsive Control, Nonlinear Impulsive System, Impulsive Control

1. Introduction

As a new control method, the idea of event-triggered control is to design the trigger time and trigger conditions according to the characteristics and purpose of the system, and establish the event-triggered mechanism. When each trigger is triggered, the controller is activated to perform the task. The difficulty lies in the balance between the design of the trigger condition and the control task. The event-triggered mechanism makes it easier to find control time and save energy than time-triggered control. The event-triggered control mechanism not only optimizes the resource utilization system, but also saves signal transmission resources and data computing resources. It is an efficient control method. At present, event-triggered control has been widely concerned and applied to various systems, such as general nonlinear systems [1] [2], networked control systems [3] [4], multi-agent systems [5]-[7] and so on. Dynamic event-triggered control can save more energy. Therefore, it is of great significance to study the event-triggered control method.

Impulsive system is a special hybrid system, including a given impulsive criterion [8] and an ordinary differential equation. It has a wide range of applications in many fields, such as robot design engineering, ecological system engineering, network communication engineering, aerospace engineering [9]-[19]. Generally speaking, pulse effect includes pulse control and pulse interference. Impulsive disturbance usually contains unstable pulses, which tests the robustness of the system; Impulsive control usually includes stable impulses, and the stabilization of the system is considered. Pulse control is a discontinuous control, which can not only improve the confidentiality, save the control cost, but also enhance the robustness of the system. Impulse control is involved in many fields, such as money supply control in aviation financial market, celestial orbital adjustment, communication security and chaos synchronization. In recent years, scholars from many different fields have devoted themselves to using impulse control when studying the stability of the system.

The combination of dynamic event triggering control [20] and pulse control is dynamic event triggering pulse control. rzén proposed the concept of event triggered control [21]. Professor Tabuada proposed the event triggering strategy for nonlinear control systems. Professor A. Girard proposed a dynamic event trigger mechanism. Professor Li proposed event triggered pulse control [22]. These documents ensure the feasibility of this study.

In this paper, the dynamic event triggered control of nonlinear systems is studied. The control mechanism of dynamic event trigger pulse and dynamic variable parameters are designed. Considering the problem of frequent trigger, the lower bound of trigger time is set. The final data simulation shows that the transmission times of dynamic event triggering mechanism is much lower than that of static event triggering mechanism.

Notations. Let $\mathbb{N}$ be a positive integer set, $\mathbb{R}$ be the set of real numbers, $\mathbb{R}^+$ be the set of nonnegative real numbers, and $\mathbb{R}$ be the n-demensional real space equipped with the Euclidean norm denoted by $\|\bullet\|$. For a locally Lipschitz continuous function $V:\mathbb{R}^n \to \mathbb{R}^+$, D^+V denotes the upper right-hand Dini derivative. $A>0$ denotes that A is a symmetric and positive definite matrix. $A<0$ ($A\le 0$) denotes that A is a symmetric and negative definite (semi-definite) matrix. I is the identity matrix with appropriate dimensions.

2. Preliminaries

Consider the following control system with time delay:

$$\begin{cases} \dot{x}(t) = f(t,x) + Bu(t), \ t \ge t_0, \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state; $B\in\mathbb{R}^{n\times m}$ is the control gain; both $f:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^n$ and the control input $u:[t_0,\infty)\to\mathbb{R}^m$ satisfy

$f(t,0) = u(0) = 0$ so that the system (1) admits the trivial solution. In this study, we consider the hybrid impulsive control

$$u(t) = u_1(t) + u_2(t),$$

with the state feedback control.

$$u_1 = kx(t), \tag{2}$$

where $k : \mathbb{R}^n \to \mathbb{R}^m$ is the feedback control law, and the impulsive control

$$u_2(t) = \sum_{i=1}^{\infty} g(x(t))\delta(t - s_i), \tag{3}$$

where $g : \mathbb{R}^n \to \mathbb{R}^m$ is the impulsive control law, $\delta(\cdot)$ denotes Delta dirac function, and the sequence of imimpulsive times $\{s_i\}_{i\in\mathbb{N}}$ satisfies $0 \le t_0 < s_i$, with $s_i < s_j$ for $i < j$, and $\lim_{i\to\infty} s_i = \infty$. Hence, the closed-loop system (1) replaced by (2) and (3) can be rewritten as an impulsive system

$$\begin{cases} \dot{x}(t) = f(t,x) + Bk(x), \ t \ne s_i, \\ \Delta x(s_i) = Bg\left(x\left(s_i^-\right)\right), \ i \in \mathbb{N} \\ x(t_0) = x_0. \end{cases} \tag{4}$$

Definition 1. We refer the reader to [23] for a detailed discussion on transformation of control system (1) into impulsive system (4).

System is said to be input-to-state stable (ISS) with respect to input u, if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that, for each initial condition $x_0 \in \mathbb{R}^n$ and input function $u \in \mathcal{PC}\left([t_0,\infty), \mathbb{R}^m\right)$, the corresponding solution to (1) exists globally and satisfies

$$\|x(t)\| \le \beta(x_0, t - t_0) + \gamma\left(\sup_{s\in[t_0,t]} \|u(s)\|\right),$$

for all $t > t_0$.

The relevant theorems can be found in the literature.

Theorem 1. *Assume that there exist functions* $V \in v_0$ *and* $\alpha_1, \alpha_2, \chi \in \mathcal{K}_\infty$, *and constants* $l > 0$, *such that, for all* $t \in \mathbb{R}^+, x \in \mathbb{R}^n$ *and* $\varphi_0 \in \mathbb{R}^n$,

(1) $\alpha_1(\|x\|) \le V(t,x) \le \alpha_2(\|x\|)$,

(2) $D^+V(t,x) \le -lV(t,x) + \chi(\|u\|)$.

Then system (1) is ISS.

To design the event-triggered implementation of u_1, we consider system with $g = 0$ and state-feedback control as follows:

$$\begin{cases} \dot{x}(t) = f(t,x) + Bu_1(t), \\ u_1(t) = k(x(t_i)), \ t \in [t_i, t_{i+1}) \\ x(t_0) = \varphi_0. \end{cases} \tag{5}$$

Let us define the state measurement error by

$$\mathfrak{e}(t) = x(t_i) - x(t)$$

for $t \in [t_i, t_{i+1})$ with $i \in \mathbb{N}$, and then rewrite

$$u_1(t) = kx(t_i) = k(\mathfrak{e}(t) + x(t)), \tag{6}$$

Substituting (6) into system (5) gives the following closed-loop system:

$$\begin{cases} \dot{x}(t) = f(t, x) + Bk(\mathfrak{e} + x), \\ x(t_0) = \varphi_0. \end{cases} \tag{7}$$

We make the following assumption on the control system (7).

Assumption 1. There exist functions $V \in v_0$ and $\alpha_1, \alpha_2, \chi \in \mathcal{K}_\infty$, and constants $l > 0$ such that all the conditions of Theorem 1 hold for system (7) with input u replaced with $\mathfrak{e}$.

It can be seen from Theorem 1 that Assumption 1 guarantees that closed-loop system (7) is ISS with respect to measurement error $\mathfrak{e}$, and system (7) is GAS provided $\mathfrak{e} = 0$. Next, an execution rule is designed to determine the updated time order $\{t_i\}_{i\in\mathbb{N}}$ of the feedback controller u_1 so that the closed-loop system after replacing u with $\mathfrak{e}$ is still GAS. To do so, we restrict $\mathfrak{e}$ to satisfy

$$\chi(\|\mathfrak{e}\|) \le \sigma\alpha_1(\|x\|)$$

for some $\sigma > 0$. Then the dynamics of V is bounded by

$$D^+V(t, x) \le -lV(t, x) + \sigma\alpha_1(\|x\|) \le -(l - \sigma)V(t, x).$$

This guarantees the control system (7) is GAS provided $\sigma < l$. The updating of the control input u_1 can be triggered by the execution rule (or event)

$$\chi(\|\mathfrak{e}\|) \ge \sigma\alpha_1(\|x\|)$$

The event times are the instants when the event happens, that is,

$$t_{i+1} = \inf\left\{t \ge t_i \mid \eta(t) + \theta(\sigma\alpha_1(\|x\|) - \chi(\|\mathfrak{e}\|)) \le 0\right\}, \tag{8}$$

$$\begin{cases} \dot{\eta}(t) = -\alpha(\eta(t)), \text{ for } t \ge t_0, \\ \eta(t_0) = \eta_0, \end{cases}$$

where $\alpha \in \mathcal{K}$ and $\theta > 0$.

According to the control law (8), the control input is updated at each t_i (the error $\mathfrak{e}$ is set to zero simultaneously) and remains constant until the next event time t_{i+1}, and then the error $\mathfrak{e}$ is reset to zero again. Therefore, the proposed event times ensures the GAS of control system (5).

Lemma 1. *Let* α *be a locally Lipschtiz continuous* $\mathcal{K}_\infty$ *and* $\eta_0 \in \mathbb{R}_+$*,Then,* $\eta \ge 0$ *for all* $t \in [0, \infty)$.

Definition 2. (*Zeno Behavior*). If there exists $T > 0$ such that $t_1 \le T$ for all $l \in \mathbb{N}$, then system (5) is said to exhibit Zeno behavior.

To proceed, let us define a sequence of event-time candidate

$$t_{i+1} = \inf\left\{t \ge t_i \mid \eta(t) + \theta(\chi(\|\mathfrak{e}\|) - \sigma\alpha_1(\|x\|)) \le 0 \wedge t_{i+1} \ge t_i + h\right\}. \tag{9}$$

We let $\Delta x(t_{i+1}) = Bg(x_{i+1}^-)$ when $t_{i+1} = t_i + h$, and update the feedback control at each t_{i+1}. According to the principle of event triggering (9), the system can be rewritten as:

$$\begin{cases} \dot{x}(t) = f(t,x) + Bu_1(t), \\ u_1(t) = k(x(t_i)),\ t \in [t_i, t_{i+1}) \\ \Delta x(t_{i+1}) = Bg(x_{i+1}^-),\ \text{if}\ t_{i+1} = t_i + h, \\ x(t_0) = \varphi_0. \end{cases} \tag{10}$$

Theorem 2 will be given below to guarantee the system (10) GAS. To prove Theorem 2 and Lemma 2 is given.

Lemma 2. *$\alpha(\eta) \ge k\eta$ and $\eta_0 \in \mathbb{R}_+$. Then, $\eta \ge 0$ and $\mathcal{W}(t) = V(t) + \eta(t) \ge 0$ for all $t \in [0, \infty)$.*

Next, we prove Theorem 2.

3. Main Results

Theorem 2. *Suppose that Assumption 1 holds on. For some $h > 0$, the event times $\{t_i\}_{i\in\mathbb{N}}$ are created by event-triggered mechanism with positive constant $\sigma < l$. If $t_{i+1} = t_i + h$, we assume there exist constants c satisfying the following conditions*

(i) for $t \in [t_i, t_{i+1})$, $D^+V(t,x) \le cV(t,x)$,

(ii) $V\left(t_{i+1}, x(t_{i+1}^-) + Bg\left(x(t_{i+1}^-)\right)\right) \le \rho V\left(t_{i+1}^-, x(t_{i+1}^-)\right)$,

(iii) $\alpha(\eta) \ge k\eta$ and $\eta(t_{i+1}) \le \rho\eta(t_{i+1}^-)$,

(iv) $\frac{1}{\rho} > \exp^{ch}$ and $k > \frac{1}{\theta}$.

Then the system (10) is GAS.

Proof. Condition (iv) indicates there exists a constant $0 < \lambda < \min\left\{l - \sigma, k - \frac{1}{\theta}\right\}$ such that

$$\frac{\exp^{\lambda\tau}}{\rho} \ge \frac{1}{\rho} \ge \exp^{(c+\lambda)h}.$$

Next, we prove that the inequality

$$\mathcal{W}(t) = V(t) + \eta(t) \le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t-t_0)} + \eta(t_0)\exp^{-\lambda(t-t_0)}\right) \tag{11}$$

holds at $[t_0, t_1)$. But we must discuss it in two cases.

Case 1: When $t_0 > t_1 + h$, the inequality (11) obviously holds at $[t_0, t_1)$. Easily obtained

$$\begin{aligned} D^+\mathcal{W} &= D^+V + \dot{\eta} \\ &\le -(l-\sigma)V - \left(k - \frac{1}{\theta}\right)\eta \\ &\le -\lambda\mathcal{W} \end{aligned}$$

for all $t \in [t_0, t_1)$.

Thus,

$$\begin{aligned}\mathcal{W}(t) &\le \left(V(t_0)+\eta(t_0)\right)\exp^{-\lambda(t-t_0)} \\ &\le \frac{1}{\rho}\left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right)\end{aligned}$$

We prove that the inequality (11) holds at $[t_0,t_1)$ in *Case 1*.

Case 2: When $t_0=t_1+h$, the inequality (11) also holds at $[t_0,t_1)$. Easily obtained

$$\begin{aligned}D^{+}\mathcal{W} &= D^{+}V+\dot{\eta} \\ &\le cV-k\eta \\ &\le c\mathcal{W}\end{aligned}$$

for all $t\in[t_0,t_1)$.

Thus,

$$\begin{aligned}\mathcal{W}(t) &\le \left(V(t_0)+\eta(t_0)\right)\exp^{c(t-t_0)} \\ &\le \frac{1}{\rho}\left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right).\end{aligned}$$

We prove that the inequality (11) holds at $[t_0,t_1)$ in *Case 2*.

Through the proof of two cases, we can get that the inequality (11) holds at $[t_0,t_1)$. Now suppose that the inequality (11) holds at any $t\in\left[t_{p-1},t_p\right)$ where $p\ge 1$. That is, the inequality (11) holds at $t\in\left[t_0,t_p\right)$ where $p\ge 1$.

Then, we prove that the inequality (11) also holds at $t\in\left[t_p,t_{p+1}\right)$ where $p\ge 1$. But we must discuss it in four cases.

Case 1: When $t_p=t_{p-1}+h$ and $t_{p+1}>t_p+h$, we prove that the inequality (11) holds at $t\in\left[t_p,t_{p+1}\right)$. We can easily get

$$\begin{aligned}\mathcal{W}(t_n) &\le \frac{1}{\rho}\left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t_n-t_0)}+\eta(t_0)\exp^{-\lambda(t_n-t_0)}\right)\rho \\ &= \left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t_n-t_0)}+\eta(t_0)\exp^{-\lambda(t_n-t_0)}\right)\end{aligned}$$

and

$$D^{+}\mathcal{W}\le -\lambda\mathcal{W}.$$

Thus,

$$\begin{aligned}\mathcal{W} &\le \mathcal{W}(t_n)\exp^{-\lambda(t-t_n)} \\ &\le \frac{1}{\rho}\left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right).\end{aligned}$$

We prove that the inequality (11) holds at $\left[t_p,t_{p+1}\right)$ in *Case 1*.

Case 2: When $t_p=t_{p-1}+h$ and $t_{p+1}=t_p+h$, we prove that the inequality (11) holds at $t\in\left[t_p,t_{p+1}\right)$. We can easily get

$$\mathcal{W}(t_n)\le\left(\alpha_2\left(\|x_0\|\right)\exp^{-\lambda(t_n-t_0)}+\eta(t_0)\exp^{-\lambda(t_n-t_0)}\right)$$

and

$$D^{+}\mathcal{W}\le c\mathcal{W}.$$

Thus,

$$\begin{aligned}\mathcal{W} &\le \mathcal{W}(t_n)\exp^{c(t-t_n)} \\ &\le \mathcal{W}(t_n)\frac{1}{\rho}\exp^{-\lambda(t-t_n)} \\ &\le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right).\end{aligned}$$

We prove that the inequality (11) holds at $\left[t_p,t_{p+1}\right)$ in *Case 2*.

Case 3: When $t_p > t_{p-1}+h$ and $t_{p+1} > t_p + h$, we prove that the inequality (11) holds at $t \in \left[t_p,t_{p+1}\right)$. Easily obtained

$$\mathcal{W}(t_n) \le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t_n-t_0)}+\eta(t_0)\exp^{-\lambda(t_n-t_0)}\right)$$

and

$$D^+\mathcal{W} \le -\lambda\mathcal{W}.$$

Thus,

$$\begin{aligned}\mathcal{W} &\le \mathcal{W}(t_n)\exp^{-\lambda(t-t_n)} \\ &\le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right).\end{aligned}$$

We prove that the inequality (11) holds at $\left[t_0,t_{p+1}\right)$.

Case 4: When $t_p > t_{p-1}+h$ and $t_{p+1} = t_p + h$, we prove that the inequality (11) holds at $t \in \left[t_p,t_{p+1}\right)$. Easily obtained

$$\mathcal{W}(t_n) \le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t_n-t_0)}+\eta(t_0)\exp^{-\lambda(t_n-t_0)}\right)$$

and

$$D^+\mathcal{W} \le c\mathcal{W}.$$

Thus,

$$\begin{aligned}\mathcal{W} &\le \mathcal{W}(t_n)\exp^{c(t-t_n)} \\ &\le \mathcal{W}(t_n)\frac{1}{\rho}\exp^{-\lambda(t-t_n)} \\ &\le \frac{1}{\rho}\left(\alpha_2(\|x_0\|)\exp^{-\lambda(t-t_0)}+\eta(t_0)\exp^{-\lambda(t-t_0)}\right).\end{aligned}$$

We prove that the inequality (11) holds at $\left[t_p,t_{p+1}\right)$ in *Case 4*.

Remark 1. Condition (i) describes the divergence rate of the V function. Conditions (ii) and (iii) describe the pulse intensity. Condition (iv) gives the corresponding parameter relationship. In the example, c is known, and the pulse intensity can be designed after setting h. The dynamic parameter η The pulse intensity of can be less than ρ, and finally only need to give the numerical $k-\frac{1}{\theta}$ less than $l-\sigma$ and $\eta_0 < \|x_0\|$, the three less than the inequality should not be greater than the case, otherwise it may lead to slow convergence rate.

Dynamic event triggering mechanism includes static event triggering mechanism. When $\eta_0 = 0$, the dynamic event trigger mechanism is the same as the static event trigger mechanism. The dynamic time trigger mechanism is unlikely to produce Zeno phenomenon, because the dynamic variable *eta* is not equal to zero, so it cannot be less than zero in a short time after the event is triggered, but can only be strictly greater than zero. The dynamic event trigger control mechanism in this paper also includes the static event trigger control mechanism and the dynamic time trigger control mechanism without pulses.

Corollary 1. *If* $\alpha_1 \ge c_1 \|x\|^q$, *then the system (10) is exponential stability.*

The proof of this theorem is simple and omitted.

4. Application

Consider the control problem of the nonlinear system

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + u(t), \; t \neq p_i \\ \Delta x(p_i) = Cx(p_i), \; i \in \mathbb{N} \\ x_{t_0} = x_0 \end{cases} \tag{12}$$

where $x(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}} \in \mathbb{R}^3$, feedback control $u(t) = Kx(t)$ with control gain $K = \gamma I$ and $\gamma = -14.35$, and initial condition $x_0 = [0.13, -0.28, -0.15]^{\mathrm{T}}$. Matrix A and function f are given as follows:

$$A = \begin{bmatrix} -\alpha(1+m_1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}, f(x) = \begin{bmatrix} -\alpha(m_1 - m_0)sat(x_1(t)) \\ 0 \\ 0 \end{bmatrix},$$

where $\alpha = 10$, $\beta = 16$, $m_0 = -8/7$, $m_1 = -5/7$, and $sat(z)$ is the saturation function defined as $sat(z) = (|z+1| - |z-1|)/2$ for $z \in \mathbb{R}$. First, we consider the event-triggered implementation of the feedback control according to (8). Note that the control system can be written in the form of (7). We next choose the Lyapunov function $V(x) = x^{\mathrm{T}}x$, where condition of Theorem 1 is clearly satisfied with. We have

$$\dot{V}(x) \le x^{\mathrm{T}}(A^{\mathrm{T}} + A + 2\gamma I)x + 2x^{\mathrm{T}} f(x(t)) + 2\gamma x^{\mathrm{T}} e$$

$$x^{\mathrm{T}}(A^{\mathrm{T}} + A + (2\gamma + 2L + 1)I)x + \gamma^2 e^{\mathrm{T}} e,$$

which implies that condition holds with

$$l = \lambda_M(A^{\mathrm{T}} + A) + 2\gamma + 2L + 1 \approx -2.3444$$

$$\chi(\mathfrak{e}) = \gamma^2 e^{\mathrm{T}} e$$

where $\lambda_M(A^{\mathrm{T}} + A)$ represents the largest eigenvalue of $A^{\mathrm{T}} + A$. Choosing $\sigma = 1.5$, the event-time candidates are determined as

$$t_{i+1} = inf\left\{t \ge t_i \mid \eta(t) + \theta(\sigma x^{\mathrm{T}} x - \gamma^2 e^{\mathrm{T}} e) \le 0 \wedge t_{i+1} \ge t_i + h\right\}, \tag{13}$$

where $\theta > 0$,

$$\begin{cases} \dot{\eta}(t) = k\eta(t), \ t \geq 0 \\ \eta(t_0) = \eta_0, \ t \leq 0 \\ \eta(t_{i+1}) = \rho\eta(t_{i+1}^-), \ t_{i+1} = t_i + h \end{cases} \tag{14}$$

and $\eta_0 > 0$ to be determined in the case of pulse.

Next, we consider the conditions that the impulsive system satisfies when $t_{i+1} = t_i + h$,

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + u(t), \ \text{for } t \in [t_i, t_{i-1}) \\ \Delta x(t_{i+1}) = Cx(t_{i+1}^-), \ \text{if } t_{i+1} = t_i + h \\ x_{t_0} = x_0, \end{cases}$$

where $C > 0$ and $h > 0$. Consider the function $\mathcal{W}(x) = x^{\mathrm{T}}x + \eta(t)$,

$$\begin{aligned} D^+\mathcal{W} &\leq x^{\mathrm{T}}\left(A^{\mathrm{T}} + A + 2\gamma I\right)x + 2x^{\mathrm{T}} f(x(t)) + 2\gamma x^{\mathrm{T}}e + \dot{\eta} \\ &\leq x^{\mathrm{T}}\left(A^{\mathrm{T}} + A + (2\gamma + 2L + \sigma m)I\right)x + 2\gamma e^{\mathrm{T}}x - me^{\mathrm{T}}e + \left(\frac{m}{\theta} - k\right)\eta, \end{aligned}$$

which implies condition (i) of Theorem 2 is satisfied with

$$\begin{bmatrix} A^{\mathrm{T}} + A + (2\gamma + 2L + \sigma m - c)I & \sigma I \\ \sigma I & -m \end{bmatrix} \leq 0$$

$$c \approx 44.9555,$$

$$m = 2.$$

When $t_{i+1} = t_i + h$, we have

$$V(x(t_{i+1})) = x^{\mathrm{T}}(t_{i+1}^-)(I + C)^{\mathrm{T}}(I + C)x(t_{i+1}^-),$$

where $I + C = 0.913 * I$.

After determining the parameters such as l, c, σ, we set $h = 0.0032$ to determine the pulse intensity. Using $k - \frac{1}{\theta}$ less than $l - \sigma$ and $\eta_0 < \|x_0\|$, we can determine $\theta = 100$, $\eta_0 = 0.44$, $k = 0.858$.

Figure 1 shows the static event trigger control mechanism, and **Figure 2** shows the dynamic event trigger control mechanism. **Figure 3** is the image of nonlinear system. Take $h = 0.0032$, $\theta = 100$, $\eta_0 = 0.44$, $k = 0.858$. It can be seen that the number of pulses triggered by dynamic events The mechanism was significantly reduced. The most outstanding point is the dynamic event trigger control strategy The convergence speed is faster than the static event trigger control strategy.

5. Conclusion

In this paper, the dynamic event triggering control mechanism of nonlinear impulsive systems is studied, the lower bound of triggering time is set, and the parameter setting of dynamic variables is given. The practice has proved that the dynamic event trigger mechanism can save more energy. However, this paper

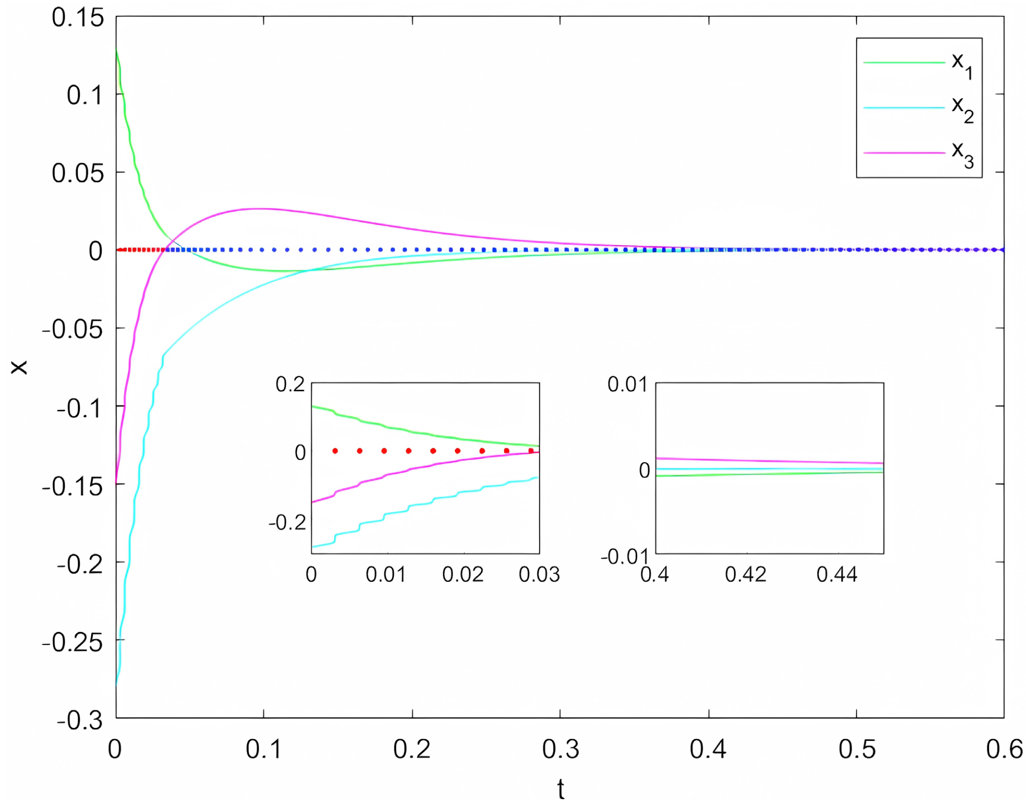

Figure 1. Static event-triggered impulsive control strategy.

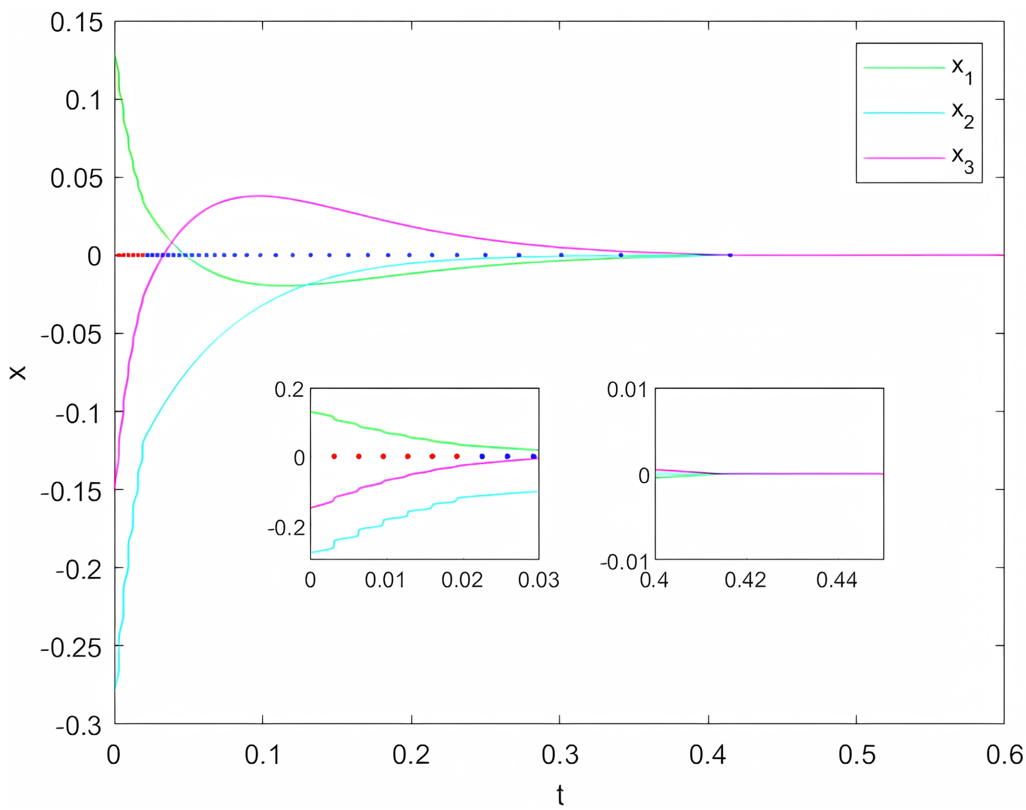

Figure 2. Dynamic event-triggered impulsive control strategy.

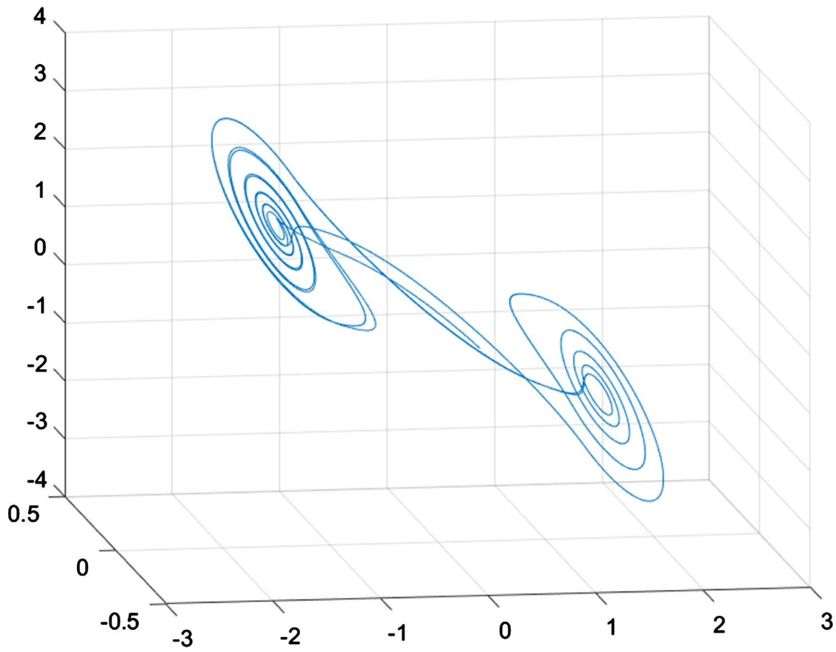

Figure 3. Nonlinear System.

does not study the influence of various parameters and the influence of noise, interference and other factors. These effects will be studied in the future.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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