Journal of Applied Mathematics and Physics, 2024, 12, 2407-2420
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Features of Recombination Radiation of GaAs Type Semiconductors with the Participation of Fine Acceptor Levels in a Magnetic Field

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How to cite this paper: Yuldashev, N.K., Yulchiev, I.I., Akhmadaliev, B.J. and Sulaymonov, K.M. (2024) Features of Recombination Radiation of GaAs Type Semiconductors with the Participation of Fine Acceptor Levels in a Magnetic Field. *Journal of Applied Mathematics and Physics*, **12**, 2407-2420.
https://doi.org/10.4236/jamp.2024.127145

Received: May 22, 2024
Accepted: July 14, 2024
Published: July 17, 2024

Abstract

Using the method of Picus and Beer invariants, general expressions are obtained for the total intensity *I* and the degree of circular polarization $P_{circ.}$ of the luminescence of GaAs-type semiconductors with the participation of shallow acceptor levels in a longitudinal magnetic field $\boldsymbol{H}$. Special cases are analyzed depending on the value and direction of the magnetic field strength, as well as on the constants of the *g*-factor of the acceptor g_1, g_2 and the conduction band electron g_e. In the case of a strong magnetic field $\boldsymbol{H}$ // [100], [111], [110], a numerical calculation of the angular dependence of the quantities *I* and $P_{circ.}$ was performed for some critical values of g_2/g_1, at which $P_{circ.}$ exhibits a sharp anisotropy in the range from −100% to +100%, and the intensity of the crystal radiation along the magnetic field tends to a minimum value.

Keywords

Semiconductor, Recombination Radiation, Shallow Acceptor Center, Magnetic Field, Zeeman Splitting, *g*-Factors, Anisotropy, Circular Polarization, Intensity

1. Introduction

The study of the polarization of recombination radiation of semiconductors in a magnetic field is of great theoretical [1]-[5] experimental [6]-[10] interest in order to obtain subtle information about their optical properties and the dynamics of crystal structures in the field of photonics and spintronics. In the pioneering work [2], a theoretical analysis was made of the case of a weak magnetic field ($\mu_0 H/kT \ll 1$) in semiconductors of the GaAs-type, when recombination radiation occurs during the electronic transition "conduction band-shallow acceptor". A simple expression is obtained for the degree of circular polarization of radiation

$$P_{circ.} = \left(g_e + 5g_h\right)\frac{\mu_0 H}{4kT}, \tag{1}$$

where g_e, g_h, g factors of a free electron and a bound hole in the acceptor, $\mu_0 = \frac{e\hbar}{2m_0 c}$, Bohr magneton, *k*-Boltzmann constant, *T*-absolute temperature on the Kelvin scale, *H*, magnetic field strength. As can be seen from (1), the polarization of recombination radiation with the participation of small acceptors is determined by the average moments of free electrons and bound holes, depends linearly and isotropically on the magnetic field, and is also inversely proportional to temperature. These conclusions of the theory [2] received their experimental confirmation in the works [6]-[8].

The authors of [7], studying the polarization of the 0.709 eV spectral line of germanium radiation over a wide range of magnetic field values (from 0 to 50 kE), showed that the degree of circular polarization depends on the direction of the magnetic field. The theory of Zeeman splitting of shallow acceptors in cubic semiconductors, taking into account cubic contributions from the band structure and strong magnetic fields, was developed in [8].

The effect of magnetic field on the physical properties of hole states of the Mn acceptor located near the (110) surface of GaAs and it was shown that the highly anisotropic wave function of the hole does not change significantly under the influence of a magnetic field up to 6 T [9]. A study of magnetoresistance in the $Ga_{0.972}Mn_{0.028}As$ epitaxial layer showed [10] that doping with Be leads to a reorientation of both the easy and hard magnetic axes in GaMnAs.

Recently, much attention has been paid to the study of the energy spectrum and wave functions of holes in the valence band of semiconductor nanostructures, such as quantum wells, quantum wires and quantum dots, in an external magnetic field [11]-[15]. Thus, a theoretical calculation of fine impurity states in semiconductor quantum wells and GaAs-(Ga, Al)As superlattices in a magnetic field along the growth direction was performed [11], which is consistent with experimental results. Circularly polarized photoluminescence of A(+) centers in GaAs/AlGaAs quantum wells has been detected, induced by a magnetic field [12], which makes it possible to determine their fine, spin, and energy structure. The anisotropy of the electronic *g*-factor for $GaAs/Al_xGa_{1-x}As$ heterostructures at liquid helium temperature was studied, corrections to the *g*-factor components linear in the magnetic field were determined, and a strong anisotropy of their values was established [13]. Suris and Semina [14] studied the dependence of the Zeeman splitting of the ground state of a hole on changes in size quantization parameters, taking into account the complex structure of the valence band and the magnetic field-induced mixing of hole states. It is shown that the hole *g*-factor is extremely sensitive to the composition of hole states and the geometry of the size quantization potential. Semina, Rodina *et al.* [15] theoretically studied the cubic anisotropy of Zeeman hole splitting in a semiconductor nanocrystal arising from crystallographic cubic-symmetric spin and kinetic energy terms in the bulk Luttinger Hamiltonian. The authors proposed possible experimental manifestations and potential methods for measuring cubic anisotropy of hole Zeeman splitting.

The purpose of this work is to obtain general expressions using the Picus and Beer invariants method for the total intensity and degree of circular polarization of photoluminescence at small acceptor centers of GaAs-type semiconductors in an arbitrary crystallographic orientation and magnetic field value. We believe that during the lifetime τ free electrons have time to reach an equilibrium spin distribution, *i.e.* $\tau \gg \tau_s$, where τ_s is the spin relaxation time.

Let us analyze the role of anisotropic Zeeman splitting of the acceptor level and the features of magnetically induced mixing of sublevels at critical values g_2/g_1 of the ratio of bound hole *g*-factor parameters in the formation of the intensity and magnetic circular polarization of photoluminescence at shallow acceptors.

2. Research Method and Theoretical Calculation

Let us consider, as in our previous work [16], a GaAs-type semiconductor placed in a uniform magnetic field, in which nonequilibrium carriers are created, and their radiative recombination occurs through levels of shallow acceptors. We assume that the directions of radiation and magnetic field coincide (Faraday geometry). Then, due to the orientation of the spins of electrons in the conduction band and holes in the acceptor levels under the influence of a magnetic field, the luminescence of the crystal turns out to be circularly polarized. For a theoretical description of such recombination radiation, we first note that the initial states of electrons and holes, taking into account the spin, are two- and four-fold degenerate and in the Luttinger-Kohn representations are described by wave functions that transform according to the representations Γ_6, Γ_8, and the Hamiltonians of the interaction of free electrons and bound holes with the magnetic field strength $\boldsymbol{H}$ in a first approximation are described by matrices [1]

$$\hat{\mathcal{H}}^{(c)} = \frac{1}{2} g_e \mu_0 \sum_i \hat{\sigma}_i H_i \,, \tag{2a}$$

$$\hat{\mathcal{H}}^{(a)} = \mu_0 \sum_i \left(g_1 \hat{J}_i + g_2 \hat{J}_i^3 \right) H_i \ \left(i = x, y, z \right). \tag{2b}$$

Here, $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$, Pauli matrices, $\hat{J}_x, \hat{J}_y, \hat{J}_z$, matrices of 4 × 4 dimensions of projections of the angular momentum operator in the basis of states $Y_n^{3/2}$, g_1, g_2 are the Zeeman splitting constants of the acceptor level, which determine the $g_h - g$ factor of the hole.

We consider perturbations (2a) and (2b) to be small quantities of the first order with respect to the intracrystalline interaction ($E_0^{(a)} \ll \frac{m_0^2 e^4}{2\varepsilon^2 h^2} \cdot \frac{m^{*2}}{m_0^2}$), which removes the spin degeneracy of the state of the conduction band electron and the hole in the acceptor. The correct wave functions of these states in the zeroth approximation are determined accordingly by the following relations:

$$\Psi_M^{(c)} = \sum_m C_m^{(M)} \psi_m^{1/2} \quad \left(m = \frac{1}{2}; -\frac{1}{2},\ M = 1; -1 \right), \tag{3a}$$

$$\Psi_N^{(a)} = \sum_n C_n^{(N)} \psi_n^{3/2} \quad \left(n = \frac{3}{2}; \frac{1}{2}; -\frac{1}{2}; -\frac{3}{2},\ N = 1;2;3;4 \right). \tag{3b}$$

Expansion coefficients $C_m^{(M)}, C_n^{(N)}$ for complete sets of orthonormal the functions $\psi_m^{1/2}$, $\psi_n^{3/2}$ and the values of the split energy levels $E_M^{(c)}, E_N^{(a)}$ can be determined from the following matrix equations

$$\left\| \hat{\mathcal{H}}^{(c)} - E_M^{(c)} \hat{I}_M \right\| \cdot \left\| \hat{C}^{(M)} \right\| = 0, \tag{4a}$$

$$\left\| \hat{\mathcal{H}}^{(a)} - E_N^{(a)} \hat{I}_N \right\| \cdot \left\| \hat{C}^{(N)} \right\| = 0, \tag{4b}$$

where $\hat{I}_M$ and $\hat{I}_N$ are 2 × 2 and 4 × 4 identity matrices, and $\hat{C}^{(M)}, \hat{C}^{(N)}$ are matrices columns 2 × 1 and 4 × 1. The first of them will allow us to find the Zeeman splitting energies in the conduction band

$$E_M^{(c)} \equiv E_m^{(c)} = m g_e \mu_0 H, \tag{5a}$$

and the second gives for the acceptor levels $E_N^{(a)} \equiv E_n^{(a)}$ expression

$$E_{(1,4)}^{(a)} = \pm \mu_0 H \sqrt{\frac{1}{8}\left[9\left(g_1 + \frac{9}{4} g_2 \right)^2 + \left(g_1 + \frac{g_2}{4} \right)^2 \right] + \left(g_1 + \frac{7}{4} g_2 \right) \sqrt{\left(g_1 + \frac{13}{4} g_2 \right)^2 - \frac{9}{4} g_2 \left(g_1 + \frac{5}{2} g_2 \right) \gamma'}}, \tag{5b}$$

$$E_{(2,3)}^{(a)} = \pm \mu_0 H \sqrt{\frac{1}{8}\left[9\left(g_1 + \frac{9}{4} g_2 \right)^2 + \left(g_1 + \frac{g_2}{4} \right)^2 \right] - \left(g_1 + \frac{7}{4} g_2 \right) \sqrt{\left(g_1 + \frac{13}{4} g_2 \right)^2 - \frac{9}{4} g_2 \left(g_1 + \frac{5}{2} g_2 \right) \gamma'}},$$

where $\gamma' = h_x^2 h_y^2 + h_y^2 h_z^2 + h_z^2 h_x^2 = \sin^4 2\theta + \sin^4 \theta \sin^2 2\varphi$ (see Figure 1).

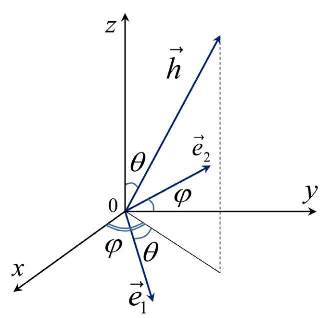

Figure 1. Choice of directions of crystallographic axes $x \| [100]$, $y \| [010]$, $z \| [001]$ and unit vectors of magnetic field strength $\boldsymbol{h} = \boldsymbol{H}/H$, radiation polarization $\boldsymbol{e}_1 \perp \boldsymbol{e}_2 \perp \boldsymbol{h}$.

Generally speaking, the Zeman splitting for the acceptor, as can be seen from (5b), has a complex anisotropic character, the analysis of which was devoted to the work [16]. At $g_2 = 0$ and $g_2/g_1 = -2/5, -4/7$, the magnitude of the splitting of the acceptor level $E_n^{(a)}$ does not depend on the direction of the vector $\boldsymbol{H}$ in the crystal. Next, we are interested in deriving general formulas for calculating the parameters of polarized luminescence. The intensity of recombination radiation associated with the conduction-acceptor quantum transition and polarized in the right or left circle in the direction of the magnetic field can be determined from the following general expression

$$I_{\sigma_\pm}\left(\boldsymbol{e}_\pm,\boldsymbol{H}\right)=\sum_{n,m} f_n^{(a)} f_m^{(c)}\left|\left\langle \Psi_n^{(a)}\left|\boldsymbol{P}\cdot\boldsymbol{e}_\pm^*\right|\Psi_m^{(c)}\right\rangle\right|^2 = C_0\sum_{n,m} f_n^{(a)} f_m^{(c)} Sp\hat{\mathfrak{M}}_{\sigma_\pm}\left(n,m\right), \quad (6)$$

where $\boldsymbol{e}$ is the polarization vector ($\boldsymbol{e}_\pm=\left(\boldsymbol{e}_1\pm i\boldsymbol{e}_2\right)/\sqrt{2}$, see **Figure 1**), $\hat{\boldsymbol{P}}$ is the quasi-momentum operator, $C_0=const$; $f_m^{(c)}, f_n^{(a)}$-energy distribution functions for conduction band electrons and holes in acceptors. We will assume that it has place Maxwell-Boltzmann distribution of electron and hole over spin levels

$$f_m^{(c)}=\frac{\exp\left(-\beta E_m^{(c)}\right)}{\sum_{m'}\exp\left(-\beta E_{m'}^{(c)}\right)},\quad f_n^{(a)}=\frac{\exp\left[-\beta E_n^{(a)}\right]}{\sum_{n'}\exp\left[-\beta E_{n'}^{(a)}\right]},\ \beta=1/kT \quad (7)$$

In (6) $\hat{\mathfrak{M}}_{\sigma_\pm}\left(n,m\right)$ denotes a 4 × 4 matrix

$$\hat{\mathfrak{M}}_{\sigma_\pm}\left(n,m\right)=\hat{B}^{(a)}\left(n\right)\hat{R}_{\sigma_\pm}\hat{B}^{(c)*}\left(m\right)\hat{R}_{\sigma_\pm}^{+}. \quad (8)$$

Here are the matrices $B_{jj'}^{(a)}\left(n\right)=C_j^{(n)}C_{j'}^{(n)+}$ and $B_{ii'}^{(c)}\left(m\right)=C_i^{(m)}C_{i'}^{(m)+}$, where $C_j^{(n)}$ and $C_i^{(m)}$ are the expansion coefficients of the wave function of a hole in state *n* and an electron in state *m*, respectively, in terms of the basis functions of the representations Γ_8 and Γ_6 (see (3a) and (3b)); $\left(R_{\sigma_\pm}\right)_{ij}$, matrix element of electron and hole recombination in state *i* = 1, 2 and *j* = 1, 2, 3, 4, independent of $\boldsymbol{H}$, *i.e.* $\hat{R}_{\sigma_\pm}$-matrix of order 4 × 2 determines the selection rule for recombination radiation "conduction band-acceptor" in the absence of a magnetic field:

j = 1/2; *m* / *j* = 3/2; *n*	1/2	−1/2
3/2	$e_\pm^* R$	0
1/2	$i\frac{2}{\sqrt{3}}e_z^* R$	$-i\frac{1}{\sqrt{3}}e_\pm^* R$
−1/2	$\frac{1}{\sqrt{3}}e_\mp^* R$	$\frac{2}{\sqrt{3}}e_z^* R$
−3/2	0	$-ie_\mp^* R$

$$\hat{R}_{\sigma_\pm}=R_0\left\|\begin{matrix} e_\pm^* & 0 \\ i\frac{2}{\sqrt{3}}e_z^* & -i\frac{1}{\sqrt{3}}e_\pm^* \\ \frac{1}{\sqrt{3}}e_\mp^* & \frac{2}{\sqrt{3}}e_z^* \\ 0 & -ie_\mp^* \end{matrix}\right\|, \quad (9)$$

where $e_\pm=\left(e_x\pm ie_y\right)/\sqrt{2}$, $R_0=const=\langle X|\hat{P}_x|S\rangle=\langle Y|\hat{P}_y|S\rangle=\langle Z|\hat{P}_z|S\rangle$.

It can be shown [17] that, for example,

$$B_{jj'}^{(a)}\left(n\right)=\prod_{\nu=1}^{3}\frac{\mathscr{H}_{jj'}^{(a)}-E_{n_\nu}^{(a)}\delta_{jj'}}{E_n^{(a)}-E_{n_\nu}^{(a)}}, \quad (10)$$

where $E_{n_\nu}^{(a)}\left(\nu=1,2,3\right)$ is the energy of a hole in three other states different from state n, $\delta_{jj'}$ - δ Dirac symbol.

Substituting (9) and (10) into (8), and then (7) and (8) into (6) after simple but cumbersome transformations, we obtain for radiation along the vector $\boldsymbol{H}$

$$I = I_{\sigma_+} + I_{\sigma_-} = r_0 F\ , \tag{11}$$

$$\begin{aligned} P_{circ.} = \frac{I_{\sigma_+} - I_{\sigma_-}}{I_{\sigma_+} + I_{\sigma_-}} = F^{-1}\Bigg\{ & (2A_0 + f_1 A_2) f_0 + \left(5 + \frac{41}{4}\frac{g_2}{g_1}\right) A_1 + A_3 f_2 \\ & - \frac{9}{2}\frac{g_2}{g_1}\left(1 + \frac{7}{4}\frac{g_2}{g_1}\right)\left[A_2 f_0 + \frac{3}{2}\left(1 + \frac{9}{4}\frac{g_2}{g_1}\right) A_3 \right]\gamma \Bigg\}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} F = {} & 4A_0 + A_2 f_3 + \left[4\left(1 + \frac{5}{2}\frac{g_2}{g_1}\right) A_1 + A_3 f_4 \right] f_0 \\ & - \frac{9}{4}\frac{g_2}{g_1}\left\{ \left[(A_1 + A_3 f_5) f_0 + \left(1 + \frac{7}{4}\frac{g_2}{g_1}\right) A_2 \right]\gamma + \frac{27}{16}\frac{g_2}{g_1}\left(1 + \frac{7}{4}\frac{g_2}{g_1}\right) A_3 f_0 \chi \right\}. \end{aligned} \tag{13}$$

The following notations are introduced here:

$$f_0 = th\left(\frac{1}{2}\beta g_e \mu_0 H\right),\ f_1 = 6\left(1 + \frac{7}{4}\frac{g_2}{g_1}\right)^2 + \frac{1}{2}\left(1 + \frac{1}{4}\frac{g_2}{g_1}\right)^2 + 9\frac{g_2}{g_1}\left(1 + \frac{7}{4}\frac{g_2}{g_1}\right),$$

$$f_2 = f_4 + \frac{1}{4}\left(1 + \frac{1}{4}\frac{g_2}{g_1}\right)^3,\ f_3 = 7\left(1 + \frac{9}{4}\frac{g_2}{g_1}\right)^2 - \frac{g_2}{g_1}\left(1 + \frac{5}{4}\frac{g_2}{g_1}\right),$$

$$f_4 = 9\left(1 + \frac{7}{4}\frac{g_2}{g_1}\right)\left(1 + \frac{9}{4}\frac{g_2}{g_1}\right)\left(1 + \frac{13}{4}\frac{g_2}{g_1}\right) + \left(1 + \frac{1}{4}\frac{g_2}{g_1}\right)^2\left(1 + \frac{5}{2}\frac{g_2}{g_1}\right),$$

$$f_5 = \frac{3}{8}\left[10 + 37\frac{g_2}{g_1} + \frac{285}{8}\left(\frac{g_2}{g_1}\right)^2\right],\ A_l = -\frac{\sum_{n=1}^{4} a_l^{(n)} \exp\left(-\beta E_n^{(a)}\right)}{\sum_{n=1}^{4} \exp\left(-\beta E_n^{(a)}\right)},$$

$$a_0^{(n)} = \frac{E_{n_1}^{(a)} E_{n_2}^{(a)} E_{n_3}^{(a)}}{(g_1 \mu_0 H)^3} a_3^{(n)},\ a_1^{(n)} = \frac{E_{n_1}^{(a)} E_{n_2}^{(a)} + E_{n_1}^{(a)} E_{n_3}^{(a)} + E_{n_2}^{(a)} E_{n_3}^{(a)}}{(g_1 \mu_0 H)^2} a_3^{(n)}$$

$$a_2^{(n)} = \frac{E_{n_1}^{(a)} + E_{n_2}^{(a)} + E_{n_3}^{(a)}}{g_1 \mu_0 H} a_3^{(n)},\ a_3^{(n)} = \frac{(g_1 \mu_0 H)^3}{\left(E_n^{(a)} - E_{n_1}^{(a)}\right)\left(E_n^{(a)} - E_{n_2}^{(a)}\right)\left(E_n^{(a)} - E_{n_3}^{(a)}\right)},$$

$$\gamma = 4\left(h_x^2 h_y^2 + h_x^2 h_z^2 + h_y^2 h_z^2\right),\ \chi = 16 h_x^2 h_y^2 h_z^2,$$

$r_0 = const$, $h_i\,(i = x, y, z)$, projections of the unit vector $\boldsymbol{h}$ onto the main symmetry axes of the crystal.

3. Discussion of the Results Obtained

3.1. Analysis of General Formulas

As can be seen from (11)-(13), the intensity I and the degree of polarization $P_{circ.}$ have a complex angular dependence due to both the anisotropy of the Zeeman splitting of the acceptor level (5b) through the quantities ($l = 0,1,2,3$) and the anisotropy of the selection rules (9) through the quantities γ and χ. It is interesting to note that at $g_2 = 0$ the angular dependence in formulas (5b) and (11)-(13) completely disappears, *i.e.* under the conditions assumed above, Zeeman splitting of a shallow acceptor and the associated luminescence of GaAs-type semiconductors do not exhibit anisotropy. In this case, the expressions for I and $P_{circ.}$ greatly simplified:

$$I(g_2 = 0) = \frac{1}{3}|R_0|^2 (1+\omega)\exp\left[\frac{1}{2}(g_1 - g_e)\beta\mu_0 H\right]\left[1 + 3\exp\left((g_1 + g_e)\beta\mu_0 H\right)\right] \quad (14)$$

$$P_{circ.}(g_2 = 0) = \frac{1-\omega}{1+\omega}, \quad \omega = \frac{1 + 3\exp\left(-\left(g_e + |g_1|\right)\beta\mu_0 H\right)}{3\exp\left(2|g_1|\beta\mu_0 H\right) + \exp\left(\left(|g_1| - g_e\right)\beta\mu_0 H\right)} \quad (15)$$

In the absence of a magnetic field, the value $\omega = 1$ and from (14), (15), as expected, we obtain $I = 8|R_0|^2/3 = const$, $P_{circ.} = 0$.

General expressions (11)-(13) are also significantly simplified for certain characteristic values of g_2/g_1 . So, for example, at $g_2/g_1 = -4/7$ there is no anisotropy for split levels $E_N^{(a)}$, and they are pairwise merge, forming two doublet states $E_{1,2}^{(a)} = -E_{3,4}^{(a)} = 3g_1\mu_0 H/7$ (see (5b)), whereas for I and P_{circ}. It is saved in the following simplified form:

$$\begin{aligned} I\left(g_2/g_1 = -\frac{4}{7}\right) &= \frac{2}{3}|R_0|^2 \exp\left(\left(\frac{g_e}{2} + \frac{3}{7}g_1\right)\beta\mu_0 H\right)\left(1 + \exp\left(-g_e\beta\mu_0 H\right)\right) \\ &\times\left(1 + \exp\left(-\frac{6}{7}g_1\beta\mu_0 H\right)\right)\left(1 + \left(1 - \frac{3}{4}\gamma\right) f_0 f_0'\right), \end{aligned} \quad (16)$$

$$P_{circ.}\left(g_2/g_1 = -\frac{4}{7}\right) = \frac{1}{2}\frac{f_0 + f_0'}{1 + \left(1 - \frac{3}{4}\gamma\right) f_0 f_0'}, \quad f_0' = th\left(\frac{3}{7}\beta g_e\mu_0 H\right). \quad (17)$$

In the general case, the $g_2/g_1 \neq 0, -4/13, -2/5, -4/7$ dependences of I and P_{circ}. On the constants g_1, g_2 (or on g_2/g_1) and on the direction of the magnetic field behave in a very complex manner.

It is of interest to analyze the general expressions for intensity (11) and circular polarization (12) in the limiting cases of weak and strong magnetic fields.

1) For a weak magnetic field ($\beta\mu_0 H < 1$) from (11) and (12) we find

$$I = \frac{4}{3}|R_0|^2\left(1 + \frac{g_e\beta\mu_0 H}{2}\right)\left[2 + g_e\left(g_1 + \frac{5}{2}g_2\gamma\right)(\beta\mu_0 H)^2\right], \quad (18)$$

$$P_{circ.} = \frac{1}{4}\left(g_e + 5g_1 + \frac{41}{4}g_2\right)\frac{\mu_0 H}{kT}. \quad (19)$$

Here we immediately note that for the degree of circular polarization induced by a weak magnetic field from (19) in the limiting case $g_2 \to 0$ we obtain the result of Dyakonov and Perel (1), and the g-factor of the hole in the acceptor coincides with the constant g_1. In formula (19), the third term $\frac{41}{4}g_2$ in brackets takes into account the more subtle paramagnetic interaction of the acceptor with the external magnetic field (see (2b)) and describes the additional hole contribution to the polarization. It can be seen that the Zeeman splitting constant g_2 will make a noticeable contribution to the polarization even at such modest values as $g_2 \approx 0.1 g_1$ (for crystals with a diamond structure the following experimental values are known: $g_1 = -1.15 \pm 0.05$, $g_2 = 0.45 \pm 0.05$, $g_e = 1.58$ [4] [7]) and it is necessary to take it into account for accurate interpretation of the corresponding results of studies of recombination radiation through small acceptors.

In the case under consideration, although formally there is an angular dependence for the Zeeman levels $E_i^{(a)}$ (5b) and intensity (18), it does not manifest itself in the polarization of the radiation (19). This is explained by the fact that, under the condition $\mu_0 H < kT$, the interval between the Zeeman sublevels of the acceptor is covered by the thermal spread of phonons ($\Delta E_{ij}^{(a)} \sim g_1 \mu_0 H < kT$), and as a result, the induced orientations of the hole spins in the acceptor, on average, are practically independent of the direction of the magnetic field. Therefore, it is natural to expect that the polarization of recombination radiation in this case is not sensitive to the merging of the magnetic sublevels of the acceptor.

Under the $\left(|g_1|,|g_e|\right)\dfrac{\mu_0 H}{kT} \ll 1$ condition, as can be seen from (18), the total radiation intensity is practically independent of the magnetic field and $I \approx 8|R_0|^2/3 = const$.

2) The condition of a strong magnetic field $\left(|g_1|,|g_e|\right)\dfrac{\mu_0 H}{kT} > 1$ is practically easy to satisfy at low temperatures with moderate values of the strength *H*. Thus, for the temperature of liquid helium we obtain $H > 3kE$. On the other hand, the theory developed here is valid when the condition $\mu_0 H \ll E_0^{(a)}$, is satisfied where $E_0^{(a)}$ is the activation energy of the ground state of the acceptor, and at $E_0^{(a)} \approx 0.05\,\text{eV}$ we have $H \ll 500kE$, which gives us grounds to consider magnetic fields with $H \geq 50kE$ strong already at the temperature of the liquid nitrogen.

In a strong magnetic field, the population of particles on split sublevels varies greatly, which leads to a significant difference in the intensities of spectral lines associated with quantum transitions between different magnetic levels of the conduction band and acceptor. Thus, the probability of an allowed transition between the lower electron and upper hole levels is much greater than the others. Therefore, considering that the luminescence under consideration occurs only due to the transition of electrons from the lower magnetic sublevel of the conduction band (m = −1/2) to the upper acceptor sublevel (n = −3/2) c $g_1 < 0$, from (11)-(13) we obtain

$$I = \frac{1}{6}|R_0|^2 \frac{\left(g_1 \mu_0 H\right)^3}{E_1^{(a)}\left(E_1^{(a)2} - E_2^{(a)2}\right)} exp\left(\left(|g_e|\mu_0 H + E_1^{(a)}\right)/2kT\right) \times F', \tag{20}$$

$$P_{circ.}=\left\{\pm\frac{E_1^{(a)}}{g_1\mu_0 H}\left[f_1-2\frac{E_2^{(a)2}}{(g_1\mu_0 H)^2}\right]+f_2-\left(5+\frac{41}{4}\frac{g_2}{g_1}\right)\frac{E_1^{(a)2}}{(g_1\mu_0 H)^2}\right.$$
$$\left.-\frac{9}{2}\frac{g_2}{g_1}\left(1+\frac{7}{4}\frac{g_2}{g_1}\right)\left[\frac{3}{2}\left(1+\frac{9}{4}\frac{g_2}{g_1}\right)\pm\frac{E_1^{(a)}}{g_1\mu_0 H}\right]\gamma\right\}\Big/F', \tag{21}$$

$$F'=\frac{E_1^{(a)}}{g_1\mu_0 H}\left(f_3-4\frac{E_2^{(a)2}}{(g_1\mu_0 H)^2}\right)\pm\left[f_4-4\left(1+\frac{5}{2}\frac{g_2}{g_1}\right)\frac{E_1^{(a)2}}{(g_1\mu_0 H)^2}\right]$$
$$-\frac{9}{4}\frac{g_2}{g_1}\left\{\left[\left(1+\frac{7}{4}\frac{g_2}{g_1}\right)\frac{E_1^{(a)}}{g_1\mu_0 H}\pm\left(f_5-\frac{E_1^{(a)2}}{(g_1\mu_0 H)^2}\right)\right]\gamma\pm\frac{27}{16}\frac{g_1}{g_2}\left(1+\frac{7}{4}\frac{g_1}{g_2}\right)\chi\right\}, \tag{22}$$

where the upper sign (+) refers to the case $g_e>0$, and the lower sign (−), to $g_e<0$, $E_1^{(a)}$ and $E_2^{(a)}$ are the Zeeman splitting energies of the two upper acceptor levels (of which $E_1^{(a)}$ is higher than $E_2^{(a)}$). Since according to (5b) $E_i^{(a)}\sim H$, then, as can be seen from (21) and (22), at $\mu_0 H/kT\gg 1$ the degree of induced circular polarization does not depend on the value of the magnetic field strength, while at the same time such a dependence for the total intensity is preserved due to the exponential multiplier $\exp\left(\left(|g_e|\mu_0 H+E_1^{(a)}\right)/2kT\right)$. At the same time, I and P_{circ} exhibit complex anisotropy and dependence on g_2/g_1 with all the above-mentioned features at the beginning of point A.

3.2. Results of Numerical Calculation

Subsequently, under a "strong" magnetic field, we performed a numerical calculation of $I(\theta, g_2/g_1)$, $P_{circ.}(\theta, g_2/g_1)$ at $g_1<0$, the results of which are presented in Figures 2-4. For these functions, regardless of the sign of the g_e factor, you can specify 3 characteristic ranges of g_2/g_1 values: $\frac{g_2}{g_1}<-\frac{4}{7}$; $\frac{g_2}{g_1}>-\frac{4}{13}$ and $-\frac{4}{7}\le\frac{g_2}{g_1}\le-\frac{4}{13}$, in which they exhibit specific characteristics.

Thus, when $g_e>0$ at $\frac{g_2}{g_1}>-\frac{4}{13}$ (region I) in arbitrary directions of the crystal, the most probable quantum transition is $-\frac{1}{2}\to-\frac{3}{2}$ and it is expanded according to the selection rule. In this region, the total radiation intensity has practically no anisotropy and is circularly polarized in the right circle (curves 1-3 in **Figure 2(a)**, **Figure 2(b)**). And at $\frac{g_2}{g_1}<-\frac{4}{7}$ (region II), the most probable transition is prohibited and it can be assumed that in GaAs-type crystals with such values of g_2/g_1 in an arbitrary direction of the magnetic field, practically no recombination radiation occurs through small acceptors. In the transition region III $-\frac{4}{7}\le g_2/g_1\le-\frac{4}{13}$, significant angular dependences of I and P_{circ} luminescence in a magnetic field are revealed, due to the anisotropy of the Zeeman splitting of the acceptor level and the direct anisotropy of the wave functions of the energy bands of the crystal.

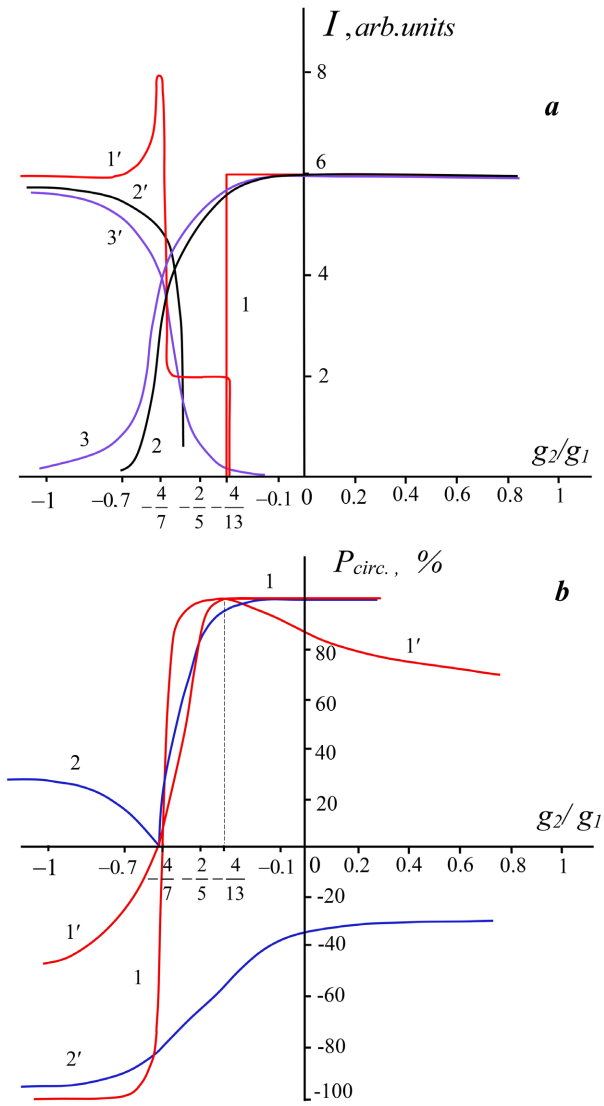

Figure 2. Dependence of the intensity (a) and degree of circular polarization (b) of radiation on g_2/g_1 in the region of critical values −4/7, −2/5 and −4/13 for directions ***H*** // [001] (curves1, 1'), [110] (2, 2') and [111] (3, 3') at $g_e > 0$ (1-3) and $g_e < 0$ (1'-3').

In the case of $g_e < 0$ ($g_1 < 0$), as can be seen from curves 1'-3' **Figure 2**, recombination through small acceptors in crystals of the GaAs type behaves completely oppositely in regions I and II of the g_2/g_1 values, compared to the case of $g_e > 0$ ($g_1 < 0$). Thus, in region I there is no radiation in the sense indicated above, and in region II there is radiation circularly polarized along the left circle. And the significant difference in the transition region in the cases where $g_e > 0$ and $g_e < 0$ is that in the first of the latter, in the directions $[001]$ and $[110]$, radiation is observed only at $\frac{g_2}{g_1} \geq -\frac{4}{13}$ and $\frac{g_2}{g_1} \geq -\frac{4}{7}$, respectively, and in the second, at $\frac{g_2}{g_1} \leq -\frac{4}{13}$ and $\frac{g_2}{g_1} \leq -\frac{4}{7}$. These results, in principle, can be easily explained by fixing the position and course of the magnetic sublevels of free electrons and the shallow acceptor depending on g_1, g_e and g_2/g_1.

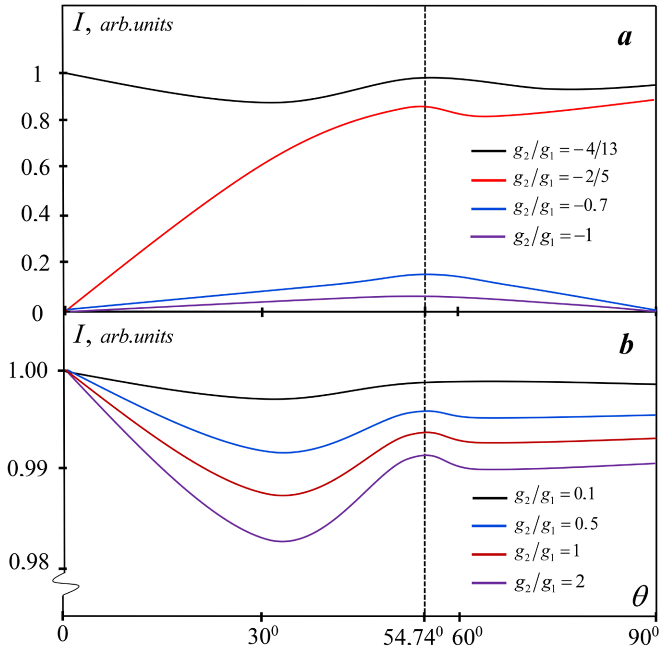

Figure 3. Angular dependence of radiation intensity for values g_2/g_1: −4/13, −2/5, −0.7, −1.0 at $g_e/g_1<0$ (a) and g_2/g_1: 0.1, 0.5, 1.0, 2.0 at $g_e/g_1>0$ (b). Vector $\boldsymbol{H}$ lies in the (110) plane, θ is the angle between $\boldsymbol{H}$ and the [001] axis. The value of $\mu_0 H/kT$ is fixed.

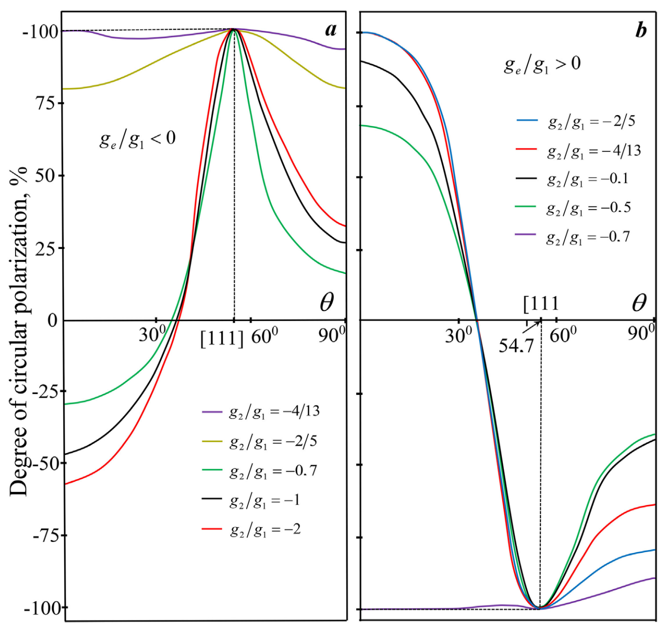

Figure 4. Angular dependence of the degree of circular polarization for values g_2/g_1: −4/13. −2/5, −0.7, −1.0, −2.0 at $g_e/g_1<0$ (a) and g_2/g_1: −2/5, −4/13, −0.1, −0.5, −0.7 at $g_e/g_1>0$ (b). $\boldsymbol{H}$ and θ are chosen in the same way as in **Figure 3**.

Figure 2(a), Figure 2(b) show the qualitative dependences of I and $P_{circ.}$ on g_2/g_1 in the vicinity of the critical values −4/7, −2/5 and −4/13 for the directions $\boldsymbol{H}$ //[001], [110], [111] at $g_e > 0$ and $g_e < 0$ with features discovered during the numerical calculation. Note that the cases $g_e > 0$ and $g_e < 0$ ($g_1 < 0$) also differ in that for them the values of the matrix elements of the allowed transitions do not completely coincide. It should also be noted that for $g_1 > 0$ the characters of the functions $P_{circ.}\left(\theta, \frac{g_2}{g_1}, H\right)$ and $I\left(\theta, \frac{g_2}{g_1}, H\right)$ do not differ qualitatively from the case of $g_1 < 0$, in particular, the graphs under study for $g_1 > 0$ ($g_e > 0$) and $g_1 < 0$ ($g_e < 0$) completely coincide.

Figure 3 and Figure 4 show the angular dependences of the total intensity and the degree of circular polarization of radiation in a "strong" magnetic field, calculated for some values of g_2/g_1 at $g_e/g_1 < 0$ (Figure 3(a) and Figure 4(a)) and $g_e/g_1 > 0$ (Figure 3(b) and Figure 4(b)). The nature of the dependence $I(\theta)$, $P_{circ.}I(\theta)$ is determined by the sign of the ratio of the g factors g_e/g_1 and g_2/g_1. It can be seen that at $g_e/g_1 < 0$, $g_2/g_1 < -4/13$ or $g_e/g_1 > 0$, $g_2/g_1 > -4/7$ the values of I and $P_{circ.}$ depend significantly on the angle θ, and the luminescence intensity decreases significantly. In the case when $g_e/g_1 < 0$, $g_2/g_1 > -4/13$, or $g_e/g_1 > 0$, $g_2/g_1 < -4/7$, in a strong magnetic field $\beta\mu_0 H \gg 1$, the radiation in the direction of the vector $\boldsymbol{H}$ is almost completely circularly polarized, regardless of the angle θ (within 2%, see Figure 4), *i.e.*, the anisotropy of Zeeman splitting in this case does not manifest itself in the polarization of luminescence.

4. Conclusions

Based on the results of this work, the following conclusions can be drawn:

1) Using the method of Picus and Beer invariants, general expressions were obtained for the total intensity and degree of circular polarization of photoluminescence at small acceptor centers of GaAs-type semiconductors in a longitudinal magnetic field.

2) In a weak magnetic field, formally there is an angular dependence for the Zeeman levels $E_i^{(a)}$ of the acceptor and the total intensity, but it does not manifest itself in the polarization of the radiation. In this case $\mu_0 H \ll kT$, the radiation intensity is practically independent of the magnetic field.

3) In a strong magnetic field $\beta\mu_0 H \gg 1$, the nature of the angular dependences $I(\theta)$, $P_{цирк.}(\theta)$ is determined by the sign of the ratio of the g-factors g_e/g_1 and g_2/g_1. In the case when $g_e/g_1 < 0$, $g_2/g_1 > -4/13$, or $g_e/g_1 > 0$, $g_2/g_1 < -4/7$ the radiation in the direction of the vector $\boldsymbol{H}$ is almost completely circularly polarized, regardless of the angle θ. At $g_e/g_1 < 0$, $g_2/g_1 < -4/13$ or $g_e/g_1 > 0$, $g_2/g_1 > -4/7$, in a strong magnetic field, the luminescence intensity decreases significantly, and the values of I and $P_{цирк.}$, significantly depend on the θ.

Thus, studying the dependence of the intensity and degree of polarization of luminescence in a magnetic field, caused by the optical transition of free electrons to the level of a shallow acceptor, on the orientation of the vector $\boldsymbol{H}$ in the crystal makes it possible, in principle, to find the values of the constants g_1 and g_2, as well as to establish some characteristic features of the functions $I(\theta, g_2/g_1, H)$, $P_{цирк.}(\theta, g_2/g_1, H)$.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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