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Implications for Antigravity Tests of Dirac's Negative Energy Antiparticles

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Abstract

P. A. M. Dirac conceived antimatter in 1928 as having *negative energy* by allowing a consistent representation of matter-antimatter annihilation into light. To achieve compatibility with special relativity, particle physics of the early 20th century made the theoretical assumption that antiparticles have *positive energy*, an assumption that remains in effect as of today. In this note we prove apparently for the first time a theorem stating that positive mass antiparticles violate Dirac's particle-antiparticle annihilation into light. We then show the consequential unsettled character of the recent gravity test of the anti-Hydrogen atom due to the positive mass of its nucleus. We conclude by suggesting that a final scientific claim on matter-antimatter gravity requires tests on particles with clear antimatter character, such as the 1994 resolutory proposal for the comparative test of the gravity of very low energy electron and positron in horizontal flight on a supercooled vacuum tube.

Keywords

Antiprotons, Anti-Hydrogen Atom, Antigravity

1. Dirac's Negative Energy Antiparticles

Following epistemological studies dating back to the 19th century, the first quantitative formulation of antimatter was achieved in 1928 by P. A. M. Dirac [1] via his celebrated equation for the electron-antielectron pair on the Minkowski space $M = M(x,\eta,I)$ with space-time coordinates $x = (x^\mu) = (x^1, x^2, x^3, x^4 = ct)$, $\mu = 1,2,3,4$, metric $\eta = Diag.(1,1,1,-1)$ and unit $I = Diag.(1,1,1,1)$ on a Hilbert space $\mathcal{H}$ with states $|\psi(x)\rangle$ over the field of complex numbers $\mathcal{C}$

$$\left(i\eta^{\mu\nu}\gamma_\mu\partial_\nu - mc\right)|\psi(x)\rangle = 0, \quad (1)$$

where the 4×4 matrices $\hat{\gamma}_\mu$

$$\hat{\gamma}_k = \begin{pmatrix} 0 & \sigma_k, \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma_4 = i\begin{pmatrix} +I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{2}$$

verify the anti-commutation rules

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \tag{3}$$

while Pauli's matrices $\sigma_k, k = 1,2,3$ represent the spin $S = 1/2$ of the particles.

According to Dirac's conception, the *electron* $e = e^-$ is represented by the top 22 component of Equation (1) with unit $+I_{2\times 2}$, thus having *positive rest energy* $m_e > 0$, while the antielectron $\overline{e} = e^+$ (now called the positron), is represented by the bottom 2 × 2 component of Equation (1) with unit $-I_{2\times 2}$, thus having *negative rest energy* $\overline{m}_e = -m_e < 0$.

An important aspect of Dirac's conception of antimatter is that it allows a consistent representation of *particle-antiparticles annihilation into light*, such as that for the electron-positron pair [2]

$$e^- + e^+ \to \gamma + \gamma, \tag{4}$$

which requires the elimination of the masses for the transition from particles to light. Such a transition is evidently possible for Equation (1) in view of the opposite values of the rest energies, while being incompatible with special and general relativities on various counts.

2. 20th Century Positive Energy Antiparticles

To achieve compatibility with special and general relativities (which was an understandable task for the time), particle physics of the early 20th century [3] assumed indeed Dirac's equation as the fundamental equation of relativistic quantum mechanics, but under the theoretical assumption that *antiparticles have the same positive mass of particles* $\overline{m}_e = m_e > 0$. This assumption remains in full effect to this day resulting in a number of problematic aspects which deserve an inspection.

The above view was implemented by assuming that *antiparticles are the image of particles under the PCT theorem*, namely, by assuming that the antiparticle image of a particle with Hilbert state $|p,s,\psi\rangle$ (where p is the linear momentum, s is the value of the spin along the third axis and ψ is the particle wave-function), is given by [4]

$$PCT|p,s,\psi\rangle = (-1)^{J-s}|p,-s,n^c\rangle, \tag{5}$$

where parity *P*, charge conjugation C and time reversal operator T are characterized by

$$\begin{aligned} C|p,s,\psi\rangle &= |p,s,-\psi^\dagger\rangle, \\ CP|p,s,\psi\rangle &= |-p,-s,-\psi^\dagger\rangle, \\ T|p,s,\psi\rangle &= |-p,-s,\psi\rangle. \end{aligned} \tag{6}$$

The 20th century interpretation of Dirac's equation can be technically seen at the level of its symmetries. Rather than identifying the symmetry of Equation (1) as the Kronecker product of the symmetries of its two distinct 2 × 2-components (as done in the next section), physicists of the early 20th century solely considered the symmetry characterized by the 4 × 4 gamma matrices. By recalling that the electron and the positron have spin 1/2, the most important space-time symmetry is the *spinorial covering of the Poincaré symmetry* [5] [6] [7]

$$\mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}(3.1), \tag{7}$$

with realization of the generators in terms of Dirac's gamma matrices

$$\begin{aligned} &\mathcal{SL}(2.C): \ S_k = \frac{1}{2}\gamma_k \times \gamma_4, \ \ R_k = \frac{1}{2}\varepsilon_{ij}^k \gamma_i \times \gamma_j, \\ &\mathcal{T}(3.1): \ P_\mu, \end{aligned} \tag{8}$$

where S_k, R_k and T_μ are the generators of spin, boosts and linear momentum, respectively, with the familiar commutation rules for $J_{\mu,\nu} = \{S_k, R_k\}$

$$\begin{aligned} &\left[J_{\mu\nu}, J_{\alpha\beta}\right] = i\left(\eta_{\nu\alpha} J_{\beta\mu} - \eta_{\mu\alpha} J_{\beta\nu} - \eta_{\nu\beta} J_{\alpha\mu} + \eta_{\mu\beta} J_{\alpha\nu}\right), \\ &\left[J_{\mu\nu}, P_\alpha\right] = i\left(\eta_{\mu\alpha} P_\nu - \eta_{\nu\alpha} P_\mu\right), \ \left[P_\mu, P_\nu\right] = 0, \end{aligned} \tag{9}$$

and Casimir invariants

$$\begin{aligned} &C_1 = I, \\ &C_2 = P^2 = P_\mu P^\mu = \left(\eta^{\mu\nu} P_\mu P_\nu\right), \\ &C_3 = W^2 = W_\mu W^\mu, \ W_\mu = \varepsilon_{\mu\alpha\beta\rho} J^{\alpha\beta} P^\rho. \end{aligned} \tag{10}$$

Unfortunately, the above 20th century theory of antimatter is incompatible with Dirac's matter-antimatter annihilation into light according to the following:

THEOREM 1: The PCT definition of antiparticles, Equation (6), under the spinorial Poincaré symmetry, Equation (7), is incompatible with Dirac's particle-antiparticle annihilation into light, Equation (4).

PROOF: Within the class of unitary equivalence of relativistic quantum mechanics, there exists no possibility of rendering null the sum of two positive masses in Equations (10).

The above Theorem suggests a moment of reflection as to whether the recent test of the gravity of the anti-hydrogen atom [8] has actually tested the gravity of antimatter because:

1) Theorem 1 creates doubts as to whether the antiproton, which is the nucleus of the anti-hydrogen atom [9], is a true antiparticle because proton-antiproton collisions in the Bose-Einstein Correlation produces a shower of *massive particles* [10], rather than annihilation into electromagnetic radiation without massive particles.

2) In regard to the production of antiprotons via the collision of high energy protons against a matter-target (usually an iridium rod), it should be noted that Rutherford's [11] synthesis of the neutron in the core of stars (as the electron "compressed" within the proton) has been confirmed at the theoretical [12] and experimental [13] levels. These results suggest the possibility that the high ener-

gy collision of protons against a matter-target (thus initially against the electron clouds of the target) may synthesize the neutron, and subsequently, the *pseudo-proton* [14]. In this case, the detection of gravity in test [8] would be correct, but solely holding for ordinary matter because the pseudoproton is a particle with a fully positive mass [15] [16].

3) The *isodual mathematica* and related *isodual theory of antimatter* [17] (which have been constructed to provide a causal description of Dirac's negative energy antiparticles) predicts antigravity between matter and antimatter at all levels of study, from Newtonian mechanics to special and general relativities [18] [19] (see also Refs 7-10 of [8]).

In particular, the isodual mathematics maintain the full validity of special and general relativities only formulated on isodual spaces over isodual numeric fields. Matter-antimatter gravitational repulsion them emerges rather forceful in the projection of the *isoidual special and general relativities* in the space-time of conventional relativities [15].

Additionally, the isodual theory of antimatter implies that, to be true antimatter and allow valid gravity tests, antiparticles should solely annihilate into electromagnetic radiation without massive residues, as apparently confirmed by very large explosions in our skies without solid debris in the ground (for brevity, see the detailed analysis of Ref. [15]).

In conclusion, Theorem 1 casts shadow on the true antimatter character of 20th century antiprotons in view of the rather natural expectation that negative energy antiparticles and positive energy particles cannot *annihilate* into light, but merely experience ordinary *scattering* with the production of massive particles as established by the proton-antiproton scattering of the Bose-Einstein correlation [10].

3. Antigravity Tests

In the preceding sections, we have presented various arguments essentially implying doubts as to whether the recent test of the gravity of the anti-Hydrogen aton [8] has actually measured the gravity between matter and antimatter due to doubts on the actual antimatter character of 20th century positive energy antiprotons and, therefore, of the nucleus of the anti-Hydrogen atom.

By keeping in mind that the sole antiparticle with proved antimatter character is the positron in view of annihilation (4), R. M. Santilli [16] [20] proposed in 1994 to conduct comparative measurements of the gravity of very low energy (thermal) electron and positron beams in horizontal flight in a supercooled vacuum tube.

In particular, the length of the tube can be selected for given eV in such a way to allow a *visible* deviation when the beams hit a terminal scientillator, and the radius of the tube can be selected to render stray field fluctuations smaller than the expected terminal deviation upward (downward) due to Earth's gravity when the positron beam hits a terminal scintillator, by therefore clearly indicating antigravity (gravity)

Detailed studies by A. P. Mills [21] [22], V. de Haan [23] [24] and other experimentalists confirmed the feasibility and resolutory character of the proposed comparative tests [16] [20].

As indicated at the 1996 international Workshop on Antimatter, Sepino, Italy, the 2011 San Marino Workshop on Antimatter, the 2016 SIPS Conference, Hainan Island, China, the 2023 SIPS Conference in Panama and other meetings, the gravity of antimatter should be tested for particles with independently proved antimatter character, such as the positrons, prior to any final claim in favor or against antigravity.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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