Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2024, 12, 1626-1646
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

# Functional Dependence of IM and HV Angle in Hallux Valgus Deformity before and after Operative

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**How to cite this paper:** Stojanović, N., Sovilj, M. and Ivanišević, V. (2024) Functional Dependence of IM and HV Angle in Hallux Valgus Deformity before and after Operative. *Journal of Applied Mathematics and Physics*, **12**, 1626-1646.
https://doi.org/10.4236/jamp.2024.125101

**Received:** March 23, 2024
**Accepted:** May 10, 2024
**Published:** May 13, 2024

## Abstract

Hallux valgus is a relatively common and multifaceted complex deformity of the front part of the foot. It is the result of multiple effects of innate (endogenous) and exogenous etiological factors with different degrees of influence. The degree of hallux valgus deformity is usually assessed by radiological values of hallux valgus (HV) and intermetatarsal (IM) angles. The aim of the paper is to justify the definition of hallux valgus deformity as a function of one angle, (HVA or IMA), and then to determine the functional connection and the most suitable function equalizing the values of the angles IMA and HVA. As hallux valgus is a double angulation deformity, the analytically determined connection between the HVA and IMA angles reduces the study of the deformity to the study of function with one argument, and makes the analysis of deformity changes before and after operative treatment simpler. For the determined connections between the angles, the values of linear proportionality coefficients and regression coefficients of corresponding linear functions of analytical equalization of the value of the IM angle and the degree of deformity for a given value of the HV angle were experimentally determined. The obtained results were checked on a sample of 396 operatively treated hallux valgus deformities. The presented analytical approach and the obtained functional links of IMA and HVA enable quantitative observation of the change in the degree of deformity based on the radiologically determined value of these angles, and the established nonlinear function will be useful for evaluating the expected value of the IM angle and the degree of deformity based only on the measured value of the HV angle.

## Keywords

Functional Connection of Deformity Angles, Analytical Equalization of IM

Angle Values, Hallux Valgus, Intermetatarsal Angle, Correction of Deformity

## 1. Introduction

Hallux valgus is a relatively common and multifaceted complex deformity of the front part of the foot. Etiologically, it is the result of multiple effects of endogenous and exogenous etiological factors with different degrees of influence, and it seems more and more that it is a combination of anomalies and acquired deformity [1] [2] [3] [4]. These are complex pathological anatomical changes that result in a double angular deformity of the first row of the foot, dominated by valgus displacement of the big toe with an increased hallux valgus angle (HVA) and an unstable metatarsophalangeal joint, and varus of the first metatarsal bone (I MT) with an increase in the intermetatarsal angle (IMA). And instability of the first metatarsocuneiforms joint [1] [4] [5].

The third aspect of the complexity of this deformity is particularly challenging and relates to the concept of its surgical treatment. It aims to correct the deformity and establish biomechanically favorable anatomical relationships of the bony and joint structures of the front part of the foot and in this way ensure a dynamically stable function of the foot. So far, over 130 operative techniques and their modifications have been described, none of which has the potential to correct all components of the deformity [6] [7] [8] [9]. This is understandable when we take into account the fact that in practice no two deformities are exactly the same because as Robinson points out [5]: "Everyone will have their own shade".

So far, several algorithms and recommendations have been published regarding the choice of the appropriate surgical technique and their combinations, which are based on the application of the principles of surgical treatment and the experiences of teams of orthopedic surgeons and podiatrists [10] [11].

The mentioned recommendations and reached consensuses made a great contribution, but at the same time they are burdened by the subjective influence of authority, which is confirmed by the research of Pinney *et al.* [12] in which over 100 orthopedic surgeons of the academic level expressed their opinion regarding the choice of surgical method of treatment for a given case. The assumption for choosing an adequate surgical method or their combined application is that the surgeon fully understands and observes the pathological anatomical changes that primarily occur at the level of the first row (medial column) of the foot, *i.e.* from the medial cuneiform bone to the distal phalanx of the big toe for each case separately [13] [14].

In order to define the severity - degree of hallux valgus deformity, a widely accepted classification was established according to the radiological values of HVA and IMA that define this double angular deformity [6] [15] [16].

- Mild deformity, in which the HVA is less than 30˚, and the IMA is less than 13˚,

- Moderate deformity, in which the HVA is less than 40˚, and the IMA is less than 20˚,
- Severe deformity, in which the HVA is greater than 40˚, and the IMA is greater than 20˚.

Since it is a double angulation deformity in which the anatomical relations at the level of two adjacent joints are disturbed and which have a mutual influence in the progression of the deformity, we consider it justified to investigate the functional connection of these two angles so that the double angular deformity could be expressed as a function of one of the angles (HVA or IMA) and thereby enable an integral examination of the change in deformity before and after surgery (correction), without the need to measure both the HV and IM angles again after surgical treatment.

## 2. Materials and Methods

The functional relationship of HVA and IMA and the choice of the function for the analytical equalization of the value of the IM angles for the given values of the measured HV angles was analyzed using the geometric-analytical method. The correctness of the obtained results was checked on a sample of surgically treated feet. At the same time, the observational research represents a descriptive analytical study that analyzed 396 operatively treated feet with pronounced hallux valgus deformity that were treated at the Institute for Orthopedic Surgical Diseases “Banjica” in Belgrade. All patients, upon admission, gave their consent that the data from their medical records can be used for research purposes and all applied aspects of the study were approved by the institution. The consent of the Ethics Committee of the Institute was also obtained for this study.

In order to carry out preoperative planning, an X-ray was taken in the AP and LL position of the foot with a load at an angle of 15 degrees and from a distance of 1 m. On the obtained X-rays before and after the operative treatment, measurements were made in accordance with the recommendations of the ad hoc committee of the American Foot and Ankle Orthopedic Association (AOFAS) [17] and in addition to other parameters, the following values were determined:

1) Hallux valgus angle (HVA, **Figure 1**), which is obtained by closing the axis of the first metatarsal bone and the proximal phalanx. A value of up to 15˚ is considered a normal finding, a mild deformity for a value of up to 30˚, a moderate deformity of 30˚ to 40˚ and a severe deformity in which the HVA is greater than 40˚.

2) Intermetatarsal angle (IMA, **Figure 1**), which is the angle between the axes of the I and II metatarsal bones. A value up to 9˚ is considered normal, mild deformity if the IMA value is up to 13˚, moderate deformity from 13˚ to 20˚ and severe if the IMA is 20 or more degrees [6] [15].

In the third part of the paper, the results of the evaluation of the linear and non-linear connection of IM and HV angles are presented. That is, in the first part, a statistically analytical approach to the evaluation of the coefficient of linear proportionality between IM and HV angles is presented, and in the second,

**Figure 1.** Basic radiological measurement parameters (HVA and IMA) of the assessment of hallux valgus deformity.

the non-linear function of the evaluation of the IMA value, if the value of HVA is known, is examined. In the last, fourth part, the analysis of the assessment of the value of the degree of deformity by a nonlinear function, for the measured values of HVA, is presented.

## 3. Results and Discussion

### 3.1. Proportionality of HV and IM Angles before and after Operative Treatment

Since hallux valgus is a double angulation deformity, research into the functional connection of HVA and IMA aims to express the double angular deformity as a function of one of these angles, in order to simplify the integral observation of the change in deformity before and after operative treatment. Considering this goal, these two questions arise. Is there a specific relationship (connection) between HVA and IMA and does the value of IMA also increase with the increase in the value of HVA? We can get a statistical answer to these questions based on the value of the Pearson linear correlation coefficient, which describes the strength and direction of the linear relationship between these two variables.

By checking, on a sample of 396 operatively treated feet, for the purposes of this work, a strong positive linear correlation between HVA and IMA, R = 0.541, Sig. = 0.000 before operative treatment and a positive relationship of medium strength R = 0.472, Sig. = 0.000 was determined after operative treatment, at the level of significance p = 0.01.

In both cases, it is expected that higher values of HVA correspond to higher values of IMA **Table 1**.

So, we can assume that the sizes of IMA and HVA before and after the operative treatment of hallux valgus deformity are proportional, with the coefficient of proportionality $k \in \mathbb{R}^+$, that is, the connection between these angles can be expressed in the form of a relation

$$\frac{\text{IMU}}{\text{HVU}} = k \Rightarrow \text{IMA} = k \cdot \text{HVA} \tag{1}$$

If such a relationship exists, then the proportionality coefficient is constant and does not depend on the choice of HVA or IMA.

We determined the value of the proportionality coefficient k before and after operative treatment experimentally, on a sample of 396 treated feet. We checked the dependence of the proportionality coefficient on the choice of operative treatment method. In the analyzed sample, two methods of operative treatment were applied, the Chevron method and the Golden method. Two hundred nine feet with deformity were treated with the Chevron method, and 187 with the Golden method.

The coefficient of proportionality of the angles, IMA, HVA, before the treatment is denoted by $k_p$, and after the operative treatment by $k_o$. Let's first calculate the average value of the proportionality coefficient of the angles before the operative treatment at the sample level.

For the measured values of HVA and IMA before the operative treatment of the deformity, the value of the proportionality coefficient was first calculated $k_i = \frac{\text{IMU}_i}{\text{HVU}_i}$.

$i = 1, 2, \cdots, 396$ for each foot with a deformity, and then the average value of the proportionality coefficient was calculated at the sample level of $N$ = 396; $\overline{k_p} = \frac{1}{N}\sum_{i=1}^{N} k_i = 0.4224$ with a standard deviation, SD = 0.0999, an error of the

**Table 1.** Values of Pearson coefficients of linear correlation between relevant angles HVA and IMA.

| | | Before surgery | After surgery |
|---|---|---|---|
| | | HVA | HVA |
| IMA | R | 0.541** | 0.472** |
| | Sig. (2-tailed) | 0.000 | 0.000 |
| | N | 396 | 396 |

**. Correlation is significant at the 0.01 level (2-tailed).

mean value, SE = 0.005 and 95% CI: from 0.4125 to 0.4323, with a range of values from Min. = 0.23 to Max = 0.81. Furthermore, the average value of the proportionality coefficient before and after the operation was calculated treatment for each of the mentioned methods and the following results were obtained.

The average value of the coefficient of proportionality of the measured angles, HVA and IMA, before operative treatment of the deformity using the Chevron method: $\overline{k_p}^C = 0$ with standard deviation SD = 0.11054, and SE = 0.0076, and 95% CI: from 0.4125 to 0.4427 and the range of values from the minimum Min = 0.23 to the maximum Max. = 0.81, and for deformities treated with the Golden method, the average value of the coefficient before surgical treatment is: $\overline{k_p}^G = 0.4166$ with SD = 0.0866, and error of assessment SE = 0.0063, and 95% CI from 0.4041 to 0.4291 and range of values from Min. = 0.23 to Max. = 0.77 (Table 2).

It was found that the calculated average values of proportionality coefficients in feet treated with the Chevron method and the Golden method before surgical treatment do not differ statistically significantly: t($n$ = 396, df = 387,411) = 1.104, Sig. = 0.270. The average value of the difference of the coefficients was M(R) = 0.01096, with a standard error, SE = 0.01006, and 95% CI (R): from −0.00882 to 0.03074, and did not show statistical significance, at the significance level of p = 0.05 Table 2 and Table 3.

Based on this, we can conclude that the average value of the proportionality coefficient of IMA and HVA in a sample of $N = 396$ feet with deformity before surgical treatment is $k_p$ = 0.4224, SD = 0.0999 and SE = 0.0005, and the 95% CI: from 0.4125 to 0.4323 and the range of values from Min. = 0.230 to Max. = 0.811 Table 2.

Therefore, the values of the $IMA_p$ angle before the operative treatment can be expressed in relation to the measured value of the hallux valgus angle using the $HVA_p$ relation

**Table 2.** Descriptive indicators of the assessment of the proportionality coefficient of IMA and HVA before and after operative treatment using the Chevron method and the Golden method.

| Coefficient proportionality | Method of treatment | N | Mean (Me) | Std. Dev. (SD) | Std. Error (SE) | 95% Confidence Interval for Mean | | Min. | Max. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound | | |
| Before the operative treatment, $k_p$ | Chevron | 209 | 0.4276 | 0.1105 | 0.0077 | 0.4125 | 0.4427 | 0.23 | 0.81 |
| | according to Golden | 187 | 0.4166 | 0.0866 | 0.0063 | 0.4041 | 0.4291 | 0.23 | 0.77 |
| | Total | 396 | 0.4224 | 0.0999 | 0.0050 | 0.4125 | 0.4323 | 0.23 | 0.81 |
| After operative treatment, $k_o$ | Chevron | 209 | 0.6034 | 0.3740 | 0.0259 | 0.5524 | 0.6544 | 0.11 | 2.60 |
| | according to Golden | 187 | 0.6476 | 0.3768 | 0.0276 | 0.5933 | 0.7020 | 0.00 | 3.00 |
| | Total | 396 | 0.6243 | 0.3755 | 0.0189 | 0.5872 | 0.6614 | 0.00 | 3.00 |

**Legend:** **N**-number of treated feet, **Me**-mean value of proportionality coefficient, **SD**-standard deviation, **SE**-standard error of evaluation, **95% CI**-95%-Interval of average values, **Min.**-minimum value, **Max.**-maximum coefficient values.

**Table 3.** Results of the comparison of the coefficient of proportionality by the test of independent samples of deformity treatment by the Chevron method and the Golden method before and after operative treatment.

| Proportionality coefficient before and after treatment | | Leven's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | F | Sig. | t | | Sig. 2-taild | Mean Difference M(R) | Std. Error Difference SE | 95% Confidence Interval of the Difference | |
| | | | | | | | | | Lower | Upper |
| $k_p$ | Equal variances | 8.642 | 0.003 | 1.089 | 394 | 0.277 | 0.01096 | 0.01006 | −0.00882 | 0.03074 |
| | Unequal variances | | | 1.104 | 387.411 | 0.270 | 0.01096 | 0.00993 | −0.00856 | 0.03048 |
| $k_o$ | Equal variances | 0.305 | 0.581 | −1.172 | 394 | 0.242 | −0.04426 | 0.03778 | −0.11853 | 0.03001 |
| | Unequal variances | | | −1.171 | 388.502 | 0.242 | −0.04426 | 0.03779 | −0.11857 | 0.03004 |

$$\mathrm{IMA}_p = k_p \cdot \mathrm{HVA}_p = 0.4224 \cdot \mathrm{HVA}_p . \tag{2}$$

The graphic representation of the linear function from (2) is shown in **Figure 2**. From relation (2), the dependence of HVA on the value of IMA is

$$\mathrm{HVA}_p = k'_p \cdot \mathrm{IMA}_p \text{, gdje je } k'_p = \frac{1}{k_p} \tag{3}$$

The value of the proportionality coefficient $k'_p$ is in the interval [1.91 - 3.10] and shows that the values of HVA are about ≈2 to ≈3 times higher than IMA. Furthermore, the calculated average value of the coefficient of proportionality of HVA and IMA after operative treatment on a sample of 396 treated feet is $k_0$ = 0.6243, with SD = 0.3755 and error of evaluation SE = 0.0189, and 95% CI: from 0.5872 to 0.6614 and a range of values from the minimum 0 to a maximum of 3, **Table 2**.

The t-test of independent samples confirmed that there is no statistically significant difference, t($n$ = 396, df = 394) = −1.172, Sig. = 0.242, between the average value of the coefficient of proportionality of those angles, for deformities treated with the Chevron method: $k_0^C = 0.6034$, with SD = 0.37399 and SE = 0.2587 and 95% CI from 0.5524 to 0.6544 and a range of values from Min. = 0.1071 to Max. = 2.6, and deformities treated according to Golden: $k_0^G = 0.6477$, SD = 0.3768, and SE = 0.2755, and 95% CI from 0.5933 to 0.7020 with values from Min. = 0 to Max. = 3, **Table 2** and **Table 3**. The average difference between the means M(R) = −0.4426, with SE = 0.03778, and 95% CI(R): from −0.11853 to 0.03001, did not show statistical significance, at the level of significance p = 0.05.

On the basis of the above, we conclude that the value of the deformity coefficient of IMA and HVA after operative treatment can be taken as the average value $k_0$ = 0.6243 calculated on a sample of 396 treated feet, with the specified 95% CI interval, at an error level of 5%.

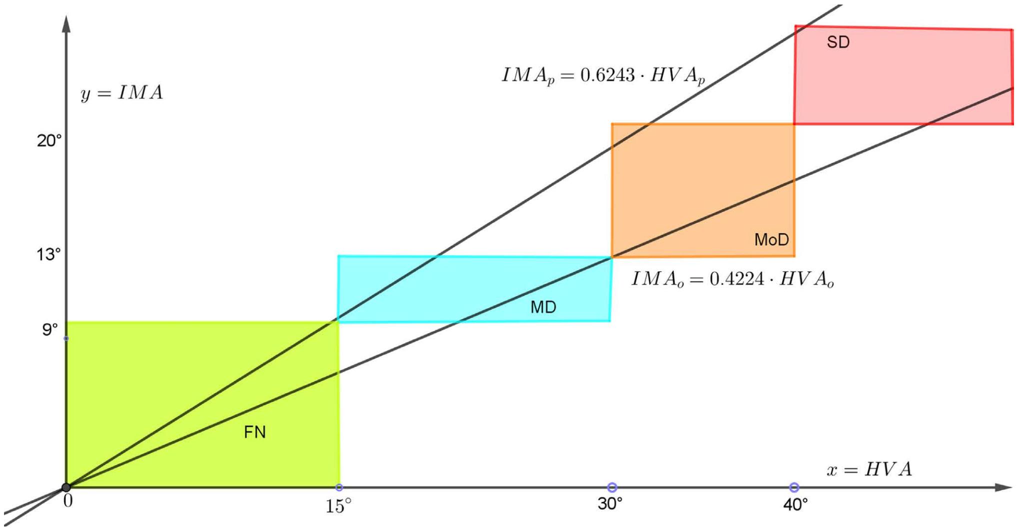


**Figure 2.** Graphic representation of proportionality coefficient and linear function of HVA and IMA before and after operative treatment, with degree of deformity. Legend: FN-normal finding, MD-mild deformity, MoD-moderate deformity, SD-severe deformity, HVA-hallux valgus angle, IMA-intermetatarsal angle.

It follows that the values of the IMAo angle after operative treatment can be expressed in relation to the measured value of the hallux valgus angle, $\text{HVA}_o$, by the relation

$$\text{IMA}_o = k_o \cdot \text{HVA}_o = 0.6243 \cdot \text{HVA}_o \tag{4}$$

The graphic representation of the functional connection (4) is shown in **Figure 2**. From relation (4) it also follows that the value of the angle $\text{HVA}_o$ in relation to the measured value of $\text{IMA}_o$ after operative treatment is given by the relation

$$\text{HVA}_o = k'_o \cdot \text{IMA}_o \text{, and } k'_o = \frac{1}{k_o} \tag{5}$$

The value of the coefficient $k'_p$ is in the interval [0.00; 4.02] Foot deformities in the plane of deformity on a sample of $N = 396$ are determined by points (HVA, IMA) for the measured value of HVA and the calculated values of $\text{IMA}_p$ before **Figure 3** and after operative treatment $\text{IMA}_o$, **Figure 4** show that relations (2) and (4) well approximate the relationship of the given angles.

The analysis of the dependence of the coefficient of proportionality of HVA and IMA on the method of treatment and degree of deformity showed that in the group with mild deformity there is no statistically significant difference between the average values of the proportionality coefficients of deformities treated with the Chevron method; ($N = 54$, Me = 0.4627, SD = 0.08924) and deformities treated with the Golden method; ($N = 17$, Me = 0.4296, SD = 0.06036): t(69) = 1.421, Sig. = 0.159, at the significance level p = 0.05, as well as in the group of feet with moderate deformity treated with the Chevron method ($N = 47$, Me = 0.4443, SD = 0.0457) and feet treated according to Golden ($N = 67$, Me = 0.4459, SD = 0.04879); t(112) = −0.178, Sig. = 0.859, **Table 4**.

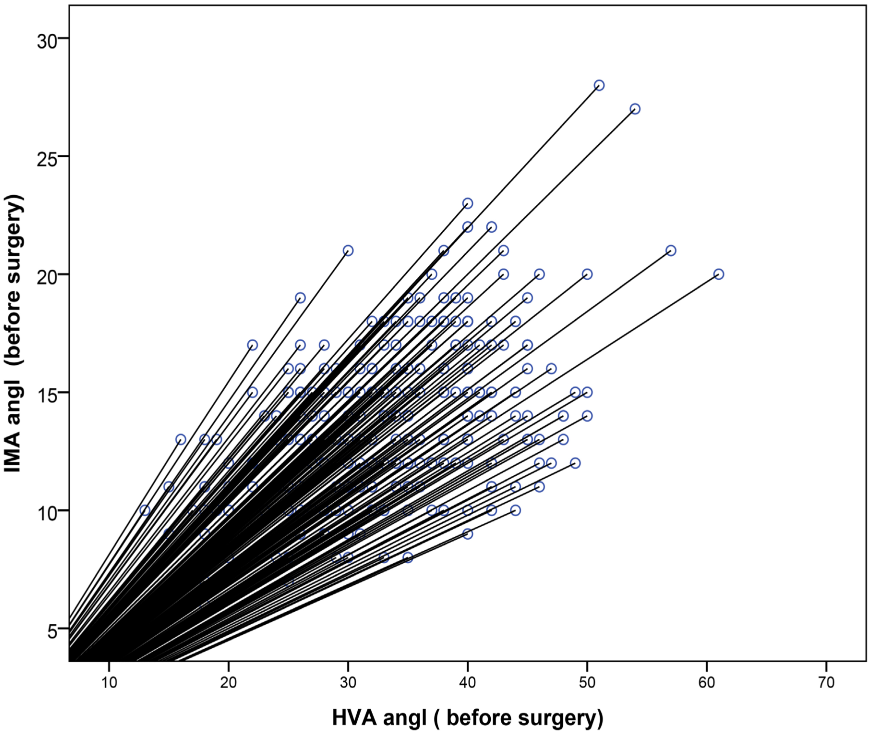


**Figure 3.** Distribution of deformities before operative treatment in relation to measured values of HV angles on a sample of N = 396 treated feet and proportionality coefficients.

**Table 4.** Proportionality coefficients of IMA and HVA in relation to the degree of deformity included in the conditions of radiological classification.

| Degree of deformity | Operational requirements | N | Mean (Me) | (Std. Error) (SE) | Std. Dev. (SD) | Min | Max | T(df) | p |
|---|---|---|---|---|---|---|---|---|---|
| MD<br>$15 \leq HVA < 30$<br>$\wedge 9 \leq IMA < 13$ | Chevron | 54 | 0.4627 | 0.0124 | 0.08924 | 0.34 | 0.73 | $t(69) = 1.423$ | 0.159 |
| | according to Golden | 17 | 0.4296 | 0.0146 | 0.06036 | 0.32 | 0.56 | | |
| | Total | 71 | 0.4548 | 0.00997 | 0.08409 | 0.32 | 0.73 | | |
| MoD<br>$30 \leq HVA < 40$<br>$\wedge 13 \leq IMA < 20$ | Chevron | 47 | 0.4443 | 0.00665 | 0.0457 | 0.34 | 0.56 | $t(112) = -0.178$ | 0.859 |
| | according to Golden | 67 | 0.4459 | 0.00596 | 0.04879 | 0.34 | 0.55 | | |
| | Total | 114 | 0.4453 | 0.00443 | 0.04729 | 0.34 | 0.56 | | |
| SD<br>$HVA \geq 40$<br>$\wedge IMA \geq 20$ | Chevron | 4 | 0.3991 | 0.03674 | 0.07348 | 0.33 | 0.50 | $t(9) = -3.037$ | 0.014 |
| | according to Golden | 7 | 0.5123 | 0.01930 | 0.05106 | 0.43 | 0.58 | | |
| | Total | 11 | 0.4711 | 0.02421 | 0.08029 | 0.33 | 0.58 | | |
| **Total** | | 196 | | | | | | | |

**Legend:** MD-mild deformity, MoD-moderate deformity, SD-severe deformity, HVA-hallux valgus angle, IMA-intermetatarsal angle.

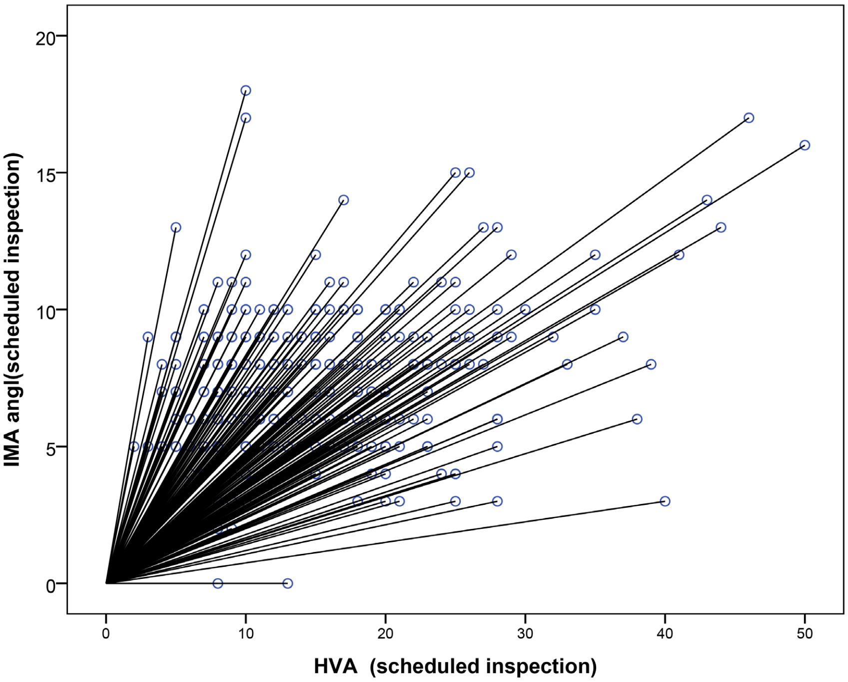


**Figure 4.** Distribution of deformities after operative treatment in relation to measured values of HV angles on a sample of N = 396 treated feet and proportionality coefficients.

However, the average value of proportionality coefficients in the group of feet with severe deformity treated with the Chevron method; (*N* = 4, Me = 0.3991, SD = 0.07348) is statistically significantly different; t(9) = −0.3.037, Sig. = 0.014, at the level of significance p = 0.05 from the average value of deformities treated by the Golden method (*N* = 7, Me = 0.5123, SD = 0.05106), **Table 4**.

Using the obtained results of the coefficient of proportionality before and after the operative treatment, the values of the lower and upper limits of the IMA angle were calculated based on the measured value of HVA and compared with the average values measured in each category of degree of deformity, **Table 5**.

The analysis included 196 feet with deformity included in the classification based on HVA and IMA values, and the other 200 cases could not be included by this classification method [18].

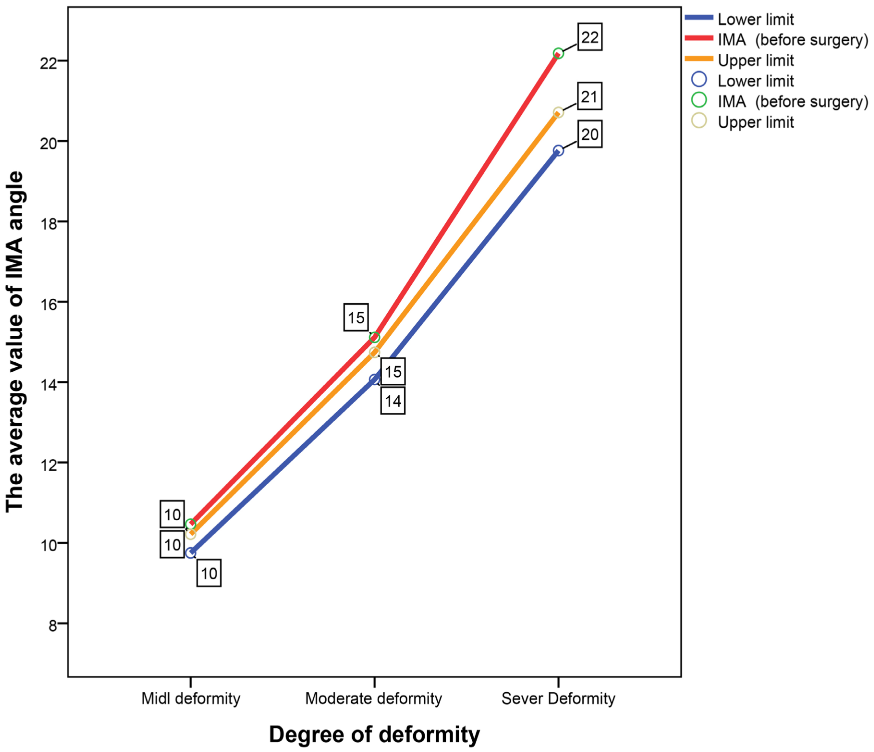
The results showed that there are no significant deviations in the average values of the lower and upper limits of the average value of IMA before operative treatment in mild and moderate deformities **Table 5**, and **Figure 5(a)**, while a statistically significant ifference was observed in the category of severe deformities. **Table 5**, and **Figure 5(b)**.

The graphic presentation of the estimated lower ($M_e$(Dg) = 10.9433, SD = 8.52663) and upper limit ($M_e$(Gg) = 12.3261, SD = 9.60408) value of the IM angle on the observed sample, **Figure 5(a)**, showed that the average value of the difference between the evaluated limits and the measured average value in the sample ($Me_o$ = 8.82, SD = 3.3737) was significantly lower before than after operative treatment **Figure 5(b)**. This difference was especially noticeable the case of severe deformities.

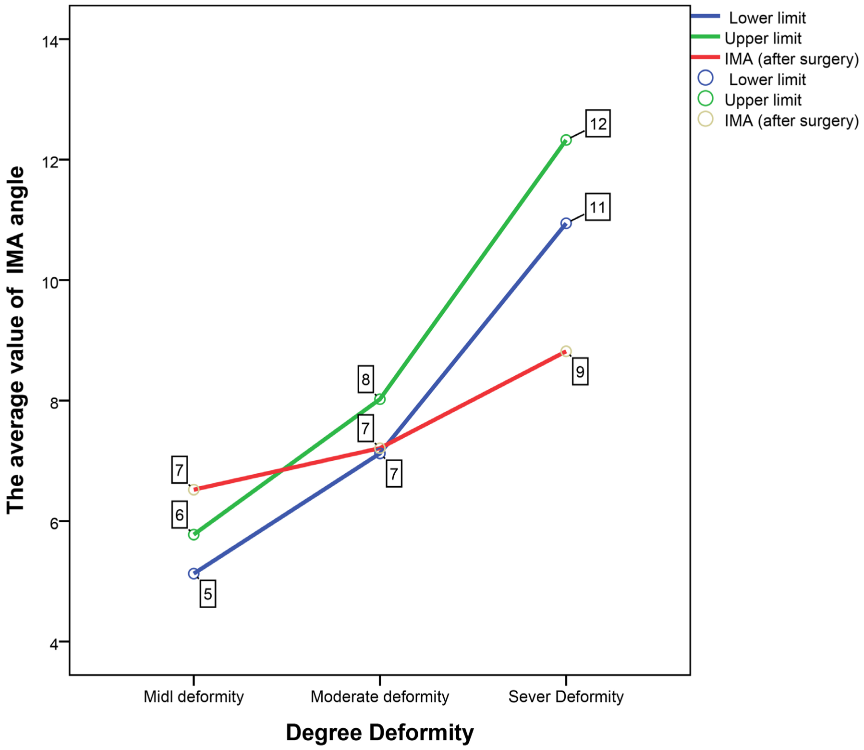
**Table 5.** Evaluation of the lower (Dg) and upper (Gg) limits of IMA values for given proportionality coefficients and measured HVA values before and after operative treatment according to deformity category.

| Degree Deformity | Evaluated statistical indicators | Assement of the value of the IMA angle before operative treatment | | IMA (before operative treatment) | Assessment of the value of the IMA angle after operative treatment | | IMA (after operative treatment) |
|---|---|---|---|---|---|---|---|
| | | Dg of IMA angle $k_d = 0.4125$ | Gg of IMA angle $k_g = 0.4323$ | | Dg of IMA angle $k_d = 0.5872$ | Gg of IMA angle $k_g = 0.6614$ | |
| Mild deformity | N | 71 | 71 | 71 | 71 | 71 | 71 |
| | Mean (Me) | 9.7489 | 10.2169 | 10.46 | 5.1277 | 5.7756 | 6.52 |
| | Std. Deviation (SD) | 1.61612 | 1.69369 | 0.939 | 4.31840 | 4.86408 | 2.568 |
| | Std. Error of Mean (SE) | 0.19180 | 0.20100 | 0.111 | 0.51250 | 0.57726 | 0.305 |
| | Min IMA | 6.19 | 6.48 | 9 | −10.57 | −11.91 | −6 |
| | Max IMA | 11.96 | 12.54 | 12 | 15.27 | 17.20 | 13 |
| Moderate deformity | N | 114 | 114 | 114 | 114 | 114 | 114 |
| | Mean (Me) | 14.0648 | 14.7399 | 15.11 | 7.1237 | 8.0238 | 7.21 |
| | Std. Deviation (SD) | 1.16927 | 1.22539 | 1.713 | 4.30476 | 4.84872 | 2.184 |
| | Std. Error of Mean (SE) | 0.10951 | 0.11477 | 0.160 | 0.40318 | 0.45412 | 0.205 |
| | Min IMA | 12.38 | 12.97 | 13 | −2.94 | −3.31 | 0 |
| | Max IMA | 16.09 | 16.86 | 19 | 20.55 | 23.15 | 15 |
| Severe deformity | N | 11 | 11 | 11 | 11 | 11 | 11 |
| | Mean (Me) | 19.7625 | 20.7111 | 22.18 | 10.9433 | 12.3261 | 8.82 |
| | Std. Deviation (SD) | 2.96573 | 3.10808 | 2.822 | 8.52663 | 9.60408 | 3.737 |
| | Std. Error of Mean (SE) | 0.89420 | 0.93712 | 0.851 | 2.57088 | 2.89574 | 1.127 |
| | Min IMA | 16.50 | 17.29 | 20 | 4.11 | 4.63 | 4 |
| | Max IMA | 25.16 | 26.37 | 28 | 29.36 | 33.07 | 16 |
| Total | N | 196 | 196 | 196 | 196 | 196 | 196 |
| | Mean (Me) | 12.8212 | 13.4366 | 13.83 | 6.6150 | 7.4509 | 7.05 |
| | Std. Deviation (SD) | 3.03968 | 3.18558 | 3.383 | 4.81401 | 5.42232 | 2.478 |
| | Std. Error of Mean (SE) | 0.21712 | 0.22754 | 0.242 | 0.34386 | 0.38731 | 0.177 |
| | Min. IMA | 6.19 | 6.48 | 9 | −10.57 | −11.91 | −6 |
| | Max. IMA | 25.16 | 26.37 | 28 | 29.36 | 33.07 | 16 |

**Legend:** Dg-lower limit of IMA value, Gg-upper limit of IMA value, Min. IMA-The minimum value of the IMA angle, Max. IMA-the maximum value of the IMA angle.



(a)



(b)

**Figure 5.** (a) Graphic representation of the lower and upper limits of the assessed and measured IMA values before operative treatment. (b) Graphic representation of the lower and upper limits of the assessed and measured IMA values after operative treatment.

### 3.2. Algebraic Equalization of IMA Values before and after Operative Treatment with a Non-Linear Function

Let's examine the dependence of the value of the IMA = $y$ angle on the value of the HVA = $x$ angle by observing the non-linear function

$$y = -\alpha + \beta \cdot \log x \tag{6}$$

where $\alpha$, $\beta$ are regression coefficients, and which has the characteristic of being linear in the XOY coordinate system; if $Y = y,\ X = \log x$. The function (6) can also be written in the form

$$y = \beta \cdot \left( -\frac{\alpha}{\beta} + \log x \right) = \beta \cdot \left( \log 10^{-\frac{\alpha}{\beta}} + \log x \right)$$

That is, in the form

$$y = \beta \cdot \log \frac{x}{10^{\frac{\alpha}{\beta}}} \tag{7}$$

In order to evaluate the model (6) or (7), the relationship between the quantities $Y = y$, $X = \log(x)$ was first investigated using the Pearson linear correlation coefficient. A strong positive correlation was calculated between these two variables $r = 0.531$, $N = 396$, p = 0.000. Based on the value of the linear correlation coefficient, the coefficient of determination, $r^2 = 0.282$ was determined, which shows that our model explains 28.2% of the variance of IMA, $Y = y$. It was determined that the model reaches statistical significance, F(1, 395) = 154.459, Sig. = 0.000 at the level of statistical significance p = 0.05.

The values of coefficients $\alpha$, $\beta$ were determined by regression analysis. The determined values of the coefficients are: $\alpha = -9.91$, $\beta = 15.528$, Table 6.

Based on the obtained values of the regression coefficients $\alpha$, $\beta$ we find that the equation for evaluating the value of IM angles for the measured values of HVA reads

$$y = -9.91 + 15.528 \cdot \log x \tag{8}$$

Or, after calculating the value of the exponent $\alpha/\beta = 0.6382$, the equation can also be written in the form

$$y = 15.528 \cdot \log \frac{x}{10^{0.6382}} \tag{8*}$$

**Table 6.** Results of the assessment of regression coefficients.

| Model | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | Correlations | | F |
|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | | | Partial | Part | |
| Constant | −9.910 | 1.881 | | −5.268 | 0.000 | | | 154.459 |
| log(HVA) | 15.528 | 1.249 | 0.531 | 12.428 | 0.000 | 0.531 | 0.531 | Sig. = 0.000 |

Dependent variable: $Y = IMA$ angle.

Since $10^{0.6382} \approx 4.645$ is the Equation (8*) of the estimated value of IMA = *y*, for the measured value of the angle HVA = *x* it has the form

$$y = 15.528 \cdot \log \frac{x}{4.645} \tag{9}$$

A graphic representation of the value of IMA evaluated by the function $y = -9.91 + 15.528 \cdot \log(x)$ for the given values of HVA according to the degree of deformity is shown in Figure 6.

Note that for a given value of IMU = *y*, we can determine the value of HVA = *x* using the inverse function of function (8) using the formula

$$\log x = 0.0644 \cdot y + 0.638 \tag{10}$$

or formulas

$$x = 10^{\frac{y+9.91}{15.528}} \tag{11}$$

The t-test of paired samples showed that there is no statistically significant difference Me(D) = 0.0003, SD = 2.69842, SE = 0.1356, with 95% CI: from −0.26629 to 0.26689, t(395) = 0.002, Sig. = 0.998, on a sample of *N* = 396 feet between the average value of the measured IMA sizes: Me = 13.41, SD = 3.184, SE = 0.160, and the average value of the IMA sizes calculated using the Formula (8) or (9) Me = 13.409, SD = 1.689, SE = 0.0849, Table 7.

From the above, we conclude that Formulas (8) or (9) approximates IMA values well, which can be seen in the graphic, Figure 7.

**Example 1.** Let's calculate the values of the IM angles and the limits of the grade of the degree of deformity classification if the value of the HVA angles is given.

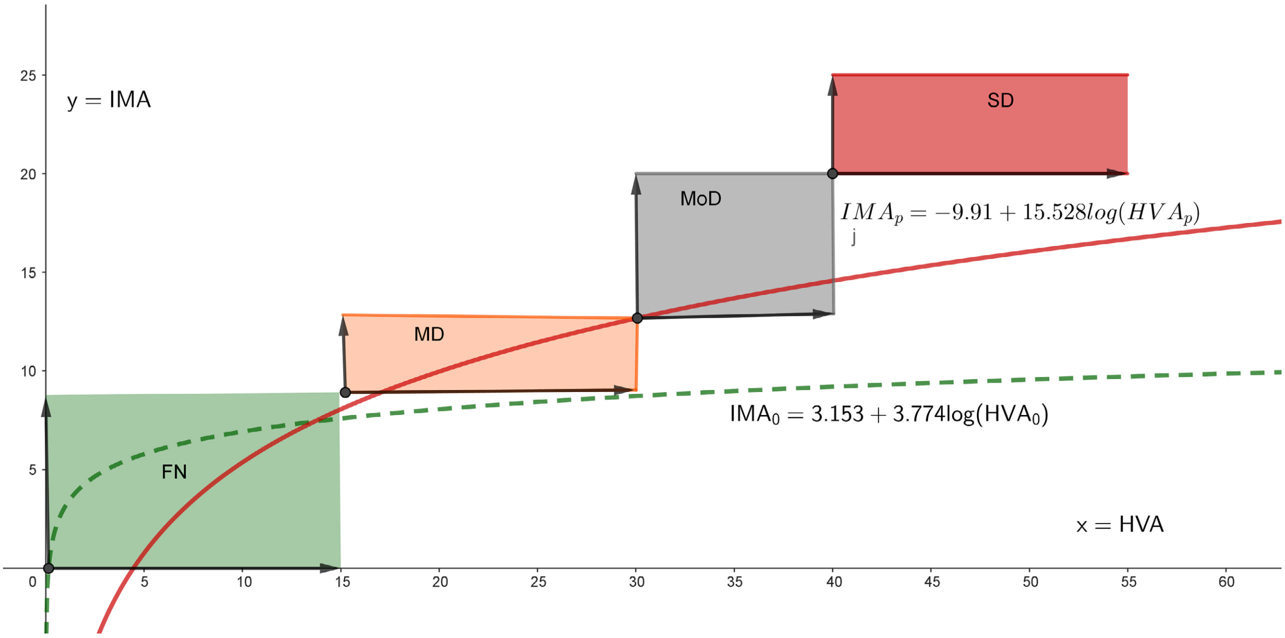


Figure 6. Graphic presentation of IMA value assessment before operative treatment with the function $y = -9.91 + 15.528 \cdot \log(x)$, and after operative treatment with the function $y = 3.153 + 3.774 \cdot \log x$ according to the degree of deformity. Legend: FN-normal finding, MD-mild deformity, MoD-moderate deformity, SD-severe deformity, HVA-hallux valgus angle, IMA-intermetatarsal angle.

Table 7. Application of the t-test in checking the statistical significance of the difference between the measured mean values of the IM angle.

| IMA angle | Paired Differences | | | | | t | *df* | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | Mean Me(D) | Std. Deviation (SD) | Std. Error Mean (SE) | 95% Confidence Interval of the Difference | | | | |
| | | | | Lower | Upper | | | |
| $IMA_1$ i $IMA_2$ | 0.0003 | 2.69842 | 0.13560 | −0.26629 | 0.26689 | 0.002 | 395 | 0.998 |

**Legend**: IMA1-measured values of the IM angle in a sample of N = 396 operatively treated feet, and IMA2-values of the IMA angle calculated using Formula (9).

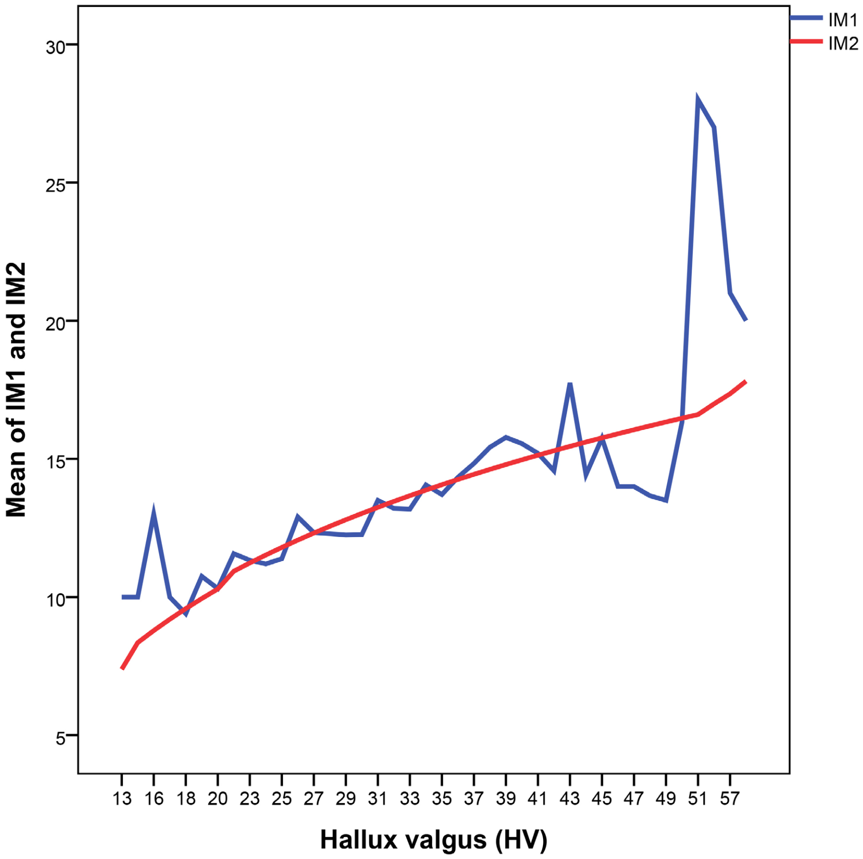


Figure 7. Approximation of the IMA value using Formula (8) or (9) on a sample of 396 surgically treated deformities. Legend: IM1- measured value of the IM angle in the experiment, IM2 estimated value of the IM angle by Formula (8).

The limits of the degree of classification of the absolute value of deformity calculated using the formula for conjugate deformity (absolute value of deformity) $d = \sqrt{x^2 + y^2}$ [18] were also calculated. The results of the calculated values are presented in Table 8 and Table 9.

Let's check the functional connection between HVA and IMA after operative treatment by observing the function

$$y = a + b \cdot \log x \tag{12}$$

Table 8. Degrees of deformity and calculated values of IM angles using Formula (22).

| **Classification of deformities** | **Normal condition** | **Mild deformity** | **Moderate deformity** | **Severe deformity** |
|---|---|---|---|---|
| **HVA** | 0˚ - 15˚ | 15˚ - 30˚ | 30˚ - 40˚ | ≥40˚ |
| **IMA** | 0˚ - 8.35˚ | 8.35˚ - 13.32˚ | 13.32˚ - 14.97˚ | ≥15˚ |
| **Absolute value of deformity(d)*** | 0˚ - 17.16˚ | 17.65˚ - 32.82˚ | 32.82˚ - 42.71˚ | ≥42.71˚ |

**Note**: In the formula for the absolute value of the deformity $d=\sqrt{x^2+y^2}$, $x$ is the value of HVA, and $y$ is the IMA angle [18].

Table 9. Classification of deformities D1 and D2 based on the estimated value of the IM angle by Formula (1) and Formula (9).

| Degree of deformities | Absolute value of deformity | The value of the IM angle | | | |
|---|---|---|---|---|---|
| | | Measured value D1 classification | | Applying Formula (22) D2 classification | |
| | | Frequency | Percent | Frequency | Percent |
| Normal condition | ≤17.50 | 2 | 0.5 | 3 | 0.8 |
| Mild deformity | 17.51 - 32.70 | 155 | 39.1 | 123 | 31.1 |
| Moderate deformity | 32.71 - 44.70 | 196 | 49.5 | 233 | 58.8 |
| Severe deformity | More than 44.71 | 43 | 10.9 | 37 | 9.3 |
| Total | | 396 | 100.0 | 396 | 100.0 |

where $a$, $b$ are regression coefficients that we will determine experimentally on a sample of $N = 396$ surgically treated feet. Note that the function (12) is linear in the coordinate system $Y = y$, $X = \log(x)$.

Pearson's linear correlation coefficient showed that there is a positive correlation of medium strength, $r = 0.346$ between the variables $Y$, $X$ at the level of statistical significance Sig. = 0.000 < 0.05, and the model implemented to be statistically significant, F(396, 1.381) = 51749, Sig. = 0.000, while the estimated regression coefficients a, b were statistically significant: $a = 3.153$, t(396) = 5.478, Sig. = 0.000, and $b = 3.774$, t(396) = 7.194, Sig. = 0.000 < 0.05. Based on the obtained values of the coefficients, the model (12) for evaluating the value of IMA when HVA is measured after operative treatment reads.

$$y = 3.153 + 3.774 \cdot \log x \tag{13}$$

where $x$ = HVA, $y$ = IMA. The graphic representation of the function (13) of the evaluated values of IMA after operative treatment, for the measured value of HVA after surgery is shown in **Figure 6**, and shows that the operative treatment of the feet was very successful. The IMA angle on the graph is less than 10 degrees even for extremely large values of the HVA angle.

### 3.3. Algebraic Equalization of the Degree of Deformity with a Non-Linear Function

The conjugated (absolute) value of the hallux valgus deformity (HVA, IMA) in relation to the values of the HVA and IMA angles was considered by the authors in the paper [18]. Here, let is examine the dependence of the absolute value of the deformity on the values of HVA and IMA, observing a non-linear function with two variables

$$d(x,y) = a + b\cdot\log(x) + c\cdot\log(y) \tag{14}$$

if we assume that the value of the IMA = $y$ angle is determined by relation (9)

$$y = 15.528\cdot\log\frac{x}{4.645},$$

and that HVA = $x$, while $a$, $b$, $c$ are regression coefficients that we will determine experimentally.

Let is note that the function (14) by applying the relation (9) can be reduced to a function of one variable by transformations,

$$\begin{aligned} d(x) &= a + b\cdot\log(x) + c\cdot\log\left(15.528\cdot\log\frac{x}{4.645}\right) \\ &= a + c\cdot\log 15.528 + b\cdot\log(x) + c\cdot\log\left(\log\frac{x}{4.645}\right) \\ &= a + 1.1911\cdot c + b\cdot\log(x) + c\cdot\log\left(\log(x) - \log(4.645)\right) \\ &= a + 1.1911\cdot c + b\cdot\log(x) + c\cdot\log\left(\log(x)\right) + 0.1759\cdot c \\ &= a + 1.3670\cdot c + b\cdot\log(x) + c\cdot\log\left(\log(x)\right) \end{aligned}$$

which can be written in the form

$$d(x) = A + b\cdot\left[\log(x) + \frac{c}{b}\cdot\log\left(\log(x)\right)\right],$$

where is $A = a + 1.3670\cdot c$ -constant, from which it follows that the function

$$d(x) = A + b\cdot\log(x) + c\cdot\log\left(\log(x)\right) \tag{15}$$

behaves as a linear function in the coordinate system XOY; in which it

$$Y = d(x)\; X = \log(x) + \frac{c}{b}\cdot\log\left(\log(x)\right) \tag{16}$$

for values $X \in$ [1.0; 1.10], *i.e.* for all values of the angle x = HVA ∈ [19, 50] while for $X >$ 1.10, *i.e.* extremely large values of hallux valgus, HVA > 50˚, the function slightly deviates from the linear function, **Figure 8**.

Experimentally determined values of regression coefficients a, b, c on a sample of $N$ = 396 surgically treated feet are: :$a$ = −75.393 with standard error SE = 0.998, $b$ = 64.391, SE = 0.755 and $c$ = 12.696 with SE = 0.804. and each of the mentioned constants has a statistically significant influence, Sig. = 0.000, and the unique contribution to the explanation of the variance of the absolute value of the deformity variable x = log(HVA) explains 74.6%, and the variable $y$ = log(IMA) uniquely explains 13.8% of the variance in model (14), which showed statistical significance: F(2,393) = 6335.984, Sig. = 0.000, **Table 10**.

**Table 10.** Results of the evaluation of regression coefficients.

| Model | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | Correlations | | F(2, 393) |
|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | | | Partial | Part | |
| Constant | −75.393 | 0.998 | | −75.537 | 0.000 | | | 6335.984<br>Sig. = 0.000 |
| log(HVA) | 64.391 | 0.755 | 0.886 | 85.264 | 0.000 | 0.974 | 0.746 | |
| log(IMA) | 12.696 | 0.804 | 0.164 | 15.800 | 0.000 | 0.623 | 0.138 | |

Dependent Variable: $Y = d(x, y)$.

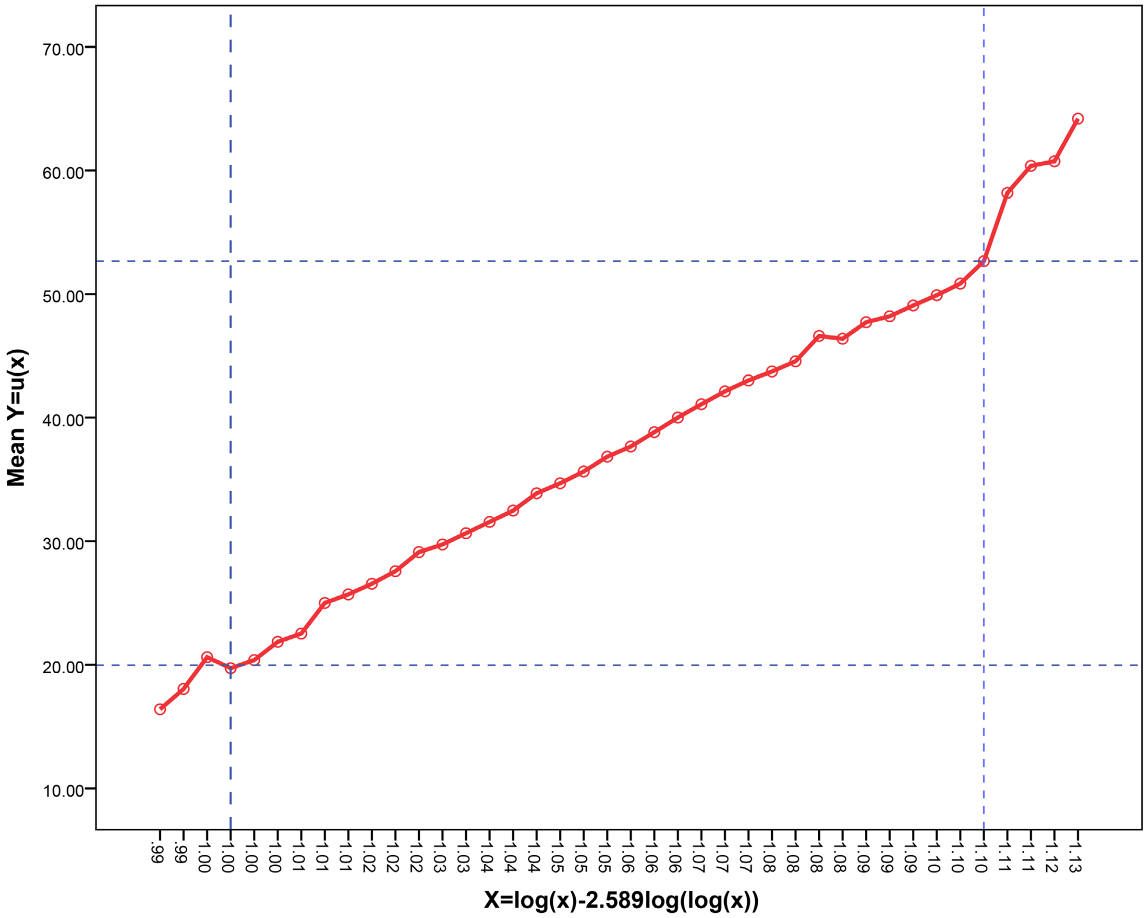


**Figure 8.** Linearity of the function $X(x) = A + b \cdot \log(x) + c \cdot \log(\log(x))$.

Thus, the equation for the evaluation of the absolute value of the deformity based on relation (14) reads

$$d(\mathrm{HVA}, \mathrm{IMA}) = -75.393 + 64.391 \cdot \log(\mathrm{HVA}) + 12.696 \cdot \log(\mathrm{IMA}) \quad (17)$$

While in the case of assessment of the absolute value of the deformity by model (15), *i.e.* only on the basis of the HVA value, then the regression coefficients have the value; $A = -285.451$, $b = 306.341\mathrm{b}$ and $c = -793.120$, so the equation of the degree of deformity is

$$d(x) = -285.451 + 306.341 \cdot \log(\text{HVA}) - 793.120 \cdot \log(\log(\text{HVA})) \tag{18}$$

**Example 2.** If the measured HVA = 32˚, evaluate the IMA value and degree of hallux valgus deformity. The expected estimated value of IMA is IMA = −9.91 + 15.528 · log32 = 13.46˚. How, by applying Formula (18), is the absolute value of the deformity

$$\begin{aligned} d(x) &= -285.451 + 306.341 \cdot \log(32) - 793.120 \cdot \log(\log(32)) \\ &= -285.451 + 461.089 - 140.842 = 34.796 \end{aligned}$$

it follows that it is a moderate deformity, Table 8, with expected IMA of 13.46 degrees.

## 4. Conclusion

The determined value of the proportionality coefficient and the non-linear functional relationship between the HV and IM angles gives the possibility of evaluating the IMA value, before and after the operative treatment if the HVA was measured. In this way, we can evaluate the absolute value of the deformity by observing the function of one argument, that is, only on the basis of the known value of HVA. This approach to the IMA evaluation contributes to a simpler observation of deformity changes and a reduction in the time of determining the radiological value of the angles, based on which determine the degree of deformity.

## Declarations

Ethics approval and consent to participate.

The authors confirm that informed consent was obtained from all subjects. The informed consent for subjects under 18 years was obtained from their parents/legal guardians. The authors confirm that all research protocols were approved by the Ethics committee of the Institute of Orthopedics "Banjica" Belgrade, Serbia.

## Availability of Data and Materials

All data and materials of the research are in possession of the corresponding author.

## Authors' Contributions

N.S. used mathematical and logical argumentation and created mathematical equation Hallux valgus deformity. M. S and V. I defined the problem of HV deformity and gave guidelines to solve the problem and reviewed the manuscript, giving suggestion for improvement.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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