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Black Hole Singularities and Planetary Formation

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Abstract

The goal of this research is to explore the effects of black hole singularities. Methodology is to start with large objects like galaxies and continue to smaller objects within our solar neighbourhood. High-redshift observations from the James Webb Space Telescope reveal that distant galaxies and their central black holes formed shortly after the Big Bang. An innovation about the speed of light explains how supermassive black holes could have formed primordially. Predictions of Hawking radiation include the possibility of black holes contributing to the energy of stars such as the Sun. Black holes have also been suggested as a source of radiation and magnetic fields in giant planets. Observations of Enceladus raise the possibility that this moon and other objects near Saturn's Rings contain small singularities. Extrapolations of this methodology indicate that black holes could exist within solar system bodies including planets. Extended discussion describes how their presence could explain mysteries of internal heat, planetary magnetic fields, and processes of solar system formation.

Keywords

Black Holes, Galaxies, Magnetic Fields, Planets, Planetary Formation, Speed of Light

1. Introduction

Black holes, or singularities, are among the most fascinating results of gravity. In their simplest form, proposed by John Michell in 1783, black holes are concentrations of mass, so dense that their escape velocity exceeds the speed of light. As singularities, they behave as point masses. They are part of the structure of galaxies and possibly smaller objects.

A black hole event horizon has the well-known Schwarzschild radius *R*:

$$R = \frac{2Gm}{c^2} \tag{1}$$

where G is the gravitational constant, m is black hole mass, and c is the speed of light.

Galaxies such as our Milky Way contain at their centres supermassive black holes. SgrA, the black hole in our galaxy, has an estimated 4 *million* solar masses. Images from the James Webb Space Telescope show that distant galaxies and their central black holes formed primordially, shortly after the Big Bang. The supermassive black hole in the Phoenix cluster of galaxies is estimated to have a mass of 100 *billion* solar masses [1] beyond the theoretical upper limit for accreting black holes [2].

Hawking concluded that black holes are not truly black, but radiate energy with high efficiency, and suggested that a singularity inside the Sun could contribute to its energy output [3]. Over time, small black holes could evaporate due to loss of mass [4]. Recent computer simulations show that a black hole of proper size could exist within the Sun without evaporating [5].

Black holes are observed to create magnetic fields. Radiation creates jets shooting out the north and south poles, following magnetic field lines. Jets from supermassive black holes have been observed coming from radio galaxies, quasars and within galaxy clusters [6]. Similar jets have been observed from Herbig-Haro objects, evidence that these embryonic stars have formed around black holes.

In addition to the problem of supermassive black holes, many issues remain about our solar system. Earth, other planetary bodies, and even some moons produce large amounts of internal heat. Some planets and moons generate internal magnetic fields. The formation of the solar system, its planets and moons, may be addressed by considering primordial black hole singularities.

2. Internal Heat

Earth's internal heat is source of wonder and mystery. Earth's interior produces energy estimated at 47 TW [7]. This heat powers volcanic eruptions, earthquakes, and the slow movement of continents. It also provides energy for subsurface organisms and the formation of hydrocarbon fuels.

Speculations about the source of Earth's internal heat have focused on radioactive uranium, thorium, and potassium [8] though these elements do not easily mix with Earth's nickel-iron core. Uranium-238 and Thorium-232 have long half-lives, but are thought to be very rare. Potassium 40, thought to be more common, has a half-life of 1.3 billion years. If K40 was part of Earth's formation 4.5 *Gyr* ago, 90% of it would be gone today.

Venus is a hot world where surface temperatures reach degrees 464 *degrees C*. The temperature was thought to be caused by carbon dioxide in the Venusian atmosphere, but CO_2 is also a product of volcanic action. The Venusian atmosphere contains large amounts of sulphur, another indicator of vulcanism. Recent spacecraft data proves that Venus is home to active volcanoes, indicating an internal source of heat.

Images from spacecraft show that Mars once had rivers oceans, and large amounts of liquid water. Evidence from Martian meteorite ALH84001 indicates that 3.5 *Gyr* ago, Mars was warm enough to harbor microbial life. Ancient volcanoes in Elysium, Olympus Mons, Tharsis Montes, and Alba Patera are proof of past internal heat. Despite a warmer past, today Mars shows no signs of liquid water or active vulcanism.

Jupiter has internal heat estimated at 4.8×10^{11} MW, about 2.6 times as much energy as the planet receives from the Sun, powering storms like the Great Red Spot. Saturn has a "hot spot" centred at its South Pole, paradoxically the warmest region of the planet's surface. Saturn produces 2.8 times as much energy as the planet receives from the Sun, approximately 8.6×10^{10} MW [9] [10]. Since these gas giants are not thought to contain significant radioactive elements, Jupiter and Saturn's internal heat has been a mystery.

3. Planetary Magnetic Fields

Earth, like the Sun and some planetary bodies, generates a bipolar magnetic field. On Earth's surface the field ranges from 25 - 65 μT. North and South magnetic poles of Earth's field are not aligned with the geographic poles, and migrate over time. Geological evidence shows that Earth's magnetic poles have flipped or changed direction in the past.

The field at Earth's outer core is calculated at 25 *gauss* [11]. Earth's magnetic field is thought to require a seed field to begin [12]. Theories of the field's origin revolve around a natural dynamo within the core. Why the enormous friction of Earth's core does not stop the dynamo is another puzzle.

Mercury was once thought to lack a magnetic field due to the planet's small size and slow rotation rate. In 1974 the Mariner 10 spacecraft found that Mercury has a surface magnetic field approximately 1.1% as strong as Earth or 300 *nT*. Mercury rotates very slowly with a period of 59 days. Dynamo theory does not explain the field of slowly rotating Mercury, or why larger Venus lacks a magnetic field.

Data from the Mars Global Surveyor and Mars Atmosphere and Volatile Evolution (MAVEN) spacecraft show that Mars presently does not have a magnetic field. Thermal remnant magnetization occurs when minerals cool below the Curie temperature while exposed to a magnetic field. TRM of carbonates in Martian meteorite ALH84001 shows that approximately 4 *Gyr* ago, Mars had a surface magnetic field of ~50 μT, nearly as powerful as Earth's field today [13]. Some mechanism caused internal heat and a magnetic field in Mars' past, but has evaporated today.

Giant planets Jupiter and Saturn have enormous magnetic fields spreading far into space. Jupiter has an immense field stretching 7 *million km* toward the Sun and outward nearly to Saturn's orbit. Equatorial field strength of Jupiter's field is 417 μT, nearly 10 times that of Earth. Saturn has a magnetic field with field strength at the equator 21 μT.

Saturn's moon Enceladus has a bipolar magnetic field with the positive pole in the South. The moon's south pole is home to hot geysers erupting water vapour into space [14] [15] [16] [17]. The warm waters within Enceladus are thought to be a possible home to life. Internal heat and a magnetic field in Enceladus could be signs that the moon formed around a black hole [18].

4. Formation

One mystery of Earth and solar system bodies is how they first formed. Philosopher Immanuel Kant suggested that the solar system's formation began with a rotating cloud of dust and gas in space [19]. Mathematician Pierre Laplace theorised that dust and gas condensed to form the Sun and planets [20]. James Clerk Maxwell calculated that such a spinning cloud would form not planets but a ring of small particles, like those surrounding Saturn. Observations of gas in space show that it does not condense, but dissipates.

Despite Maxwell's criticism, the *nebular hypothesis* of Kant and Laplace is the foundation of planetary formation theories. Problems of the nebular hypothesis have led to new models. Chamberlin and Moulton proposed that planets formed by combining from smaller *planetesimals* [21]. Over time the tidal model of Jeans [22], the protoplanet theory of McCrea [23] and the capture theory of Woolfson [24] have been proposed but found incomplete.

Recent models suggest that dust grains within a gas cloud collide and stick together, creating "pebbles" with diameter from 1 *mm* to 1 *m* [25] [26]. However, growth of "pebbles" would tend to stop at this mass [27]. Further growth would require that pebbles somehow accumulate into 1 *km* and larger planetesimals [28] [29] [30]. Formation of planets from pebbles is another mystery [31].

New research involving silicon isotopes suggests that planets formed within just 3 million years [32] [33] placing a severe constraint on theories. Observations of other solar systems indicate that protoplanetary disks have lifetimes of only a few million years [34]. Formation of even 1 *km* planetesimals is unlikely even in a longer time period. A process is needed that can form planets in a short time frame.

5. Black Hole Singularities

As an aside, I shed light on a novel solution to the problem of supermassive black holes. Size of a black hole is limited by a "horizon distance", related to the speed of light. Primordial singularities were once thought to be tiny due to today's speed of light. If c in the early universe was much greater, black holes could have formed in a variety of sizes, even to the supermassive.

The horizon distance r is given by:

$$r = ct \tag{2}$$

where c is speed of light, t is age of the universe.

The speed of light is also part of a very simple equation [35]:

$$GM = tc^3 \tag{3}$$

where G is the gravitational constant, M and t are mass and age of the universe.

By this equation, when t was small c was extremely large, enabling formation of black holes of great size. The speed of light also explains a problem of exploding stars, the appearance of acceleration in the redshifts of Type Ia supernovae. Variation in c today is shown by comparing data from the Lunar Laser Ranging Experiment with independent experiments [36]. Billions of supermassive black holes are each a signal that the speed of light has changed [37].

A black hole can be described as having a surface area:

$$A = \frac{16\pi G^2 m^2}{c^4} \tag{4}$$

Here, A is area of a spherical surface $4\pi R^2$.

A black hole is calculated to have entropy given by:

$$S = \frac{kc^3}{4\hbar G} A \tag{5}$$

where k is the Stefann-Boltzmann constant, S is entropy [38].

A black hole would then produce radiation with peak temperature given by:

$$T = \frac{\hbar}{2\pi kc} \kappa \tag{6}$$

where T is peak temperature, κ is surface gravity [39].

The black hole's surface gravity may be given by:

$$\kappa = \frac{Gm}{R^2} = \frac{4\pi Gm}{A} \tag{7}$$

where A is surface area, κ is gravity at event horizon R.

Radiation temperature T would be inversely proportional to black hole mass m:

$$T = \frac{\hbar c^3}{8\pi kGm} = \frac{1.2\times 10^{23}\ \text{K}\cdot\text{kg}}{m} \tag{8}$$

As an example, a black hole of mass $m = 10^{12}$ kg radiates at temperature $T = 1.2\times 10^{11}$ K.

Black hole radiation has power P proportional to the inverse-square of mass:

$$P = f(t)\frac{4.8\times 10^{33}\ \text{W}}{m^2} \tag{9}$$

where P is radiated power, $f(t)$ is a factor of order 1 [40].

The giant planets could be home to singularities. A black hole within Jupiter would have mass of approximately 4×10^{19} kg producing 4.8×10^{11} MW energy. Saturn's central black hole would have mass of 7×10^{18} kg and would generate approximately 8.6×10^{10} MW [41]. They would explain the internal heat of giant planets without speculative radioactive elements.

Jupiter and Saturn could alternately contain swarms of smaller black holes. A mass of 10^{15} kg would radiate approximately 5×10^{6} MW of energy [42]. A collection of 10^{5} such black holes orbiting within Jupiter would account for the planet's internal heat. About 10^{4} singularities of this mass would account for Saturn's heat.

6. Discussion

Observations of giant planets like Saturn and moons like Enceladus are clues that Earth could also have formed around a primordial singularity. This tiny object would consume approximately 1.6×10^{4} kg/yr of Earth to produce 47 TW of radiation, keeping the core hot indefinitely. Outward radiation pressure would prevent more of Earth from being absorbed, creating an equilibrium. In the early solar system, Earth may have formed around this small singularity as a pearl forms around a grain of sand.

Detection of neutrinos from Earth's interior has been seen as an indicator of radioactive elements, but neutrinos may also be produced by black holes. A black hole would produce radiation at a variety of frequencies, particularly those typical of neutrinos. The hypothesis of radioactive decay is disproven by the giant planets, which produce enormous heat but have no radioactive elements.

Earth's Antarctic regions contain subsurface lakes of liquid water, indicating an internal heat source. Antarctica is known to be home to many volcanoes, possible signs of a "hot spot". The singularity would rotate independently of Earth, generating a magnetic field where the poles are not aligned with Earth's rotation axis. These magnetic poles would migrate over time, as observed.

Mercury, which has a magnetic field, may also be home to a black hole. Venus could also contain a black hole, as source of the planet's internal and atmospheric heat. The planet's slow rotation may explain why some planets generate heat but not a magnetic field. Venus could lack a magnetic field because her black hole does not rotate.

Mars has ancient volcanoes, but shows no sign of current volcanic activity. Mars' magnetic field is today very weak, but evidence shows it was once much greater. The southern hemisphere of Mars shows signs of remnant magnetization of ~22 nT, while the northern hemisphere does not. If Mars once contained a black hole, it may have evaporated over time.

Giant planets are prime locations of black holes, which could cause their internal heat and powerful magnetic fields. As Jupiter's magnetic poles are not aligned with the planet's rotation axis, the black hole would also rotate independently. Radiation from the black hole would be manifested in storms like the Great Red Spot.

Saturn's magnetic poles are closely aligned to the planet's rotation axis, indicating that the black hole's rotation axis is also closely aligned. As with the moon

Enceladus, the black hole rotates anticlockwise when viewed from the North. This rotation powers a magnetic field with the "positive" pole in the South. On Saturn's north pole, cloud layers are crowded together in a hexagon pattern.

Charged particles within Saturn's core spiral out magnetic field lines toward the poles. Negatively charged electrons follow the north magnetic pole and are absorbed by Saturn's interior. Positively charged protons, heavier, penetrate the atmosphere and cause Saturn's south pole to be warmer.

From Saturn's south pole, positively charged ions follow the magnetic field lines in a donut-shaped path before reaching Saturn's surface again at the north pole. There they crowd against the negative field lines, creating a temperature gradient. Crowding of magnetic field lines could create the hexagon pattern of Saturn's north pole.

Saturn's Rings show other clues to singularities. Moons within the Rings exist inside a "Roche Limit" where liquid bodies were once thought to be impossible due to tidal forces. The A Ring sports propellor-shaped features indicating masses of 100 m objects where no objects have been seen [43] [44]. Massive unseen bodies, behaving as point masses, have also been deduced to exist in the B Ring [45].

Emission of X-rays within the Rings may be another sign of radiating black holes [46]. Cassini observed the Rings emitting both radio and plasma waves [47]. Moons Tethys and Dione, which orbit within the Rings, produce outwardly radiating plasma [48]. Prometheus, a moon of mass 1.6×10^{17} kg, has been observed exchanging materiel and creating structure in the F Ring [49].

Earth's moon shows no sign of a magnetic field or current vulcanism. The most popular theory is that the Moon was formed when a Mars-sized object struck Earth. The resulting debris particles were large enough to quickly condense into the Moon. Enceladus and the Saturn system show that black holes could exist within smaller moons.

Existence of black holes within Earth and other planetary bodies was proposed by Trofimenko [50], then independently by this author [51] and Zhilyaev. A black hole seems surprising, but solves multiple mysteries of the planet. It would explain Earth's internal heat, magnetic field, and even its formation from dust in space. Data from spacecraft like Cassini, along with observatories like JWST, shine light upon these mysteries.

As a practical application, black holes could someday be an enormous resource of energy, dwarfing even nuclear power. Though nuclear fusion converts about 0.7% of fuel to energy, efficiency of a black hole can be 2 orders of magnitude greater. Black holes within Earth's solar system could someday be harvested, providing energy to fulfill all humanity's needs and reach toward the stars.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Brockamp, M., *et al.* (2016) Unveiling Gargantua: A New Search Strategy for the Most Massive Central Cluster Black Holes. *Astronomy & Astrophysics*, **585**, Article No. A153. https://doi.org/10.1051/0004-6361/201526873

[2] Carr, B., *et al.* (2021) Constraints on Stupendously Large Black Holes. *Monthly Notices of the Royal Astronomical Society*, **501**, 2029-2043. https://doi.org/10.1093/mnras/staa3651

[3] Hawking, S.W. (1971) Gravitational Radiation from Colliding Black Holes. *Physical Review Letters*, **26**, 1344-1346. https://doi.org/10.1103/PhysRevLett.26.1344

[4] Hawking, S.W. (1975) Particle Creation by Black Holes. *Communications in Mathematical Physics*, **43**, 199-220. https://doi.org/10.1007/BF02345020

[5] Bellinger, E.P., *et al.* (2023) Solar Evolution Models with a Central Black Hole. *The Astrophysical Journal*, **959**, Article 113. https://doi.org/10.3847/1538-4357/ad04de

[6] Beall, J.H. (2014) A Review of Astrophysical Jets. *Acta Polytechnica CTU Proceedings*, **1**, 259-264. https://doi.org/10.14311/APP.2014.01.0259

[7] Davies, J.H. and Davies, D.R. (2010) Earth's Surface Heat Flux. *Solid Earth*, **1**, 5-24. https://doi.org/10.5194/se-1-5-2010

[8] Korenaga, J. (2011) Earth's Heat Budget: Clairvoyant Geoneutrinos. *Nature Geoscience*, **4**, 581-582. https://doi.org/10.1038/ngeo1240

[9] Ingersoll, A.P., *et al.* (1975) Pioneer 11 Infrared Radiometer Experiment: The Global Heat Balance of Jupiter. *Science*, **188**, 472-473. https://doi.org/10.1126/science.188.4187.472

[10] Reese, E.J. (1971) Recent Photographic Measurements of Saturn. *Icarus*, **15**, 466-479. https://doi.org/10.1016/0019-1035(71)90124-2

[11] Buffett, B.A. (2010) Tidal Dissipation and the Strength of the Earth's Internal Magnetic Field. *Nature*, **468**, 952-954. https://doi.org/10.1038/nature09643

[12] Merrill, R.T., *et al.* (1996) The Magnetic Field of the Earth. Academic Press, Cambridge.

[13] Weiss, B.P., *et al.* (2002) Records of an Ancient Martian Magnetic Field in ALH84001. *Earth and Planetary Science Letters*, **201**, 449-463. https://doi.org/10.1016/S0012-821X(02)00728-8

[14] Yeager, A. (2008) Enceladus Shoots Supersonic Jets of Water. *Nature*. https://doi.org/10.1038/news.2008.1254

[15] Hansen, C.J., *et al.* (2006) Enceladus' Water Vapor Plume. *Science*, **311**, 1422-1425. https://doi.org/10.1126/science.1121254

[16] Brown, R.H., *et al.* (2006) Composition and Physical Properties of Enceladus' Surface. *Science*, **311**, 1425-1428. https://doi.org/10.1126/science.1121031

[17] Dong, Y., *et al.* (2011) The Water Vapor Plumes of Enceladus. *Journal of Geophysical Research*: *Space Physics*, **116**, A10204. https://doi.org/10.1029/2011JA016693

[18] Riofrio, L.M. (2023) Singularities in Formation of Solar System Bodies. *International Journal Innovative Research in Science & Engineering*, **3**, 7-9.

[19] Kant, I. (1755) Universal Natural History and Theory of the Heavens. Richer Resources Publications, Arlington.

[20] Laplace, P. (1796) Exposition of a World System. Bibliothèque Nationale de France, Paris.

[21] Chamberlin, T.C. (1916) The Origin of the Earth. University of Chicago Press, Chicago.

[22] Jeans, J.H. (1917) The Part played by Rotation in Cosmic Evolution. *Monthly Notices of the Royal Astronomical Society*, **77**, 186-199. https://doi.org/10.1093/mnras/77.3.186

[23] McCrea, W.H. (1960) The Origin of the Solar System. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **256**, 245-266. https://doi.org/10.1098/rspa.1960.0108

[24] Woolfson, M.M. (1993) Solar System—It's Origin and Evolution. *Quarterly Journal of the Royal Astronomical Society*, **34**, 1-20.

[25] Blum, J., *et al.* (2000) Growth and Formation of Planetary Seedlings: Results from a Microgravity Aggregation Experiment. *Physical Review Letters*, **85**, 2426-2429.

[26] Poppe, T., *et al.* (2000) Experiments on Collisional Grain Charging of Micron-Sized Preplanetary Dust. *The Astrophysical Journal*, **533**, 472-480. https://doi.org/10.1086/308631

[27] Brauer, F., *et al.* (2008) Coagulation, Fragmentation and Radial Motion of Solid Particles in Protoplanetary Disks. *Astronomy & Astrophysics*, **480**, 859-877. https://doi.org/10.1051/0004-6361:20077759

[28] Johansen, A., *et al.* (2007) Rapid Planetesimal Formation in Turbulent Circumstellar Disks. *Nature*, **448**, 1022-1025. https://doi.org/10.1038/nature06086

[29] Cuzzi, J.N., *et al.* (2008) Toward Planetesimals: Dense Chondrule Clumps in the Protoplanetary Nebula. *The Astrophysical Journal*, **687**, 1432-1447. https://doi.org/10.1086/591239

[30] Chambers, J.E. (2016) Pebble Accretion and the Diversity of Planetary Systems. *The Astrophysical Journal*, **825**, Article 63. https://doi.org/10.3847/0004-637X/825/1/63

[31] Halliday, A.N. and Canup, R.M. (2023) The Accretion of Planet Earth. *Nature Reviews Earth & Environment*, **4**, 19-35. https://doi.org/10.1038/s43017-022-00370-0

[32] Schiller, M., *et al.* (2020) Iron Isotope Evidence for Very Rapid Accretion and Differentiation of the Proto-Earth. *Science Advances*, **6**, eaay7604. https://doi.org/10.1126/sciadv.aay7604

[33] Onyett, I.J., *et al.* (2023) Silicon Isotope Constraints on Terrestrial Planet Accretion. *Nature*, **619**, 539-544. https://doi.org/10.1038/s41586-023-06135-z

[34] Haisch, K.E., *et al.* (2001) Disk Frequencies and Lifetimes in Young Clusters. *The Astrophysical Journal*, **553**, L153-L156. https://doi.org/10.1086/320685

[35] Riofrio, L.M. (2019) A Simple Experiment Testing Quantum Values. *Journal of Cosmology*, **1**, Article No. 1.

[36] Riofrio, L.M. (2012) Calculation of Lunar Orbit Anomaly. *Planetary Science*, **1**, Article No. 1. https://doi.org/10.1186/2191-2521-1-1

[37] Riofrio, L.M. (2021) Status of Light as Solution to Dark Energy. *Journal of Applied Mathematics and Physics*, **9**, 2579-2591. https://doi.org/10.4236/jamp.2021.911166

[38] Bekenstein, J.D. (1973) Black Holes and Entropy. *Physical Review D*, **7**, 2333-2346. https://doi.org/10.1103/PhysRevD.7.2333

[39] Hawking, S.W. (1972) Black Holes in General Relativity. *Communications in Mathematical Physics*, **25**, 152-166. https://doi.org/10.1007/BF01877517

[40] Semiz, I. (1995) Black Hole as the Ultimate Energy Source. *American Journal of Physics*, **63**, 151-156. https://doi.org/10.1119/1.17973

[41] Zhilyaev, B.E. (2003) Singular Sources of Energy in Stars and Planets. *Kinematics and Physics of Celestial Bodies*, **4**, 211-216.

[42] Fukuda, Y., *et al.* (1998) Evidence for Oscillation of Atmospheric Neutrinos. *Physical Review Letters*, **81**, 1562-1567.

[43] Tiscareno, M.S., *et al*. (2006) 100-Metre-Diameter Moonlets in Saturn's a Ring from Observations of 'propeller' Structures. *Nature*, **440**, 648-650. https://doi.org/10.1038/nature04581

[44] Sremcevic, M., *et al*. (2007) A Belt of Moonlets in Saturn's a Ring. *Nature*, **449**, 1019-1021. https://doi.org/10.1038/nature06224

[45] Spitale, J.N. and Porco, C.C. (2010) Free Unstable Modes and Massive Bodies in Saturn's Outer B Ring. *The Astronomical Journal*, **140**, 1747-1757. https://doi.org/10.1088/0004-6256/140/6/1747

[46] Bhardwaj, A., *et al*. (2005) Discovery of Oxygen Kα X-Ray Emission from the Rings of Saturn. *The Astronomical Journal*, **627**, L73-L76. https://doi.org/10.1086/431933

[47] Gurnett, D.A., *et al*. (2005) Radio and Plasma Wave Observations at Saturn from Cassini's Approach and First Orbit. *Science*, **307**, 1255-1259. https://doi.org/10.1126/science.1105356

[48] Burch, J.L., *et al*. (2007) Tethys and Dione as Sources of Outward-Flowing Plasma in Saturn's Magnetosphere. *Nature*, **447**, 833-835. https://doi.org/10.1038/nature05906

[49] Murray, C.D., *et al*. (2005) How Prometheus Creates Structure in Saturn's F Ring. *Nature*, **437**, 1326-1329. https://doi.org/10.1038/nature04212

[50] Trofimenko, A.P. (1990) Black Holes in Cosmic Bodies. *Astrophysics and Space Science*, **168**, 277-292. https://doi.org/10.1007/BF00636873

[51] Riofrio, L.M. (2005) Sources of Mass/Energy Observed in Saturn's Rings. In: *American Geophysical Union* 2005 *Fall Meeting*, San Francisco, 5-8 December 2005, P33B-0240.