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Active Cancelation of Acoustic Noise in Waveguides by Standard Control Task Decision Usage

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Abstract

Cybernetic decision variants were analyzed in order to use for physical task of active noise cancelation. 10 dB mean active noise cancellation is demonstrated in two decades frequency band by usage of cybernetic decision for acoustical duct physical scale model. The used decision was found on minimization of acoustical field power transfer function from the beginning of waveguide to their end.

Keywords

Waveguide, Noise, Active Cancelation, Control, Synthesis Procedure H_2

1. Introduction

Heating, ventilation and air conditioning systems are used in various types of equipment. As a rule, powerful systems include waveguides with a high intensity of the sound field. As a result, sanitary and hygienic standards are violated, and environmental safety is not ensured. Traditional noise attenuation methods associated with passive absorption of sound vibrations are not effective at low frequencies due to the fact that absorption is proportional to the displacement velocity, and at these frequencies, the displacement velocities of particles in the wave are not large. Therefore, it is necessary to use active noise control, which provides damping at low frequencies.

To implement active control, the method is proposed and algorithms for its implementation are developed, which are described in the following order. In Section 2, the classification of active noise control systems is given in order to determine the location of the approach used below. Section 3 describes the acoustic field models used in the implementation of active control, from the full description to simplifications using specific boundary conditions of waveguides. Further description is given for the acoustic field with transfer characteristics commonly used in cybernetics. Then, a description is given for methods for identifying models of transfer characteristics of acoustic fields. Those methods are based on experimentally observed signals, which are excited by auxiliary fields. At the end of section, there is the description of physical part of experimental setup. The models of the transfer characteristics of acoustic fields are identified in this setup. The examples of practical identification implementation are demonstrated. Section 4 describes the criteria by which controllers can be synthesized. Identified models of transfer characteristics for acoustic fields are used in the synthesis. The controller is adapted to the sound wave in the duct by identified models' usage. Section 5 describes the electronic part of experimental setup and specific features of the observed acoustic signal spectra. Section 6 presents experimental results of active noise control, in Section 7, a brief conclusion is given.

The analysis showed that the active method of noise canceling is formalized as control of the transfer characteristic of the acoustic field from the waveguide input to its output by introducing a damping acoustic field by secondary emitters in order to minimize the controlled transfer characteristic. Secondary emitter is fed from reference microphone by spatially feed-forward control by controller that was synthesized in H_2 metrics to guarantee the stability.

2. Classification of Active Noise Control Systems

An extended model of the object $G(s)$ is used (**Figure 1**), to describe active noise control. This state space model links the available for observation signal y and the output signal z to be minimized. A scheme that is close to reality seems to include two more sources of interference: n—the controller playback error, d—the error of the monitoring sensors.

Active Noise Control (ANC) refers to the observation of certain processes y in an object $G(s)$, which is under the influence of a disturbance w, with output noise z, the formation of control u based on the results of observation y, and the introduction through the actuator of the action in order to minimize the output signal z. The signal observations of y can be made in different ways [1].

In the literature on ANC, the term feed-forward control is used [2], if it is possible to obtain information about the processes in the source of cancelled noise, sometimes not by acoustic methods. The reference microphone located higher along the propagation path of the cancelled field represents information about the source, but distorted by the cancelling field. For this reason, for a scheme with a spatially prior reference microphone, it is more correct to use the term "spatially feed-forward control" [2]. In most cases, only the signal generated by both the primary and secondary sources is available for observation. Then, we are talking about systems with feed-back communication [1]. With active noise cancelation, secondary sources of structural vibrations or acoustic fields are introduced, which create vibrations conformal to the existing ones and opposite to them in sign. If we remove the signals z, w, n in **Figure 1**, we get a functional scheme of a resonant absorber with a perforated panel [3], in which the role of the noise to be cancelled is performed by the signal d, in this case, a feedback algorithm is implemented. In [3], an active method of increasing absorption with a working band determined by the integration time is described. The implementation was carried out in a square-section waveguide overlapped at the input and output by speakers and was an interferometer used to evaluate the realized absorption coefficient at the operating frequency. The processing algorithm [3] was a modification of the compensation algorithm [4], in which the spectral samples of the reference signal were calculated, they were recalculated into spectral samples of the control signal using a well-known operator of the electro acoustic compensation path, the spectral samples of the control signal were recalculated into a time signal. Such processing in the spectral region is effective only at very low frequencies. The algorithm of spatially feed-forward control in the time domain is given below, which significantly expands the working band.

For the purposeful formation of the control signal, information about the models of both primary and secondary fields is needed.

3. Models for Describing the Acoustic Field

In many cases, the correct description of fields by solutions of partial differential equations can be significantly simplified. By the time the development of active sound attenuation systems began, there was a well-developed theory of adaptive antenna arrays [5]. The principles of signal processing in such arrays were the basis for the algorithms of adaptive systems of active sound cancelation [6]. To implement them, it is necessary to know the solutions of partial differential equations in the form of Green's functions describing models of transformation of signals emitted in order to compensate for unwanted noise. Determining the Green's function for sufficiently complex boundary conditions is a difficult task in itself, which is no easier than directly determining the field for an arbitrary distribution of the sound potential or its normal derivative given on the surface [7]. In addition to the fact that Green's functions are not easy to find, they describe the behavior of an object only at a given frequency. To cancel noise in the frequency band, it is necessary additionally for each receiver to implement a filter with a finite impulse response, or more economically, a filter with an infinite

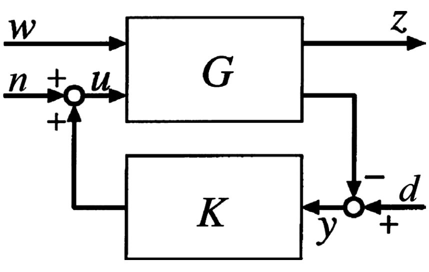

Figure 1. Block diagram of the extended object.

impulse response to represent the transfer characteristic. The placement of sensitive and actuating elements during identification has much in common with their use in procedures for bringing the impedances of an object without feedback to the desired values of the impedances in the presence of feedback [8]. However, the used identification methods make it possible to synthesize a controller that generates an electrical control signal based on observed sensor signals in a wide band, whereas the recommended methods for measuring impedances use a description only in terms of forces, oscillatory velocities and impedances, without specifying ways to implement them through physical devices controlled by electrical signals [9].

An air duct is a device in which the simplest models can be used to describe the acoustic field, especially in the frequency range below the first critical frequency of the waveguide. However, certain difficulties arise in this most simple device.

There are air environment displacements in the plane waves propagating in the duct, they are directed along the duct axis, and at high frequencies there are waves with fronts of a much more complex shape. The loudspeaker as a source of the secondary damping signal cannot be positioned in the center of the pipe section so that the created loudspeaker field will be conformal to the wave propagating in the duct. Usually the loudspeakers are placed around the perimeter of the duct. If the secondary signal sources are located along the perimeter, the displacements in the secondary signal sources are going on orthogonally to the duct axis.

In [10], the solution of the continuity and motion equations in partial differential is given. It is implemented in COMSOL Multiphysics. The example of a circular tube with absolutely rigid walls and three harmoniously pulsating annular inserts, *i.e.* cylindrical radiators, is analyzed. The propagation of sound frequencies and harmonic vibrations is modeled [11]. Since vibrations are introduced on the tube surface, the values of pressures and velocities are maximum on it, and as they approach the axis, the values gradually decrease. However, the distribution of oscillatory velocity and pressure along the radius of the duct acquires a uniform appearance [10] already a few centimeters from the annular radiators, although it looks very uneven in the vicinity of the radiators.

Such a rapid formation of a plane wave in the duct when the observation point is shifted along the duct axis from the radiator section provides a basis for simplifying the situation and describing the wave propagation process in the duct using the telegraphic equation [11] [12].

Let us restrict ourselves to the consideration of devices which described by a system of simple differential equations of a variable y with constant coefficients and excited by the process w. The task is to achieve the desired value of the disturbances transmission coefficient in the object from the input to the output. To solve this task additional observations y over the object and additional impact u on the object are used (see **Figure 1**). Monitoring by energy density sensors is much preferable to pressure sensors in noise cancelling [13].

The set of transfer characteristics of a generalized object is a 3 × 3 matrix of vector $|w;n;d|$ to vector $|z;u;y|$ transformation [14]. In any system, excitation may occur if the feedback becomes positive, instability will occur. Lemma III.1 [14] proves that for the stability of the object model $G(s)$, the stability of the model part which represent the transformation $|n;d|$ into a vector $|u;y|$ is necessary and sufficient. This circumstance is taken into account when synthesizing a controller that converts observation y into control u.

The scheme is used (see **Figure 1**) to cancel the noise. One needs to take in scheme:

$$G(s)=\begin{vmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{vmatrix}=\begin{vmatrix} P & S \\ I & R \end{vmatrix}, \tag{1}$$

where $P=G_{zw}/G_{yw}$ —transfer characteristic of the primary path between the reference and check microphones, $S=G_{zu}$ —transfer characteristic of the secondary path from the source of the cancelling field to the check microphone, $R=G_{yu}$ —transfer characteristic of the acoustic feedback path from the source of the cancelling field to the reference microphone. All these transfer characteristics can be practically identified.

The scheme in **Figure 1** can be specified using the transfer characteristics identified at the stage that precede the controller synthesis operation. This scheme is shown in **Figure 2**, it is supplemented with weighing functions W_e, W_u, W_y, which allow [15] [16] to control the characteristics of the controller during its synthesis.

Blocks *D*, indicated in **Figure 2**, correspond to a direct transfer from the input to the output without modification. These blocks are technologically necessary due to the description peculiarities of block diagrams in the MATLAB software, the designations i№ and o№ correspond to the numbering of inputs and outputs that used for in the MATLAB software description. It is necessary that all

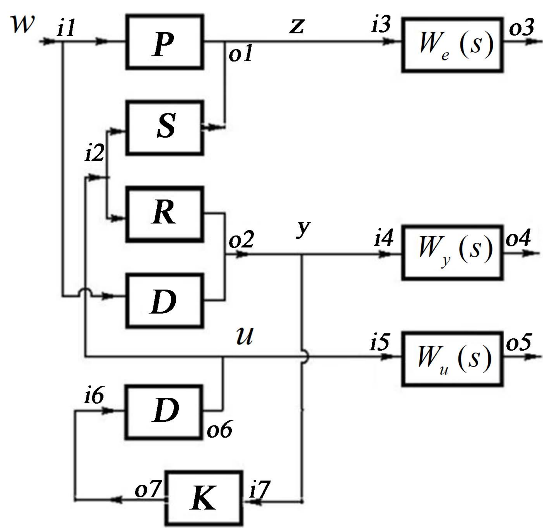

Figure 2. A block diagram of a generalized object supplemented with weighing functions W_e, W_u, W_y [15], when synthesizing a controller *K* for noise cancellation *w*.

descriptions will be observable and controllable to be used in the synthesis of the controller. Any system can be transformed into an observable and controlled system by similarity transformations. State variables that do not meet the requirements are removed from the description while transformation is implemented. Such a transformation is called the Kalman decomposition [15] [17].

For objects described by differential or difference equations, the polynomial representation arises naturally when using differential or difference operators in the Laplace transform or *z*-transform [18]. For systems with many inputs and outputs, polynomial matrices are a natural generalization [19]. In the synthesis of controllers, preference is given to the description in state variables (internal), which easily implements the assignment of any initial states of the object under consideration, whereas the fractional-rational representation (external) assumes zero initial conditions.

The formation of the model and the evaluation of its parameters in full-scale objects have to be carried out in the identification process [20]. Identification is carried out by comparing the probing signals at the input of the identified element with the synchronous recording of the output signals of the same element.

The problem of active cancellation of broadband noise on the prototype of a trunk duct with rigid walls is considered below [21]. The prototype is formed by an element of a cylindrical duct with a diameter of 150 mm. The first critical frequency of the duct is 1340 Hz, below which the simplest situation is observed in such pipes—the propagation of a plane sound wave. The sketch of the prototype of the duct is shown in **Figure 3**. The reference microphone was No. 4, the check microphone was No. 8, the path between them was the primary path. The transfer characteristic from the electric input of the cancelling signal (secondary) speaker to the microphone No. 8 was the transfer characteristic of the secondary path. The transfer characteristic from the electric input of the cancelling signal speaker to microphone No. 4 was the transfer characteristic of the acoustic feedback path.

An identification procedure was used in this article. The Generalized Binary

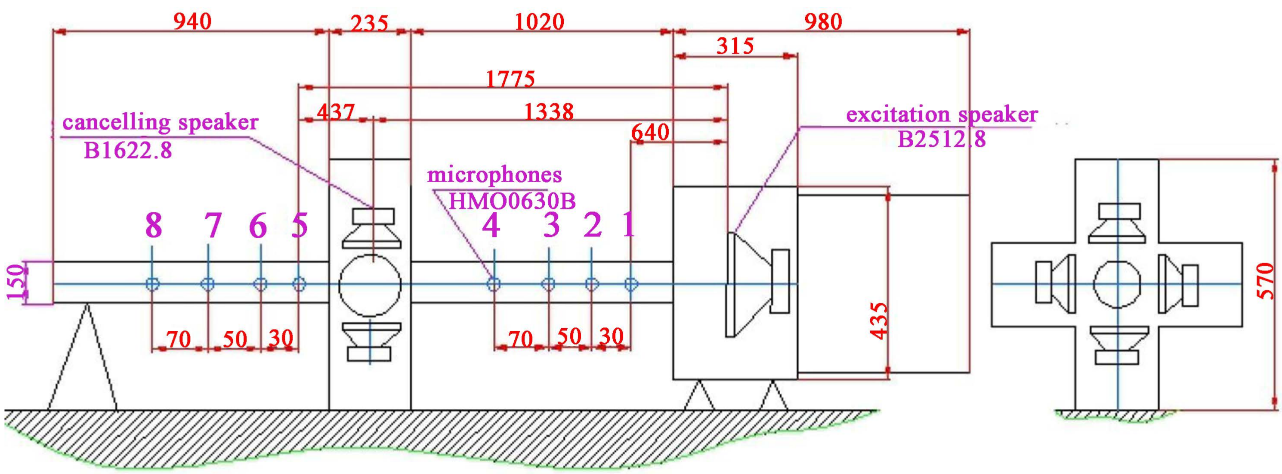

Figure 3. The sketch of the prototype of the duct with the main dimensions.

Noise (GBN) [22] was used as excitation in the procedure. The mathematical expectation of a random switching time has the value of 5 msec in exciting noise. The microphone records were analyzed during the identification procedure. The GBN was fed to the excitation speaker, when identifying the primary path.

The identification signals were recorded on microphones No. 4 and No. 8. The fractional rational model of the primary path was based on the prediction of the identification signal on microphone No. 8. The input of identifiable model was supplied with an identification signal on microphone No. 4. The identification quality was characterized by the energy of the mismatch signal. The mismatch signal was a difference between the identification signal registered on microphone No. 8 and the signal at the output of the identifiable model, the input of which was supplied with the identification signal registered on microphone No. 4.

Identification was carried out in 2 stages. At the first stage, the subspace method [23] based on the Predictor Based Subspace IDentification (PBSID) [24] method was used. This method does not require a preliminary selection of model parameters, except for specifying its order. The Predicted Error Method (PEM) [20] was used at the second stage, for which the vector obtained at the first stage was the initial vector of the model parameters.

The GBN was fed to the cancelling speaker, and identification signals were recorded on microphones No. 8 when identifying the secondary path from the cancelling speaker to microphone No. 8. In much the same way, GBN was fed to the cancelling speaker, and identification signals were recorded on microphones No. 4 when identifying the acoustic feedback path from the cancelling speaker to microphone No. 4.

Identification also included two stages. The secondary path model was based on the prediction of the identification signal on microphone No. 8 by an output signal of identifiable fractional rational model, to the input of which a GBN signal was applied similar to the cancelling speaker signal. In much the same way, the acoustic feedback path model was based on the prediction of the identification signal on microphone No. 4 by an output signal of identifiable fractional rational model, to the input of which a GBN signal was applied similar to the cancelling speaker signal. The quality of identification was characterized by the energy of the mismatch signal between the identification signal registered on the microphone No. 8 or No. 4 and the signal at the output of the identifiable model, to the input of which the GBN signal was applied similar to the cancelling speaker signal. The misalignment energy was less than 2% of the energy recorded by the microphones.

The internal representation of the full-scale object transfer characteristics are shown in **Figure 4** as a result of identification.

The identification results are the initial ones for solving the problem of controller synthesis $K(q)$. The task is to synthesize a controller transfer characteristic which transform the observed signal $y(k)$ to a control signal $u(k)$, which would minimize the specified norm of the transfer characteristic from the

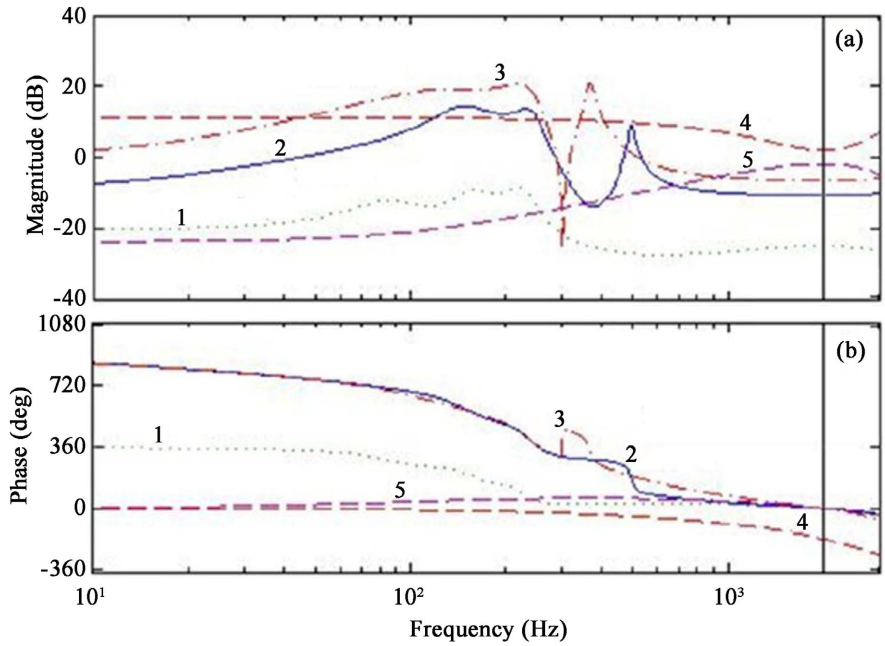

Figure 4. Amplitude (a) and phase (b) of the transfer characteristics: 1—primary path (P), 2—secondary path (S), 3—acoustic feedback path (R), 4—low-frequency weighing filter W_0, 5—high-frequency weighing filter W_1.

input signal $w(k)$ to the signal $z(k)$ coming out of the duct.

4. Minimization of the Transfer Characteristic by Introduction of Feedback

If the transfer characteristic can be minimized by introducing feedback, then the output signal level will be decreased compared to the case of no feedback, and, thus, the unwanted noise will be cancelled. For the transfer characteristic from the uncontrolled source w to the output of the system z (G_{zw}), we are looking for a controller that not only leads to a stable system, but also minimizes H_2 [16] [25] [26] [27] or H_∞ [16] [26] [27] [28] the norm of the transfer characteristic G_{zw}. The norm H_2 supervises the process power, the norm H_∞ supervises the maximum deviation from the zero value of the process. Such controls are called H_2 [16] or H_∞ [16], respectively [1]. Linear Quadratic Gaussian (LQG) control can minimize the transfer characteristic, however, it requires independence of both distorting noises from each other, subordination by both to the Gaussian distribution density with zero mathematical expectations, and their known covariance matrices: Ξ—distorting the state process and Θ—distorting the observation process. To construct control in the problems of LQG and LQR, two algebraic matrix Riccati equations have to be solved [29]. H_2 (H_∞) technologies [16] do not require information about distorting noises, use A.M. Lyapunov stability theory. The acoustic specifics of the solving task leads to a recursive characteristic of the controller, and for such task H_2 (H_∞) technologies naturally guarantee the solution stability and also include the possibility of robust evaluation [30] [31]. Control construction in H_2 or H_∞ problems can be realized by solving two algebraic matrix Riccati equations [32] [33] for an observer and a controller. The most effective procedure for controller synthesis is associated with solving a system of linear matrix inequalities, which, however, requires linearization of conditions [34], which in their original form are not linear, like the Riccati equations.

The choice of H_2 control is due to the fact that gradient methods, such as: Filter-X LMS [5] [35] [36], Filter-U LMS [37] [38], Filter-V LMS [38] [39], they do not guarantee stability, and the Hyper-stable Adaptive Recursive Filter (HARF) [40] [41] [42] is too difficult to implement due to the adaptive Strictly Positive Real (SPR) filter. Such SPR filter ought to be used for the approximation error averaging, in order to guarantee stability.

An example of the H_∞ technology was carried out by solving two related Ricatti equations [26] [28] [43] which was the only method at the time of publication.

The task of synthesizing H_2 (H_∞) controller K does not require a statistical description of distorting noise. The task is formulated as a search for certain transfer function of the controller. Mentioned transfer function described feedback to a system consisting of an object $G = \left[G_{zw}, G_{zu}; G_{yw}, G_{yu}\right]$ so that modified system is stable and at the same time has minimum the H_2 (H_∞) norm. The problem is formulated using an external description of the transfer characteristics, but the solution most often uses an internal description of the object and controller [44].

The synthesized controller is implemented using digital electronics due to the complexity of the algorithms. At the same time, the digital implementation imposes additional requirements on the controller. It should not only provide a stable feedback circuit, but the controller itself should be stable [45].

5. Experimental Setup

The sketch of the prototype of the duct is in **Figure 3**. The signal from microphone No. 4 is fed to the controller via the Analog to Digital Converter (ADC). The controller transfers input signal into another one, which is transmitted through the Digital to Analog Converter (DAC) to the cancelling speaker. The controller was implemented on a personal computer using LabVIEW-RT technology [46]. DAC and ADC placed on the NI PCIe-6323 board were used. The sampling rate is 4 kHz. The speaker B2512.8 served as the excitation emitter, four speakers of type B1622.8 were used as cancelling actuators. HMO0603B type microphones were used. The feed-forward control scheme is implemented. Acoustic field transformations are described by transfer characteristics. The synthesis was carried out by adapting H_2 to a fragmentary a priori description. The controller was designed to reflect sound. The optimization problem was settled by solving a system of linear matrix inequalities [34].

The finite length waveguide output section, the leftmost in **Figure 3**, goes out into the open space without a flange. This is practically very close to the conditions of an acoustically soft boundary. The pressure in such a waveguide at a distance x from the leftmost section is proportional to the ratio of the two sinuses $p \approx \sin\left(\xi\left(l - x\right)\right) / \sin\left(\xi l\right)$, where ξ is the wave number, l is the length of the waveguide segment with absolutely rigid walls. As the length of the waveguide, you can take either the length of the output section, in this case $l = 0.94\ \text{m}$, or the length of the entire waveguide, in this case $L = 2.195\ \text{m}$. The frequencies of resonances and zeros in the mentioned waveguides for the location of microphone No. 8, whose distance from the leftmost section is $x = 0.4735\ \text{m}$, are shown in **Table 1**. The illustration of the signal energy spectra on microphone No. 8 are shown in **Figure 5**. The frequencies of resonances and zeros for this illustration are shown in **Table 2**.

Broadband resonance, tentatively assigned to the frequency of 180 Hz, coincides with resonances at a frequency of 177.7 Hz for a short and at a frequency of 152.2 Hz for a long duct. Some theoretical values of resonances at frequencies 304.3, 380.4, 710.6 are close to those observed at frequencies 300, 380 and 710 Hz, and the theoretical value of zero at a frequency of 679.1 Hz is close to the observed zero at a frequency of 680 Hz. The resonance at a frequency of 355.3 Hz practically coincides with zero at a frequency of 358.0 Hz, which in the calculation

Table 1. Theoretical values of frequencies (Hz) of resonances and zeros on microphone No. 8.

	Frequency (Hz)									
Reson. l		177.7			355.3		533.0		710.6	
Zeros l					358.0				716.0	
Reson. L	76.1	152.2	228.2	304.3	380.4	456.5	532.6	608.7	684.7	760.8
Zeros L	97.0		194.0	291.0	388.0	485.0		582.1	679.1	776.1

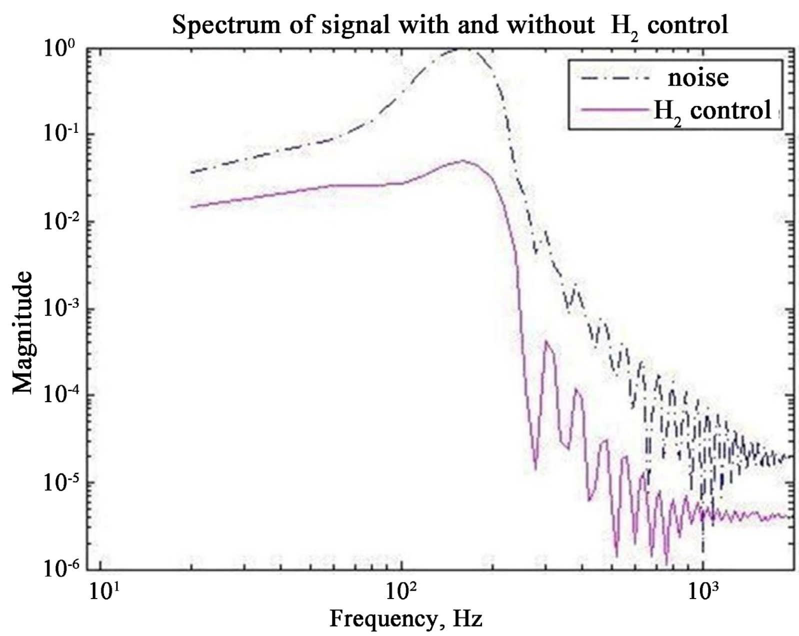

Figure 5. The spectrum of the signal on microphone No. 8 before and after switching the controller into the feedback circuit. The dashed line is the spectrum of the initial noise, the solid line is the spectrum when the controller implementing H_2 control is turned on.

Table 2. Observed values of resonances and zeros frequencies (Hz) on microphone No. 8.

	Frequency (Hz)								
Reson.	180	300	380	460	560	630	710	800	890
Zeros	280	350	400	510	600	680	760	840	900

leads to uncertainty disclosed by the Lopital rule, but the phenomenon in the physical prototype hardly reflects this mathematical action. So the computational model is too crude to describe the behavior of the field in the vicinity of resonances and zeros. To describe the transfer characteristics shown in Figure 4, we need a more complex model for describing the transformation of the acoustic field during propagation from one point of the waveguide to another. The evaluation practice shows that such models are not suitable even for the initial points of estimating the parameters of the so-called gray models, when the functional values are determined by analytical models, and the parameters of these models are estimated by the signals observed in the experiment.

6. Results of Active Sound Control in the Waveguide

The effective implementation of active noise cancelling is expected in the band no more than 300 Hz, on the stipulation that, sampling frequency is 4 kHz. Therefore, in exciting noise in the band from 20 Hz to 2 kHz, the spectral density above the frequency of 250 Hz decreased at a rate of 12 dB per octave. The results of the spectrum analysis on the control microphone (No. 8) are shown in Figure 5. It can be seen that there has been a cancelling of about 10 dB in the frequency band for two decades.

7. Conclusion

The physical problem of active cancelation of a broadband acoustic field is solved using cybernetic methods of identification and control of sound fields. The relevant features of possible alternatives of cybernetic solutions to the physical problem of active noise field cancelation are compared. The problem of active cancelation has been defined and solved in a generalized form, but individual fragments have been studied using the example of active cancelation of a broadband noise field in a cylindrical sound duct. In the preliminary tuning, the used technology is more complicated than that suggested by the Filter-X LMS algorithm, but it consumes frugally the computing resource of the processor on which the canceling out controller is implemented. The presence of poles in the transmission characteristic of the optimal controller is due to the fact of the acoustic feedback presence. The Filter-V LMS technology appeared precisely because of this circumstance, but, unfortunately, it cannot guarantee the stability of the active canceling out system, in contrast to the H_2 (H_∞) technology. Unfortunately, the H_2 (H_∞) technology is not so efficient in adaptability as the Filter-X LMS algorithm, but it has undoubted advantages in that it allows control of all elements of the physical components description of the noise field—the wave transfer characteristics of the primary and secondary paths, as well as the acoustic feedback path—and their influence on the synthesis of the controller. In the Filter-X LMS technology, the same elements are combined in such a way that it is impossible to isolate and check up individual elements. It follows from the obtained results that broadband low-frequency noise fields propagating in the trunk sound ducts can be suppressed using the stated technology.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Nomenclature

d—the error of the monitoring sensors,
D—the direct transfer from the input to the output without modification,
H_2—Hardy subspace of Hilbert space L_2 of complex valued functions,
H_∞—Hardy subspace of Banach space L_∞ of complex valued functions,
$G(s)$—matrix valued transfer characteristic of the object model,
G_{yw}—transfer characteristic of the object model from disturbance w to observing process y,
G_{zw}—transfer characteristic of the object model from disturbance w to the output signal z to be minimized,
k—number of time interval in discrete-time system,
$K(q)$—controller for noise cancellation,
l—the length of the waveguide segment with absolutely rigid walls,
L—the length of the entire waveguide,
n—the controller playback error,
$P = G_{zw}/G_{yw}$—transfer characteristic of the primary path between the reference and check microphones,
q—the argument of z-transform of discrete-time system,
$R = G_{yu}$—transfer characteristic of the acoustic feedback path from the source of the cancelling field to the reference microphone,
s—the argument of Laplace transform,
$S = G_{zu}$—transfer characteristic of the secondary path from the source of the cancelling field to the check microphone,
u—the control signal,
w—the influence of a disturbance,
W_e—error process weighing filter,
W_u—control signal weighing filter,
W_y—observing process weighing filter,
W_0—low-frequency weighing filter,
W_1—high-frequency weighing filter,
x—distance,
y—the available for observation signal,
z—the output signal to be minimized,
ξ—the wave number,
Θ—distorting the observation process (measurement noise) covariance matrix,
Ξ—distorting the state process (plant noise) covariance matrix.