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How to Predict the Temperature of the CMB Directly Using the Hubble Parameter and the Planck Scale Using the Stefan-Boltzmann Law

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Abstract

Based on recent progress in quantum gravity and quantum cosmology, we are also presenting a way to estimate the temperature in the cosmos, the Hubble sphere, from a relation between the Planck temperature and the Hubble scale. Our analysis predicts the Hubble sphere temperature of 2.72 K with the one standard deviation confidence interval between 2.65 K and 2.80 K, which corresponds well with the measured temperature observed from the cosmic microwave background (CMB) of about 2.72 K. This adds evidence that there is a close connection between the Planck scale, gravity, and the cosmological scales as anticipated by Eddington already in 1918.[1]

Keywords

CMB Temperature, Hubble Parameter, Stefan-Boltzmann Law, Planck Scale

1. The Hawking Temperature of the Hubble Sphere

Quantum cosmology has garnered increased attention in recent years, for example [2]-[9]. In this paper, we will demonstrate the existence of a link between the Planck temperature and the temperature within the Hubble sphere, which is further corroborated by the measured temperature of the cosmic microwave background.

In this section, we will first establish the mathematical relationship for the Hawking temperature across the entire Hubble sphere in the critical Friedmann

[1]This paper is a strongly improved version of our pre-print posted November 13th, 2023, at Research Square. [1]

universe. Then, in the next section, we will establish the connection between the Hawking temperature and the Planck temperature. By employing the Stefan-Boltzmann law [10] [11], we will predict the temperature of the Hubble sphere, which we demonstrate to closely align with the measured temperature of the cosmic microwave background (CMB). The CMB temperature is often associated with a black body temperature; Muller et al. [12], for example, states (see also [13]):

"*Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature* T_0 *at* $z=0$, *which is measured with very high accuracy,* $T_0 = 2.72548 \pm 0.00057\ \mathrm{K}$."

It is well known that the Stefan-Boltzmann law can be used to investigate and predict black body temperatures; however, no formula has been derived to predict the CMB temperature now (T_0). The reason why the approach we will look at in this paper has been missed in the past is that one has to discover certain connections between the Planck temperature, the Hawking temperature (which is another black body temperature), and the CMB temperature, and their connections throughout the Stefan-Boltzmann law. As we will soon see, this means that the CMB temperature can not only be measured but also predicted from quantum cosmology by utilizing the Stefan-Boltzmann law. We will limit ourselves in this paper to derive predictions for the CMB temperature close to $z=0$ that means the present CMB temperature. In addition, comes the well-known adjustment of $T_t = T_0(1+z)$ for z considerably higher than zero, as we will shortly also mention more about at the end of the paper.

The Hawking [14] temperature of a black hole is given by:

$$T_{\mathrm{Hw}} = \frac{\hbar c^3}{k_b 8\pi GM} \tag{1}$$

where M represents the mass of the black hole, k_b is the Boltzmann constant, and G is Newton's gravitational constant and $\hbar$ is the reduced Planck constant, also known as the Dirac constant. The Friedmann [15] equation is given by:

$$H_0^2 = \frac{8\pi\rho + \Lambda c^2}{3}, \tag{2}$$

where $\rho = \frac{M}{\frac{4}{3}\pi R_H^3}$ (the critical density of the critical Friedmann universe), and the Hubble radius $R_H = \frac{c}{H_0}$, where H_0 is the Hubble constant. In the special case where the cosmological constant, Λ, is set to zero, and we then solve the Friedmann equation for mass:

$$M_c = \frac{c^2 R_H}{2G}. \tag{3}$$

This is well known as the mass (mass equivalent) of the critical Friedmann universe. Be aware that when we say "mass" here, it can be considered equivalent to the mass in the Friedmann model, as it does not distinguish between effects from mass or energy. This means that energy is treated as mass equivalent since we naturally have $M = \frac{E}{c^2}$. We solve (Equation (3)) for R_H, which gives:

$$R_H = \frac{2GM_c}{c^2}. \tag{4}$$

This means the Hubble radius is mathematically identical to the Schwarzschild [16] radius of a black hole with a mass equal to the critical mass of the Hubble sphere. Patheria [17] and Stuckey [18] have even suggested that we possibly live inside a black hole, a controversial idea discussed even in recent literature [19]-[21]. We will not delve into a discussion about whether we could live inside a black hole or not, but we will focus on the mathematics and demonstrate that starting from the critical Friedmann universe, we can surprisingly predict the correct temperature of the cosmic microwave background, in full alignment with observations, we will later discuss also why this can be.

Now, let us input the critical mass of the Hubble sphere into the Hawking temperature formula. This gives:

$$T_{HW,H} = \frac{\hbar c^3}{k_b 8\pi G M_c} = \frac{\hbar c^3}{k_b 8\pi G \frac{c^3}{2GH_0}} = \frac{\hbar H_0}{k_b 4\pi} \approx 1.38 \times 10^{-30}\ \text{K}. \tag{5}$$

However, it's important to note that this temperature has never been observed. Nevertheless, if we take this literally, this is the radiation emitted from the Hubble sphere. As energy flows out of the Hubble sphere, there should also be a possibility of predicting what this means for the interior of the Hubble sphere. This is what we aim to explore in the next section.

2. The Hubble Sphere and Its Temperature Derived from the Planck Temperature

Planck mass particles have been suggested by multiple authors to be the most fundamental particles in the universe possibly, for example, Motz [22] and Haug [23]. It is also assumed that the Planck scale somehow will play a central role in quantum gravity. Einstein [24] already in 1916 suggested that the next step in gravity theory would be a unified quantum gravity theory, something he worked on much of the rest of his life, but unfortunately, without a big breakthrough. Eddington [25], in 1918, was likely the first to suggest that such a quantum gravity theory had to be somehow linked to the Planck scale through the Planck length. Most researchers working on quantum gravity today seem to be of the opinion that the Planck scale will play an important role in such a theory; see, for example, [26]-[28].

It is worth noting that as early as 1987, Cohen [29] pointed out that it would likely be impossible to find the Planck length independent of deriving it from G, c, and $\hbar$. This view was held until at least 2016, as seen in the interesting paper by McCulloch [30]. However, in recent years, it has been demonstrated that we can find the Planck length independent of any knowledge of G or $\hbar$; see [31]

[32]. It has also been shown that the Planck length can be derived from cosmological redshift without knowledge of G or $\hbar$; see [33]. Haug [34] has additionally recently demonstrated that a series of cosmological phenomena and entities, such as the critical mass of the universe divided by the Hubble radius, is identical to the Planck mass divided by twice the Planck length. Furthermore, the Hubble constant can be expressed as $H_0 = \frac{\bar{\lambda}_c c}{2 l_p^2}$, where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical mass of the universe. The reduced Compton wavelength of the critical mass in the universe can also be found without knowledge of the kilogram of the critical mass or the Planck constant; see [33].

If there is indeed a link between the Planck scale and the cosmic scale, then one could expect, or at least hope, that other observed phenomena of the cosmos, such as the cosmic microwave background temperature, possibly also have a link to the Planck scale. In this section, we will demonstrate that we can indeed predict the CMB temperature of about 2.72 Kelvin from the Planck temperature when utilizing a combination of the Hawking temperature with the Stefan-Boltzmann law.

The luminosity of the Hubble sphere, according to the Stefan-Boltzmann law, must be given by

$$L_H = 4\pi R_H^2 \sigma T_H^4 \tag{6}$$

where T_H is what we will call the Hubble Sphere Temperature. Next, we will utilize the idea that the most elementary of all particles are likely Planck mass particles. The Planck mass is normally considered to have a mass of $m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-8}$ kg. This is much higher than any known atom. However, Haug [23] has recently suggested that the Planck mass particle only lasts the Planck time and that the particle then has a mass of $m_p t_p \approx 1.17 \times 10^{-51}$ if observed over a second, which corresponds well with a survey of existing and proposed classical and quantum approaches to the photon mass, as seen in Spavieri *et al.* [35]. Only if observed inside the Planck time window is the mass the Planck mass; that is, this mass is special as it is observer window time-dependent, as discussed in [36]. This is connected to a deeper quantum theory in which mass becomes time-dependent when approaching Compton time observational windows. However, understanding this is not essential for this paper. We mention it merely so that readers can refer to the cited papers for more information on possible interpretations of the Planck mass. Going into depth about the different views on the potential photon mass is outside the scope of this article; we mention this to avoid any automatic rejection of the idea that the Planck mass particle can be the most elementary particle in the universe. It could actually be linked to photons. Even if photons are assumed to be massless, it could be that photons acquire this mass during collisions with other photons. It is well known within the standard literature that photon-photon collisions likely give rise to mass, as shown in, for example, Pike *et al.* [37]. For the moment, simply assume there is a Planck mass particle playing a central role in the universe and possibly as the ultimate building block of all matter.

The energy passing through a sphere with a radius equal to the Planck length is given by:

$$E = \frac{L_H}{4\pi l_p^2}. \tag{7}$$

The radiant flux absorbed by the Planck sphere's cross-section πr^2 is thus expressed as:

$$\Phi_{abs} = \pi l_p^2 E = \pi l_p^2 \frac{L_H}{4\pi l_p^2} = \pi l_p^2 \frac{4\pi R_H^2 \sigma T_H^4}{4\pi l_p^2} = \pi R_H^2 \sigma T_H^4. \tag{8}$$

The Planck temperature, as given by Max Planck [38] [39], is $T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b} = \frac{\hbar c}{l_p k_b}$. Furthermore, the Hawking temperature of a Planck mass is determined by Hawking radiation and is given by:

$$T_{Hw,p} = \frac{\hbar g}{2\pi c k_b}. \tag{9}$$

The acceleration at the Planck mass at the Schwarzschild radius of the Planck mass is $g = \frac{Gm_p}{r_s^2} = \frac{Gm_p}{(2l_p)^2}$, which leads to:

$$T_{Hw,p} = \frac{\hbar g}{2\pi c k_b} = \frac{\hbar \frac{Gm_p}{r_s^2}}{2\pi c k_b} = \frac{\hbar c}{8\pi l_p k_b} = \frac{T_p}{8\pi}. \tag{10}$$

This is also identical to: $\frac{\hbar c^3}{8\pi G m_p k_b} = \frac{T_p}{8\pi}$. The same result can be obtained from the Unruh [40] temperature when applied to a Planck mass particle:

$$T_{Unn,p} = \frac{\hbar a}{2\pi c k_b} = \frac{\hbar \frac{c^2}{4l_p}}{2\pi c k_b} = \frac{\hbar c}{8\pi l_p k_b} = \frac{T_p}{8\pi}. \tag{11}$$

Since the Stefan-Boltzmann law involves a fourth power, it has a stabilizing effect on the exchange, and the flux emitted by Planck particles (Planck spheres) should be approximately equal to the flux absorbed, especially close to the steady state, where we have:

$$4\pi l_p^2 \sigma T_{Hw,p}^4 = \pi l_p^2 E = \pi l_p^2 \frac{4\pi R_H^2 \sigma T_H^4}{4\pi l_p^2} = \pi R_H^2 \sigma T_H^4. \tag{12}$$

This gives:

$$T_{Hw,p}^4 = T_H^4 \frac{R_H^2}{4l_p^2}$$

$$T_{Hw,p} = T_H \sqrt{\frac{R_H}{2l_p}} \tag{13}$$

and

$$T_H = T_{Hw,p}\sqrt{\frac{2l_p}{R_H}}$$

$$T_H = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} \approx 2.72\ \text{K} \tag{14}$$

and

$$T_H^2 R_H = T_p^2 \frac{1}{32\pi^2} l_p. \tag{15}$$

As there is considerable uncertainty in both the Hubble constant, considerable effort has been put into measuring it as accurately as possible, for example, [41]-[47]. If we use the recent Hubble constant value given by Kelly *et al.* [48] of $66.6^{+4.1}_{-3.3}$ km/s/Mpc (see also [49] [50]), we get a predicted Hubble temperature using (Equation (14)) to be 2.72 K with a one standard deviation confidence interval of 2.65 K to 2.8 K. This is well in line with the measured cosmic microwave background temperature. For example, Fixsen [51] used the Wilkinson Microwave Anisotropy Probe data to obtain a cosmic microwave background (CMB) temperature of 2.7260 K ± 0.0013 K. Similarly, in a recent 2023 study, Dahl *et al.* [52] found a CMB temperature of 2.725007 K ± 0.000024 K (see also [53]).

Our findings should naturally be investigated to determine to what degree they could be consistent with the Λ-CDM model. Most importantly, it is essential that the predictions from this formula be consistent with observations. Even if it should not be consistent with the Λ-CDM model, there are multiple other types of cosmological models that should be investigated in relation to this. This seems to be fully consistent with at least some cosmological models known as $R_h = ct$, which are actively discussed as an alternative to Λ-CDM. See [54]-[59] for more information. Additionally, there are black hole cosmology models, likely first introduced in 1972 by [60], that continue to be actively discussed despite challenges. For example, see [18] [61]-[63]. To investigate a series of different cosmological models is outside the scope of this paper. The aim of this paper is to demonstrate for the first time that a formula to predict the CMB temperature now can be derived from the Stefan-Boltzman law, this has never been shown before. It is practically compatible with the Planck collaboration H_0 measurements within the Λ-CDM model.

We have utilized the critical universe solution (The Friedmann equation when the cosmological constant is set equal to zero). The Λ-CDM model features a positive cosmological constant due to accelerating expansion that is assumed to be caused by dark energy, prompting us to inquire about how our model appears to predict the temperature of the cosmic microwave background so precisely. The reason for this may be that the temperature measured thus far for the cosmic microwave background (CMB) is more closely related to the early universe, as discussed by [64] [65]. Alternatively, it could indicate a shift toward a new cosmology. However, further investigation by multiple researchers over time will be required to confirm this.

Interestingly, we can apply a similar law between the Hawking temperature of the Hubble sphere and the Hubble temperature. We get:

$$T_H = T_{Hw,H}\sqrt{\frac{R_H}{2l_p}} \approx 2.72\ \text{K}. \tag{16}$$

This means we also have:

$$T_H = T_{Hw,H}\sqrt{\frac{R_H}{2l_p}} = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} \approx 2.72\ \text{K}. \tag{17}$$

Tatum *et al*. [66] have independently somewhat heuristically suggested that the Hawking temperature can be used to find the Hubble temperature by providing the formula:

$$T_H = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}}. \tag{18}$$

That is, they have altered the M in the denominator to $\sqrt{M_c m_p}$ in the Hawking temperature formula. From a deeper analysis, we can see that his formula is identical to Equation (17), as we have:

$$\begin{aligned} T_H &= T_{Hw,H}\sqrt{\frac{R_H}{2l_p}} = T_{Hw,H}\sqrt{\frac{\frac{2GM_c}{c^2}}{\frac{2Gm_p}{c^2}}} = T_{Hw,H}\sqrt{\frac{M_c}{m_p}} \\ &= \frac{\hbar c^3}{k_b 8\pi G M_c}\sqrt{\frac{M_c}{m_p}} = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}}. \end{aligned} \tag{19}$$

However, it is important to notice also that there was no independent way to find the Planck mass independent off G when they published this in 2015. This means they could not have used the result (for predictions) we soon will show (Equation (21)) that likely can be seen as the true depth of quantum cosmology in relation to the CMB temperature. It is first in recent years been demonstrated how to find the Planck units totally independent of knowledge off G and $\hbar$, see [31] [32].

This means that the formula initially somehow heuristically suggested by Tatum et al. is fully consistent with our more formal analysis, where the formula is derived based on the Stefan-Boltzmann law. Many of the greatest ideas in physics began somewhat heuristically or speculatively before being fully formalized mathematically and rooted in physical "laws". For example, FitzGerald [67] merely described length contraction in words and stated it as a possible explanation for the null result in the Michelson and Morley experiment [68]. Later, Lorentz [69] formalized length contraction in his transformations, and naturally, Einstein [70] further naturally improved the theory of relativity.

Haug [31] has demonstrated that the Schwarzschild radius, from a deeper perspective is always identical to $r_s = 2l_p\frac{l_p}{\bar{\lambda}_c}$. Furthermore, since the Hubble constant always equals $H_0 = \frac{\bar{\lambda}_c c}{2l_p^2}$, and the Hubble radius is equal to $R_H = \frac{c}{H_0}$, we must also have $R_H = 2l_p \frac{l_p}{\bar{\lambda}_c}$, where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical universe mass. This means we can also express the Hubble temperature as:

$$T_H = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} = \frac{T_p}{8\pi}\sqrt{\frac{\bar{\lambda}_c}{2l_p}} \approx 2.72\ \text{K}. \tag{20}$$

Further since the Planck temperature is given as $T_p = \hbar \frac{c}{l_p}\frac{1}{k_b}$, we can also rewrite this as:

$$T_H = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_H}} = \frac{1}{k_b}\hbar\frac{c}{8\pi l_p}\sqrt{\frac{\bar{\lambda}_c}{2l_p}} \approx 2.72\ \text{K}. \tag{21}$$

Again, both the Planck length and the reduced Compton wavelength of the critical mass of the universe can be found independently of G and c. Equation (21) can be seen as the deepest quantum form describing the CMB temperature, it is only dependent on the Planck constant, the Planck length and the reduced Compton wavelength, all related to the quantum scale of the world.

That there is a link to the Planck scale even for cosmological observable phenomena also seems to be in line with a recent but simple Planck quantization of Einstein's field equation (see [71] [72]):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi l_p^2}{\hbar c}T_{\mu\nu}. \tag{22}$$

This rewritten field equation is simply the deeper level of Einstein's field equation as it gives all the same predictions as Einstein's original field equation, but delving into it further is outside the scope of this paper. This paper simply demonstrates that there is also a connection between the Planck temperature and the temperature in the Hubble sphere, which again corresponds very well to the measured CMB temperature.

3. Interesting Relations

There are multiple interesting relations following this. We get:

$$T_{Hw,H} R_H = T_{Hw,p} 2l_p$$

$$\frac{\hbar c^3}{k_b 8\pi G M_c} R_H = \frac{\hbar c^3}{k_b 8\pi G m_p} 2l_p \tag{23}$$

where $T_{Hw,H}$ is the Hawking temperature of the Hubble sphere in the critical Friedmann universe, and $T_{Hw,p}$ is the Hawking temperature of a Planck mass black hole (Planck mass particle). This is fully consistent with Equation (14) and is also fully consistent with a result derived for the critical Friedmann universe. Haug [34] has derived from the critical Friedmann universe the following:

$$\frac{M_c}{R_H} = \frac{m_p}{2l_p} \tag{24}$$

That one can easily get from Equation (25) and can also naturally easily be checked by numerical input. Haug proves that this is not just an approximation, but only in the critical Friedmann universe.

We also have

$$T_H \sqrt{2l_p R_H} = T_{Hw,p} 2l_p \tag{25}$$

where $T_H = T_{CMB} \approx 2.72\text{ K}$. This means that the Hubble temperature is now equal to the CMB temperature. Naturally, if we look at the CMB temperature from far away, we are looking back into what the Hubble and CMB temperatures were in the past. It is well known that it follows the rule $T_t = T_0(1+z)$, which has been well tested up to a cosmological redshift of approximately $z = 6$. However, the uncertainty in the value is very large. Riechers *et al.* [73] report a CMB temperature from back in the cosmic epoch at $z = 6.34$. However, the one standard deviation reported was 16.4 - 30.2 K, so the uncertainty in observations so far back in time is within one standard deviation uncertainty. So, there is clearly more experimental and theoretical work to do here. However, it is already clear that the CMB temperature derived from the Stefan-Boltzman law is very accurate for predicting the CMB temperature now, that is, for low z values.

4. Practical and Theoretical Implications

The findings have several important implications. The formula initially suggested by Tatum *et al.* now has a solid theoretical foundation in the Stefan-Boltzmann law. Since we first released a pre-print of this paper, significant further progress has been made. Tatum, Haug, and Wojnow [74] have taken advantage of that the CMB temperature now T_0 is measured with much greater precision than the Hubble constant and used the relationship between the T_0 and H_0 to dramatically reduce the uncertainty in the Hubble constant.

The Hubble tension is an unsolved problem in Λ-CDM cosmology, see Valentino *et al.* [75]. Krishnan *et al.* [76] have even indicated that the Hubble tension could signal a breakdown in FLRW cosmology. Haug and Tatum [77] have further connected the CMB H_0 relation discussed in this paper with a new cosmological redshift and proposed a potential solution to the Hubble tension within $R_h = ct$ cosmology and tested it out on the full distance ladder of supernovas SN Ia, which can even be proven with a closed-form mathematical solution as recently done by Haug [78]. Additionally, Haug and Wojnow [79] have recently demonstrated a possible but somewhat speculative relationship between the Casimir effect and the CMB temperature.

The solid foundation of the relationship between the CMB and the Hubble constant provided by the Stefan-Boltzmann law in this paper suggests promising avenues for further exploration. It seems like the Λ-CDM model now has a strong competitor. The Λ-CDM model cannot predict T_0, as pointed out, for example, by Narlikar and Padmanabhan [80] and it is considered a free parameter related to Ω_R, they also point out that:

"*Although* T_0 *is probably the best-determined cosmological parameter today, an interpretation relating the present background temperature to other physical processes in the universe, when available, would clearly mark an improvement over the standard interpretation.*"

With the relationship between the CMB temperature and the Hubble constant now derived from the Stefan-Boltzmann law in addition to the ongoing research development along these lines, one can achieve in $R_h = ct$ cosmology what has not yet been achieved in the Λ-CDM model on this point. Further investigation is naturally required for both models.

5. Conclusions

We have demonstrated a close connection between the Planck temperature and the Hubble sphere temperature. By combining the Planck temperature with the Stefan-Boltzmann law and insights from the Hawking or Unruh temperature, we can predict the temperature within the Hubble sphere to fall between 2.65 K and 2.8 K with a 68.3% confidence interval. This prediction is based on the Hubble constant, which is reported as $66.6^{+4.1}_{-3.3}$ km/s/Mpc in a recent study by Kelly *et al.* [48].

Our finding, which suggests that the Hubble sphere temperature can be predicted from the Planck temperature, aligns with recent advancements in quantum gravity and quantum cosmology. These developments provide additional evidence that gravity, at a deeper level, is intricately linked to the Planck scale, as foreseen by Eddington already in 1918.

Data Availability Section

All data generated or analysed during this study are included in this published article.

Conflicts of Interest

The authors declare no conflict of interest.

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