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Modelling Orthometric Heights from a Combination of Ellipsoidal Heights and Gravimetric Geoid Model in Rivers State, Nigeria

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Abstract

Many applications in geodesy, hydrography and engineering require geoid-related heights. Spirit leveling which is the traditional means of obtaining geoid- or mean sea level-related heights is slow, time-consuming and costly. Global Navigation Satellite Systems (GNSS) offer faster and relatively cheaper way of obtaining geoid-related heights when geoidal undulation is applied to ellipsoidal heights. However, difficulties involved in determining acceptable geoid height have seriously hampered the application of GNSS for leveling in Rivers State, thus necessitating the need to develop an acceptable geoid model which will serve as a means of conversion of GNSS-delivered ellipsoidal heights to their orthometric heights equivalent. In pursuance of this objective, a detailed gravimetric geoid has been evaluated for Rivers State, Nigeria. The computation of the geoid was carried out by the traditional remove-restore procedure. The Earth Geopotential Model 2008 (EGM08) was applied as the reference field for both the remove and restore parts of the procedures; spherical Fast Fourier Transform (FFT) was employed for the evaluation of the Molodenskii's integral formula for the height anomaly, (ζ) to yield the quasi-geoid; while the Residual Terrain Modelling (RTM) was done by prism integration. The classical gravimetric geoid over Rivers State was obtained from the rigorously evaluated quasi-geoid by adding the quasi-geoid to geoid ($N - \zeta$) correction it. The minimum and maximum geoid height values are 18.599 m and 20.114 m respectively with standard deviation of 0.345 m across the study area. Comparison of the gravimetric geoidal heights with the GPS/Leveling-derived geoidal heights of 13 stations across Rivers State, Nigeria showed that the absolute agreement with respect to the

GPS/leveling datum is generally better than 7 cm root mean squares (r.m.s) error. Results also showed that combining both GPS heights and the computed Rivers State geoid model can give orthometric heights accurate to 3 cm post-fit using a 4-parameter empirical model. The geoid model can thus serve as a good alternative to traditional leveling when used with GPS leveling, particularly for third order leveling in the study area.

Keywords

Geoid Modelling, Remove-Compute-Restore, Fast Fourier Transform, Residual Terrain Model, Ellipsoidal Heights, Orthometric Heights

1. Introduction

Measurements derived from Global Navigation Satellite Systems such as Global Positioning System (GPS) provide position of points which are commonly evaluated in a terrestrial three-dimensional Cartesian Coordinates. To obtain the equivalent geodetic coordinates in terms of latitude (φ), longitude (λ), and, ellipsoidal height (h), the resulting X, Y, and Z co-ordinates of the GPS points are transformed, employing the parameters of the reference ellipsoid. While ellipsoidal heights (h) are well known as heights reckoned from a defined reference ellipsoid, orthometric heights which are required in most engineering and hydrographic applications are reckoned from the geoid. The separation between the two heights system hinges on the difference between the reference ellipsoid and the geoid. This difference is referred to as geoidal height (N). If the ellipsoidal height (h) derived from GPS observations and the geoid-ellipsoid separation (N) of a station is known, then the orthometric height (H) of the station can be readily be computed directly from Equation (1) [1]:

$$H = h - N \tag{1}$$

By combining the computed differences in geoid heights and ellipsoidal heights of two points, ΔN and Δh, respectively, ΔN and Δh determined by GPS in a relative mode, the orthometric height changes between two benchmarks can be realized in the absence of spirit leveling from the relation [1]:

$$\Delta h = \Delta H - \Delta N \tag{2}$$

Determination of the geoid of a locality is important for many reasons. In the transformation of a local datum to world datum and verification of global datums, geoid heights are greatly required. Also in order to obtain high accuracy leveling results, the combination of an accurate GPS-derived heights and geoid heights plays principal role. Spirit leveling is not only time consuming, it is tedious and a costly conventional surveying practice. The knowledge of the geoid is also highly imperative in height control, in geophysical explorations (reconnaissance survey), in control surveys, and, in large scale mapping for engineering surveys and, related surveys. The study area is the hub of the oil and gas industry of Nigeria where many different International and local companies are operating, the determination of a precise geoid model will ensure proper data integration and the use of the GPS for orthometric height determination. The geoid solution was based on Earth Gravitational Model 2008 (EGM08) model coefficient set complete to degree and order 2160, point gravity anomalies obtained from Bureau Gravimetrique International (BGI) and digital elevation model (DEM) from Shuttle Radar Topography Mission (SRTM) heights. This study was carried out to generate an accurate geoid file for Rivers State by integrating the above dataset with EGM08 model coefficients set, so as to satisfy current geodetic requirements in the study area. Different methods of geoid determination have been developed over the years, with each having its merits and drawbacks. Each method has its specific procedure of evaluation with technique-definite input variables. For instance, Least Squares Collocation (LSC) employs point data as directly observed, whereas Stokes/Molodenskii integral uses mean values for gridded blocks or compactments regularly spaced data [2]. Stokes/Molodenskii integration by fast Fourier Transform (FFT) require 100% zero padding of the input data, while the analytic integration does not [3]. In LSC, stochastic model in form of covariance function is required and has to be well-defined [3], whereas in Stokes integration all data have to be of equal weight in the evaluation of the geoid undulation solution [2].

The combination of the Global Gravity Model (GGM) dataset with terrestrial gravity data so as to condense the latter to a localized area for geoid height computation applying the Remove-Compute-Restore technique has been done by several researchers [4] and [5]. In this study, the combination of the global gravity model (GGM) set with terrestrial gravity data was employed to evaluate the geoid. The modified spherical Stokes's kernel was used in the geoid computation as an alternative to the conventional Stokes's kernel after [6] tampered 100% zero padding so as to overcome cyclic effects. This is because it is established that spherical function tapers off more rapidly than the ellipsoidal function for cumulative spherical distances [3]. Therefore, we can anticipate that a truncation of the spherical (modified) integration at a definite spherical distance result to lesser truncation errors in relation to the truncation of the ellipsoidal (original) Stokes's integration.

At present, there is no officially adopted and published National geoid model or even regional geoid acceptable in any region of the country as posited by [7]. Since geoid heights are indispensable tool in the conversion of orthometric heights (H) of points established by leveling, gravity and GPS methods, the modelling of a single local geoid for the entire Rivers State is anticipated to eradicate use of diverse height systems within the study area as currently practiced by different Oil and Gas companies in the area which are in most cases not compatible with one another and this will in effect unify height systems within the study area and provide the tool to quickly develop orthometric heights all over Rivers State. Reference [8] maintained that the absence of a generally and officially published geoid model has made it difficult, among other problems, to create a link between Land and Sea Datum as analogous to Vdatum in the US and Canada and Vertical Offshore Reference Frame (VORF) in the United Kingdom for seamless bathymetry in the near and offshore zone of our coastal waters. This is necessary because there are so many offshore activities taking place in Rivers State, Nigeria today as a result of exploration and exploitation of gas, oil and minerals deposits. A model of the geoid will help in the appropriate integration of height data over land and sea. This study, in considering the important role the study area plays in the economy of Nigeria, was intended to bridge the gap by evaluating a fit-for-purpose geoid across Rivers State, Nigeria through tailoring the gravimetric geoid to the GPS/leveling data of the area, as well as developing a computer-based graphic user interface (GUI) program for easy conversion of GPS-delivered ellipsoidal heights to orthometric heights.

1.1. Aim and Objectives of the Study

This study was aimed at modelling orthometric heights from a combination of ellipsoidal heights and gravimetric geoid model in Rivers State, Nigeria, with the following objectives:

1) Compute height anomalies (ζ) using Molodenskii integral evaluated by FFT technique and then converting the height anomalies (ζ) to geoid undulation (N) values with which to generate regular geoid undulation grid file.

2) Fit or tailor the geoid undulation file to the GPS/leveling data.

3) Evaluate the relative accuracy of the geoid model resulting from this procedure.

4) Use the tailored geoid file as a basis of computing orthometric height of any desired point within the area.

1.2. The Study Area

The study area is the hub of the oil and hydrocarbon industry in the Niger Delta area of Nigeria. Rivers State has a mostly flat terrain in the Niger Delta area of Southern Nigeria. The inland part of the State is made up of tropical rainforest, and towards the coast, the typical Niger Delta environment geographies of many mangrove swamps [9]. Wikipedia [9] has it that Rivers State has a total area of 11,077 km^2, making it the 26th largest State in Nigeria. The State is surrounded by Imo, Abia and Anambra States to the north, Akwa Ibom State to the east and Bayelsa and Delta States to the west. On the south, it is bounded by the Atlantic Ocean. Its topography ranges from flat plains, with a network of rivers to estuaries and tributaries. Exploration and exploitation of crude oil as well as engineering activities related to it in the area include, but not limited to, seismic surveys, oil well-heads location surveys, pipeline surveys and construction pipelines of various sizes from oil wells to flow stations and then to oil terminals. There are many creek crossings, involving hydrographic surveys. All these activities require accurate height information. Determination of the geoid is an important component in obtaining accurate sea-level referenced heights.

For this study, the geoid computation covers an area lying between latitude 4.2811˚N to 5.7655˚N and Longitude 6.3304˚E to 7.6221˚E as shown in Figure 1.

2. Materials and Methods

2.1. Data Used for the Study

The evaluation of a local geoid gravimetrically and the tailoring process require four datasets. These are terrestrial gravity data, digital terrain model (DTM), Global Gravity Model (GGM) in form of spherical harmonic coefficients and GPS/leveling data. For this study, 50 points of terrestrial land gravity and over two thousand marine gravity points obtained from Bureau Gravimetrique International (BGI) [10] were used. These data which were contributed by different organizations and individuals as obtained for different applications, were accessed from BGI [10] and the fill-in gravity data were computed using software from the International Centre for Global Earth Models (ICGEM) [11]. The elevation data in form of digital terrain model is the Shuttle Radar Transmission Mission (SRTM) heights accessed from the United States Geological Surveys (USGS) [12]. The spherical harmonic coefficients EGM08 were downloaded from [13]. Sixteen GPS/leveling data were obtained from the Office of the Surveyor General of Rivers State, Nigeria. Thirteen of these points were used for the external assessment of the geoid and three points for cross-validation.

2.2. Method

Among the different approaches used in the determination of the gravimetric geoid either at regional or local scale, the best known method in the literatures is the Remove-Compute-Restore (R-C-R) approach as argued by [14]. Although there is no consensus as to the best approach because proponents of each method

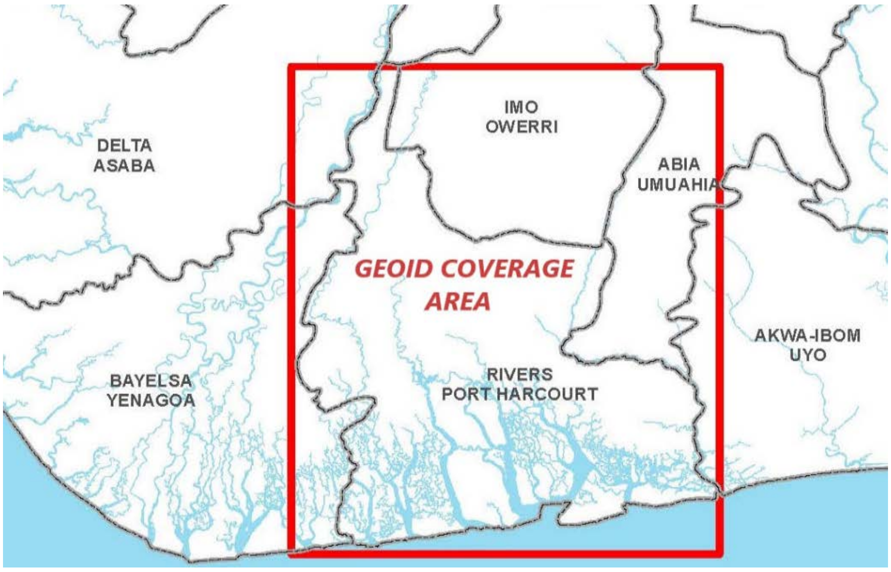

Figure 1. The study area (source: office of the surveyor general of rivers state).

prefer theirs over the others [15], the R-C-R method is the method adopted for this study. Also by employing R-C-R through FFT, there is no need of the time-consuming point-wise numerical summations of the Stokes'/Molodenskii's integral, since the evaluation of convolution integrals is substituted by very proficient multiplications. The spectral techniques based on the FFT overcome very efficiently the problem of slow evaluation speed and provide a homogenous coverage of results, which is very suitable for graphical plotting interpolation and/or prediction [3]. The R-C-R approach through the FFT computation tool for evaluation of the Molodenskii's integral formula was employed as implemented in the GRAVSOFT software suite [16]. The Remove-compute-restore (R-C-R) procedure accounts for the long, medium and short wavelength components of the height anomaly as contributed by the GGM, Terrestrial gravity data and DTM respectively [2]. This is expressed in Equation (3) as obtained from [15]:

$$\zeta = \zeta_{GGM} + \zeta_{RES} + \zeta_{RTM} \tag{3}$$

where:

ζ_{GGM} : The height anomaly derived from the global gravity field.

ζ_{RES} : The residual height anomaly derived from the Molodenskii integral employing residual gravity anomalies.

ζ_{RTM} : The geoid restore effects derived from the topography.

and each of these signals is, evaluated respectively using Equations (4)-(6) as given by [17] [18] and [19] respectively:

$$\zeta_{GGM} = \frac{T - \Delta g_{GGM} H}{\gamma} \tag{4}$$

where:

H is the orthometric height of the evaluation point.

T is the anomalous potential.

Δg_{GGM} is the gravity anomaly from the global gravity field model (GGM).

$$\zeta_{RTM} = \frac{G\rho}{\gamma} \int_{h_{ref}}^{h} \frac{1}{r} \mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{5}$$

where:

$$r = \left[\left(x_P - x \right)^2 + \left(y_P - y \right)^2 + \left(H_P - z \right)^2 \right]^{1/2}$$

$$\zeta_{RES} = \frac{R}{4\pi\gamma} \iint_{\sigma} \left(\Delta g_{sa} + G_1 \right) S(\psi) \mathrm{d}\sigma \tag{6}$$

where:

Δg_{sa} = Molodenskii surface gravity anomaly.

$$G_1 = \text{indirect effect} = \frac{R^2}{2\pi} \iint \frac{\left(H - H_P \right)}{l_0^3} \Delta g \mathrm{d}\sigma$$

where:

H_P: The evaluation point and H being the reference point.

R: The mean radius of the Earth.

l: The distance between the evaluation point and reference point.

The surface gravity anomaly Δg_{sa} was gridded by Least Squares Collocation using the TCGRID subroutine in GRAVSOFT before the computation of equation 6. The results of Equations (4)-(6) were gridded to the same resolution of $0.05^\circ \times 0.05^\circ$ across the study area before summing them by the GCOMB subroutine in GRAVSOFT to obtain the height anomaly as given by Equation (3).

The quasi-geoid which is equivalent to the height anomaly reckoned from the ellipsoid is then converted to the classical geoid using Equation (7) as given by [20]:

$$N = \zeta - \frac{\Delta g_B H}{\gamma} \tag{7}$$

where:

H_P: The evaluation point and H being the reference point.

Δg_B: The Bouguer anomaly.

γ: The normal gravity of the reference ellipsoid.

Presently, GPS is commonly used for height determination. According to [2], the R-C-R procedure refers to the geocentric reference system implicit in the geopotential model used. On the other hand, the local leveling datum to which the orthometric heights are referenced normally do not refer to the geocentric reference system. To overcome this difference, the different height data are combined. To achieve this, the gravimetric geoid heights were evaluated at the GPS/leveling-derived geoid undulations points which are considered as independent and external datasets that one can use for the validation of a gravimetric geoid model only if the GPS/leveling results are not included in the gravimetric geoid model solution [21].

To validate the gravimetric geoid, the computed value is compared with the GPS/leveling-derived geoid height on benchmark points in two ways. In the first case, each geoid undulation value on each benchmark from the gravimetric geoid model is compared with the GPS/leveling-derived geoid undulation. This is performed on all the benchmarks included in the analysis after the removal of the outliers. This type of comparison is referred to as absolute comparison [21] and the mathematical model is given as:

$$l_i = h_{GPS_{(i)}} - H_{(i)} - N_{grav_{(i)}} \tag{8}$$

or

$$l_i = N_{GPS/leveling_{(i)}} - N_{grav_{(i)}} \tag{9}$$

where:

l_i is the residual.

This technique or approach is usually employed when comparing the combined gravimetric geoid models. Both the GPS/leveling and the gravimetric geoid undulations of the same point cover the entire spectrum bandwidth. The residuals l_i at all benchmarks were analysed using a four-parameter model for the fitting process as given by [22]. In practice, $h_{GPS} - H - N_{grav}$ is not equal to zero because it contains the errors in the geoid itself and also the errors in the GPS and leveling measurements. These errors are considered as the systematic datum differences between the gravimetric geoid and the GPS/leveling data, and possible long-wavelength errors of the geoid and were removed by applying a correction model obtained from the fitting process. This helped to make the gravimetric model fit better to the GPS/leveling data [23]. Reference [3] stated that the long wavelength errors can be reduced by constraining the gravimetric geoid solution to the GPS/leveling-derived undulations, which is sometimes called geoid fitting or tailoring to the GPS/leveling benchmarks.

The computation of relative accuracy used in assessing the gravimetric models was done using the following equation [21]:

$$\Delta l_{ij} = \left(N_{GPS/leveling_{(j)}} - N_{GPS/leveling_{(i)}}\right) - \left(N_{grav_{(j)}} - N_{grav_{(i)}}\right) \tag{10}$$

where:

i and j are GPS/leveling benchmark points in the network of any two points.

$N_{GPS/leveling}$ is the geoid height from GPS/leveling.

N_{grav} is the gravimetric geoid heights from modelling.

The relative accuracy assessments for the baseline distances S_{ij} as computed are expressed in parts per million (ppm) as follows by [21]:

$$\Delta l_{ij}^{rel}\left(\text{ppm}\right) = \frac{\Delta l_{ij}\left(\text{mm}\right)}{\left[S_{ij}\right]\left(\text{km}\right)} \tag{11}$$

The distances were computed using the mathematical relationship as given below:

$$\left[S_{ij}\right]_{km} = \sqrt{\left(\Delta X\right)^2 + \left(\Delta Y\right)^2 + \left(\Delta Z\right)^2} \tag{12}$$

where:

ΔX: The difference between the two cartesian coordinates at j and i of X.

ΔY: The difference between the two cartesian coordinates at j and i of Y.

ΔZ: The difference between the two cartesian coordinates at j and i of Z.

3. Results and Discussions

3.1. Results

The results as shown in **Tables 1-4** and **Figure 2** satisfy the objectives of the research.

3.2. Discussion of Results

In any typical geoid determination, fitting and transformation process from 3D

Table 1. Statistics of the Geoid over the study area.

Gravity Field/Unit: m	Max.	Min.	Mean	Std. Dev.
Classical Geoid (N)	20.180	18.500	19.150	0.345

Table 2. External assessment of gravimetric Geoid with GPS/leveling.

Attribute	Lat. (dd)	Long. (dd)	h (m)	H (m)	N^{GPS} (m)	N^{GRAV} (m)	Diff. (m)
AP1	4.8695	6.9779	33.7200	14.8081	18.9119	18.8199	0.092
HS8	4.7651	7.0166	26.0280	6.9860	19.0420	19.0350	0.007
PT 4 Emma	4.7984	7.0056	30.6930	11.6906	19.0024	18.9524	0.050
PT 8 Emma	4.8338	7.0070	26.7890	7.8509	18.9381	18.8381	0.100
PT 5 Emma	4.8069	7.0094	29.3740	10.3801	18.9939	18.9339	0.060
PT 9 Emma	4.8366	7.0153	29.1410	10.1660	18.9750	18.8860	0.089
PT 2 Abdul	4.8443	7.0395	32.6400	13.6539	18.5861	18.5161	0.070
PT 3 Abdul	4.8408	7.0313	26.7500	7.7697	18.9803	18.9093	0.071
AP4	4.8683	6.9899	35.8490	16.9261	18.9229	18.8349	0.088
PP5	4.8703	7.1089	38.8020	19.7522	19.0498	19.0028	0.047
PT 4 Abdul	4.8372	7.0229	32.8420	13.8392	19.0028	18.9558	0.047
PT 3 Emma	4.7902	7.0023	25.1950	6.2283	18.9667	18.8907	0.076
PT 6 Emma	4.8155	7.0098	34.5140	15.4366	19.0774	19.1044	0.027
	dd = Degree Decimal				RMS (Error)		7 cm

Table 3. Results of the cross-validation (Post-Fit).

Attribute	Lat. (dd)	Long. (dd)	h (m)	H (m)	N^{GPS} (m)	N^{GRAV} (m)	Diff.
PT 7 Emma	4.8239	7.006	33.3790	14.3716	19.0074	18.9524	0.055
Uniport	4.8937	6.9144	29.7120	10.8670	18.8450	18.8600	−0.015
PP9	4.8883	7.1445	33.5700	14.4602	19.1098	19.1248	−0.015
						RMS (Error)	3 cm

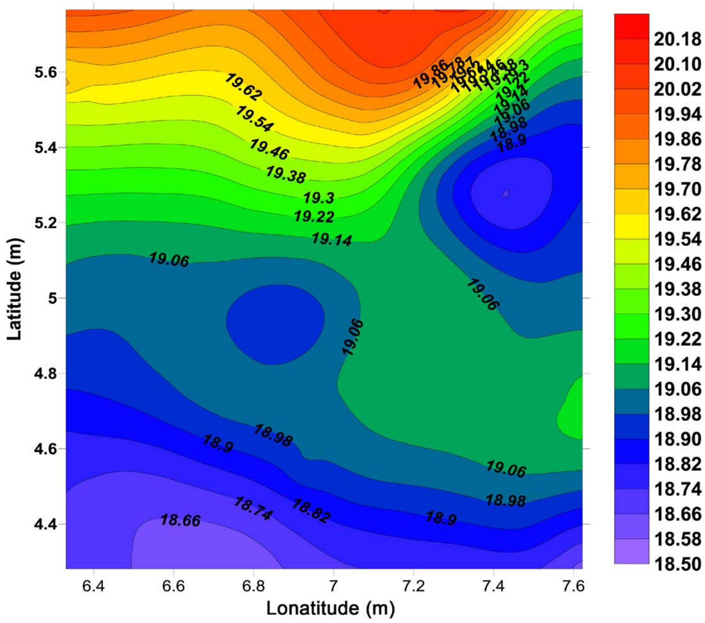

Figure 2. Geoid over the study area.

Table 4. Results of relative accuracy of selected baselines over the study area.

Baselines		$(N^{GPS} - N^{GRAV})_{ij}$ (m)	Distance (km)	Rel. Accuracy (ppm)
AP1	PT 8 Emma	0.008	5.099	1.5688
AP1	PT 9 Emma	0.003	5.518	0.5437
AP1	PT 2 Abdul	0.022	7.379	2.9813
AP1	AP4	0.004	1.338	2.9903
AP1	PT 3 Emma	0.016	9.177	1.7434
HS8	PP5	0.040	15.497	2.5811
PT 4 Emma	PP5	0.003	13.947	0.2151
PT 4 Emma	PT 4 Abdul	0.003	4.700	0.6383
PT 8 Emma	AP4	0.012	4.261	2.8165
PT 5 Emma	PT 2 Abdul	0.010	5.315	1.8813
PT 5 Emma	PT 3 Abdul	0.011	4.467	2.4625
PT 5 Emma	PP5	0.013	13.076	0.9942
PT 9 Emma	AP4	0.001	4.497	0.2224
PT 9 Emma	PT 3 Emma	0.013	5.330	2.4391
PT 2 Abdul	PT 3 Abdul	0.001	0.989	1.0116
PT 2 Abdul	PP5	0.023	8.217	2.7989
PT 2 Abdul	PT 3 Emma	0.006	7.268	0.8256
PT 3 Abdul	PP5	0.024	9.205	2.6073
PT 3 Abdul	PT 3 Emma	0.005	6.454	0.7747
AP4	PT 3 Emma	0.012	8.745	1.3722
PP5	PT 3 Emma	0.029	14.774	1.9628

GPS derived ellipsoidal height to gravity-related 1D orthometric height, distortions or residuals are always inherent. These have been associated to datum inconsistency, systematic errors introduced from the different contributors to the geoid and the transformation process through interpolation. This level of accuracy achieved may be attributed also to the un-modelled errors from the DEM used for this research, terrestrial gravity data and from the interpolation of the surface gravity anomalies used in the Molodenskii integral. The geoid in this research was first evaluated as a quasi-geoid and was subsequently converted to geoid by applying the quasi-geoid to geoid correction (ζ-to-N). The geoid values within the computation area has maximum and minimum of 20.180 m and 18.500 m and standard deviation of ±0.345 m, as already presented in **Table 1**. The largest values in absolute sense are in northern part of the computation area and the smallest values around the coast as this is expected due to the flat topography. The range of the geoid value is 1.68 m across the study area.

In this research, the overall accuracy of the gravimetric geoid was assessed using GPS/leveling data collected over the study area and it shows good agreement at 7 centimetres (pre-fit) and 3 centimetres (post-fit) as deduced in Table 2 and Table 3 respectively. The geoid values across the study area revealed that the geoid is above ellipsoid. The geoid file developed in this research is capable of providing orthometric heights at centimetre-level when compared to the ellipsoidal heights within the study area. Besides, this geoid file can be used in providing gravity-related heights for engineering and hydrographic applications at unobserved points. For example, the orthometric height of point HS8 was transferred by leveling from the established tide-gauge installed at Port Harcourt Port Authority, Rivers State. The observed orthometric for HS8 is 6.986 m while the derived orthometric height is 6.993 m, a residual of 0.007 m. The implication of this result is that the geoid file can be used for height information for port development and reference for bathymetry information in the inland waters of the study area. The results of the relative accuracy Table 4 revealed that the geoid can serve as a tool for the conversion of 3D derived-ellipsoidal height to orthometric heights using Equation (2) in place of spirit-leveled height differences over long baselines and the heights so derived can serve the purposes of third order mapping and leveling applications which is 2.0 mm $\sqrt{K}$. where K is distance between benchmarks in kilometer.

The height transformation described in this work is purely a geometrical process. The output is used to define the parameters for mapping of any local area for engineering and related applications. It must also be pointed out that the fitting and transformation process as presented in this work has not removed the errors inherent in the geodetic leveling data. It has simply harmonized the corresponding points on the two surfaces (N^{GPS} and N^{GRAV}) with repeatable and consistent results that are compatible across the study area.

4. Conclusion

The gravimetric geoid model over Rivers State, Nigeria was determined by rigorously evaluating Molodenskii's Integral for the quasi-geoid, applying Remove-Compute-Restore procedure using terrestrial gravity values, EGM08 and SRTM height data. The resulting quasi-geoid was next converted to a classical geoid by adding the N – ζ correction to it. The geoid model at short distance reflects the details and strength of the gravimetric geoid and at long distances the trends of the leveling networks. The geoid surface from the fitting process is not an equipotential surface but rather a corrective surface. A geoid file was created for the area and can be accessed through a graphic user interface (GUI) in an interpolation program Height Transformation Model developed for this, which allows height users to interpolate geoid values, and subsequently transform ellipsoidal heights to orthometric heights or vice versa. The Program/Software when provided to end users can rapidly provide gravity-related heights which are required for engineering and related applications across the computation area.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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